

MATHEMATICAL PROGRAMMING-1

EO-1

Operations Research -

Operations Research is the systematic, method-oriented study of the basic structure, characteristics, functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decision making.

Linear Programming Problem (LPP) -

A linear form is meant a mathematical expression of the type $a_1x_1 + a_2x_2 + \dots + a_nx_n$, where a_1, a_2, \dots, a_n are constants and x_1, x_2, \dots, x_n are variables. The term 'programming' refers to the process of determining a particular programme or plan of action.

Linear programming deals with the optimization (Maximization/Minimization) of a function of variables known as objective function, subject to a set of linear equations and/or inequalities known as constraints or restrictions.

Like & Subscribe

General Form of Linear Programming Problem (LPP) -

Definition:- The general LPP calls for optimising (Maximizing or minimizing) a linear function of variables called the 'objective function' subject to a set of linear equations and/or inequalities called the 'constraints' or 'restrictions'.

General Form:

Objective Function (Max or Min) $Z = C_1x_1 + C_2x_2 + \dots + C_nx_n$

subject to constraints $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq \text{or } \geq) b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq \text{or } \geq) b_2$

\vdots

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq \text{or } \geq) b_m$

and $x_1, x_2, \dots, x_n \geq 0$

Like & Subscribe

MATHEMATICAL PROGRAMMING-1

EO-1

Formulation of LP problems -

Eg: A firm manufactures two types of products A and B sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day. Formulate the problem as LPP.

Solⁿ: x_1 be the number of products of type A

x_2 be the number of products of type B

Machine	Type of products (minutes)		Available time (minutes)
	Type A (x_1 units)	Type B (x_2 units)	
G	1	1	400
H	2	1	600
profit per unit	Rs. 2	Rs. 3	

x_1 be the number of products of type A

x_2 be the number of products of type B

Machine	Type of products (minutes)		Available time (minutes)
	Type A (x_1 units)	Type B (x_2 units)	
G	1	1	400
H	2	1	600
profit per unit	Rs. 2	Rs. 3	

$2x_1$

$3x_2$

$$\text{Max } Z = 2x_1 + 3x_2 \quad (\text{objective Function})$$

$$\left. \begin{array}{l} x_1 + x_2 \leq 400 \\ 2x_1 + x_2 \leq 600 \end{array} \right\} \text{ constraints}$$

