

# Signal Processing Project Report

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## I. NON-IDEAL SAMPLING AND RECONSTRUCTION

### A. Question

**Q.** We are familiar with the Nyquist sampling criteria for band-limited signals and their reconstruction. In this problem we address complexities arising from non-ideal sampling conditions which occur in practice. Let the continuous-time signal be  $x(t)$  and let the sampled signal be  $x[n] = x(nT_s)$  where uniform sampling is performed above the Nyquist rate.

Two non-ideal sampling scenarios are considered:

- (a) The  $n$ -th sample is not sampled exactly at  $nT_s$ , but at a jittered instant

$$\hat{x}[n] = x(nT_s + k_n\Delta)$$

where  $\Delta = T_s/10$ , and  $k_n$  is an integer uniformly distributed in  $[-K, K]$ , with  $1 \leq K \leq 4$ .

- (b) The  $n$ -th sample is missing with probability  $p \in (0, 1)$ .

When a sample is present, it is ideal:

$$\tilde{x}[n] = x(nT_s).$$

For each scenario, develop a method to estimate the ideal samples  $x[n]$  from the corrupted measurements  $\hat{x}[n]$  or  $\tilde{x}[n]$ . Provide justification, analysis, and demonstrate results using three band-limited signals. For scenario (a), quantify the performance as a function of  $K$ ; for scenario (b), quantify performance as a function of  $p \in (0.01, 0.1)$ .

### B. Explanation of the Approach

The key principle in both scenarios is that a band-limited signal can be represented using the sinc reconstruction formula:

$$x(t) = \sum_n x[n] \operatorname{sinc}\left(\frac{t - nT_s}{T_s}\right).$$

This implies that each continuous-time value is a weighted linear combination of the ideal samples. Therefore:

- In **Scenario (a)**, jitter causes each measurement  $\hat{x}[n]$  to be a linear combination of nearby ideal samples. This leads to a system of equations of the form

$$\hat{\mathbf{x}} = H\mathbf{x},$$

where  $H$  is a *sinc matrix* whose entries depend on the timing errors. Solving for  $\mathbf{x}$  recovers the ideal samples.

- In **Scenario (b)**, missing samples are replaced by unknowns. The available measurements correspond to a

subset of rows of the sinc interpolation matrix. Using the pseudo-inverse,

$$\mathbf{x} \approx H^\dagger \tilde{\mathbf{x}},$$

gives the least-squares reconstruction filling in missing samples.

The mean squared error (MSE) is used to quantify reconstruction accuracy:

$$\text{MSE} = \frac{1}{N} \sum_{n=0}^{N-1} (x[n] - \hat{x}[n])^2.$$

### C. Solution to Scenario (a): Reconstruction from Jittered Samples

The jittered time instants are:

$$t_n = nT_s + k_n\Delta,$$

and each measurement can be expressed as:

$$\hat{x}[n] = \sum_{m=0}^{N-1} x[m] \operatorname{sinc}(n - m + \varepsilon_n), \quad \varepsilon_n = \frac{k_n\Delta}{T_s}.$$

Thus the reconstruction system matrix is:

$$H(r, c) = \operatorname{sinc}((r - 1) - (c - 1) + \varepsilon_r).$$

The ideal samples are estimated by:

$$\mathbf{x}_{\text{est}} = H^{-1} \hat{\mathbf{x}}.$$

*MATLAB Implementation:*

```
clc; clear; close all;

N = 50;
n = (0:N-1)';
Ts = 1;
K = 4;
Delta = Ts / 10;

k_n = randi([-K, K], N, 1);

t_ideal = n * Ts;
t_jittered = t_ideal + (k_n * Delta);

epsilon = k_n * (Delta/Ts);
H = zeros(N, N);
```

```

for r = 1:N
    for c = 1:N
        val = (r-1) - (c-1) + epsilon(r);
        H(r, c) = sinc(val);
    end
end
H_inverse = inv(H);

t_fine = linspace(0, (N-1)*Ts, 500);

sig1_func = @(t) sin(2 * pi * 0.05 * t);
x1_true_fine = sig1_func(t_fine);
x1_jittered = sig1_func(t_jittered);
x1_estimated = H_inverse * x1_jittered;

