Quiz 2

(MA6.102) Probability and Random Processes, Monsoon 2025

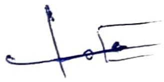
28 October, 2025

Max. Duration: 45 Minutes

Question 1 (5 marks). Consider two jointly continuous random variables X and Y with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 2 & \text{if } x > 0, y > 0, \text{ and } x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Compute the probability $P(X < Y)$.
(b) Determine the conditional CDF $F_{X|\{X<Y\}}$, and then obtain the conditional PDF $f_{X|\{X<Y\}}$.



Question 2 (5 marks). Let X and Y be jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = x + y, \quad (x,y) \in [0,1]^2.$$

Define new random variables $Z = X^2$ and $W = X(1+Y)$.
(a) Determine the range (support) of the pair (Z, W), i.e., the subset of $\mathbb{R}^2$ where (Z, W) can take values.
(b) Find the joint PDF $f_{Z,W}$.

Question 3 (5 Marks). Let $M_X(s) = \mathbb{E}[e^{sX}]$ be finite for $s \in (-c, c)$, for some $c > 0$. Show that

$$\lim_{n\to\infty} \left(M_X\left(\frac{s}{n}\right)\right)^n = e^{s\mathbb{E}[X]}.$$

Hint: $n \log M_X(\frac{s}{n}) = \frac{\log M_X(\frac{s}{n})}{\frac{1}{n}}$.