non-negative constraints are

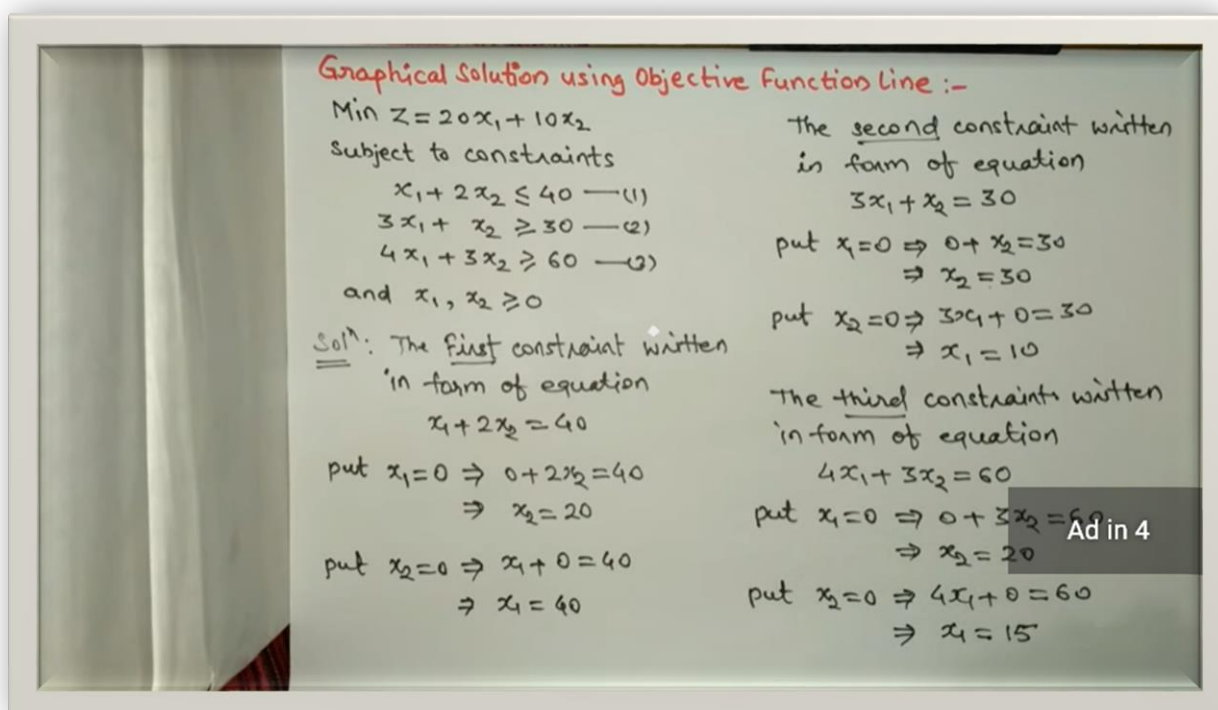
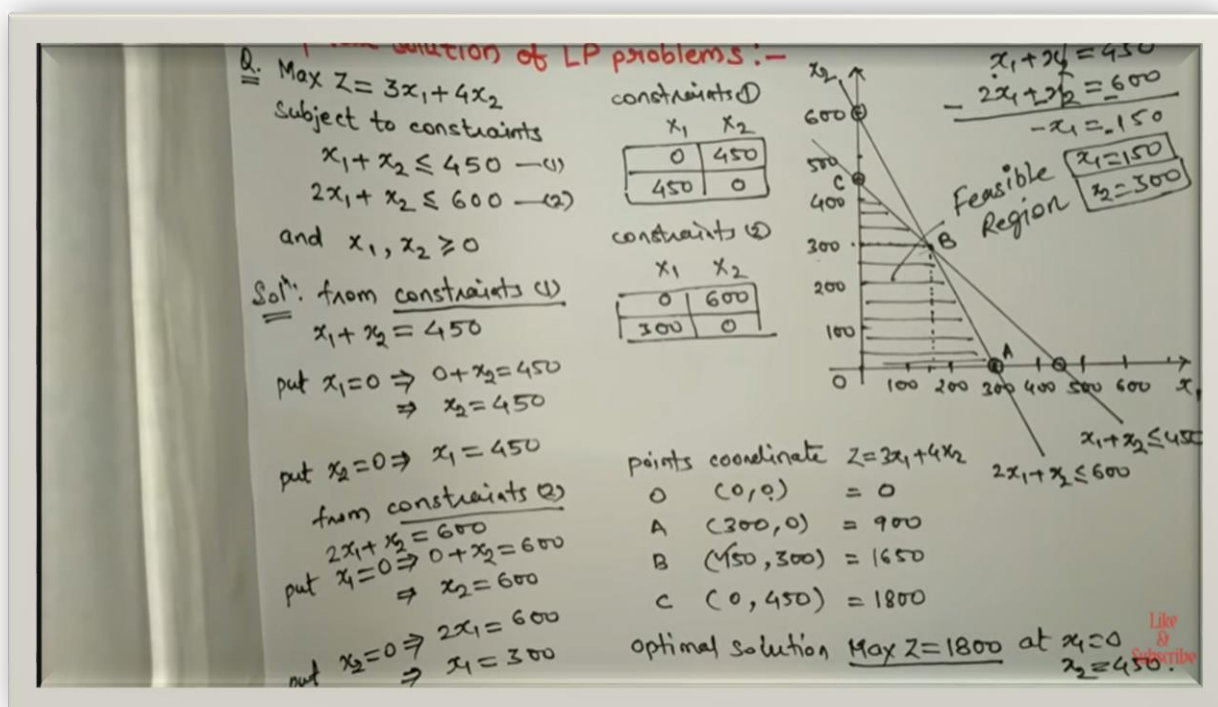
$$x_1 \geq 0, x_2 \geq 0$$

