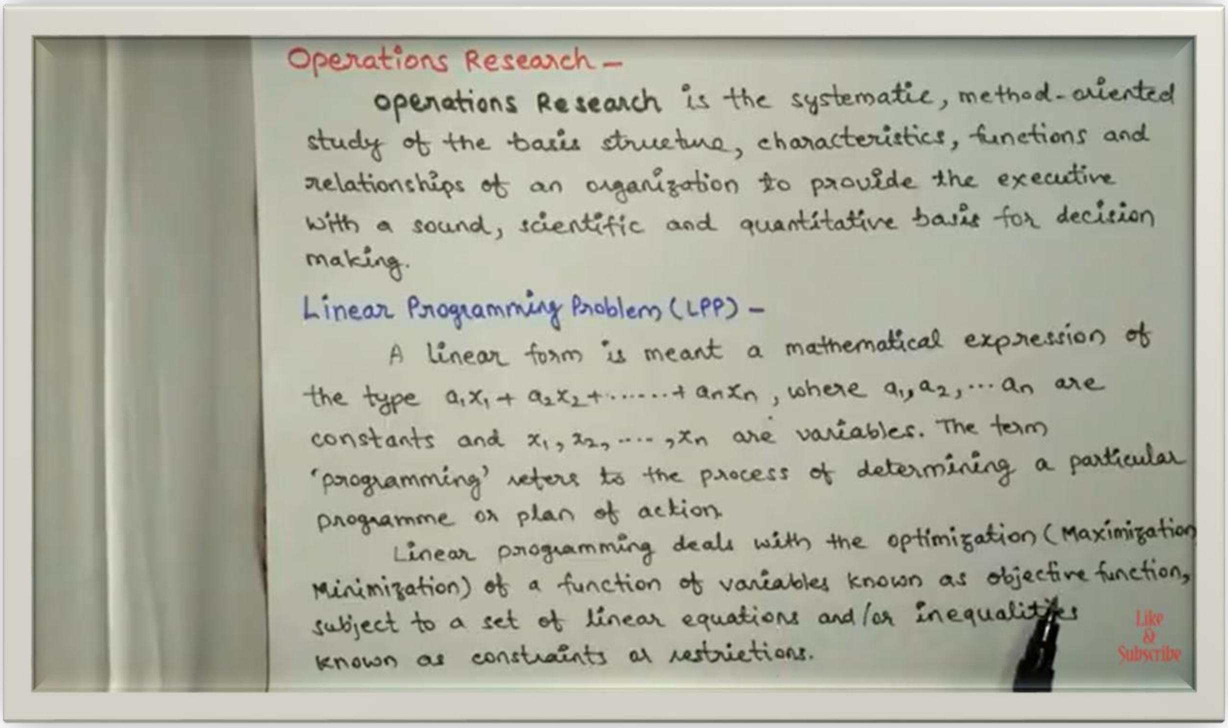
MATHEMATICAL PROGRAMMING-1

CO-1



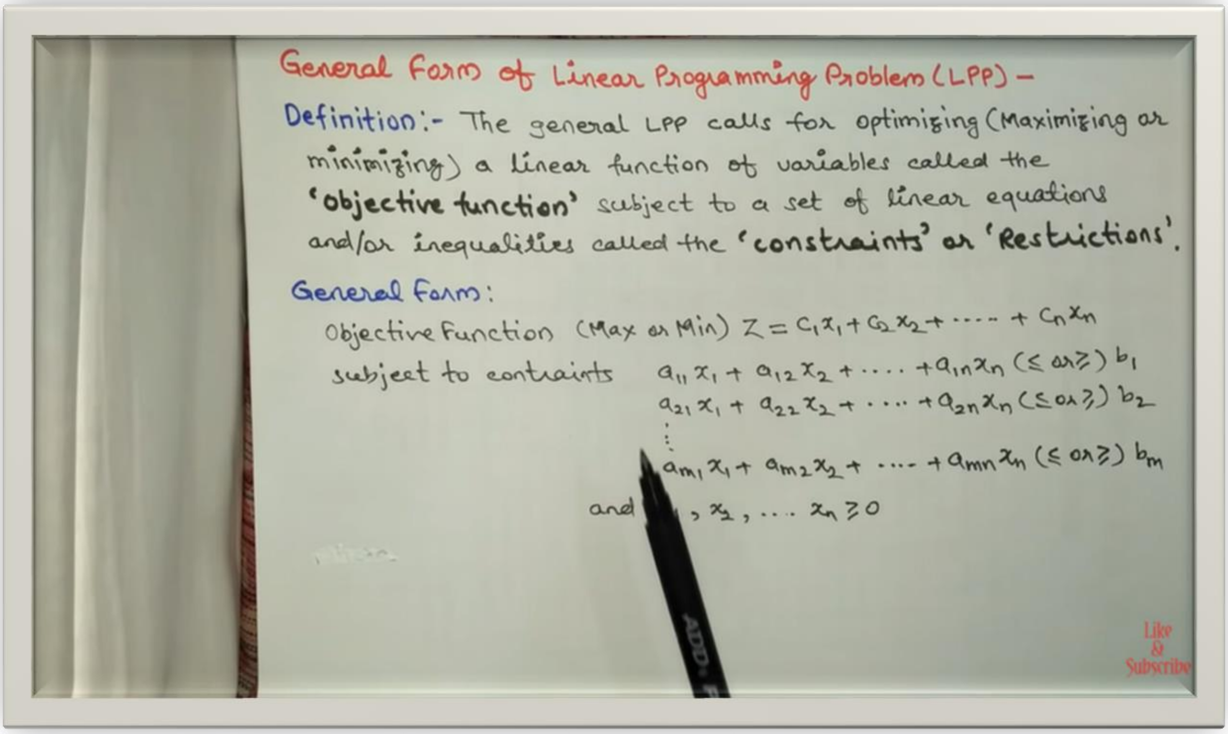
Operations Research –

operations Research is the systematic, method-oriented study of the basic structure, characteristics, functions and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decision making.

Linear Programming Problem (LPP) –

A linear form is meant a mathematical expression of the type $a_1x_1 + a_2x_2 + \cdots + a_nx_n$, where $a_1, a_2, \cdots a_n$ are constants and $x_1, x_2, \cdots, x_n$ are variables. The term 'programming' refers to the process of determining a particular programme or plan of action.

Linear programming deals with the optimization (Maximization Minimization) of a function of variables known as objective function, subject to a set of linear equations and/or inequalities known as constraints or restrictions.



General Form of Linear Programming Problem (LPP) –

Definition:- The general LPP calls for optimizing (Maximizing or minimizing) a linear function of variables called the **'objective function'** subject to a set of linear equations and/or inequalities called the **'constraints'** or **'Restrictions'**.

General Form:

Objective Function (Max or Min) $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

subject to contraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\leq \text{or} \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\leq \text{or} \geq)\ b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq \text{or} \geq)\ b_m$$

and $x_1, x_2, \ldots x_n \geq 0$

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Formulation of LP problems –

Eg: A firm manufactures two types of products A and B sells them at a profit of Rs.2 on type A and Rs. 3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day. formulate the problem as LPP.

