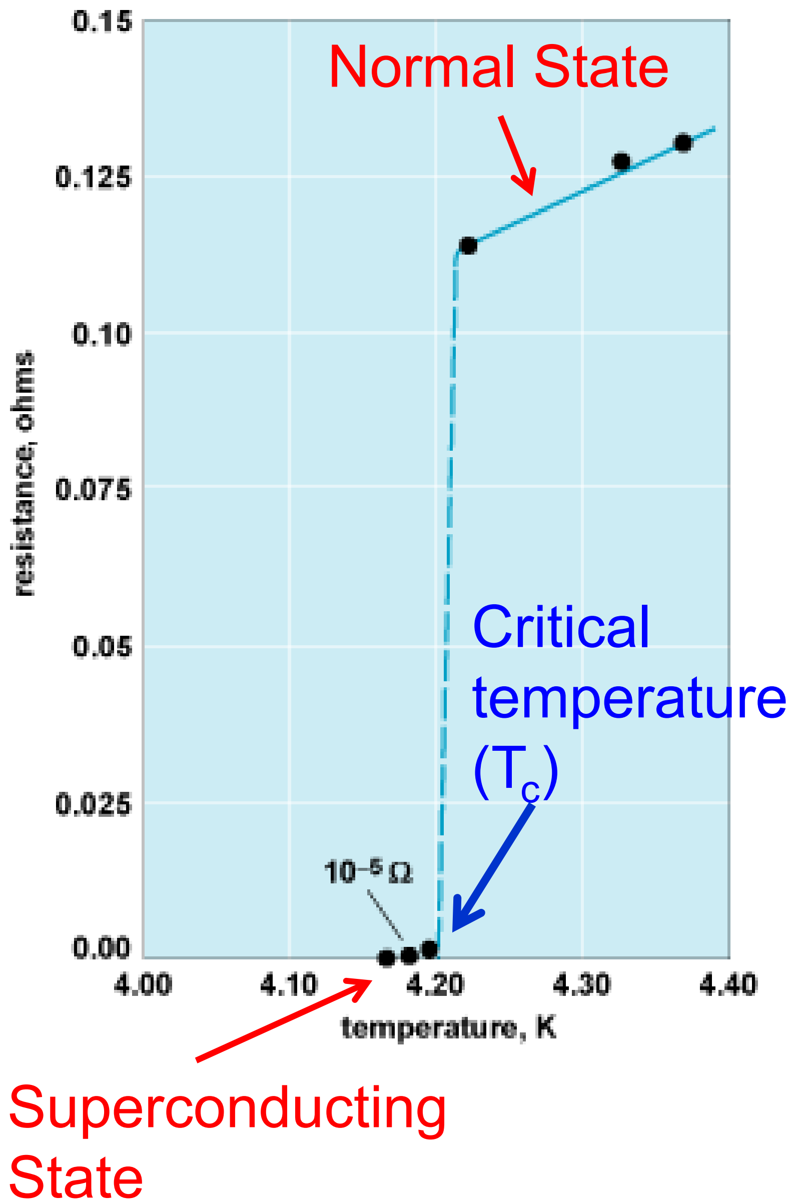
SUPERCONDUCTIVITY



Observed in many metal and alloys

First observed by H. Kamerlingh Onnes, 1911

Solid Hg at 0^{o}C , R = 39.7 Ω

At 4.3 K, R = 0.084 Ω (0.0021 times low than that 0 ^{o}C)

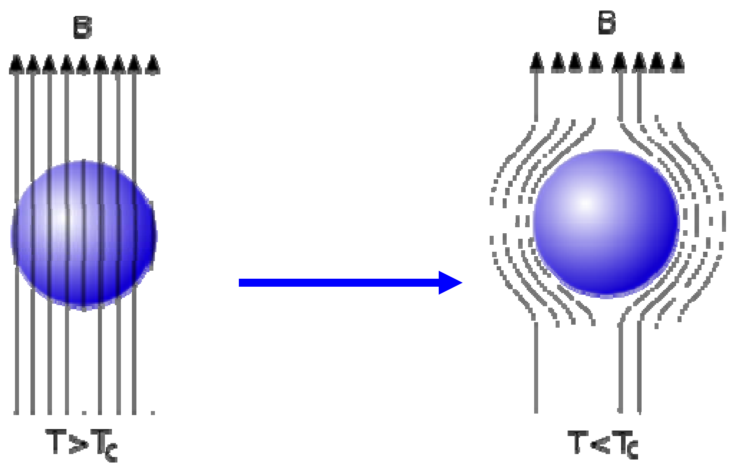
At 3.0 K, R < 3x10^{-6} Ω (one ten-millionth of the value at 0 ^{o}C)

EXPERIMETAL SURVEYS

Bulk superconductor in a weak magnetic field acts as a perfect diamagnet.

