Problem 1

A point is chosen at random (according to a uniform PDF) within a semicircle of the form

$$\{(x, y) \mid x^2 + y^2 \leq r^2,\ y \geq 0\},$$

for some given $r > 0$.

(a) Find the joint PDF of the coordinates X and Y of the chosen point.

(b) Find the marginal PDF of Y and use it to find $\mathbb{E}[Y]$.

Problem 2

A professor schedules two student appointments for the same time. The appointment durations are independent and exponentially distributed with mean thirty minutes. The first student arrives on time, but the second student arrives five minutes late.

Question: What is the expected value of the time between the arrival of the first student and the departure of the second student?

Problem 3

Let the random variables X and Y have a joint PDF which is uniform over the triangle with vertices at $(0, 0)$, $(0, 1)$, and $(1, 0)$.

(a) Find the joint PDF of X and Y.

(b) Find the marginal PDF of Y.

(c) Find the conditional PDF of X given Y.

(d) Find $\mathbb{E}[X \mid Y = y]$, and use the total expectation theorem to find $\mathbb{E}[X]$ in terms of $\mathbb{E}[Y]$.

Problem 4

One of the two wheels of fortune, A and B, is selected by the toss of a fair coin, and the wheel chosen is spun once to determine the value of a random variable X.

The PDF of X given A is selected is

$$f_{X|A}(x) = 1, \quad 0 \leq x \leq 1.$$

The PDF of X given B is selected is

$$f_{X|B}(x) = 3, \quad 0 \leq x \leq \frac{1}{3}.$$

Question: Find the probability that A was selected given that $X \leq \frac{1}{4}$.

Problem 5

Let

$$f(x, y) = 24xy, \quad 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le x + y \le 1,$$

and let it equal 0 otherwise.

(a) Show that $f(x, y)$ is a joint probability density function.

(b) Find $\mathbb{E}[X]$.

(c) Find $\mathbb{E}[Y]$.

Problem 6

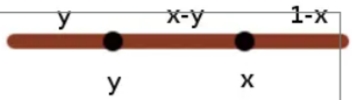
Along a road 1 mile long are 3 people "distributed at random." Assume that "distributed at random" means that the positions of the 3 people are independent and uniformly distributed over the road.

Question: Find the probability that no two people are less than a distance of d miles apart, where $d \leq \frac{1}{2}$.

Problem 7

A stick is broken into 3 parts, by choosing 2 points randomly along its length. With What probabilty can it form a triangle?

Problem 8



- A stick is broken into 3 parts, by choosing 2 points randomly along its length. With what probability can it form a triangle?