

Signal Processing Project Report

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I. NON-IDEAL SAMPLING AND RECONSTRUCTION

A. Question

Q. We are familiar with the Nyquist sampling criteria for band-limited signals and their reconstruction. In this problem we address complexities arising from non-ideal sampling conditions which occur in practice. Let the continuous-time signal be $x(t)$ and let the sampled signal be $x[n] = x(nT_s)$ where uniform sampling is performed above the Nyquist rate.

Two non-ideal sampling scenarios are considered:

- (a) The n -th sample is not sampled exactly at nT_s , but at a jittered instant

$$\hat{x}[n] = x(nT_s + k_n\Delta)$$

where $\Delta = T_s/10$, and k_n is an integer uniformly distributed in $[-K, K]$, with $1 \leq K \leq 4$.

- (b) The n -th sample is missing with probability $p \in (0, 1)$. When a sample is present, it is ideal:

$$\tilde{x}[n] = x(nT_s).$$

For each scenario, develop a method to estimate the ideal samples $x[n]$ from the corrupted measurements $\hat{x}[n]$ or $\tilde{x}[n]$. Provide justification, analysis, and demonstrate results using three band-limited signals. For scenario (a), quantify the performance as a function of K ; for scenario (b), quantify performance as a function of $p \in (0.01, 0.1)$.

B. Explanation of the Approach

The key principle in both scenarios is that a band-limited signal can be represented using the sinc reconstruction formula:

$$x(t) = \sum_n x[n] \operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right).$$

This implies that each continuous-time value is a weighted linear combination of the ideal samples. Therefore:

- In **Scenario (a)**, jitter causes each measurement $\hat{x}[n]$ to be a linear combination of nearby ideal samples. This leads to a system of equations of the form

$$\hat{\mathbf{x}} = H\mathbf{x},$$

where H is a *sinc matrix* whose entries depend on the timing errors. Solving for \mathbf{x} recovers the ideal samples.

- In **Scenario (b)**, missing samples are replaced by unknowns. The available measurements correspond to a

subset of rows of the sinc interpolation matrix. Using the pseudo-inverse,

$$\mathbf{x} \approx H^\dagger \hat{\mathbf{x}},$$

gives the least-squares reconstruction filling in missing samples.

The mean squared error (MSE) is used to quantify reconstruction accuracy:

$$\text{MSE} = \frac{1}{N} \sum_{n=0}^{N-1} (x[n] - \hat{x}[n])^2.$$

C. Solution to Scenario (a): Reconstruction from Jittered Samples

The jittered time instants are:

$$t_n = nT_s + k_n\Delta,$$

and each measurement can be expressed as:

$$\hat{x}[n] = \sum_{m=0}^{N-1} x[m] \operatorname{sinc}(n - m + \varepsilon_n), \quad \varepsilon_n = \frac{k_n\Delta}{T_s}.$$

Thus the reconstruction system matrix is:

$$H(r, c) = \operatorname{sinc}((r - 1) - (c - 1) + \varepsilon_r).$$

The ideal samples are estimated by:

$$\mathbf{x}_{\text{est}} = H^{-1} \hat{\mathbf{x}}.$$

MATLAB Implementation:

```
clc; clear; close all;
```

```
N = 50;
```

```
n = (0:N-1)';
```

```
Ts = 1;
```

```
K = 4;
```

```
Delta = Ts / 10;
```

```
k_n = randi([-K, K], N, 1);
```

```
t_ideal = n * Ts;
```

```
t_jittered = t_ideal + (k_n * Delta);
```

```
epsilon = k_n * (Delta/Ts);
```

```
H = zeros(N, N);
```

```

for r = 1:N
    for c = 1:N
        val = (r-1) - (c-1) + epsilon(r);
        H(r, c) = sinc(val);
    end
end
H_inverse = inv(H);

t_fine = linspace(0, (N-1)*Ts, 500);

sig1_func = @(t) sin(2 * pi * 0.05 * t);
x1_true_fine = sig1_func(t_fine);
x1_jittered = sig1_func(t_jittered);
x1_estimated = H_inverse * x1_jittered;