MEISSNER EFFECT (Meissner and Ochsenfeld, 1933):

When a specimen is placed in a magnetic field and is cooled through the transition temperature for superconductivity, the magnetic flux originally present is ejected from the specimen.



Compound	T_c in K
Nb_3Sn	18.05
Nb_3Ge	23.2
Nb_3Al	17.5
NbN	16
V_3Ga	16.5
V_3Si	17.1
Ti_2Co	3.44
La_3In	10.4

We know

$$B = \mu_0 (H + M)$$

Thus, at T < Tc

$$\Rightarrow 0 = \mu_0 H + \mu_0 M \qquad \Rightarrow \frac{M}{H} = \chi = -1$$

According to Ohm's law, $E = \rho j$

$$\Rightarrow \rho \to 0, E \to 0$$

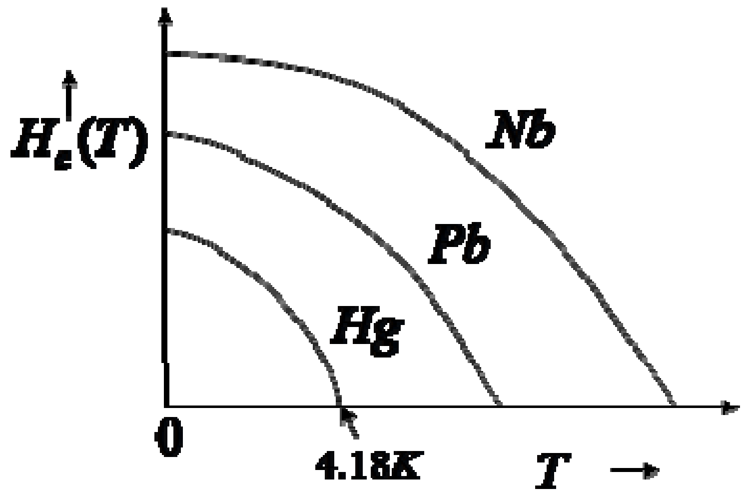
$$\frac{d\vec{B}}{dt} = \vec{\nabla} \times \vec{E} \qquad \Rightarrow \frac{d\vec{B}}{dt} = 0$$

$$\Rightarrow B = \text{const}$$

Meissner effect contradicts this result and suggests that diamagnetism is essential property of superconducting state.

DESTRUCTION OF SUPERCONDUCTIVITY BY MAGNETIC FIELDS

A sufficiently strong magnetic field destroys the superconductivity.



The critical value of the magnetic field for destruction is denoted by $H_c(T)$ – a function of temperature.

$$H_c(T) = H_c(0)[1 - (\frac{T}{T_c})^2]$$

Where $H_c(0)$ is the critical field at T = 0 K.

At $T = T_c$, $H_c(T_c) = 0$

Critical field that destroys superconductivity need not to be external. It may arise due to the electric current flowing through the superconducting specimen itself.

$$I_c = 2\pi r H_c$$

r = radius of current carrying wire.

Example 1: A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0K. Find the critical field at 2K.

Solution:

$T_c(0) = 3.7\ \mathrm{K}$ $\quad H_c(0) = 0.0306\ \mathrm{T}$ $\quad T = 2\ \mathrm{K}$

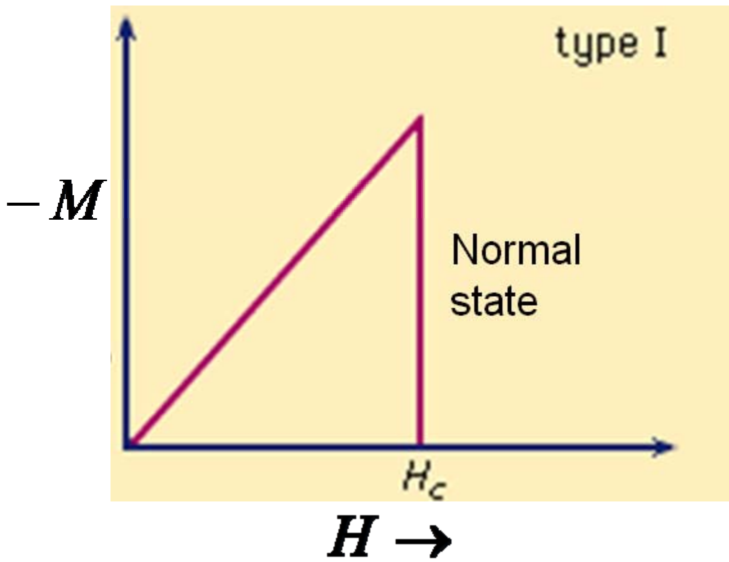
$$H_c(T) = H_c(0)[1 - (\frac{T}{Tc})^2] = 0.0306\,[1 - (\frac{2}{3.7})^2] = 0.0216$$

