Quiz 1

(MA6.102) Probability and Random Processes, Monsoon 2025

29 August, 2025

Max. Duration: 45 Minutes

Question 1 (5 marks). Consider a family that has two children. Assume that every birth results in a boy with probability $\frac{1}{2}$, independent of other births and also that the parents in the family had decided to have exactly two children. Assume that if a child is a girl, her name will be Lilly with probability α independently from other child's name. If the child is a boy, his name will not be Lilly.
(a) Given that both children are girls, what is the probability that at least one of them is named Lilly?
(b) What is the probability that at least one child is a girl named Lilly.
(c) Given that the family has at least one child named Lilly, what is the probability that both children are girls?

Question 2 (5 Marks). Let the sample space be $\Omega = \{-3, -2, -1, 0, 1, 2, 3\}$, and let $X : \Omega \to \mathbb{R}$ be a function defined as $X(\omega) = |\omega|$.
(a) For each $x \in \mathbb{R}$, list the events $\{X \leq x\} \triangleq \{\omega \in \Omega : X(\omega) \leq x\}$.
(b) Determine the cardinality of the smallest σ-field with respect to which X is a random variable.
(c) Find that smallest σ-field.

Question 3 (5 Marks). Let $F : \mathbb{R} \to [0, 1]$ be a function satisfying:

- If $x < y$, then $F(x) \leq F(y)$.
- $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$.
- $\lim_{\epsilon \downarrow 0^+} F(x + \epsilon) = F(x)$, for every $x \in \mathbb{R}$.

Show that there exists a probability space $(\Omega, \mathcal{F}, P)$ and a random variable $X : \Omega \to \mathbb{R}$ such that the cumulative distribution function (CDF) of X is equal to F, i.e., $P(X \leq x) = F(x)$, for all $x \in \mathbb{R}$.
Hint: $\Omega = \mathbb{R}$.