Sol^n: x_1 be the number of products of type A

x_2 be the number of products of type B

Machine	Type of products (minutes) Type A (x_1 units)	Type B (x_2 units)	Available time (minutes)
G	1	1	400
H	2	1	600
profit per unit	Rs. 2	Rs. 3	

x_2 be the number of products of type B

Machine	Type of products (minutes) Type A (x_1 units)	Type B (x_2 units)	Available time (minutes)
G	1	1	400
H	2	1	600
profit per unit	Rs. 2	Rs. 3	
	$2x_1$	$3x_2$	

Max $Z = 2x_1 + 3x_2$ (objective Function)

$$\left.\begin{array}{l} x_1 + x_2 \leq 400 \\ 2x_1 + x_2 \leq 600 \end{array}\right\} \text{constraints}$$

non-negative constraints are

$x_1 \geq 0$, $x_2 \geq 0$

LPP

Max $Z = 2x_1 + 3x_2$

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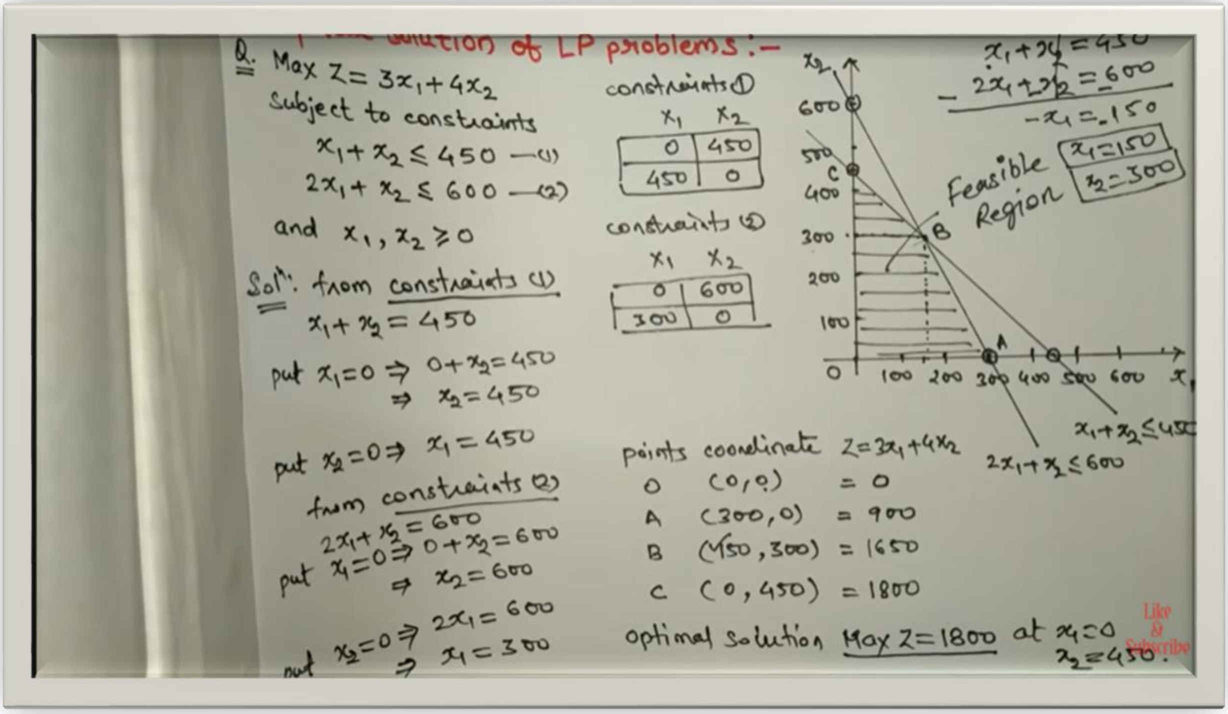