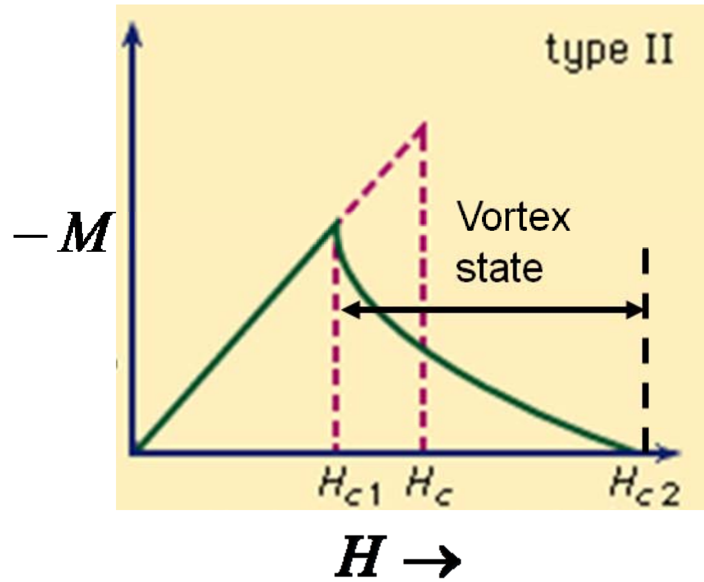
Example 2: Calculate critical current for a wire of lead having a diameter of 1mm at 4.2 K. The critical temperature for lead is 7.18 K and $H_c(0) = 6.5 \times 10^4$ A/m.

$$H_c(T) = H_c(0)[1 - (\frac{T}{Tc})^2] = 6.5 \times 10^4 [1 - (\frac{4.2}{7.18})^2] = 4.28 \times 10^4\ A/m$$

$$I_c = 2\pi r H_c = 134.46 \quad \mathrm{A}$$

TYPE OF SUPERCONDUCTORS





Type-I: Pb, T_c = 7 K, H_c = 4.8x10^4 Amp/ m, Al, Zn, Hg, Sn etc

Type-II: Pb + 2% by weight Nb, H_{c1} = 3x10^4 Amp/ m and H_{c2} = 8x10^4 Amp/m

In the vortex state Meissner effect is said to be incomplete as B ≠ 0.

ISOTOPE EFFECT

$$M^{\alpha} T_c = \text{const.}$$

Where α is isotope effect coefficient. It depends on material and its value ranges from 0.4 – 0.5.

Example: Hg (199.5) has T_c = 4.185 K while that Hg (203.4) has T_c = 4.146 K

Substance	α
Zn	0.45±0.05
Cd	0.32±0.07
Sn	0.47±0.02
Hg	0.50±0.03
Pb	0.49±0.02
Tl	0.61±0.10

The dependence of T_c on isotopic mass i.e. on the number of neutrons in the nucleus shows that lattice vibrations and hence electron-lattice interactions are deeply involved in superconductivity.

The value of α is taken as 0.5 in the above equation.

$$\left(\frac{M_1}{M_2}\right)^{\frac{1}{2}} = \frac{T_{c2}}{T_{c1}}$$

LONDON EQUATION

Assumptions of London's Theory:

The conduction electrons in superconducting materials are classified in two categories **(Gorter and casimir, 1934:**

(1) **Super-electrons** and (2) **Normal electrons**.

The super-electrons don't experience scattering from the vibrating lattice like normal electrons. They have perfect order (zero entropy) etc.

(i) At T = 0 K, all electrons are super-electrons.

(ii) As the temperature increases from 0 K, a fraction of electrons is super-electron and remaining normal.

(iii) At $T = T_c$, all electrons behave as normal electrons.

Let n = Total no of conduction electrons per unit volume in normal state

$$n = n_s + n_n$$

n_s = super-electron density and
n_n = normal electron density

The total current density may be written as,

$$J = J_s + J_n = en_s v_s + en_n v_n$$

Here, J_s and J_n are current densities due to super-electrons and normal electrons. v_s and v_n are velocities electrons in superconducting and normal phases.