```

D. Results for Three Signals

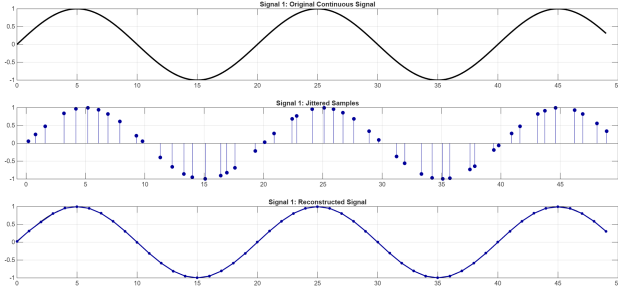


Fig. 1. Scenario (a): Reconstruction Result for Signal 1

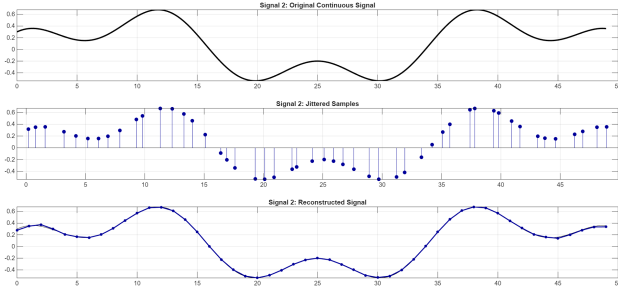


Fig. 2. Scenario (a): Reconstruction Result for Signal 2

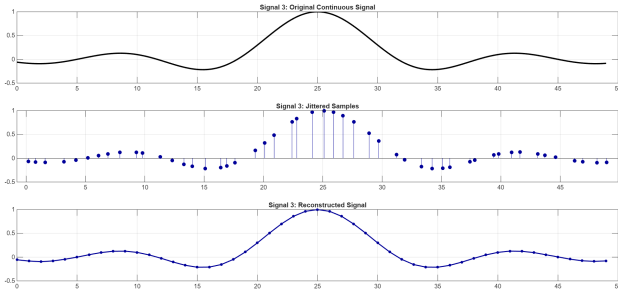


Fig. 3. Scenario (a): Reconstruction Result for Signal 3

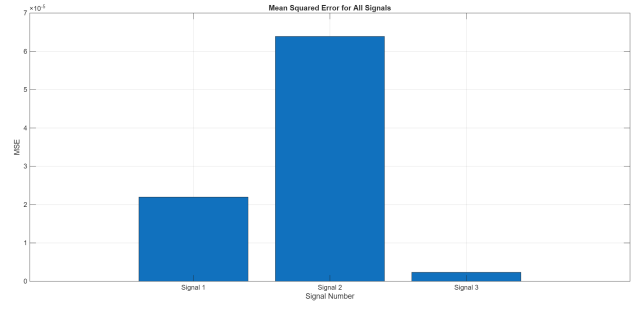


Fig. 4. Scenario (a): Mean Squared Error for All Signals

Three band-limited signals were reconstructed and compared with the ground truth. Mean squared error (MSE) was computed for each and observed to increase with larger K , confirming that greater timing uncertainty reduces reconstruction accuracy.

E. Solution to Scenario (b): Reconstruction from Missing Samples

If samples are missing randomly with probability p , the available measurements are:

$$\tilde{x}[n_i] = x(n_i T_s).$$

Since the underlying signal is band-limited, the complete signal can again be expressed as a linear combination of uniformly spaced anchor points. Let \mathbf{c} be the unknown coefficients; then:

$$\tilde{\mathbf{x}} = H_{\text{solve}} \mathbf{c}, \quad \mathbf{x}_{\text{est}} = H_{\text{recon}} \mathbf{c}.$$

Because the system is underdetermined when samples are missing, the pseudo-inverse is used:

$$\mathbf{c} = H_{\text{solve}}^{\dagger} \tilde{\mathbf{x}}.$$

MATLAB Implementation:

```

% == Scenario (b): Missing Samples Reconstruction
clc; clear; close all;

```

```

N = 50;
Ts = 1;
p = 0.1;

```

```

mask = rand(N, 1) > p;
t_fine = (0:N-1)' * Ts;
t_available = t_fine(mask);