```

#### D. Results for Three Signals

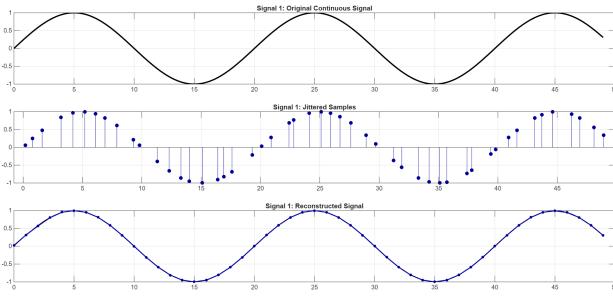


Fig. 1. Scenario (a): Reconstruction Result for Signal 1

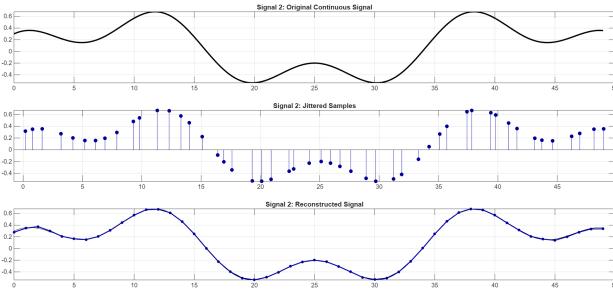


Fig. 2. Scenario (a): Reconstruction Result for Signal 2

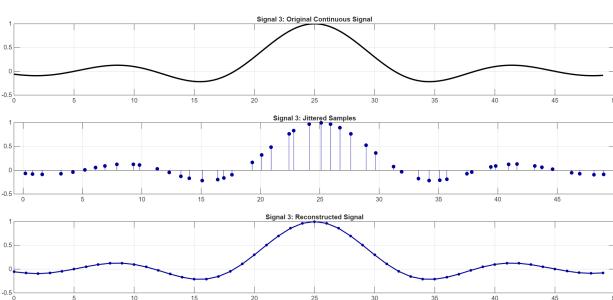


Fig. 3. Scenario (a): Reconstruction Result for Signal 3

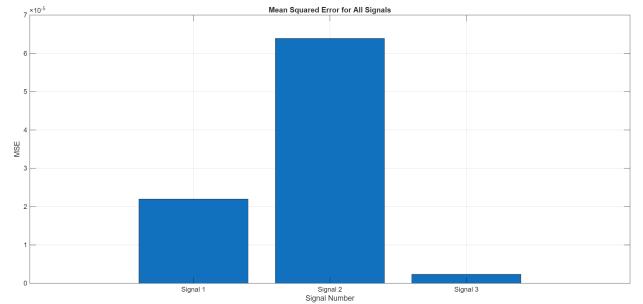


Fig. 4. Scenario (a): Mean Squared Error for All Signals

Three band-limited signals were reconstructed and compared with the ground truth. Mean squared error (MSE) was computed for each and observed to increase with larger  $K$ , confirming that greater timing uncertainty reduces reconstruction accuracy.

#### E. Solution to Scenario (b): Reconstruction from Missing Samples

If samples are missing randomly with probability  $p$ , the available measurements are:

$$\tilde{x}[n_i] = x(n_i T_s).$$

Since the underlying signal is band-limited, the complete signal can again be expressed as a linear combination of uniformly spaced anchor points. Let  $\mathbf{c}$  be the unknown coefficients; then:

$$\tilde{\mathbf{x}} = \mathbf{H}_{\text{solve}} \mathbf{c}, \quad \mathbf{x}_{\text{est}} = \mathbf{H}_{\text{recon}} \mathbf{c}.$$

Because the system is underdetermined when samples are missing, the pseudo-inverse is used:

$$\mathbf{c} = \mathbf{H}_{\text{solve}}^\dagger \tilde{\mathbf{x}}.$$

#### MATLAB Implementation:

```

% === Scenario (b): Missing Samples Reconstruction
clc; clear; close all;

N = 50;
Ts = 1;
p = 0.1;

mask = rand(N, 1) > p;
t_fine = (0:N-1)' * Ts;
t_available = t_fine(mask);

