

MODULE *BitcoinChain*
 EXTENDS *Naturals*, *FiniteSets*, *Sequences*

CONSTANTS

Nodes, Set of nodes in the network
MaxBlocks, Maximum number of blocks that can be created. We need this to cap the model run.
GenesisHash Hash of the genesis block

VARIABLES

blocksByNode, *blocksByNode*[*n*] is the set of blocks known by node *n*
chainsByNode, *chainsByNode*[*n*] is the *DAG* of blocks for node *n*
tipsByNode, *tipsByNode*[*n*] is the current tip of the chain for node *n*
workByNode, *workByNode*[*n*] is a function mapping each block to its cumulative work
confirmedByNode, *confirmedByNode*[*n*] is the set of confirmed blocks for node *n*
network, *network*[*n*] is the set of blocks in transit to node *n*
nextBlockId Counter used to generate unique block *IDs*

vars \triangleq \langle *blocksByNode*, *chainsByNode*, *tipsByNode*, *workByNode*, *confirmedByNode*, *network*, *nextBlockId* \rangle

Type definitions

Block \triangleq [*id* : *Nat*, *prevHash* : *Nat*, *nonce* : *Nat*, *timestamp* : *Nat*]

Chain \triangleq [*blocks* : SUBSET *Nat*, *edges* : SUBSET (*Nat* \times *Nat*)]

Helper functions

WorkFor(*b*) \triangleq 1 Simplified work calculation, each block contributes 1 unit of work

Initial state

Init \triangleq
 \wedge *blocksByNode* = [*n* \in *Nodes* \mapsto {[*id* \mapsto 0, *prevHash* \mapsto *GenesisHash*, *nonce* \mapsto 0, *timestamp* \mapsto 0]}]
 \wedge *chainsByNode* = [*n* \in *Nodes* \mapsto [*blocks* \mapsto {0}, *edges* \mapsto {}]]
 \wedge *tipsByNode* = [*n* \in *Nodes* \mapsto 0]
 \wedge *workByNode* = [*n* \in *Nodes* \mapsto [*i* \in {0} \mapsto 1]] Initialize work for genesis block to 1
 \wedge *confirmedByNode* = [*n* \in *Nodes* \mapsto {}]
 \wedge *network* = [*n* \in *Nodes* \mapsto {}]
 \wedge *nextBlockId* = 1

Action: Create a new block

CreateBlock(*n*) \triangleq
 \wedge *nextBlockId* < *MaxBlocks*
 \wedge LET
 newBlock \triangleq [*id* \mapsto *nextBlockId*,
 prevHash \mapsto *tipsByNode*[*n*],
 nonce \mapsto *nextBlockId*, Simplified nonce
 timestamp \mapsto *nextBlockId*] Simplified timestamp
 updatedChain \triangleq [
 blocks \mapsto *chainsByNode*[*n*].*blocks* \cup {*nextBlockId*},
 edges \mapsto *chainsByNode*[*n*].*edges* \cup {(*tipsByNode*[*n*], *nextBlockId*)}

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]
newWork  $\triangleq$  workByNode[n][tipsByNode[n]] + WorkFor(newBlock)
updatedWork  $\triangleq$  [workByNode[n] EXCEPT ![nextBlockId] = newWork]
prevTip  $\triangleq$  tipsByNode[n] Previous tip before creating the new block
IN
  Update the blocks known by the local node
   $\wedge$  blocksByNode' = [blocksByNode EXCEPT ![n] = @  $\cup$  {newBlock}]
  Update the chain for the local node
   $\wedge$  chainsByNode' = [chainsByNode EXCEPT ![n] = updatedChain]
  Update the tip for the local node
   $\wedge$  tipsByNode' = [tipsByNode EXCEPT ![n] = nextBlockId]
  Track total work for the new block at this node, even if it is a constant for now
   $\wedge$  LET
    newWorkMap  $\triangleq$  [b  $\in$  DOMAIN workByNode[n]  $\cup$  {nextBlockId}  $\mapsto$ 
      IF b = nextBlockId THEN newWork ELSE workByNode[n][b]]
    IN
      workByNode' = [workByNode EXCEPT ![n] = newWorkMap]
      The new block will be received by all other nodes in the network
       $\wedge$  network' = [m  $\in$  Nodes  $\mapsto$  IF m  $\neq$  n THEN network[m]  $\cup$  {newBlock} ELSE network[m]]
      Update the global nextBlockId
       $\wedge$  nextBlockId' = nextBlockId + 1
      Local node immediately confirms the previous tip
       $\wedge$  confirmedByNode' = [confirmedByNode EXCEPT ![n] = confirmedByNode[n]  $\cup$  {prevTip}]