Under the action of applied electric field, E, the super-electrons follow the equation of motion:

$$m\frac{dv_s}{dt} = eE \qquad \Rightarrow \frac{dv_s}{dt} = \frac{eE}{m}$$

Differentiating equation for J_s $\quad (J_s = en_s v_s)$

$$\frac{dJ_s}{dt} = n_s e \frac{dv_s}{dt}$$

$$\Rightarrow \frac{dJ_s}{dt} = n_s e \frac{eE}{m}$$

$$\boxed{\Rightarrow \frac{dJ_s}{dt} = \frac{n_s e^2 E}{m}}$$

London's first equation.

Taking curl of this equation,

$$\vec{\nabla} \times \frac{d\vec{J}_s}{dt} = \vec{\nabla} \times \frac{n_s e^2 \vec{E}}{m}$$

$$\frac{d}{dt}\left(\vec{\nabla} \times \vec{J}_s\right) = \frac{n_s e^2}{m}\left(\vec{\nabla} \times \vec{E}\right)$$

$$\frac{d}{dt}\left(\vec{\nabla} \times \vec{J}_s\right) = \frac{n_s e^2}{m}\left(\vec{\nabla} \times \vec{E}\right) = \frac{n_s e^2}{m}\left(-\frac{d\vec{B}}{dt}\right) \qquad \left(\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}\right)$$

$$\Rightarrow \vec{\nabla} \times \frac{d}{dt}\vec{J}_s = -\frac{n_s e^2}{m}\frac{d\vec{B}}{dt}$$

Integrating this equation w. r. t. time and taking the constant of integration to be zero consistent with Meissner effect,

$$\vec{\nabla} \times \vec{J}_s = -\frac{n_s e^2 \vec{B}}{m}$$

This equation is known as **London's second equation**.

According to Maxwell's equation,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$$

Taking curl of both sides

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \mu_0 \left(\vec{\nabla} \times \vec{J}_s\right)$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \mu_0 \left(\vec{\nabla} \times \vec{J}_s\right)$$

$$\Rightarrow \vec{\nabla}\left(\vec{\nabla} \bullet \vec{B}\right) - \nabla^2 B = -\mu_0 \frac{n_s e^2 \vec{B}}{m} \qquad \left[\vec{\nabla} \times \vec{J}_s = -\frac{n_s e^2 \vec{B}}{m}\right]$$

$$\Rightarrow \nabla^2 B = \mu_0 \frac{n_s e^2 \vec{B}}{m} \qquad \boxed{\Rightarrow \nabla^2 \vec{B} = \frac{\vec{B}}{\lambda_L^2}}$$

Where, $\lambda_L^2 = \dfrac{m}{\mu_0 n_s e^2}$ $\qquad \boxed{\Rightarrow \lambda_L = \left(\dfrac{m}{\mu_0 n_s e^2}\right)^{\frac{1}{2}}}$

λ_L is known as London's penetration depth.

From this equation if B = B_0, $\nabla^2 \vec{B} = 0$ But, $\dfrac{\vec{B}}{\lambda_L^2} \neq 0$

Therefore above will be satisfied when B = 0 inside the specimen which in turn satisfies Meissner effect.

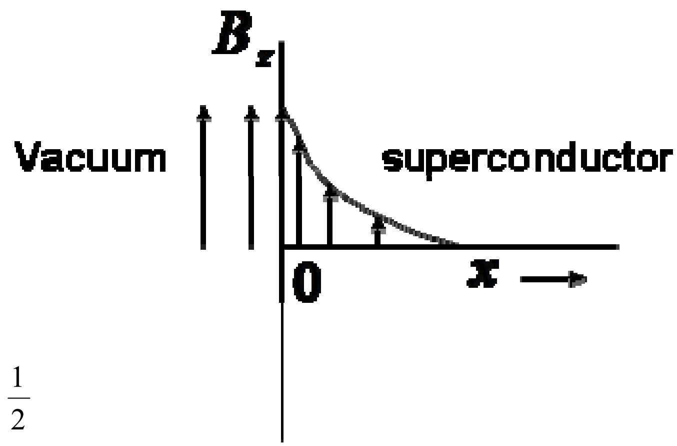
General solution of equation $$\nabla^2 \vec{B} = \frac{\vec{B}}{\lambda_L^2}$$

$$B_z(x) = B_z(0) e^{-\frac{x}{\lambda_L}}$$

(supposing that applied field is in z-direction).

At x = λ_L, $$B_z(x) = \frac{B_z(0)}{e}$$

Thus λ_L is a distance from the surface of the metal in which the field decays to 1/e of its value outside the metal.