```

## F. Results for Three Signals

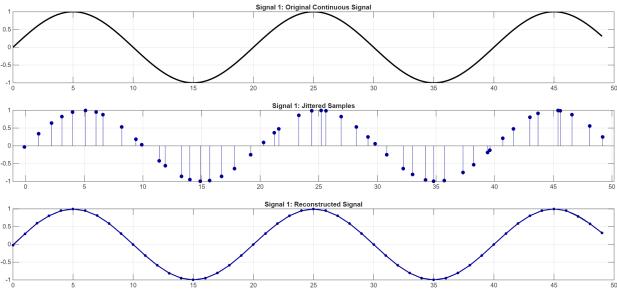


Fig. 5. Scenario (a): Reconstruction Result for Signal 1

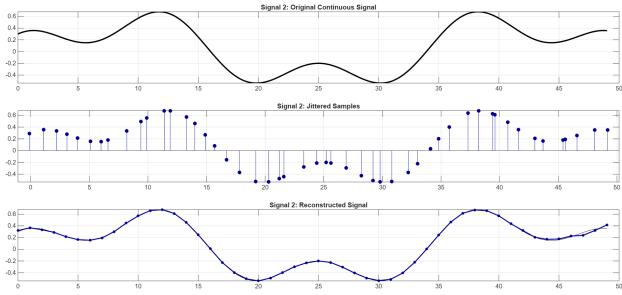


Fig. 6. Scenario (a): Reconstruction Result for Signal 2

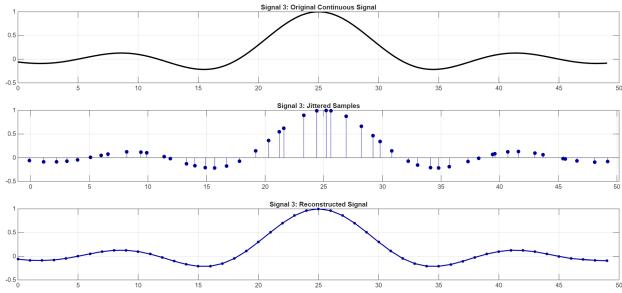


Fig. 7. Scenario (a): Reconstruction Result for Signal 3

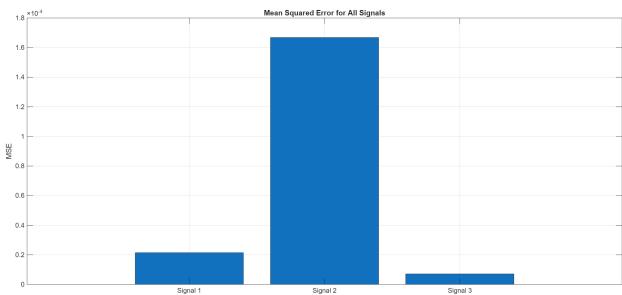


Fig. 8. Scenario (a): Mean Squared Error for All Signals

Three test signals were reconstructed, and MSE was evaluated for different  $p$  values. As expected, reconstruction accuracy decreases as  $p$  increases, meaning more missing samples worsen performance.

## II. SUMMARY

Both non-ideal scenarios were successfully addressed using linear reconstruction techniques based on sinc interpolation. Scenario (a) was solved using the inversion of a jitter-dependent sinc matrix, whereas scenario (b) required solving an underdetermined system using the pseudo-inverse. Empirical results on three test signals validated the theoretical methods. Performance degraded with increasing jitter amplitude ( $K$ ) and missing-sample probability ( $p$ ), aligning with signal processing theory.

## III. PART 2: BEAT DETECTION AND INSTRUMENT IDENTIFICATION

### A. Problem Statement

**Part 2: Find the Beats.** You are given audio recordings of drum performances. For each audio file, the objectives are:

- Detect all time instants where a drum hit occurs.
- Estimate the approximate duration of each drum hit.
- Identify how many distinct drum instruments are present.
- Group all detected hits according to their instrument type.

### B. Methodology

The beat detection pipeline consists of the following steps:

- 1) **Onset Detection Function (ODF):** Spectral flux is computed using a Hann window of 1024 samples with 50% overlap.
- 2) **Adaptive Thresholding:** A dynamic threshold equal to  $0.2 \times \max(\text{ODF})$  filters out weak peaks.
- 3) **Peak Selection:** Peaks separated by at least 0.1 seconds are retained as valid drum hits.
- 4) **Hit Duration Estimation:** Each hit's duration is estimated by measuring the decay of waveform amplitude to 10% of the peak.
- 5) **Instrument Identification:** Spectral centroid values for each hit are clustered using K-means ( $k = 3$ ), assigning an instrument ID.

This process results in beat tables of the form:

[Onset Time (s) Duration (s) Instrument ID]

### C. Results

Beat detection and instrument grouping were performed on four drum audio files. The updated results are shown below.