LPP

$$\text{Max } Z = 2x_1 + 3x_2$$

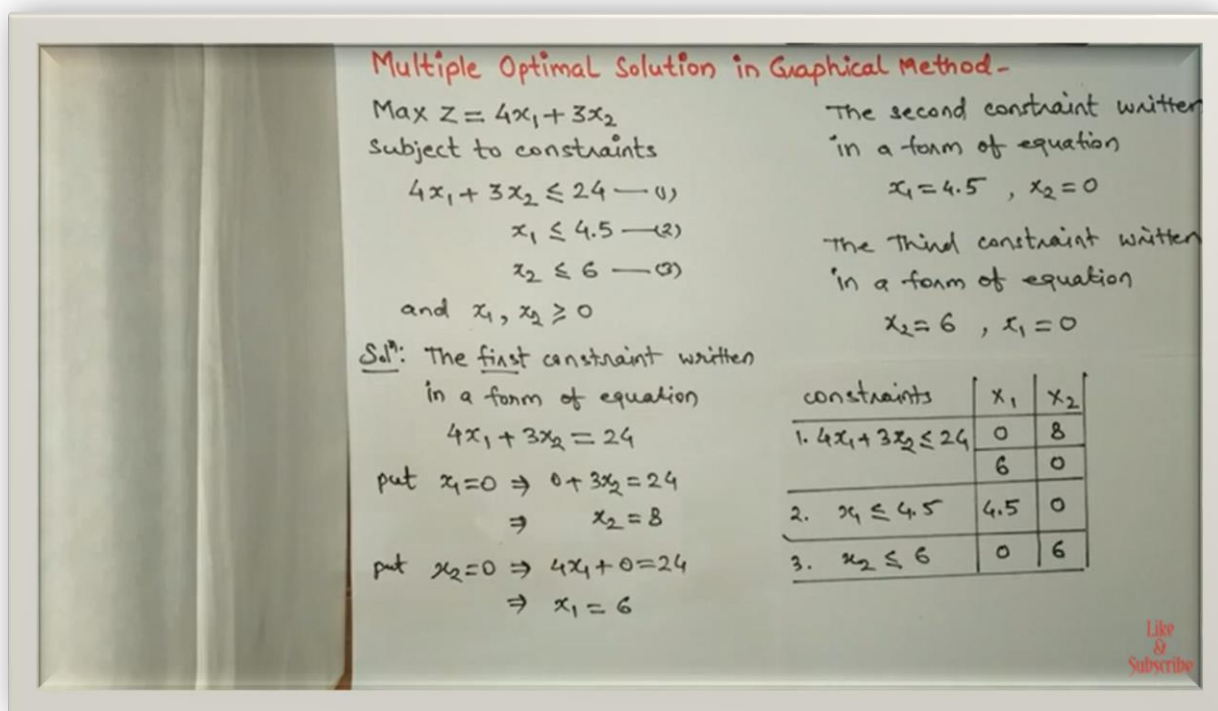
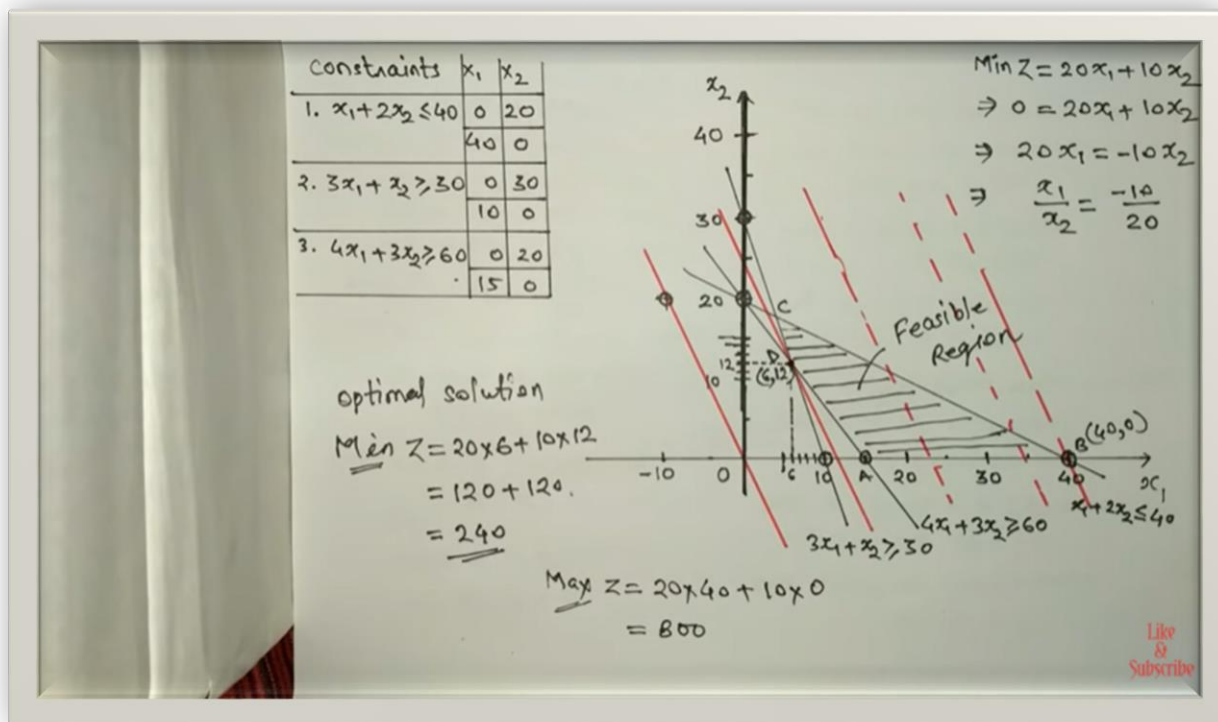
MATHEMATICAL PROGRAMMING-1

