Tutorial 06

MA6.102 Probability and Random Processes, Monsoon 2025

Date: 18-10-2025

Problem 1

A defective coin minting machine produces coins whose probability of heads is itself a random variable P with PDF:

$$f_P(p) = \begin{cases} pe^p, & p \in [0,1] \\ 0, & \text{otherwise} \end{cases}$$

A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent.

(a) Find the probability that a coin toss results in heads.

(b) Given that a coin toss resulted in heads, find the conditional PDF of P.

(c) [Exercise] Given that the first coin toss resulted in heads, find the conditional probability of heads on the next toss.

Problem 2

A particle leaves the origin under the influence of the force of gravity and its initial velocity v forms an angle ϕ with the horizontal axis. The path of the particle reaches the ground at a distance

$$d = \frac{v^2}{g} \sin 2\phi$$

from the origin. Assuming that ϕ is a random variable uniform between 0 and $\pi/2$, determine: (a) the density of d, and (b) the probability that $d \leq d_0$.

Problem 3

Given RVs such that

$$Y = \tan X$$

Find the density of Y, given an arbitrary distribution f_X followed by X.

[Exercise] Now solve this for the special case of $X \sim U[-\pi/2, \pi/2]$

Problem 4

Given two RVs X and Y, such that (assume independent):

$$Z = \max(X, Y); \qquad W = \min(X, Y)$$

Find the densities of W and Z.

[Exercise] Find the density for $Z = \frac{\min(X,Y)}{\max(X,Y)}$

Problem 5

Let $x \sim U(0,1)$ and $y \sim U(0,1)$ be independent random variables. Find the density function of z. *Note: This is a practical procedure to generate Gaussian random variables from two independent uniformly distributed random variables!*

$$z = (-2 \ln x)^{1/2} \cos(2\pi y)$$

[Exercise] Try to prove the same, but start with the auxiliary random variable $w = y$