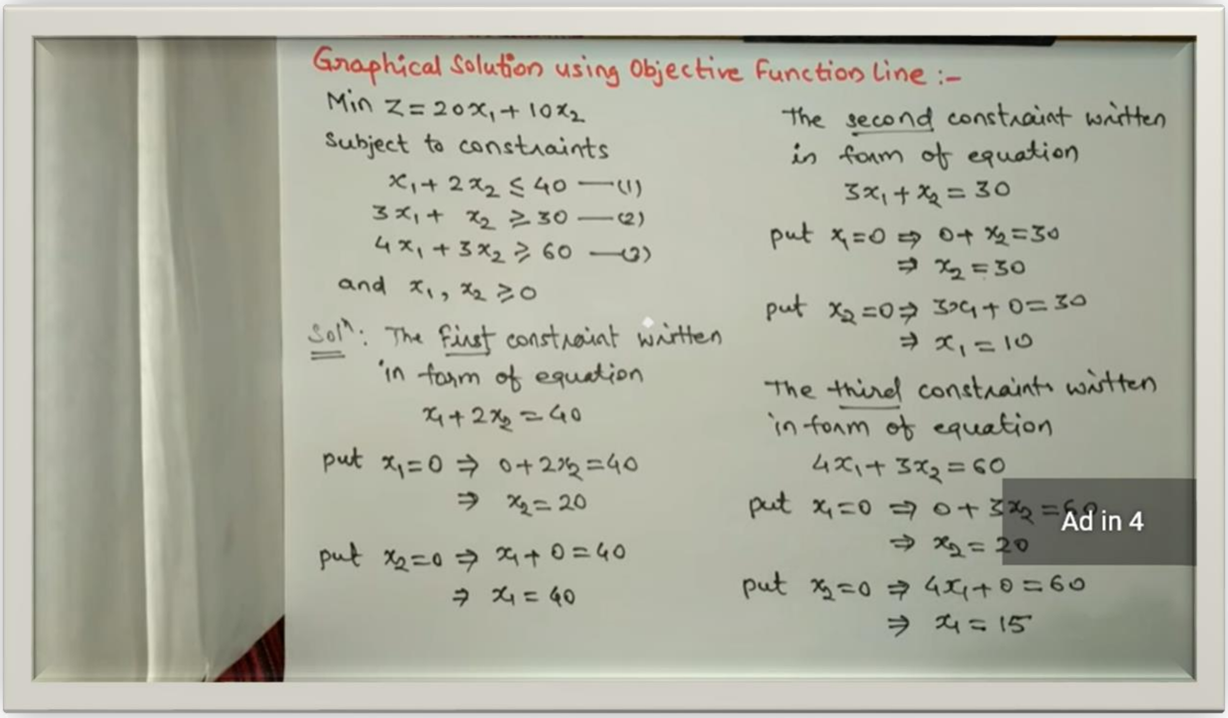

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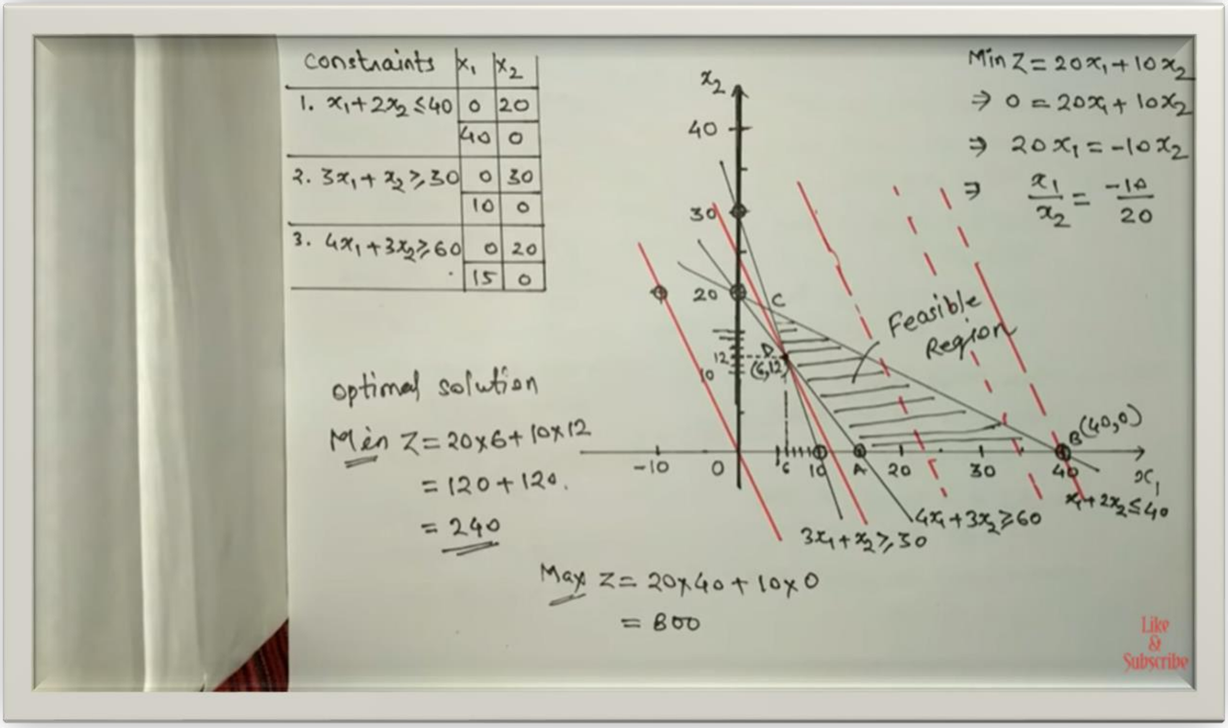

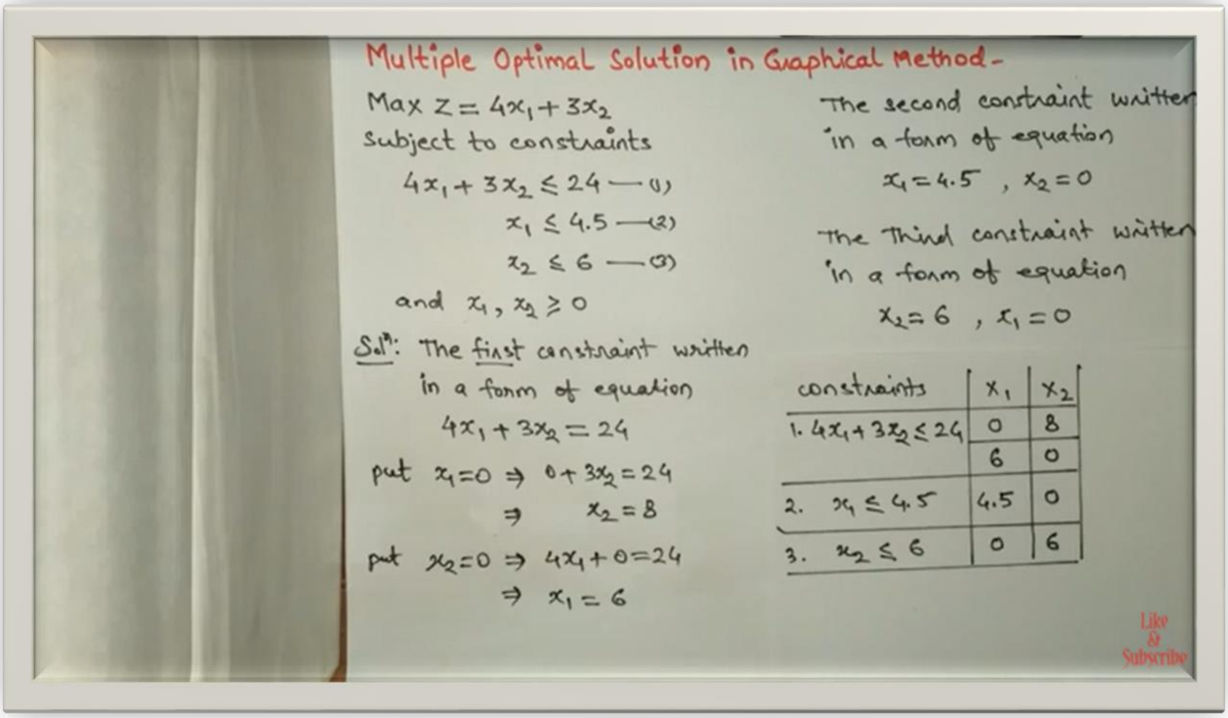

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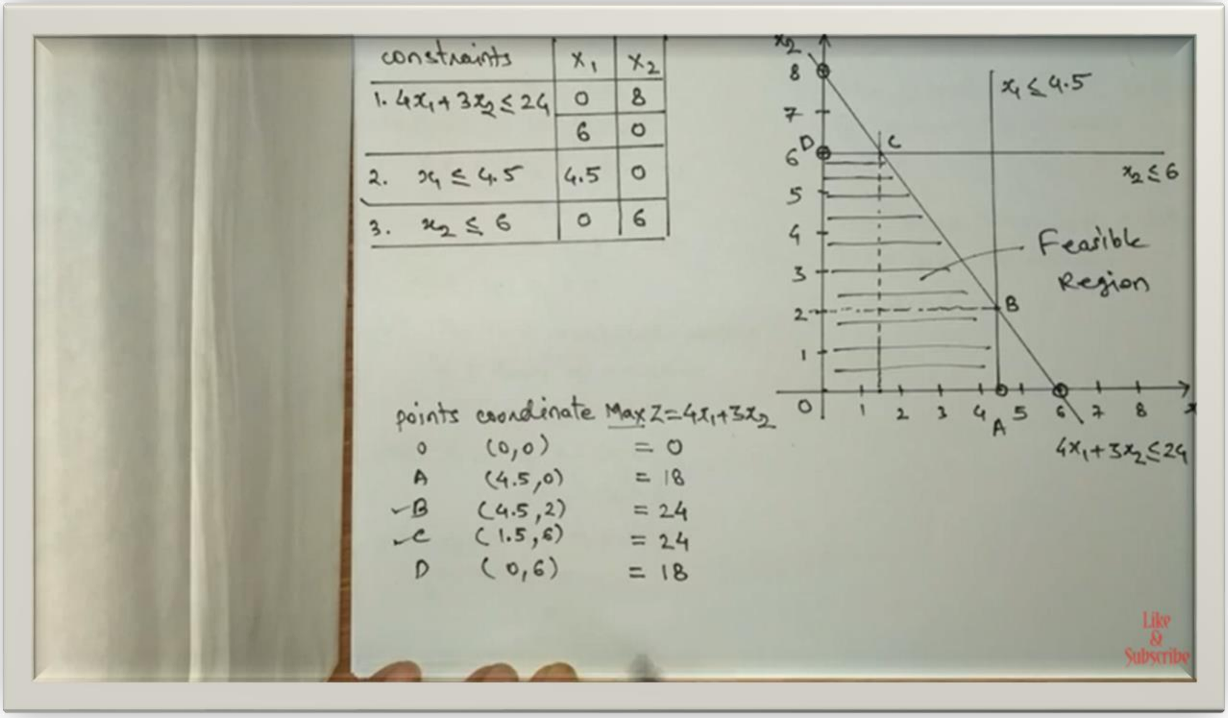

