Practice Problem Set 1

(MA6.102) Probability and Random Processes, Monsoon 2025

Problem 1. Let $X \sim \text{Binomial}(m, p)$ and $Y \sim \text{Binomial}(n, p)$ be independent random variables. Show that $X + Y \sim \text{Binomial}(m + n, p)$.

Problem 2. Let $X \sim \text{Binomial}(n, p)$. Find $\mathbb{E}\left[\frac{1}{X+1}\right]$.

Problem 3. For a non-negative integer valued random variable N, show that

$$\sum_{i=0}^{\infty} iP(N > i) = \frac{1}{2}(\mathbb{E}[N^2] - \mathbb{E}[N]).$$

Problem 4. Let $X = \sum_{i=1}^{n} X_i$, where $X_1, X_2, \ldots, X_n$ are Bernoulli random variables (need not be independent). Show that $\mathbb{E}[X^2] = \sum_{i=1}^{n} P_{X_i}(1)\mathbb{E}[X|X_i = 1]$.

Problem 5. Let X be a Poisson random variable with parameter λ. Show that $P_X(k)$ increases monotonically and then decreases monotonically as k increases reaching its maximum when k is the largest integer not exceeding λ.

Problem 6. For two discrete random variables X and Y, prove the triangle inequality:

$$\sqrt{\mathbb{E}[(X+Y)^2]} \leq \sqrt{\mathbb{E}[X^2]} + \sqrt{\mathbb{E}[Y^2]}.$$

[*Hint:* Use the Cauchy-Schwarz inequality.]

Problem 7. Let Y be a non-negative integer-valued random variable. Is it true that

$$\mathbb{E}[Y] = \int_0^{\infty} P(Y > y)\; dy.$$

Problem 8. Let X be a Gaussian random variable with mean 0 and variance 1, and $Y = e^X$. Find $\mathbb{E}[Y]$.

Problem 9. Let X be a Laplace random variable with PDF

$$f_X(x) = \frac{\lambda}{2}\mathrm{e}^{-\lambda|x|},\; x \in \mathbb{R},$$

where $\lambda > 0$. Compute the mean $\mathbb{E}[X]$ and variance $\text{Var}(X)$.

Problem 10. Calculate the CDF of the two-sided exponential random variable that has PDF given by

$$f_X(x) = \begin{cases} p\lambda\mathrm{e}^{\lambda x}, & \text{if } x < 0 \\ (1-p)\lambda\mathrm{e}^{-\lambda x}, & \text{if } x \geq 0, \end{cases}$$

where $\lambda > 0$ and $0 < p < 1$.

All the best for mid-semester examinations