EO-1



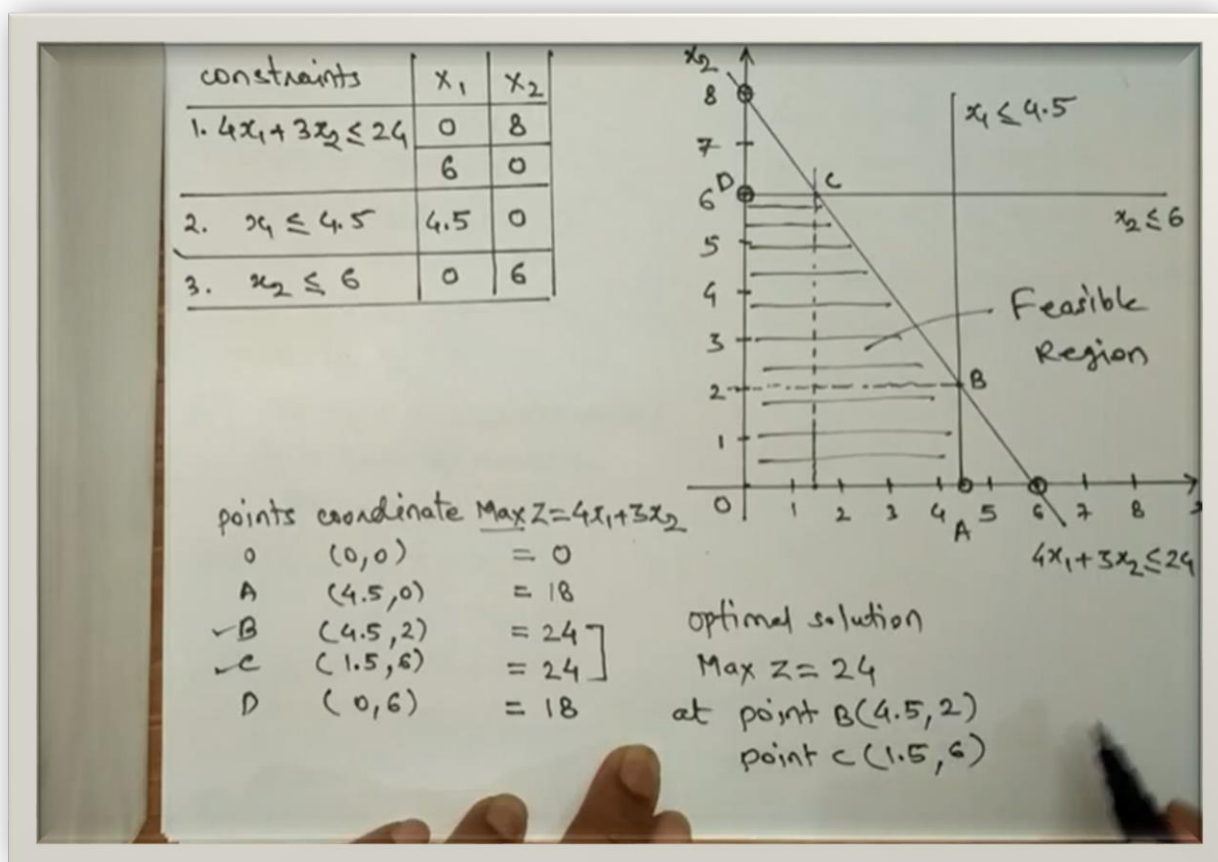
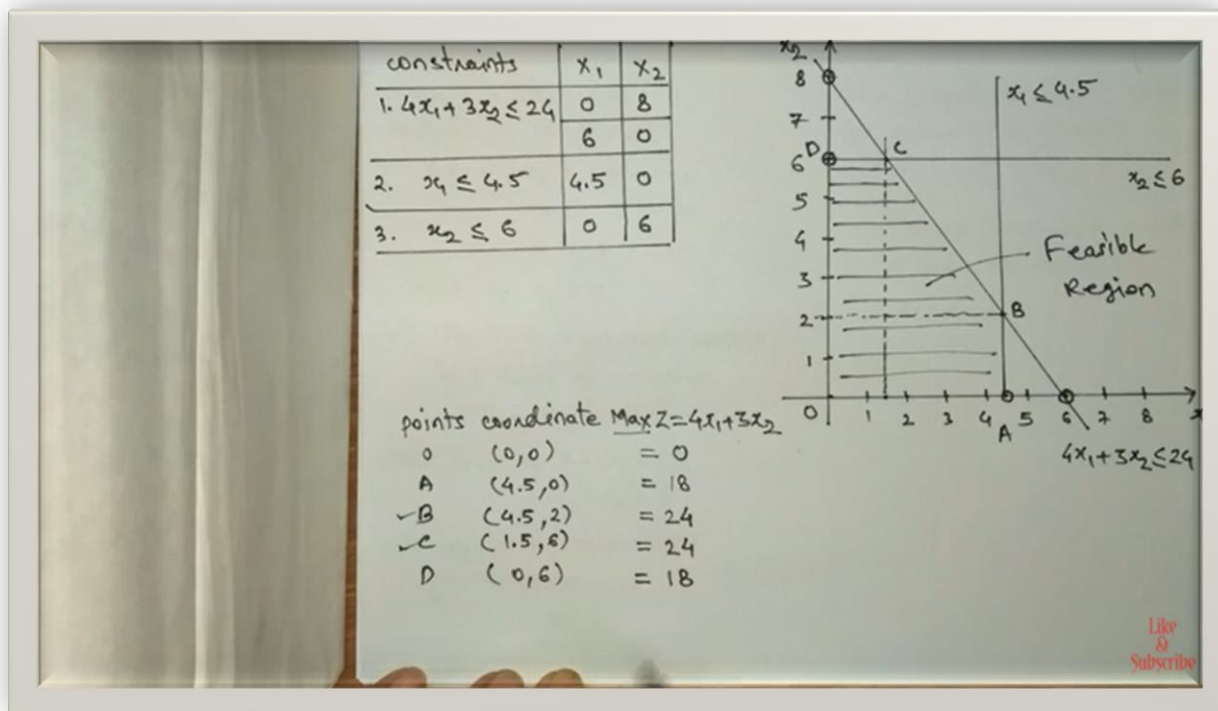
MATHEMATICAL PROGRAMMING-1