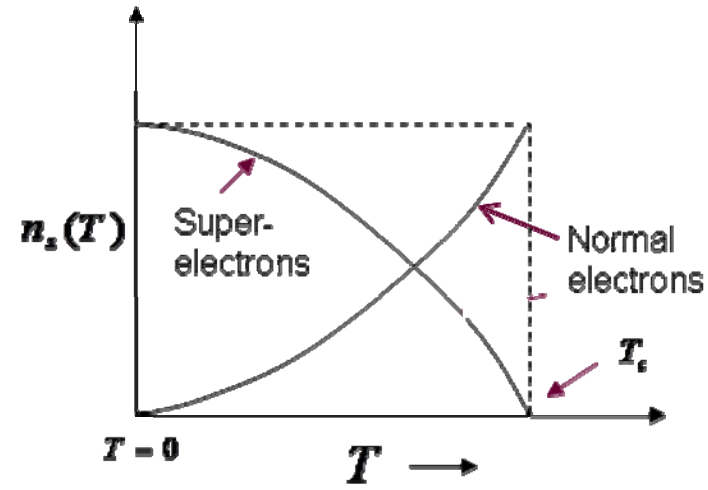
$$\lambda_L = \left(\frac{m}{\mu_0 n_s e^2} \right)^{\frac{1}{2}}$$

Substituting m = 9.1x10^{-31} kg, μ_0 = 4π x 10^{-7} H/m, e = 1.6 x 10^{-19} C and n_s = 10^{28} /m^3, we have,

$$\lambda_L = \left(\frac{9.1 \times 10^{-31}}{4\pi \times 10^{-7} \times 10^{28} (1.6 \times 10^{-19})^2} \right)^{\frac{1}{2}} \Rightarrow \lambda_L = 5.32 \times 10^{-8} m = 532 \ \mathring{A}$$

Variation of λ_L with temperature

λ_L is given as
$$\lambda_L = \left(\frac{m}{\mu_0 n_s e^2}\right)^{\frac{1}{2}}$$



Concentration of super-electrons (n_s) varies with temperature and follows the equation

$$n_s(T) = n_s(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right]$$

Where n_s (0) is no. of super-electrons at 0 K.

The ratio: n_s (T)/ n_s(0) = order parameter.

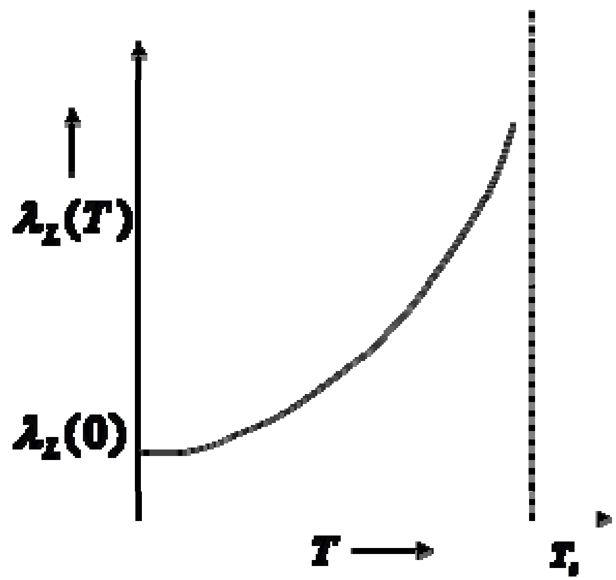
Substituting n_s in the equation for λ_L,

$$\lambda_L(T) = \left(\frac{m}{\mu_0 n_s(0)\left[1 - \left(\frac{T}{T_c}\right)^4\right] e^2}\right)^{\frac{1}{2}}$$

Let us define,

$$\lambda_L(0) = \left(\frac{m}{\mu_0 n_s(0) e^2} \right)^{\frac{1}{2}} \qquad \lambda_L(T) = \left(\frac{m}{\mu_0 n_s(0) \left[1 - \left(\frac{T}{T_c} \right)^4 \right] e^2} \right)^{\frac{1}{2}}$$

Then, $$\lambda_L(T) = \frac{\lambda_L(0)}{\left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{\frac{1}{2}}}$$



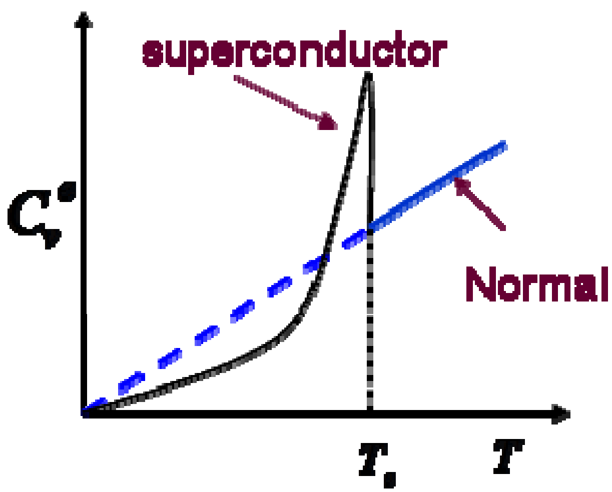
Thus At $T = T_c$, $\lambda_L = \infty$

and at $T = 0$, $\lambda_L = \lambda_L(0)$

At $T = T_c$, $\lambda_L = \infty$ means field has completely penetrated inside the material and therefore the material is no longer a superconductor. The superconductivity is destroyed at T_c.