  Action: Receive a block from the network
  ReceiveBlockWithPreviousKnown(n, block)  $\triangleq$ 
     $\wedge$  block.prevHash  $\in$  chainsByNode[n].blocks
     $\wedge$ 
      LET
        prevBlock  $\triangleq$  block.prevHash
        updatedChain  $\triangleq$  [
          blocks  $\mapsto$  chainsByNode[n].blocks  $\cup$  {block.id},
          edges  $\mapsto$  chainsByNode[n].edges  $\cup$  {<prevBlock, block.id>}
        ]
        newWork  $\triangleq$  workByNode[n][tipsByNode[n]] + WorkFor(block)
        newTip  $\triangleq$  IF newWork > workByNode[n][tipsByNode[n]]
          THEN block.id
          ELSE tipsByNode[n]
      IN
         $\wedge$  blocksByNode' = [blocksByNode EXCEPT ![n] = @  $\cup$  {block}]
        Take the next block in the receive queue
         $\wedge$  network' = [network EXCEPT ![n] = @  $\setminus$  {block}]
         $\wedge$  chainsByNode' = [chainsByNode EXCEPT ![n] = updatedChain]
         $\wedge$  LET
          newWorkMap  $\triangleq$  [b  $\in$  DOMAIN workByNode[n]  $\cup$  {block.id}  $\mapsto$ 

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$$\text{IF } b = \text{block.id} \text{ THEN } \text{newWork} \text{ ELSE } \text{workByNode}[n][b]$$

$$\text{IN}$$

$$\text{workByNode}' = [\text{workByNode} \text{ EXCEPT } ![n] = \text{newWorkMap}]$$

$$\text{Confirm the block if it is a new tip}$$

$$\wedge \text{IF } \text{newWork} > \text{workByNode}[n][\text{tipsByNode}[n]] \text{ THEN}$$

$$\wedge \text{confirmedByNode}' = [\text{confirmedByNode} \text{ EXCEPT } ![n] = \text{confirmedByNode}[n] \cup \{\text{tipsByNode}[n]\}]$$

$$\text{ELSE}$$

$$\wedge \text{confirmedByNode}' = \text{confirmedByNode}$$

$$\text{Update the tip for the local node}$$

$$\wedge \text{tipsByNode}' = [\text{tipsByNode} \text{ EXCEPT } ![n] = \text{block.id}]$$

$$\wedge \text{UNCHANGED } \langle \text{nextBlockId} \rangle$$

$$\text{Next state relation}$$

$$\text{Next} \triangleq$$

$$\vee \exists n \in \text{Nodes} : \text{CreateBlock}(n)$$

$$\vee \exists n \in \text{Nodes} : \exists \text{block} \in \text{network}[n] : \text{ReceiveBlockWithPreviousKnown}(n, \text{block})$$

$$\text{vars} \triangleq \langle \text{blocksByNode}, \text{chainsByNode}, \text{tipsByNode}, \text{workByNode}, \text{confirmedByNode}, \text{network}, \text{nextBlockId} \rangle$$

$$\text{Invariants}$$

$$\text{TypeInvariant} \triangleq$$

$$\wedge \forall n \in \text{Nodes} :$$

$$\wedge \text{nextBlockId} \in \text{Nat}$$

$$\wedge \text{tipsByNode}[n] \in \text{Nat}$$

$$\wedge \text{chainsByNode}[n].\text{blocks} \subseteq \text{Nat}$$

$$\wedge \text{chainsByNode}[n].\text{edges} \subseteq (\text{Nat} \times \text{Nat})$$

$$\wedge \forall \text{blockId} \in \text{DOMAIN } \text{workByNode}[n] :$$

$$\wedge \text{blockId} \in \text{Nat}$$

$$\wedge \text{workByNode}[n][\text{blockId}] \in \text{Nat}$$

$$\wedge \text{confirmedByNode}[n] \subseteq \text{Nat}$$

$$\wedge \text{network}[n] \subseteq \text{Block}$$

$$\wedge \forall b \in \text{blocksByNode}[n] :$$

$$\wedge b.\text{id} \in \text{Nat}$$

$$\wedge b.\text{prevHash} \in \text{Nat}$$

$$\wedge b.\text{nonce} \in \text{Nat}$$

$$\wedge b.\text{timestamp} \in \text{Nat}$$

$$\text{Properties}$$

$$\text{TipHasMostWork} \triangleq$$

$$\forall n \in \text{Nodes} :$$

$$\forall b \in \text{chainsByNode}[n].\text{blocks} :$$

$$\text{workByNode}[n][\text{tipsByNode}[n]] \geq \text{workByNode}[n][b]$$

$$\text{Complete specification}$$

$$\text{Spec} \triangleq \text{Init} \wedge \square[\text{Next}]_{\text{vars}}$$
