

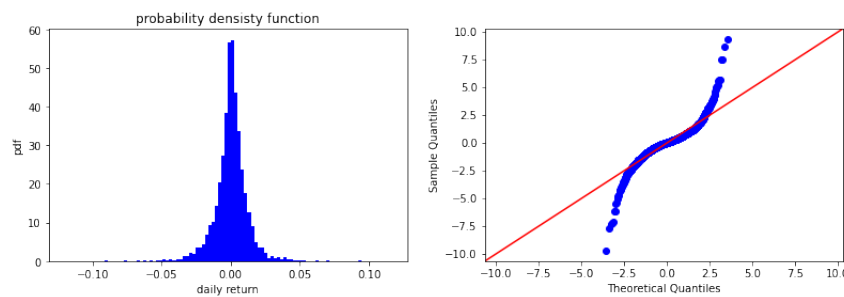
Problem 1. Asset Prices

Returns of risky cash assets (such as stocks) are commonly assumed to be normally distributed, while their prices log-normally distributed. Let's investigate these empirically.

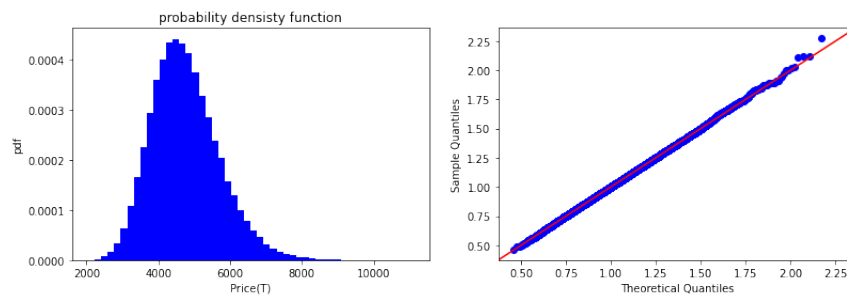
SP500_randomized.csv contains daily closing prices of the SP-500 index over the past 20-plus years. Please note that historical records in this file are not in any chronological order.

Let's assume that daily returns of the SP-500 index are independent and identically distributed.

- [a] Estimate the probability density function of the daily return and plot it.
- [b] Estimate the mean, variance, skewness and kurtosis of the daily return and report them numerically.
- [c] Provide a QQ-plot of the estimated return distribution versus the normal distribution.



- [d] Assume SP500 index is currently 4430. Based on our return distribution assumption, predict the index value one year (252 trading days) from now, using a Monte Carlo simulation with 100,000 paths. Report your predicted value and its variance.
- [e] Plot the estimated probability density function of the index value one year from now.
- [f] Does [e] look log-normal to you? Provide a QQ-plot of the estimated price distribution versus the log-normal distribution.



You may find the following functions useful:

https://pandas.pydata.org/pandas-docs/stable/reference/api/pandas.to_datetime.html

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.skew.html>

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.kurtosis.html>

<https://www.statsmodels.org/stable/generated/statsmodels.graphics.gofplots.ProbPlot.html>