EO-1



MATHEMATICAL PROGRAMMING-1

CO-1



MATHEMATICAL PROGRAMMING-1

EO-1

No optimal solution in Graphical Method -

Q: Max $Z = 3x_1 + 2x_2$
 Subject to constraints
 $x_1 + x_2 \leq 1$ — (1)
 $x_1 + x_2 \geq 3$ — (2)
 and $x_1, x_2 \geq 0$

Solⁿ: from constraint (1)

$$x_1 + x_2 = 1$$

put $x_1 = 0 \Rightarrow x_2 = 1$ (0, 1)

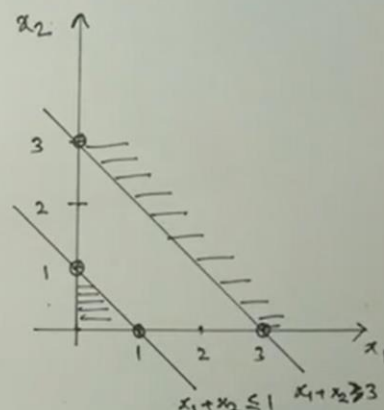
put $x_2 = 0 \Rightarrow x_1 = 1$ (1, 0)

from constraint (2)

$$x_1 + x_2 = 3$$

put $x_1 = 0 \Rightarrow x_2 = 3$ (0, 3)

put $x_2 = 0 \Rightarrow x_1 = 3$ (3, 0)



There is no feasible region.

Hence there is 'No optimal' solution.

