

**Problem 1. Gradient Descent**

The file data.csv contains a small sample from the unobserved true data generating model,

$$y = 10 + 20x_1 + 30x_2 + \epsilon, \quad \epsilon \sim N(\mu = 0, \sigma^2 = 4)$$

You decide to use the data to fit a linear regression model, assuming homoscedasticity:

$$y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \epsilon$$

A. Use code to calculate and report the theoretical LSE value of  $\beta = (\beta_0, \beta_1, \beta_2)^T$  as well as the sum of squared residuals  $\sum_i (y_i - \hat{y}_i)^2$

B. Use a simple neural network with no hidden layers to perform the same regression involving the loss function

$$R = \sum_i (y_i - \hat{y}_i)^2$$

Please write your own code from scratch and refrain from using existing deep learning packages.

Use gradient descent or stochastic gradient descent, with the initial weights  $\beta^{(0)} = (0.01, 0.01, 0.01)^T$  as well as a fixed learning rate of your choice, to estimate the weights  $\beta = (\beta_0, \beta_1, \beta_2)^T$  of the neural network.

Report the following at convergence:

- b1. The smallest loss your network is able to achieve
- b2. The corresponding learning rate
- b3. The corresponding estimated "optimal" weights
- b4. The corresponding number of updates when your network reaches the "optimal" estimates
- b5. A plot of the final 50 (or a reasonable number of) updates of the loss values vs the update number, in the following format

