Tutorial- 15/11/25

PRP

Question

Consider the random process defined as

$$X_n = V_n \cos(\omega_0 n) + W_n \sin(\omega_0 n),$$

where $\{V_n\}$ and $\{W_n\}$ are zero-mean random processes that are independent of each other. Both processes $\{V_n\}$ and $\{W_n\}$ have the same autocorrelation function denoted by $r[k]$. Here, ω_0 is a known constant.

(a) Find the mean and autocorrelation function of the random process $\{X_n\}$.

(b) Comment on the wide-sense stationarity of this process. How does it depend on the value of $\omega_0 \in [0, 2\pi)$?

Show that if φ is a random variable with

$$\Phi(\lambda) = \mathbb{E}\{e^{j\lambda\varphi}\}$$

and $\Phi(1) = \Phi(2) = 0$, then the process

$$x(t) = \cos(\omega t + \varphi)$$

is WSS. Find $\mathbb{E}\{x(t)\}$ and $R_x(\tau)$ if φ is uniform in the interval $(-\pi, \pi)$.

question

Let Y_1, Y_2, Y_3, $\cdots$ be a sequence of i.i.d. random variables with mean $EY_i = 0$ and $\mathrm{Var}(Y_i) = 4$. Define the discrete-time random process $\{X(n), n \in \mathbb{N}\}$ as

$$X(n) = Y_1 + Y_2 + \cdots + Y_n, \quad \text{for all } n \in \mathbb{N}.$$

Find $\mu_X(n)$ and $R_X(m,n)$, for all $n, m \in \mathbb{N}$.

solution

We have

$$\begin{aligned}\mu_X(n) &= E[X(n)] \\ &= E[Y_1 + Y_2 + \cdots + Y_n] \\ &= E[Y_1] + E[Y_2] + \cdots + E[Y_n] \\ &= 0.\end{aligned}$$

Let $m \leq n$, then

$$\begin{aligned}R_X(m,n) &= E[X(m)X(n)] \\ &= E\left[X(m)\big(X(m) + Y_{m+1} + Y_{m+2} + \cdots + Y_n\big)\right] \\ &= E[X(m)^2] + E[X(m)]E[Y_{m+1} + Y_{m+2} + \cdots + Y_n] \\ &= E[X(m)^2] + 0 \\ &= \mathrm{Var}\big(X(m)\big) \\ &= \mathrm{Var}\big(Y_1\big) + \mathrm{Var}\big(Y_2\big) + \cdots + \mathrm{Var}\big(Y_m\big) \\ &= 4m.\end{aligned}$$

Similarly, for $m \geq n$, we have

$$\begin{aligned}R_X(m,n) &= E[X(m)X(n)] \\ &= 4n.\end{aligned}$$

We conclude

$$R_X(m,n) = 4\min(m,n).$$

question

Let $X(t)$ be a continuous-time WSS process with mean $\mu_X = 1$ and

$$R_X(\tau) = \begin{cases} 3 - |\tau| & -2 \leq \tau \leq 2 \\ 1 & \text{otherwise} \end{cases}$$

a. Find the expected power in $X(t)$.

b. Find $E\left[\left(X(1) + X(2) + X(3)\right)^2\right]$.

solution

a. The expected power in $X(t)$ at time t is $E[X(t)^2]$, which is given by

$$R_X(0) = 3.$$

b. We have

$$\begin{aligned} E\left[\left(X(1)+X(2)+X(3)\right)^2\right] &= E\Big[X(1)^2+X(2)^2+X(3)^2 \\ &\quad + 2X(1)X(2)+2X(1)X(3)+2X(2)X(3)\Big] \\ &= 3R_X(0)+2R_X(-1)+2R_X(-2)+2R_X(-1) \\ &= 3\cdot 3+2\cdot 2+2\cdot 1+2\cdot 2 \\ &= 19. \end{aligned}$$

Question-

Let $\{N(t), t \in [0, \infty)\}$ be a Poisson Process with rate λ. Find its covariance function:

$$C_N(t_1, t_2) = \mathrm{Cov}(N(t_1), N(t_2)), \quad \text{for } t_1, t_2 \in [0, \infty)$$

Question-

Let $\{N(t), t \in [0, \infty)\}$ be a Poisson process with rate λ.
Find the probability that there are two arrivals in $(0, 2]$ and three arrivals in $(1, 4]$.