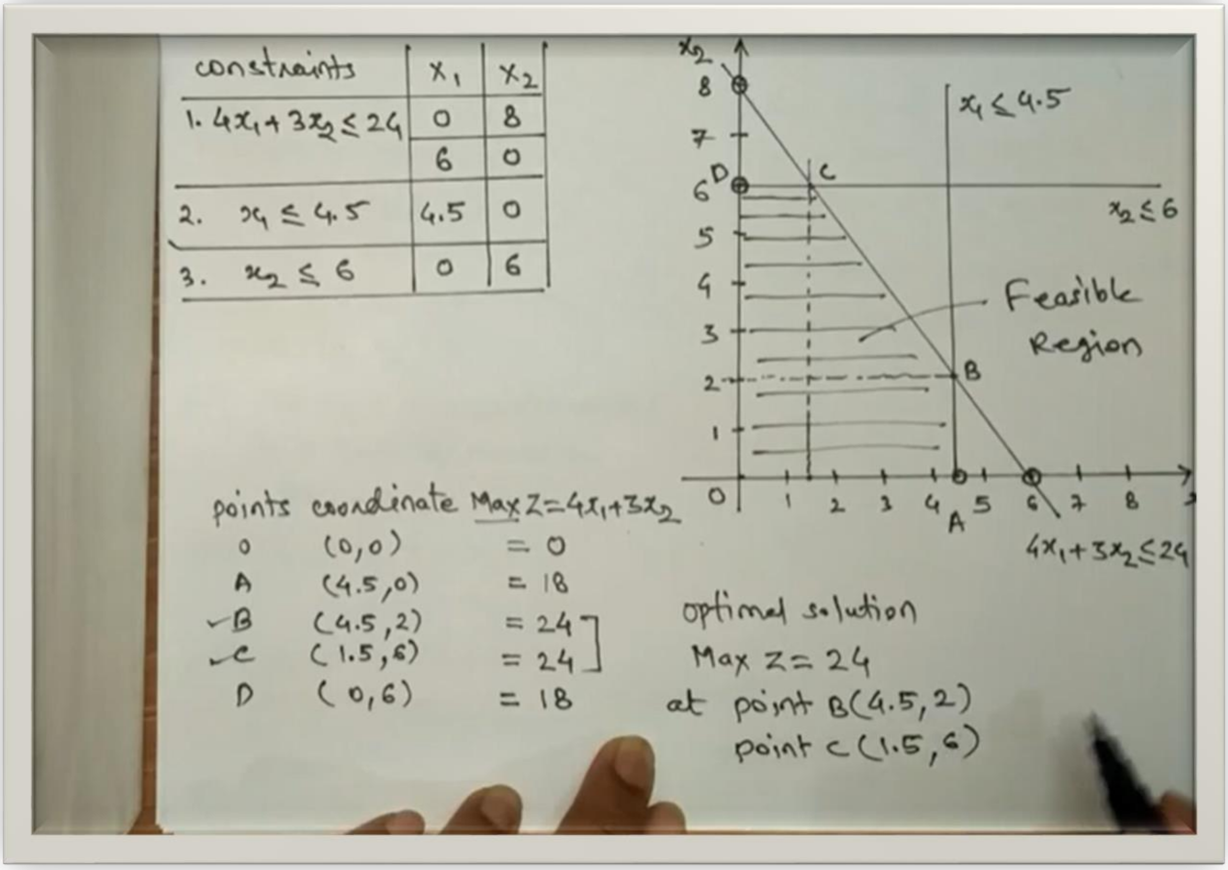

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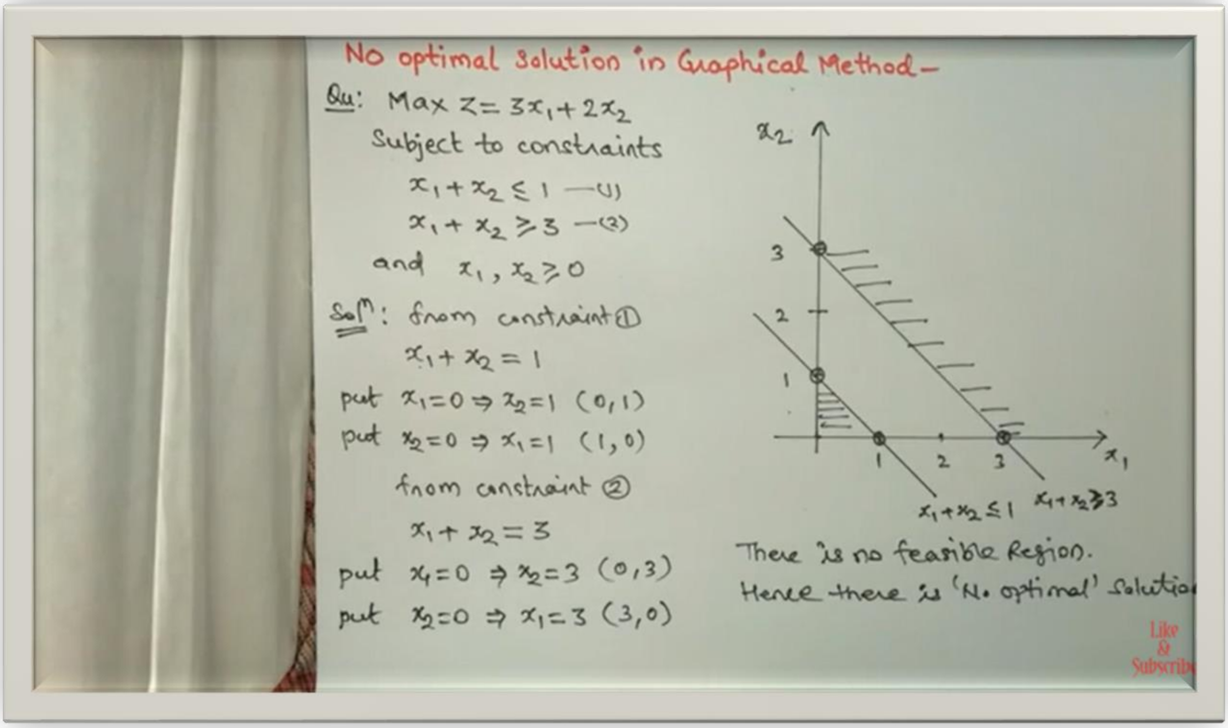