Like & Subscribe

Unbounded Solution in Graphical Method -

Q: Max $Z = 3x_1 + 2x_2$
 Subject to constraints
 $x_1 - x_2 \leq 1$ — (1)
 $x_1 + x_2 \geq 3$ — (2)
 and $x_1, x_2 \geq 0$

Solⁿ: from constraint (1)

$$x_1 - x_2 = 1$$

put $x_1 = 0 \Rightarrow x_2 = -1$ (0, -1)

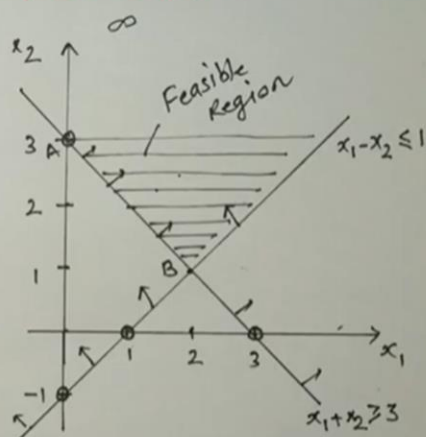
put $x_2 = 0 \Rightarrow x_1 = 1$ (1, 0)

from constraint (2)

$$x_1 + x_2 = 3$$

put $x_1 = 0 \Rightarrow x_2 = 3$ (0, 3)

put $x_2 = 0 \Rightarrow x_1 = 3$ (3, 0)



The maximum value of the objective function occurs at a point at ∞ .
 Hence the given problem has Unbounded solution.

Like & Subscribe

MATHEMATICAL PROGRAMMING-1

CO-1

General LPP to Standard LPP -

1. Write the objective function in the maximization form.
2. Convert all inequalities as equations.
3. The right side element of each constraint should be made non-negative.
4. All variables must have non-negative values.

Like
&
Subscribe

General LPP to Standard LPP -

1. Write the objective function in the maximization form.
 $\text{Max } Z \quad \text{Min } Z \quad \boxed{\text{Max } Z' = \text{Min } (-Z)}$ Eg: $\text{Min } Z = -3x_1 + x_2$
 $\text{Max } Z' = 3x_1 - x_2$
2. Convert all inequalities as equations.
 \leq + slack Eg: $x_1 + 2x_2 \leq 12$ & $2x_1 + x_2 \geq 15$
 \geq - surplus $x_1 + 2x_2 + s_1 = 12$ $2x_1 + x_2 - s_2 = 15$
3. The right side element of each constraint should be made non-negative.
 Eg: $2x_1 + x_2 - s_2 = -15$ (Multiplying by -1)
 $\Rightarrow -2x_1 - x_2 + s_2 = 15$
4. All variables must have non-negative values.
 Eg: $x_1 + x_2 \leq 3$, $x_1 \geq 0$, x_2 is unrestricted in sign
 $x_1 + (x_2' - x_2'') \leq 3$ $x_1, x_2', x_2'' \geq 0$
 $x_1 + x_2' - x_2'' + s_1 = 3$

Like
&
Subscribe

MATHEMATICAL PROGRAMMING-1

CO-1

Examples: GLPP to SLPP -

① Max $Z = 3x_1 + x_2$
 Subject to
 $2x_1 + x_2 \leq 2$
 $3x_1 + 4x_2 \geq 12$
 and $x_1, x_2 \geq 0$

SLPP

Max $Z = 3x_1 + x_2$
 s.t.
 $2x_1 + x_2 + s_1 = 2$
 $3x_1 + 4x_2 - s_2 = 12$
 and $x_1, x_2, s_1, s_2 \geq 0$

② Min $Z = 4x_1 + 2x_2$
 subject to
 $3x_1 + x_2 \geq 2$
 $x_1 + x_2 \geq 21$
 $x_1 + 2x_2 \geq 30$
 and $x_1, x_2 \geq 0$

SLPP

Max $Z' = -4x_1 - 2x_2$
 subject to
 $3x_1 + x_2 - s_1 = 2$
 $x_1 + x_2 - s_2 = 21$
 $x_1 + 2x_2 - s_3 = 30$
 and $x_1, x_2, s_1, s_2, s_3 \geq 0$

③ Min $Z = x_1 + 2x_2 + 3x_3$
 subject to
 $2x_1 + 3x_2 + 3x_3 \geq -4$
 $3x_1 + 5x_2 + 2x_3 \leq 7$
 and $x_1, x_2 \geq 0$,
 x_3 is unrestricted in sign

SLPP

Max $Z' = -x_1 - 2x_2 - 3x_3$
 subject to
 $-2x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 4$
 $3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$
 and $x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$

Simplex Method -

Q. Solve the following LPP
 Max $Z = 80x_1 + 55x_2$
 Subject to
 $4x_1 + 2x_2 \leq 40$
 $2x_1 + 4x_2 \leq 32$
 and $x_1, x_2 \geq 0$

