Probability and Random Processes

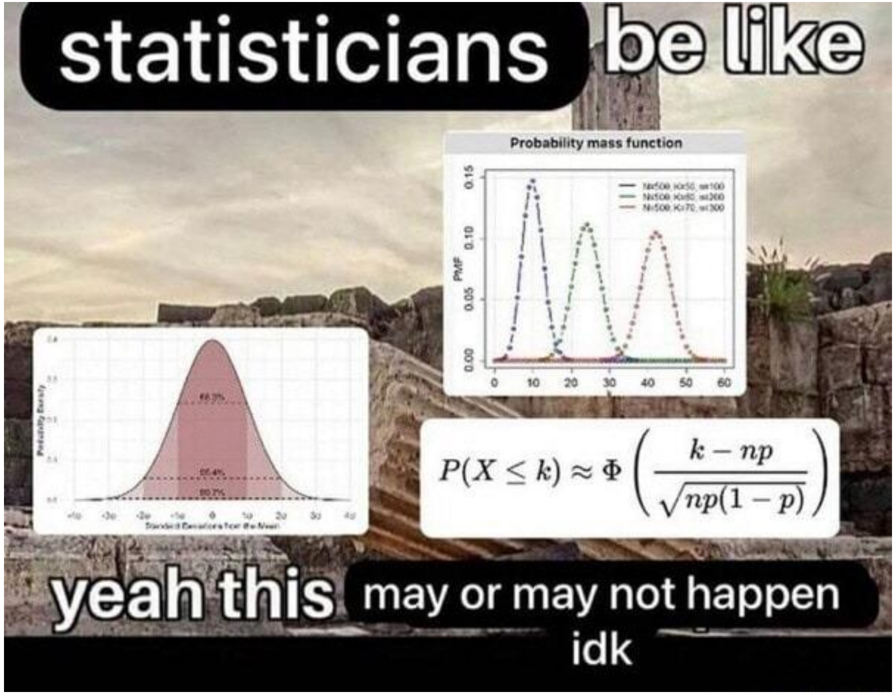
Tutorial-1

09-08-2025

Agenda

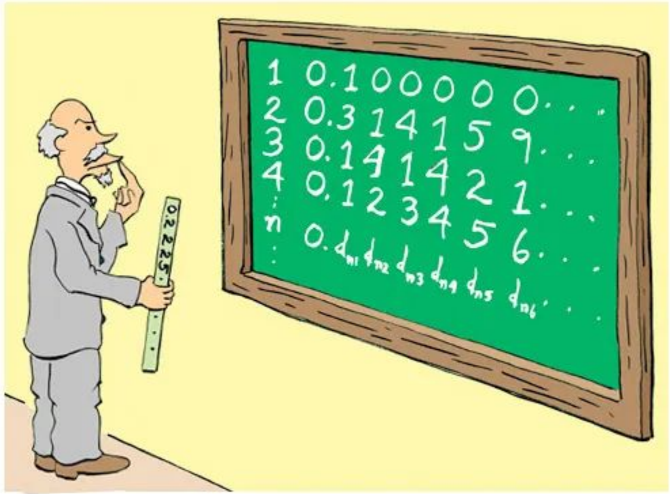
- Cantor’s Diagonalization Argument
- Inclusion-Exclusion Principle
- Continuity of Probability
- Some Problems!

- Primers
  - Exclusive, Exhaustive, Independence



Uncountability of $\{0, 1\}^{\infty}$

- What is the difference between Finiteness and Infiniteness?
    - How does countability and uncountability come into the picture?
- Is the set of infinite sequences of 0s and 1s Finite?
    - Is it countable?
- Cantor's Diagonalization Argument
    - It is not enumerable!
- But what if you treat them as binary numbers?
    - Isn't that Counter-intuitive?

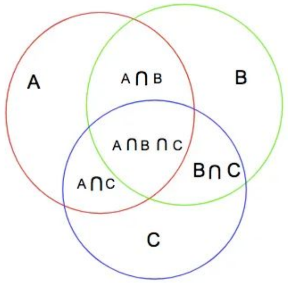


Problem 1 - Set Theory

- In a school of 100 students:
  - 35 students play cricket, 40 play football, 45 play tennis.
  - 10 play both cricket and football, 15 play both football and tennis, 7 play both cricket and tennis.
  - 6 play all three sports.

  Find the number of people who do not play sports.

- Inclusion-Exclusion Principle
  - Intuition
  - Proof



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

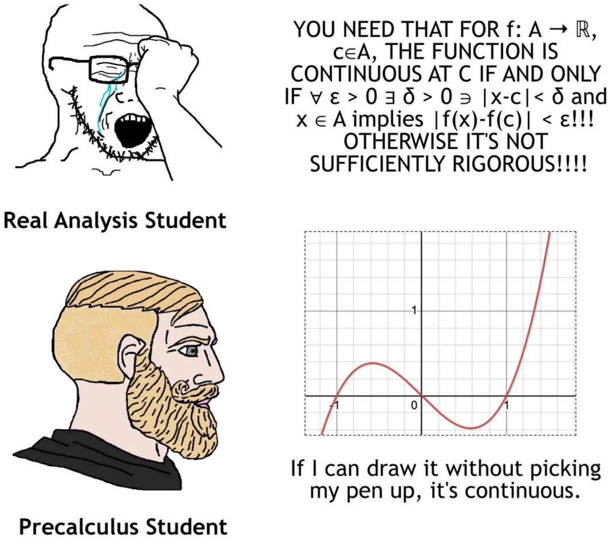
Problem 2 - Probability Law

- Suppose that $P(A) = 0.3$, and $P(B) = 0.8$. Find the bounds on $P(A \cap B)$.

- $P(A) = 0.4$, $P(B) = 0.7$, $P(A \cup B) = 0.9$
  - Find $P(A \cap B)$
  - Find $P(A^c \cap B)$
  - Find $P(A-B)$
  - Find $P(A^c-B)$
  - Find $P(A^c \cup B)$
  - Find $P(A \cap (B \cup A^c))$

Continuity of Probability

- Probability is a function!
- Continuity of a function - Definition



<u>Problem 3</u>

Let $\Omega = [0,1]$, $\mathcal{F} = \mathcal{B}([0,1])$, and $\mathbb{P}([a,b]) = \mathbb{P}((a,b]) = \mathbb{P}([a,b)) = \mathbb{P}((a,b)) = b - a$ for $a, b \in [0,1]$, $a \leq b$ (this is called the *Lebesgue measure*).

Let

$$B_n = \left[0, \frac{n}{n+1}\right], \quad n \in \mathbb{N}.$$

What is $\lim_{n\to\infty} \mathbb{P}(B_n)$?

Bonus Question

- How do you place 50 good candies and 50 rotten candies in two boxes such that if you choose a box at random and take out a candy at random, it better be good!

  We need to maximize the probability of getting a good candy when selecting a random box and a random candy from it.

- Additionally, you can check out the famous Monty Hall problem!