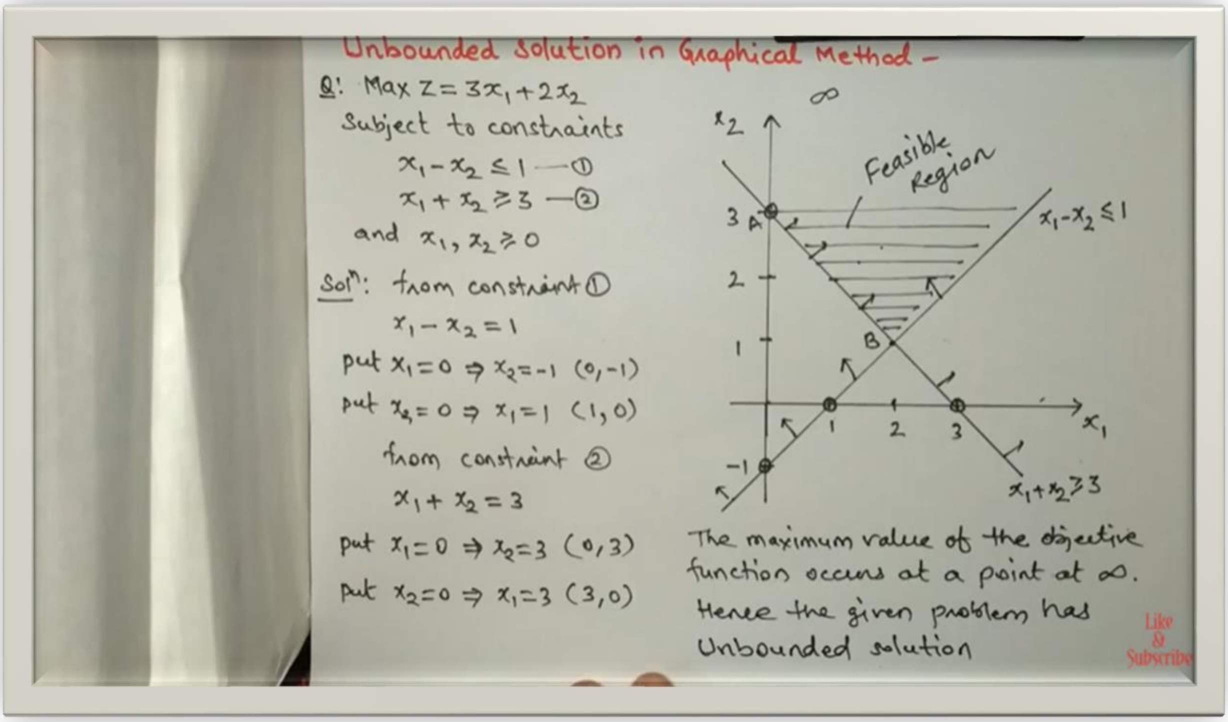

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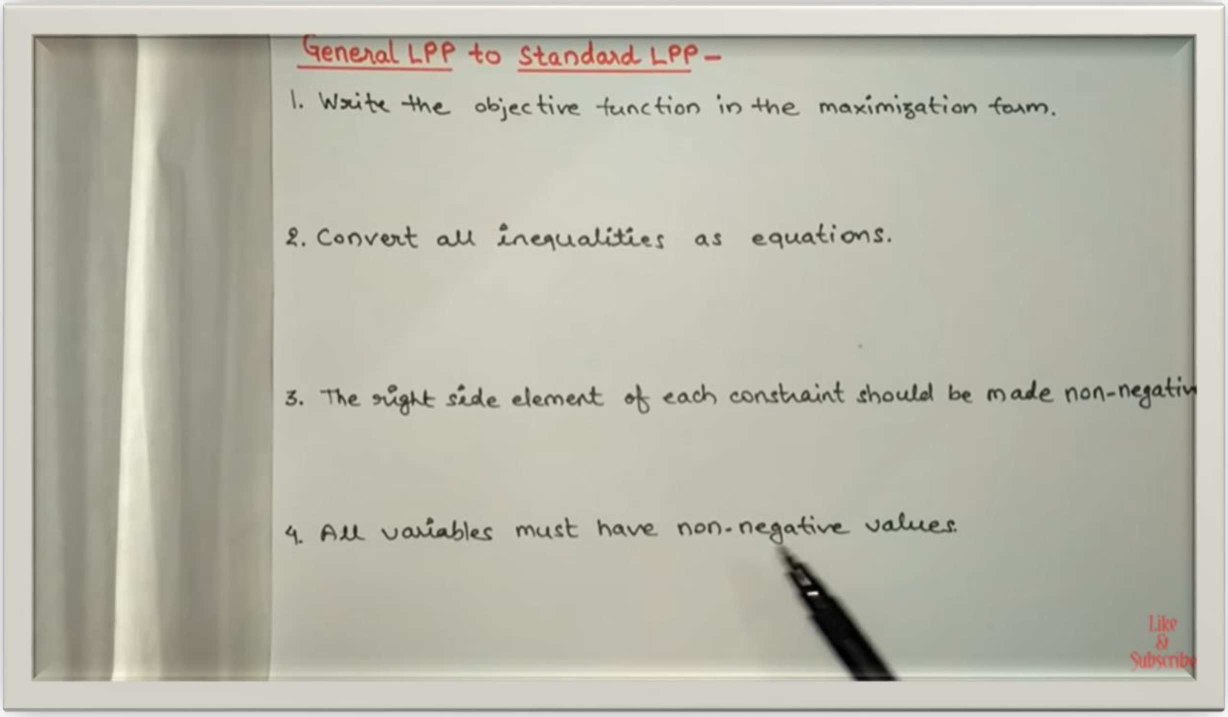