Solⁿ: step ① SLPP

Max $Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$
 Subject to
 $4x_1 + 2x_2 + s_1 = 40$
 $2x_1 + 4x_2 + s_2 = 32$
 and $x_1, x_2, s_1, s_2 \geq 0$

step ② Represent in Matrix form

$$AX = B$$

$$\begin{bmatrix} x_1 & x_2 & s_1 & s_2 \\ 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

step ③ construct starting simplex table

Basic Variable	C_B	X_B	C_j 80 55 0 0			
			x_1	x_2	s_1	s_2
s_1	0	40	4	2	1	0
s_2	0	32	2	4	0	1

MATHEMATICAL PROGRAMMING-1

CO-1

Step 4: Calculation of Z and A_j (Net Evaluation) and test the basic feasible solution for optimality by the rules given.

$$Z = C_B X_B \quad \text{and} \quad A_j = Z_j - C_j = C_B X_j - C_j$$

Rule 1 - If all $A_j \geq 0$, the solution under the test will be optimal.

Rule 2 - If atleast one A_j is negative, the solution is not optimal and then proceeds to improve the solution in the next step.

Rule 3 - If corresponding to any negative A_j , all elements of the column x_j are negative or zero, then the solution under test will be unbounded.

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: step (1) SLPP

step (2) construct starting simplex table

		C_j	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	
s_1	0	40	4	2	1	0	
s_2	0	32	2	4	0	1	
$Z = 0$		$A_j \rightarrow$	-80	-55	0	0	

Step 5: To improve the basic feasible solution, the incoming and outgoing vectors are determined.

(i) Incoming (Entering) Vector (x_k) :- most negative value of A_j

(ii) Outgoing (Leaving) Vector :- $\min [X_B/x_k, x_k > 0]$

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: step (1) SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

step (2) construct starting simplex table

		C_j	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	Min X_B/x_k
$\leftarrow s_1$	0	40	4	2	1	0	40/4 = 10
s_2	0	32	2	4	0	1	32/2 = 16
$Z = 0$		$A_j \rightarrow$	-80	-55	0	0	

(x_1) incoming vector.

outgoing vector

Like & Subscribe

MATHEMATICAL PROGRAMMING-1

EO-1

Step 6: Mark the key element (pivot element) at the intersection of outgoing vector and incoming vector.

(i) key element should be one.

(ii) Remaining values of x_k should be zero.
(column)

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: step 1 SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

step 2 Construct starting simplex table

		Cj: 80 55 0 0						MinR
Basic Variable	Cb	Xb	x ₁	x ₂	s ₁	s ₂	x _b /x _k	
← S ₁	0	40	4	2	1	0	40/4=10	outgoing vector
S ₂	0	32	2	4	0	1	32/2=16	
Z = 0			Δj → -80	-55	0	0		(x ₁) incoming vector

		Cj: 80 55 0 0						
Basic Variable	Cb	Xb	x ₁	x ₂	s ₁	s ₂		

Like & Subscribe

Step 7: Now repeat step 4 to step 6 until an optimal solution is obtained.

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: step 1 SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

$$\text{and } x_1, x_2, s_1, s_2 \geq 0$$

step 2 Construct starting simplex table

		Cj: 80 55 0 0						MinR
Basic Variable	Cb	Xb	x ₁	x ₂	s ₁	s ₂	x _b /x _k	
← S ₁	0	40	4	2	1	0	40/4=10	outgoing vector
S ₂	0	32	2	4	0	1	32/2=16	
Z = 0			Δj → -80	-55	0	0		(x ₁) incoming vector

		Cj: 80 55 0 0						
Basic Variable	Cb	Xb	x ₁	x ₂	s ₁	s ₂		
x ₁	80	10	1	1/2	1/4	0		
S ₂	0	12	0	3	-1/2	1		

Like & Subscribe

MATHEMATICAL PROGRAMMING-1

EO-1

Simplex Method -

Q. Solve the following LPP

$$\text{Max } Z = 80x_1 + 55x_2$$

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: step ① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

$$\text{and } x_1, x_2, s_1, s_2 \geq 0$$

step ② Represent in matrix form

$$AX = B$$

$$\begin{bmatrix} x_1 & x_2 & s_1 & s_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

step ③ Construct starting simplex table

		C _j	80	55	0	0	Min R
Basic Variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂	X _B /X _k
← s ₁	0	40	4	2	1	0	40/4=10
s ₂	0	32	2	4	0	1	32/2=16
Z = 0							