Variation of specific heat of superconductor with temperature

In normal conductor: $$C_v = C_v^{\,e} + C_v^{\,l} = AT + BT^3$$



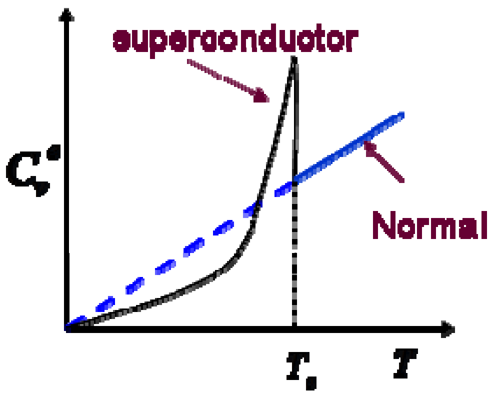
C_v^e = contribution to specific heat by electrons in the conduction band

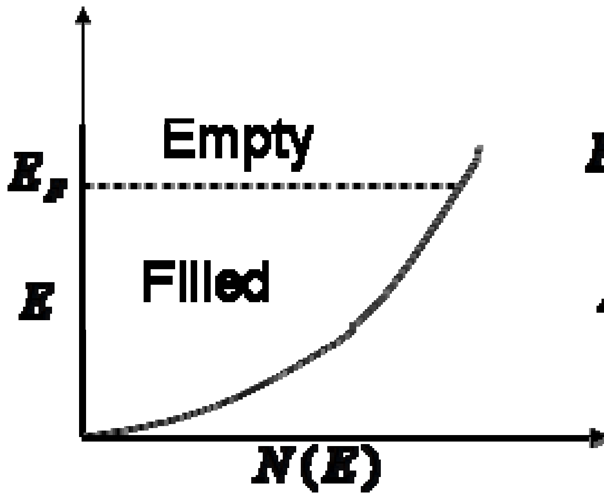
C_v^l= contribution to specific heat from lattice

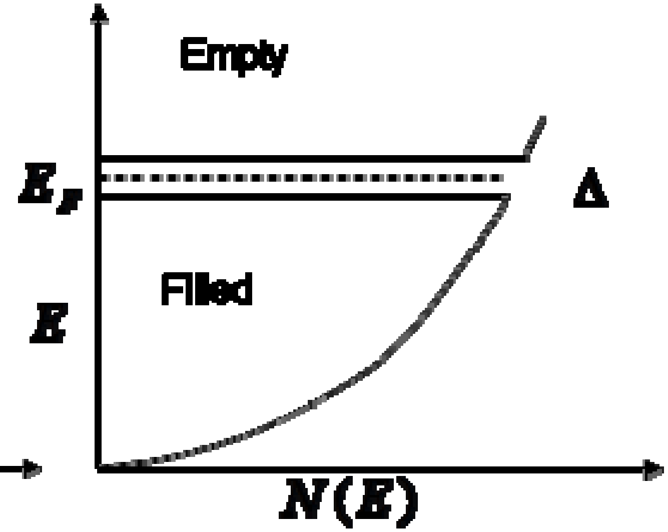
In superconductor:

$$C_v^{\,e} = Ae^{-\frac{BT_C}{T}}$$

The exponential behavior of the variation of specific heat with temperature suggests the existence of an energy gap in the excitation spectrum of the conduction electrons in the metal.







$$\Delta = kT_c$$

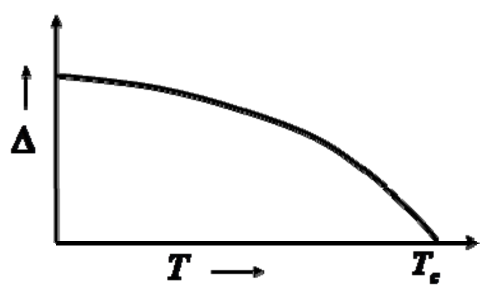
Metal

Superconductor

Thus at 5 K, $\Delta \approx 10^{-4} eV$

The energy gap varies with temperature:

$$\frac{\Delta(T)}{\Delta(0)} = 1.74(1 - \frac{T}{T_c})^{\frac{1}{2}}$$



El.	Δ (m eV)	T_c (k)	Δ/K_BT_c
Nb	3.05	9.5	3.8
Ta	1.40	4.48	3.6
Sn	1.15	3.72	3.5
Al	0.34	1.20	3.3
Pb	2.90	7.19	4.3
Hg	1.65	4.15	4.6

BCS (BARDEEN-COOPER–SCHRIEFFER) THEORY

Under certain situations, the electrons may attract each other. If they attract then they form a pair of two electrons known as Cooper pair.