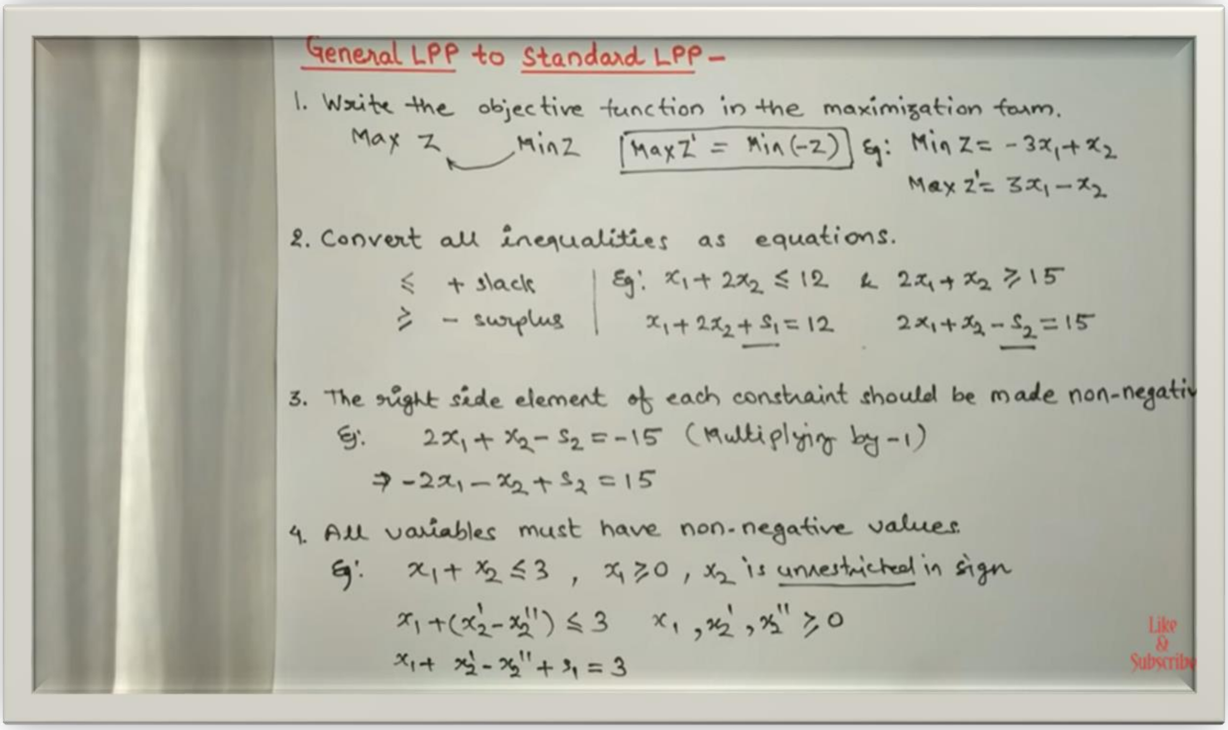
General LPP to Standard LPP –

1. Write the objective function in the maximization form.

2. Convert all inequalities as equations.

3. The right side element of each constraint should be made non-negativ

4. All variables must have non-negative values.



General LPP to Standard LPP –

1. Write the objective function in the maximization form.

   Max Z ⟵ Min Z $\boxed{\text{Max } Z' = \text{Min}(-Z)}$ Eg: Min $Z = -3x_1 + x_2$

   Max $Z' = 3x_1 - x_2$

2. Convert all inequalities as equations.

   $\leq$ + slack | Eg: $x_1 + 2x_2 \leq 12$ & $2x_1 + x_2 \geq 15$

   $\geq$ – surplus | $x_1 + 2x_2 + \underline{s_1} = 12$ $2x_1 + x_2 - \underline{s_2} = 15$

3. The right side element of each constraint should be made non-negativ

   Eg: $2x_1 + x_2 - s_2 = -15$ (Multiplying by -1)

   $\Rightarrow -2x_1 - x_2 + s_2 = 15$

4. All variables must have non-negative values.

   Eg: $x_1 + x_2 \leq 3$, $x_1 \geq 0$, x_2 is <u>unrestricted</u> in sign

   $x_1 + (x_2' - x_2'') \leq 3$ $x_1, x_2', x_2'' \geq 0$

   $x_1 + x_2' - x_2'' + s_1 = 3$

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Examples: GLPP to SLPP –

① $\text{Max } z = 3x_1 + x_2$

Subject to

$2x_1 + x_2 \overset{+s_1}{\leq} 2$

$3x_1 + 4x_2 \underset{-s_2}{\geq} 12$

and $x_1, x_2 \geq 0$

SLPP

$\text{Max } z = 3x_1 + x_2$

s.t.

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 = 12$

and $x_1, x_2, s_1, s_2 \geq 0$

② $\text{Min } Z = 4x_1 + 2x_2$

subject to

$3x_1 + x_2 \geq 2$

$x_1 + x_2 \geq 21$

$x_1 + 2x_2 \geq 30$

and $x_1, x_2 \geq 0$

SLPP

$\text{Max } z' = -4x_1 - 2x_2$

subject to

$3x_1 + x_2 - s_1 = 2$

$x_1 + x_2 - s_2 = 21$

$x_1 + 2x_2 - s_3 = 30$

and $x_1, x_2, s_1, s_2, s_3 \geq 0$

③ $\text{Min } Z = x_1 + 2x_2 + 3x_3$

Subject to

$2x_1 + 3x_2 + 3x_3 \geq -4$ ($\leq$)

$3x_1 + 5x_2 + 2x_3 \leq 7$

and $x_1, x_2 \geq 0$,

x_3 is unrestricted in sign

SLPP

$\text{Max } z' = -x_1 - 2x_2 - 3x_3$

Subject to

$-2x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 4$

$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$

and $x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$

Simplex Method –

Q. Solve the following LPP

$\text{Max } Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \leq 40$

$2x_1 + 4x_2 \leq 32$

and $x_1, x_2 \geq 0$

Solⁿ: step① SLPP

$\text{Max } Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$

Subject to

$4x_1 + 2x_2 + s_1 = 40$

$2x_1 + 4x_2 + s_2 = 32$

and $x_1, x_2, s_1, s_2 \geq 0$

step② Represent in matrix form

$AX = B$

$$\begin{array}{cccc} x_1 & x_2 & s_1 & s_2 \\ \end{array}$$
$$\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 40 \\ 32 \end{bmatrix}$$

step③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	s_1	s_2	
s_1	0	40	4	2	1	0	
s_2	0	32	2	4	0	1	

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Step 4: Calculation of Z and Δ_j (Net Evaluation) and test the basic feasible solution for optimality by the rules given.

$\boxed{Z = C_B X_B}$ and $\boxed{\Delta_j = Z_j - C_j = C_B X_j - C_j}$

Rule 1 – If all $\Delta_j \geq 0$, the solution under the test will be optimal.

Rule 2 – If atleast one Δ_j is negative, the solution is not optimal and then proceeds to improve the solution in the next step.

Rule 3 – If corresponding to any negative Δ_j, all elements of the column X_j are negative or zero, then the solution under test will be unbounded.

Simplex Method –

Q. Solve the following LPP

Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \leq 40$

$2x_1 + 4x_2 \leq 32$

and $x_1, x_2 \geq 0$

Sol^n: step① SLPP

step② construct starting simplex table

Basic Variable	C_B	$C_j \rightarrow$ X_B	80 x_1	55 x_2	0 s_1	0 s_2
S_1	0	40	4	2	1	0
S_2	0	32	2	4	0	1
Z = 0		$\Delta_j \rightarrow$	-80	-55	0	0

Step 5: To improve the basic feasible solution, the incoming and outgoing vectors are determined.