(x₂) incoming vector

		C _j	80	55	0	0	
Basic Variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂	
x ₁	80	10	1	1/2	1/4	0	
s ₂	0	12	0	3	-1/2	1	
Z = 800							

Like & Subscribe

Subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: step ① SLPP

$$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$$

Subject to

$$4x_1 + 2x_2 + s_1 = 40$$

$$2x_1 + 4x_2 + s_2 = 32$$

$$\text{and } x_1, x_2, s_1, s_2 \geq 0$$

step ② Represent in matrix form

$$AX = B$$

$$\begin{bmatrix} x_1 & x_2 & s_1 & s_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

		C _j	80	55	0	0	Min R
Basic Variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂	X _B /X _k
← s ₁	0	40	4	2	1	0	40/4=10
s ₂	0	32	2	4	0	1	32/2=16
Z = 0							

(x₂) incoming vector

		C _j	80	55	0	0	Min R
Basic Variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂	X _B /X _k
x ₁	80	10	1	1/2	1/4	0	20
← s ₂	0	12	0	3	-1/2	1	4
Z = 800							

incoming

		C _j	80	55	0	0	
Basic Variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂	
x ₁	80	8	1	0	1/3	-1/6	
x ₂	55	4	0	1	-1/6	1/3	
Z = 860							

Like & Subscribe

MATHEMATICAL PROGRAMMING-1

CO-1

$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$
 Subject to
 $4x_1 + 2x_2 + s_1 = 40$
 $2x_1 + 4x_2 + s_2 = 32$
 and $x_1, x_2, s_1, s_2 \geq 0$

Step 2 Represent in Matrix form

$$AX = B \quad \begin{bmatrix} x_1 & x_2 & s_1 & s_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

x_1	80	10	1	1/2	0	20
s_2	0	12	0	3	-1/2	1
$Z = 800$	Δ_j	0	-15	20	0	

outgoing vector ← s_2 incoming

	C_j	80	55	0	0	
Basic Variable	X_B	x_1	x_2	s_1	s_2	
x_1	80	8	1	0	1/3	-1/6
x_2	55	4	0	1	-1/6	1/3
$Z = 860$	Δ_j	0	0	35/2	5	

Since all $\Delta_j \geq 0$, optimal basic feasible solⁿ is obtained.
 $\text{Max } Z = 860, x_1 = 8, x_2 = 4.$

Simplex Method - Minimization Problem

Qu: GLPP! $\text{Min } Z = x_1 - 3x_2 + 2x_3$
 Subject to
 $3x_1 - x_2 + 3x_3 \leq 7$
 $-2x_1 + 4x_2 \leq 12$
 $-4x_1 + 3x_2 + 8x_3 \leq 10$
 and $x_1, x_2, x_3 \geq 0$

Like & Subscribe

MATHEMATICAL PROGRAMMING-1

CO-1

Simplex Method - Unbounded Solution

$$\text{Max } Z = 3x_1 + 4x_2$$

Subject to

$$x_1 - x_2 \leq 1$$

$$-x_1 + x_2 \leq 2$$

$$\text{and } x_1, x_2 \geq 0$$

Solⁿ: SLPP

$$\text{Max } Z = 3x_1 + 4x_2 + 0s_1 + 0s_2$$

Subject to

$$x_1 - x_2 + s_1 = 1$$

$$-x_1 + x_2 + s_2 = 2$$

$$\text{and } x_1, x_2, s_1, s_2 \geq 0$$

Initial Basic feasible solution

$$x_1 = x_2 = 0 \quad s_1 = 1, s_2 = 2$$

Matrix form $AX = B$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

construct starting simplex table

		C _j 3 4 0 0					Min Ratio X _B /X _k , X _k
Basic variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂	
s ₁	0	1	1	-1	1	0	-
s ₂	0	2	-1	1	0	1	2/1 ←
Z = 0		A _j -3 -4 0 0					

outgoing vector

(x₂) incoming vector

		C _j 3 4 0 0				
Basic variable	C _B	X _B	x ₁	x ₂	s ₁	s ₂
s ₁	0	3	0	0	1	1
x ₂	4	2	-1	1	0	1
Z = 8		A _j -7/4 0 0 4				

∴ all element of the column x₁ are negative or zero then the solution is Unbounded.

Like & Subscribe