PRP Tutorial 23rd August

Bayes' Theorem, Total probability Theorem

LIKELIHOOD
The probability of "B" being True, given "A" is True

PRIOR
The probability "A" being True. This is the knowledge.

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

POSTERIOR
The probability of "A" being True, given "B" is True

MARGINALIZATION
The probability "B" being True.

Total Probability Theorem:

$$P(B) = \sum_{i=1}^{n} P(B \mid A_i)\, P(A_i), \quad \text{if } P(A_i) > 0,\ i \in [1 :$$

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

One way to design a spam filter is to look at the words in an email. In particular, some words are more frequent in spam emails. Suppose that we have the following information:

- 50% of emails are spam;
- 1% of spam emails contain the word "refinance";
- 0.001% of non-spam emails contain the word "refinance".

Suppose that an email is checked and found to contain the word "refinance". What is the probability that the email is spam?

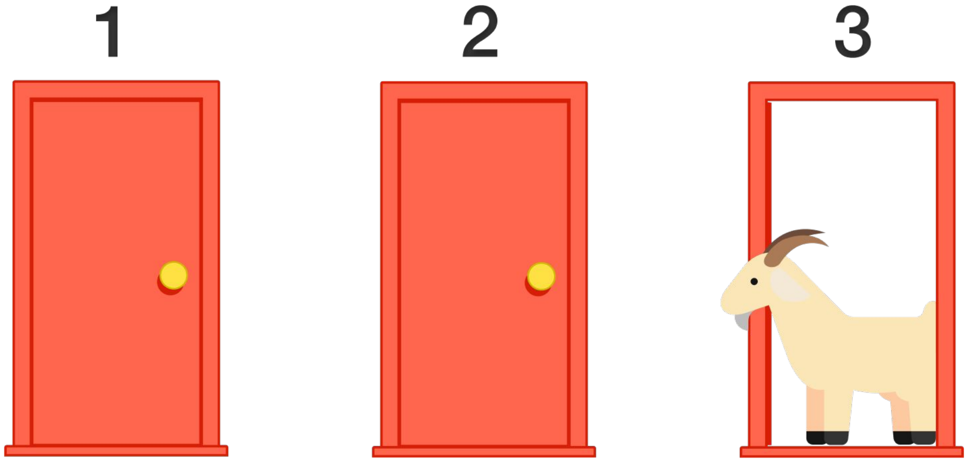
13. The Prisoner's Dilemma

Three prisoners, A, B, and C, with apparently equally good records have applied for parole. The parole board has decided to release two of the three, and the prisoners know this but not which two. A warder friend of prisoner A knows who are to be released. Prisoner A realizes that it would be unethical to ask the warder if he, A, is to be released, but thinks of asking for the name of *one* prisoner *other than himself* who is to be released. He thinks that before he asks, his chances of release are $\frac{2}{3}$. He thinks that if the warder says "B will be released," his own chances have now gone down to $\frac{1}{2}$, because either A and B or B and C are to be released. And so A decides not to reduce his chances by asking. However, A is mistaken in his calculations. Explain.

The Monty Hall Problem-

You are in a game show, and the host gives you the choice of three doors. Behind one door is a car and behind the others are goats. You pick a door, say Door 1. The host who knows what is behind the doors opens a different door and reveals a goat (the host can always open such a door because there is only one door behind which is a car). The host then asks you: "Do you want to switch?" The question is, is it to your advantage to switch your choice?

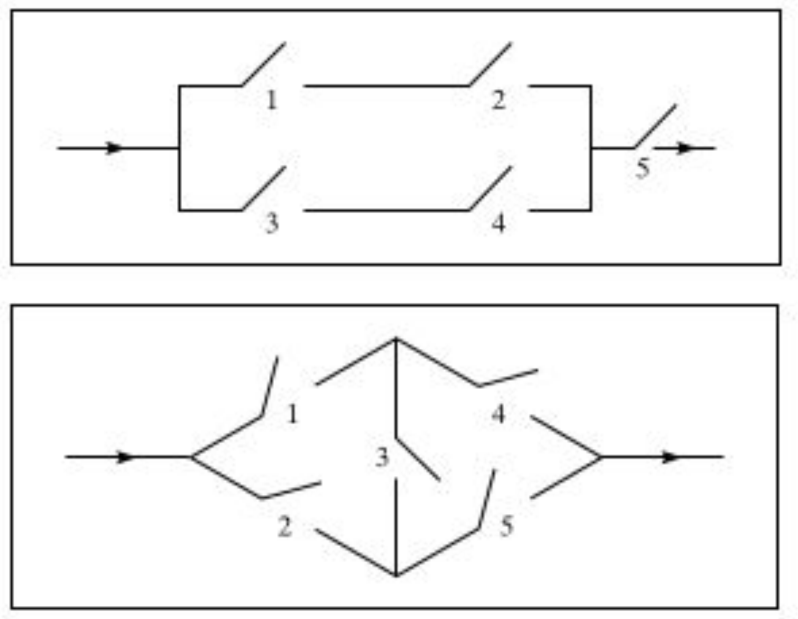
Go through- [Monty Hall problem - Wikipedia]



Independence and Conditional Independence

The probability of the closing of the ith relay in the circuits shown in Figure 3.4 is given by pi , i = 1, 2, 3, 4, 5. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits?

(a) (b)



There are $k + 1$ coins in a box. When flipped, the ith coin will turn up heads with probability i/k, $i = 0, 1, \dots, k$. A coin is randomly selected from the box and is then repeatedly flipped. If the first n flips all result in heads, what is the conditional probability that the $(n + 1)$st flip will do likewise.

Find the limit when k tends to infinity.

Are the outcomes of successive flips independent?

Properties of CDF

Construct the CDFs for the following

(7) Example. Constant variables. The simplest random variable takes a constant value on the whole domain Ω. Let $c \in \mathbb{R}$ and define $X : \Omega \to \mathbb{R}$ by

$$X(\omega) = c \quad \text{for all} \quad \omega \in \Omega.$$

(8) Example. Bernoulli variables. Consider Example (1.3.2). Let $X : \Omega \to \mathbb{R}$ be given by

$$X(H) = 1, \quad X(T) = 0.$$

(9) Example. Indicator functions. A particular class of Bernoulli variables is very useful in probability theory. Let A be an event and let $I_A : \Omega \to \mathbb{R}$ be the *indicator function* of A; that is,

$$I_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A, \\ 0 & \text{if } \omega \in A^c. \end{cases}$$