(i) Incoming (Entering) Vector (X_K): – most negative value of Δ_j

(ii) Outgoing (Leaving) Vector: – min $[X_B/x_K \,,\ x_K > 0]$

Simplex Method –

Q. Solve the following LPP

Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \leq 40$

$2x_1 + 4x_2 \leq 32$

and $x_1, x_2 \geq 0$

Sol^n: step① SLPP

Max $Z = 80x_1 + 55x_2 + 0S_1 + 0S_2$

Subject to

$4x_1 + 2x_2 + S_1 = 40$

step② construct starting simplex table

Basic Variable	C_B	$C_j \rightarrow$ X_B	80 x_1	55 x_2	0 s_1	0 s_2	Min X_B/X_K
outgoing vector ← S_1	0	40	4	2	1	0	40/4=
S_2	0	32	2	4	0	1	32/2=
Z = 0		$\Delta_j \rightarrow$	-80	-55	0	0	

(X_K) incoming vector.

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Step 6: Mark the key element (pivot element) at the intersection of outgoing vector and incoming vector.

(i) Key element should be one.

(ii) Remaining values of x_k (column) should be zero.

Simplex Method –

Q. Solve the following LPP

Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \le 40$

$2x_1 + 4x_2 \le 32$

and $x_1, x_2 \ge 0$

Sol^n: step① SLPP

Max $Z = 80x_1 + 55x_2 + 0S_1 + 0S_2$

Subject to

$4x_1 + 2x_2 + S_1 = 40$

$2x_1 + 4x_2 + S_2 = 32$

step③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	Min R
Basic Variable	C_B	X_B	x_1	x_2	S_1	S_2	X_B/X_k
outgoing vector $\leftarrow S_1$	0	40	4	2	1	0	40/4=
S_2	0	32	2	4	0	1	32/2=
$Z = 0$		$\Delta_j \rightarrow$	-80	-55	0	0	

$(x_k)\uparrow$ incoming vector

		C_j					
Basic Variable	C_B	X_B	x_1	x_2	S_1	S_2	

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Step 7: Now repeat step 4 to step 6 until an optimal solution is obtained.

Simplex Method –

Q. Solve the following LPP

Max $Z = 80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \le 40$

$2x_1 + 4x_2 \le 32$

and $x_1, x_2 \ge 0$

Sol^n: step① SLPP

Max $Z = 80x_1 + 55x_2 + 0S_1 + 0S_2$

Subject to

$4x_1 + 2x_2 + S_1 = 40$

$2x_1 + 4x_2 + S_2 = 32$

and $x_1, x_2, S_1, S_2 \ge 0$

step③ construct starting simplex table

		$C_j \rightarrow$	80	55	0	0	Min R
Basic Variable	C_B	X_B	x_1	x_2	S_1	S_2	X_B/X_k
outgoing vector $\leftarrow S_1$	0	40	4	2	1	0	40/4=
S_2	0	32	2	4	0	1	32/2=
$Z = 0$		$\Delta_j \rightarrow$	-80	-55	0	0	

$(x_k)\uparrow$ incoming vector

		C_j	80	55	0	0	
Basic Variable	C_B	X_B	x_1	x_2	S_1	S_2	
x_1	80	10	1	1/2	1/4	0	
S_2	0	12	0	3	-1/2	1	

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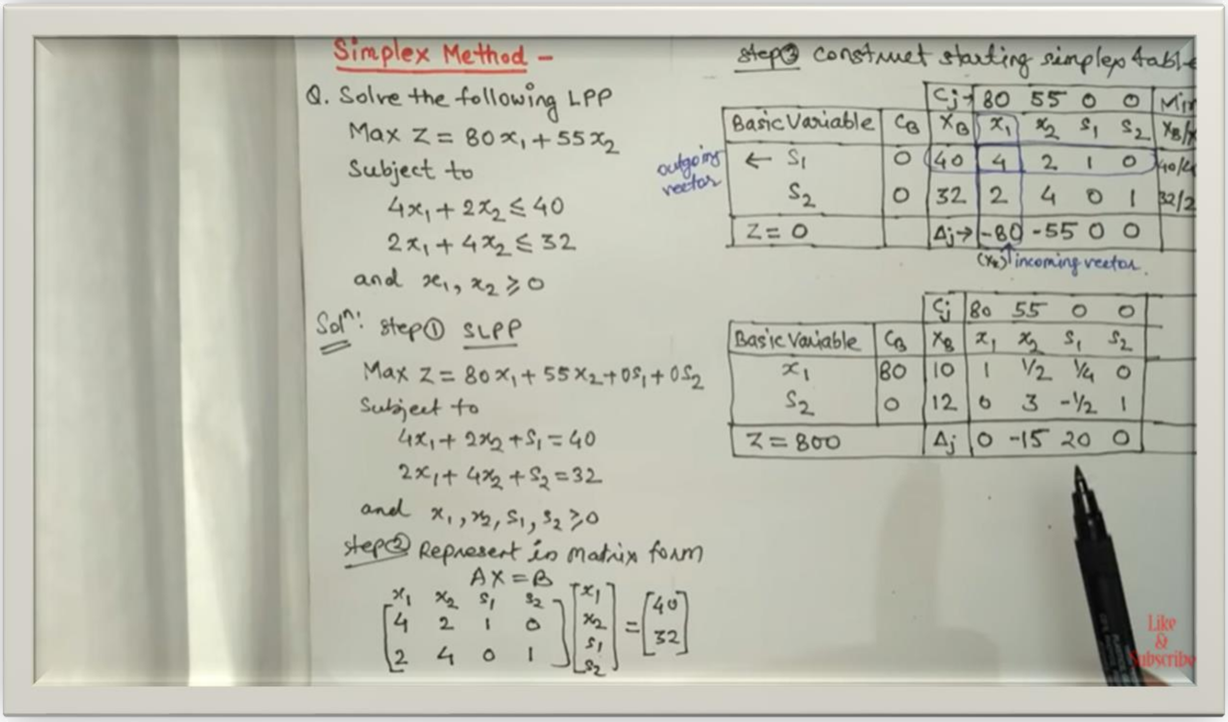