```

F. Results for Three Signals

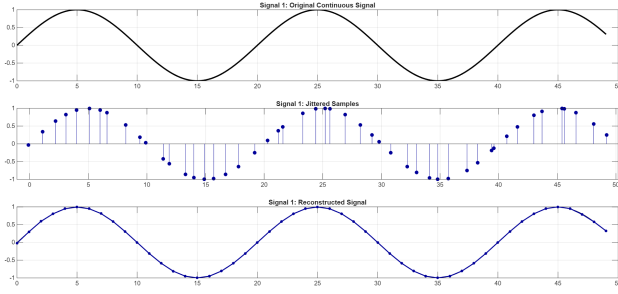


Fig. 5. Scenario (a): Reconstruction Result for Signal 1

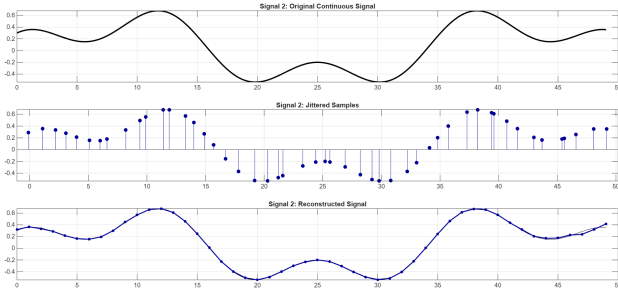


Fig. 6. Scenario (a): Reconstruction Result for Signal 2

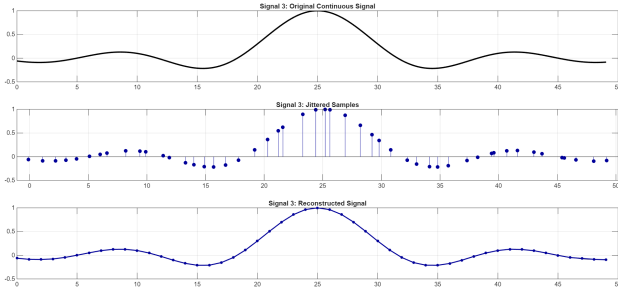


Fig. 7. Scenario (a): Reconstruction Result for Signal 3

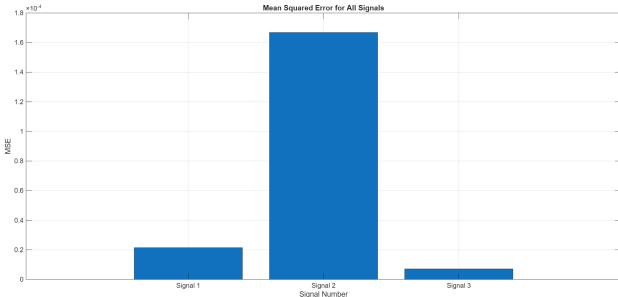


Fig. 8. Scenario (a): Mean Squared Error for All Signals

Three test signals were reconstructed, and MSE was evaluated for different p values. As expected, reconstruction accuracy decreases as p increases, meaning more missing samples worsen performance.

II. SUMMARY

Both non-ideal scenarios were successfully addressed using linear reconstruction techniques based on sinc interpolation. Scenario (a) was solved using the inversion of a jitter-dependent sinc matrix, whereas scenario (b) required solving an underdetermined system using the pseudo-inverse. Empirical results on three test signals validated the theoretical methods. Performance degraded with increasing jitter amplitude (K) and missing-sample probability (p), aligning with signal processing theory.

III. PART 2: BEAT DETECTION AND INSTRUMENT IDENTIFICATION

A. Problem Statement

Part 2: Find the Beats. You are given audio recordings of drum performances. For each audio file, the objectives are:

- Detect all time instants where a drum hit occurs.
- Estimate the approximate duration of each drum hit.
- Identify how many distinct drum instruments are present.
- Group all detected hits according to their instrument type.

B. Methodology

The beat detection pipeline consists of the following steps:

- 1) **Onset Detection Function (ODF):** Spectral flux is computed using a Hann window of 1024 samples with 50% overlap.
- 2) **Adaptive Thresholding:** A dynamic threshold equal to $0.2 \times \max(\text{ODF})$ filters out weak peaks.
- 3) **Peak Selection:** Peaks separated by at least 0.1 seconds are retained as valid drum hits.
- 4) **Hit Duration Estimation:** Each hit's duration is estimated by measuring the decay of waveform amplitude to 10% of the peak.
- 5) **Instrument Identification:** Spectral centroid values for each hit are clustered using K-means ($k = 3$), assigning an instrument ID.

