Problem 1. Logistic Regression, FFNN
The file Default.csv contains data regarding loan defaults:
default: individual's default status labeled as "Yes" or "No"
balance: loan balance income: individual's income

You deciced to use the data to fit a logistic regression model.

$$\ln\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 \times \text{loan.balance} + \beta_2 \times \text{income} \tag{1}$$

A. Use your preferred package to fit the logistic regression model and report the following:
a1. The MLE estimates of $\beta_0, \beta_1, \beta_2$
a2. The maximum log-likelihood achieved at the MLE estiamtes

B. Use a simple neural network with no hidden layers to estimate the default probability π to classify default, involing the sigmoid activation function

$$S(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{1+e^x}$$

and the binary cross-entropy loss function

$$R = -\frac{1}{n}\sum_{i=1}^{n}(1-y_i)\ln(1-\hat{y}_i) + y_i \ln \hat{y}_i$$

Note that by adopting the sigmoid activation function, your neural network output $\hat{y}_i$ is just the estimate for the default probability $\hat{\pi}_i$

Please write your own code to implement a full Gradient Descent algorithm to fit the neural network.
To facilitate convergence, you should standardize features (loan balance and income). For example, with standardized features and a learning rate of 25, you can get to within $1 \times 10^{-8}\times$ the log-likelihood reported in a2.
Please start you fit with a initial weight of $(0.0, 0.0, 0.0)^T$

Let your code report
b1. the smallest binary cross-entropy your alogrithm has achieved over 200 updates (epochs)
b2. the associated weghts ($\beta_j s$ fitted with standardized features)
b3. the corresponding $\beta_j s$ for the model in equation (1) fitted with non-standardized features.