Cooper pair is a new particle with a mass of 2m and charge 2e.

Best condition for the formation of Cooper pair is

(i) The two electrons have equal and opposite momentum

(ii) Equal and opposite spin.

Potential energy of the two electron system consists of two terms :

Repulsive term (V_r) due to columbic repulsion between two charges

Attractive term (V_{ph}) in the superconducting phase.

Thus total potential energy is

Attractive

$$V = V_r + V_{ph} = +\frac{ke^2}{r} - V_{ph}$$

repulsive

If $V < 0$ This will mean that net attraction between two electrons and their energy is lowered. Under this condition Cooper pairs will be formed.

If $V > 0$ In this condition, repulsion will dominate and Cooper pairs will not be formed and both electrons will remain separated.

$$T_c = 1.14\,\theta_D e^{\frac{-1}{N(0)V}}$$

Net attractive interaction

Density of states at Fermi surface

Debye temperature

Binding energy (Energy Gap):

It is the energy required to break the Cooper pair into two separated electrons:

If, E_c = Energy of Cooper pairs and E_F = Energy of free state,

$$E_g = E_c - E_F \approx 10^{-4} eV$$

Mechanism of attraction between two electrons:

(i) The first electron e_1 distorts the ion and creates a dipole in it i.e. it polarizes the ion.

(ii) Suppose a nearby second electron sees this distorted ion, it gets attracted towards this ion.

(iii) The net result is that the two electrons e_1 and e_2 come closer to each other and their energy is lowered.

(iv) They form pairs.

Why superconductivity in materials

Resistance in a material arises due to scattering of electrons from imperfections/ impurities/ thermal vibrations.

In superconductor, net momentum of the pair is 0 (because momentum of Cooper pairs is equal and opposite). Therefore, de-Broglie wavelength of Cooper pairs which is given as

$$\lambda = \frac{h}{p}$$

will be very large. Thus for Cooper pairs,

λ >> a (size of the obstacle)

In this situation, the scattering of the Cooper pairs would be negligible by imperfections/ impurities/ thermal vibrations and hence $\rho = 0$.

COHERENCE LENGTH

The maximum distance up to which the motion of Cooper pairs remain correlated to give superconductivity is known as coherence length (ξ).

Since the electron states responsible for superconductivity lie within $k_B T_c$ of the Fermi surface, by uncertainty principle

$$\Delta E \tau = (k_B T_c)\tau \approx \hbar \qquad \Rightarrow \tau \approx \frac{\hbar}{k_B T_c}$$

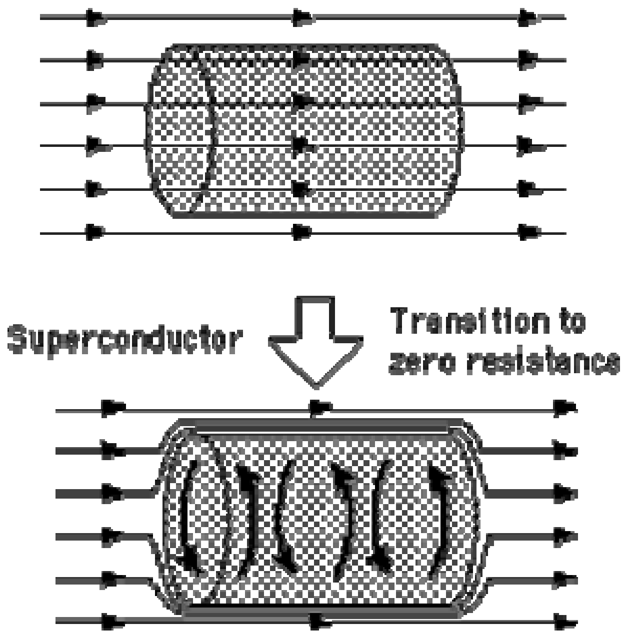
If velocity of electrons at Fermi surface is v_F, the wave function must extend over a distance $\xi = v_F \tau$, Thus,

