

Experiment NO-5

Aim: Implement Fast Fourier Transform

Theory:

The Direct Fourier Transform (DFT) can be written as follows:

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-j2\pi k \frac{n}{N}}$$

To determine the DFT of discrete signal $x[n]$, we multiply each of its value of e raised to cosine function of n . We then sum the results obtained for a given n . As the name implies, the Fast Fourier Transform (FFT) is an algorithm that determines DFT of an input significantly faster than computing it directly. FFT reduces number of computations needed for a problem of size N from $O(N^2)$ to $O(N \log N)$.

N	1000	10^6	10^9
N^2	10^6	10^{12}	10^{18}
$N \log_2 N$	10^4	20×10^6	30×10^9

Suppose, we separated the Fourier Transform into even and odd indexed sub-sequence.

$$\begin{aligned} n &= 2r, & \text{if even} \\ n &= 2r+1, & \text{if odd} \end{aligned}$$

where,

$$r = 1, 2, \dots, \frac{N}{2}$$

Later, we end up with summation of the terms. The advantage of this approach lies in the fact that the even and odd indexed sub-sequences can be computed concurrently.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi k n}{N}}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] e^{-j \frac{2\pi k (2r)}{N}} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] e^{-j \frac{2\pi k (2r+1)}{N}}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r] e^{-j \frac{2\pi k r}{N/2}} + e^{-j \frac{2\pi k}{N}} \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] e^{-j \frac{2\pi k r}{N/2}}$$

$$X[k] = X_{\text{even}}[k] + e^{-j \frac{2\pi k}{N}} X_{\text{odd}}[k]$$