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open import Relation.Binary.PropositionalEquality
open Relation.Binary.PropositionalEquality.≡-Reasoning
data ℕ : Set where
  zero : ℕ
  suc   : ℕ → ℕ
_+_    : ℕ → ℕ → ℕ
zero + y = y
suc x + y = suc (x + y)
data Vec (A : Set) : ℕ → Set where
  nil    : Vec A zero
  cons   : {n : ℕ} → A → Vec A n → Vec A (suc n)
head    : {A : Set} {n : ℕ} → Vec A (suc n) → A
head (cons x xs) = x
tail    : {A : Set} {n : ℕ} → Vec A (suc n) → Vec A n
tail (cons x xs) = xs
_++_    : {A : Set} {n m : ℕ} → Vec A n → Vec A m → Vec A (n + m)
nil ++ ys = ys
cons x xs ++ ys = cons x (xs ++ ys)
+-assoc : ∀ n m l → n + (m + l) ≡ (n + m) + l
+-assoc zero m l = refl
+-assoc (suc n) m l = cong suc (+-assoc n m l)
+-suc   : ∀ n m → suc (m + n) ≡ m + suc n
+-suc n zero = refl
+-suc n (suc m) = cong suc (+-suc n m)
+-comm  : ∀ n m → n + m ≡ m + n
+-comm zero zero = refl
+-comm zero (suc m) = cong suc (+-comm zero m)
+-comm (suc n) m = begin
  suc (n + m)
  ≡⟨ cong suc (+-comm n m) ⟩
  suc (m + n)
  ≡⟨ +-suc n m ⟩
  m + suc n ■

```