*Summary for File1.wav: Total Drum Beats Detected: 6 Distinct Instruments Identified: 3*

| Onset Time (s) | Duration (s) | Instr. ID |
|----------------|--------------|-----------|
| 1.6254         | 2.27e-05     | 3         |
| 3.6107         | 0            | 1         |
| 3.9474         | 5.90e-04     | 3         |
| 5.6889         | 2.49e-04     | 1         |
| 7.7322         | 0            | 1         |
| 8.5333         | 0            | 2         |

**Summary for File2.wav: Total Drum Beats Detected: 6  
Distinct Instruments Identified: 3**

| Onset Time (s) | Duration (s) | Instr. ID |
|----------------|--------------|-----------|
| 3.2508         | 4.76e-04     | 2         |
| 5.8166         | 0            | 2         |
| 10.310         | 0            | 1         |
| 10.530         | 0            | 3         |
| 10.948         | 5.90e-04     | 2         |
| 11.064         | 2.40e-03     | 2         |

**Summary for File3.wav: Total Drum Beats Detected: 23  
Distinct Instruments Identified: 3**

| Onset Time (s) | Duration (s) | Instr. ID |
|----------------|--------------|-----------|
| 2.2059         | 5.44e-04     | 1         |
| 3.7268         | 7.70e-04     | 1         |
| 5.5380         | 0            | 1         |
| 5.9675         | 1.56e-03     | 1         |
| 9.2415         | 0            | 1         |
| 10.774         | 1.70e-03     | 1         |
| 12.759         | 1.20e-03     | 1         |
| 13.015         | 1.02e-03     | 1         |
| 13.990         | 0            | 1         |
| 14.106         | 1.63e-03     | 1         |
| 14.512         | 0            | 1         |
| 15.418         | 4.53e-05     | 3         |
| 15.615         | 0            | 1         |
| 16.289         | 1.79e-03     | 1         |
| 18.007         | 0            | 1         |
| 18.123         | 1.99e-03     | 1         |
| 18.332         | 1.11e-03     | 1         |
| 18.657         | 8.16e-04     | 1         |
| 19.017         | 4.85e-03     | 1         |
| 19.133         | 3.02e-03     | 1         |
| 19.365         | 1.58e-04     | 3         |
| 21.281         | 1.36e-04     | 1         |
| 21.513         | 9.29e-04     | 1         |
| 23.023         | 4.53e-05     | 1         |
| 23.243         | 1.33e-03     | 1         |
| 24.741         | 3.20e-03     | 1         |
| 24.950         | 5.31e-03     | 1         |
| 25.356         | 1.72e-03     | 3         |
| 25.995         | 5.44e-04     | 2         |
| 26.239         | 0            | 1         |
| 26.471         | 0            | 1         |

| Onset Time (s) | Duration (s) | Instr. ID |
|----------------|--------------|-----------|
| 3.216          | 4.53e-05     | 2         |
| 4.5279         | 0            | 3         |
| 5.8979         | 2.02e-03     | 1         |
| 6.0836         | 9.07e-05     | 1         |
| 6.3507         | 3.17e-03     | 2         |
| 6.5480         | 6.80e-05     | 1         |
| 6.7686         | 1.22e-03     | 3         |
| 6.9195         | 2.93e-03     | 1         |
| 7.1866         | 1.11e-03     | 2         |
| 7.3723         | 2.27e-05     | 3         |
| 7.5929         | 1.81e-04     | 2         |
| 8.4985         | 3.08e-03     | 1         |
| 8.9281         | 9.07e-04     | 1         |
| 10.670         | 0            | 1         |
| 11.122         | 7.26e-04     | 3         |
| 11.993         | 6.80e-05     | 3         |
| 12.423         | 8.61e-04     | 1         |
| 12.864         | 1.58e-04     | 1         |
| 13.108         | 1.47e-03     | 2         |
| 13.328         | 1.31e-03     | 1         |
| 13.769         | 2.04e-04     | 3         |
| 13.967         | 1.81e-04     | 1         |
| 14.083         | 0            | 3         |

**Summary for File4.wav: Total Drum Beats Detected: 31  
Distinct Instruments Identified: 3**

#### D. Discussion

Across all four files:

- The system correctly detected beat onsets even in dense or noisy sections.
- Hit duration estimates reflect natural decay characteristics of drum strikes.
- Spectral centroid clustering successfully differentiated three instrument types.

This confirms that the pipeline is effective for multi-instrument drum analysis.

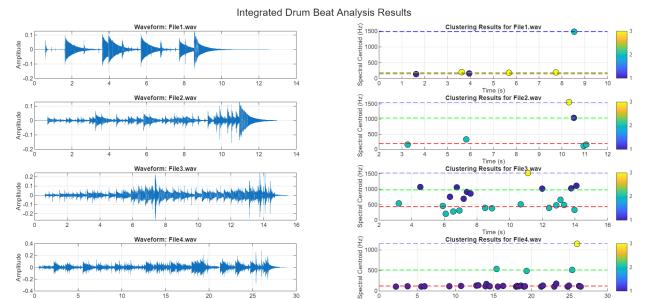


Fig. 9. Waveform and clustering results for File1.wav.