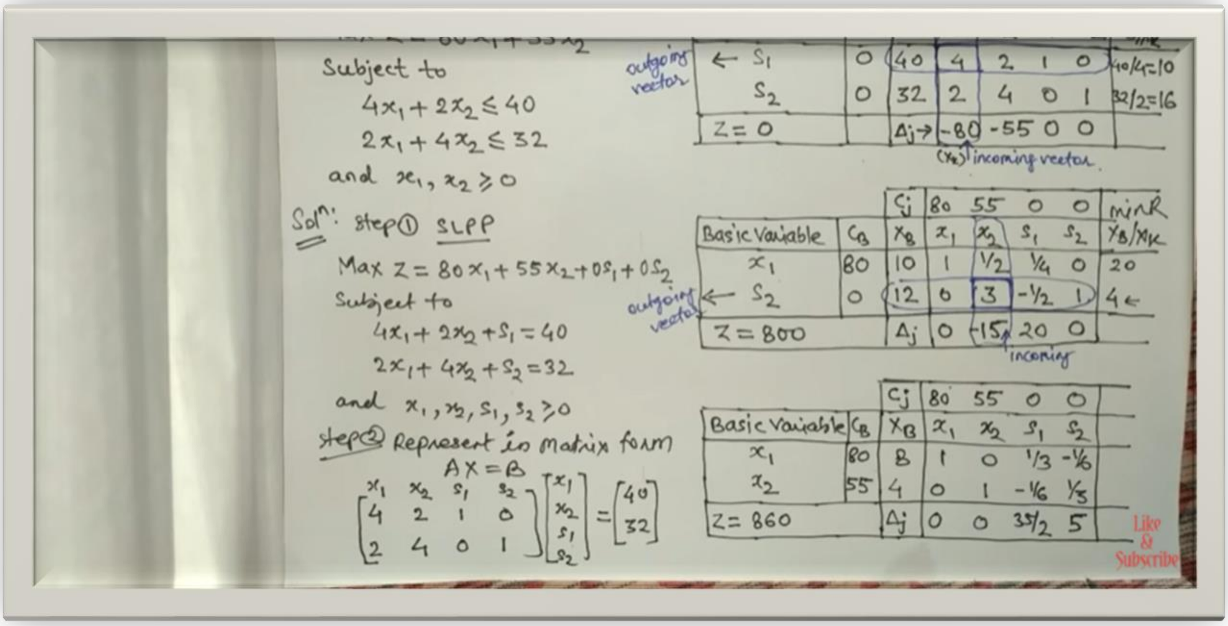

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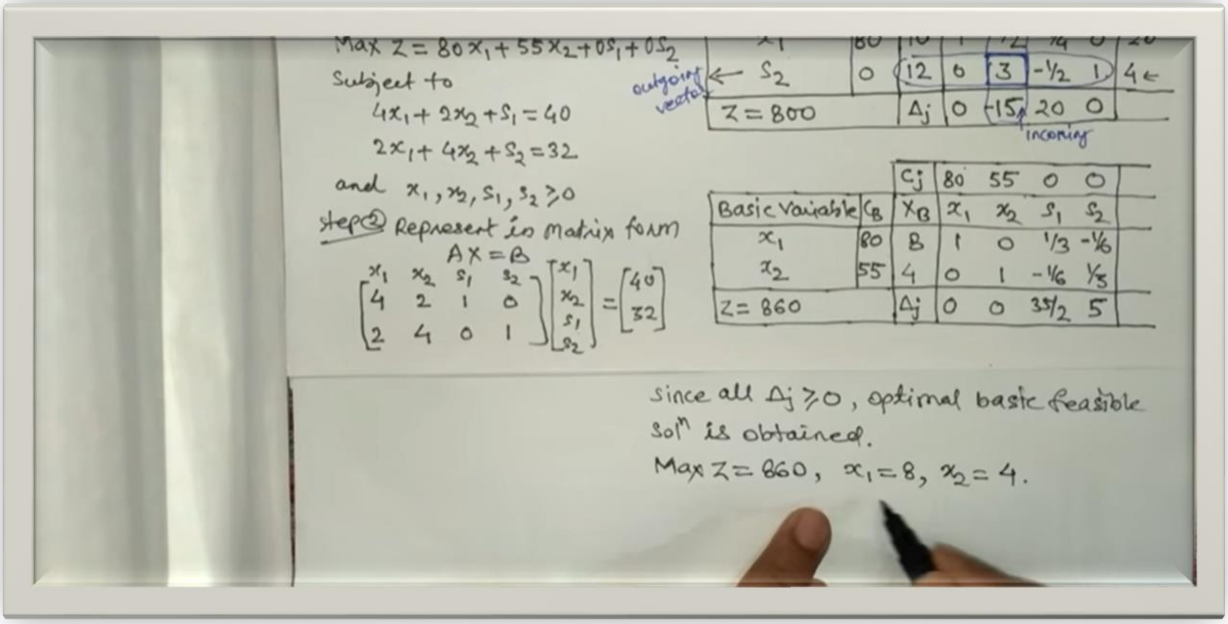

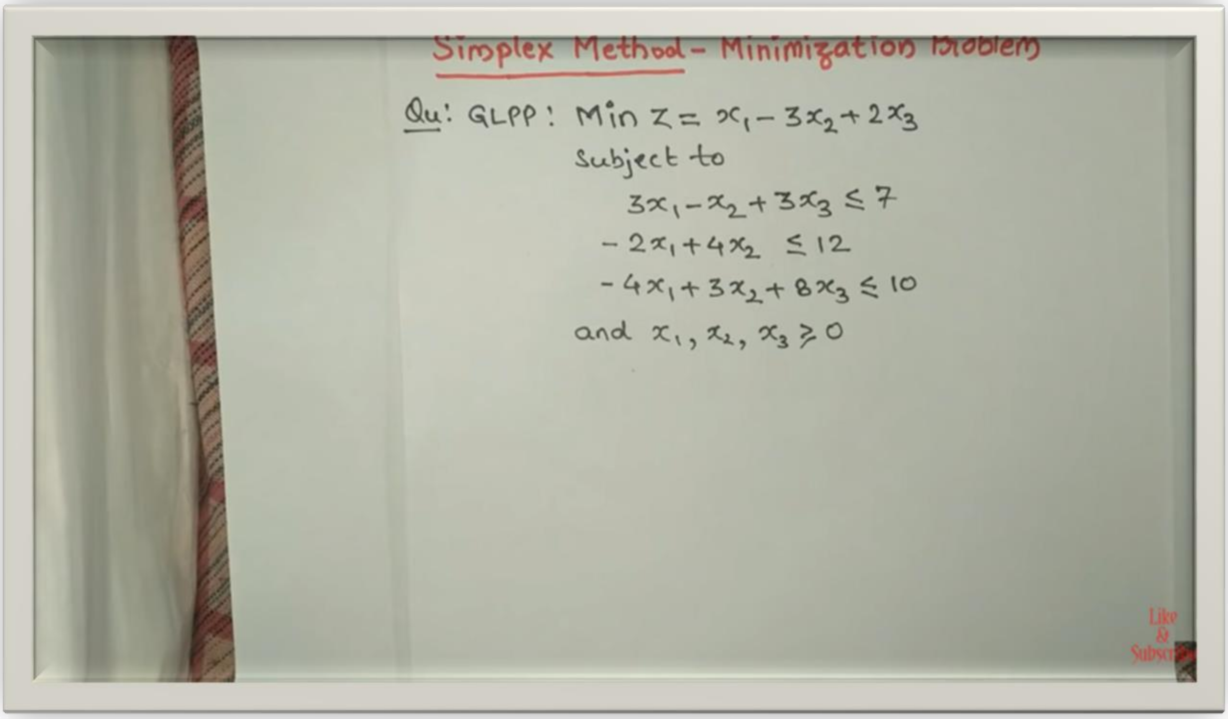

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