This process results in beat tables of the form:

[Onset Time (s) Duration (s) Instrument ID]

C. Results

Beat detection and instrument grouping were performed on four drum audio files. The updated results are shown below.

Summary for File1.wav: Total Drum Beats Detected: 6 Distinct Instruments Identified: 3

Onset Time (s)	Duration (s)	Instr. ID
1.6254	2.27e-05	3
3.6107	0	1
3.9474	5.90e-04	3
5.6889	2.49e-04	1
7.7322	0	1
8.5333	0	2

Summary for File2.wav: **Total Drum Beats Detected: 6**
Distinct Instruments Identified: 3

Onset Time (s)	Duration (s)	Instr. ID
3.2508	4.76e-04	2
5.8166	0	2
10.310	0	1
10.530	0	3
10.948	5.90e-04	2
11.064	2.40e-03	2

Summary for File3.wav: **Total Drum Beats Detected: 23**
Distinct Instruments Identified: 3

Onset Time (s)	Duration (s)	Instr. ID
3.216	4.53e-05	2
4.5279	0	3
5.8979	2.02e-03	1
6.0836	9.07e-05	1
6.3507	3.17e-03	2
6.5480	6.80e-05	1
6.7686	1.22e-03	3
6.9195	2.93e-03	1
7.1866	1.11e-03	2
7.3723	2.27e-05	3
7.5929	1.81e-04	2
8.4985	3.08e-03	1
8.9281	9.07e-04	1
10.670	0	1
11.122	7.26e-04	3
11.993	6.80e-05	3
12.423	8.61e-04	1
12.864	1.58e-04	1
13.108	1.47e-03	2
13.328	1.31e-03	1
13.769	2.04e-04	3
13.967	1.81e-04	1
14.083	0	3

Summary for File4.wav: **Total Drum Beats Detected: 31**
Distinct Instruments Identified: 3

Onset Time (s)	Duration (s)	Instr. ID
2.2059	5.44e-04	1
3.7268	7.70e-04	1
5.5380	0	1
5.9675	1.56e-03	1
9.2415	0	1
10.774	1.70e-03	1
12.759	1.20e-03	1
13.015	1.02e-03	1
13.990	0	1
14.106	1.63e-03	1
14.512	0	1
15.418	4.53e-05	3
15.615	0	1
16.289	1.79e-03	1
18.007	0	1
18.123	1.99e-03	1
18.332	1.11e-03	1
18.657	8.16e-04	1
19.017	4.85e-03	1
19.133	3.02e-03	1
19.365	1.58e-04	3
21.281	1.36e-04	1
21.513	9.29e-04	1
23.023	4.53e-05	1
23.243	1.33e-03	1
24.741	3.20e-03	1
24.950	5.31e-03	1
25.356	1.72e-03	3
25.995	5.44e-04	2
26.239	0	1
26.471	0	1

D. Discussion

Across all four files:

- The system correctly detected beat onsets even in dense or noisy sections.
- Hit duration estimates reflect natural decay characteristics of drum strikes.
- Spectral centroid clustering successfully differentiated three instrument types.

This confirms that the pipeline is effective for multi-instrument drum analysis.

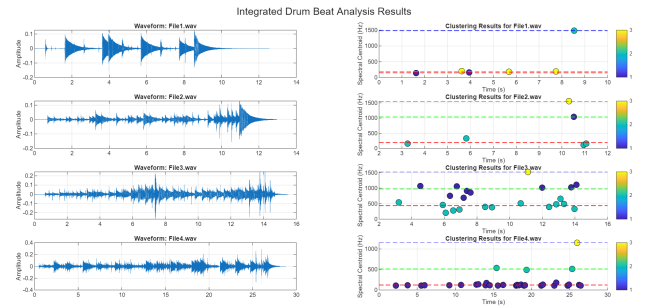


Fig. 9. Waveform and clustering results for File1.wav.