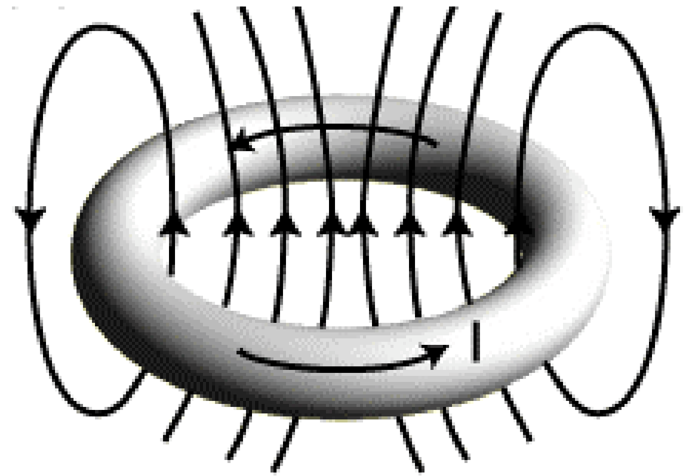
$$\xi = \frac{\hbar v_F}{k_B T_c} \qquad \Rightarrow \xi = \frac{\hbar v_F}{2\Delta} \qquad\qquad (\xi \approx 10^{-6}\ \text{m})$$

$\Delta =$ Energy gap $\qquad v_F =$ Fermi velocity

gap parameter $\quad 2\Delta_0 = 3.5 k_B T_c \quad$ at 0K

FLUX QUANTIZATION



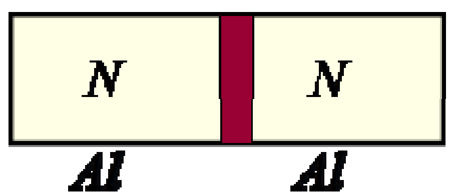


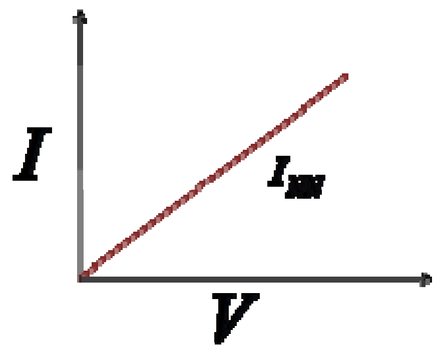
The total magnetic flux that passes through a superconducting ring may assume only quantized values i.e. integral multiples of flux quantum ħ/2e (fluxoid).

Thus, $$\phi = \frac{n\hbar}{2e}$$

Electron Tunneling

Consider two normal metals (N) separated by an insulator junction.

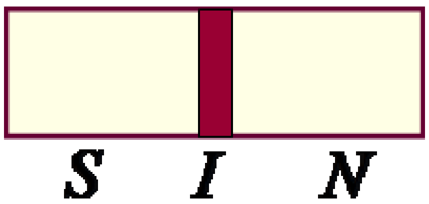


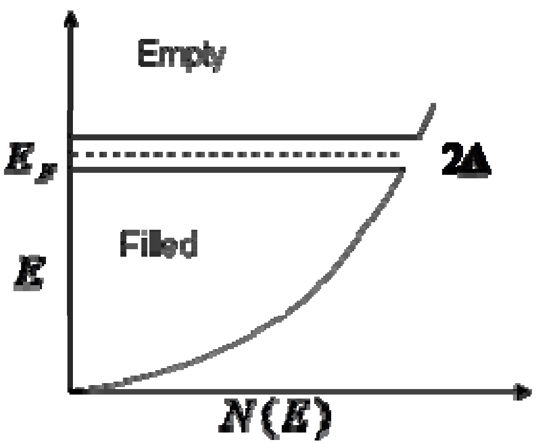


If thickness of the junction >> 100 Å, no conduction electron will pass through this junction from metal 1 to metal 2.

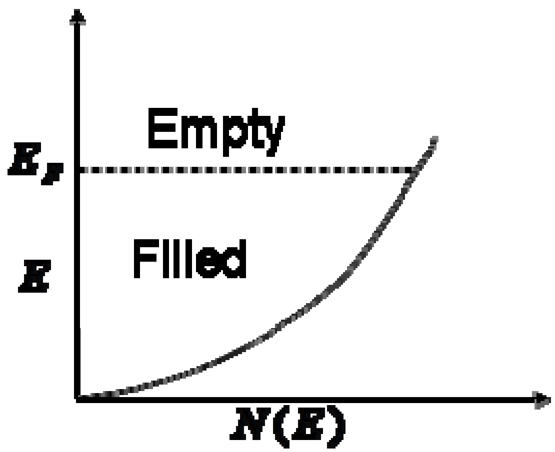
But, if thickness of the junction ≤ 10 Å, then electron from metal 1 will start tunneling through the insulator to the metal 2. This is called quantum tunneling.

If one metal is changed by a superconductor then V-I characteristic changes.

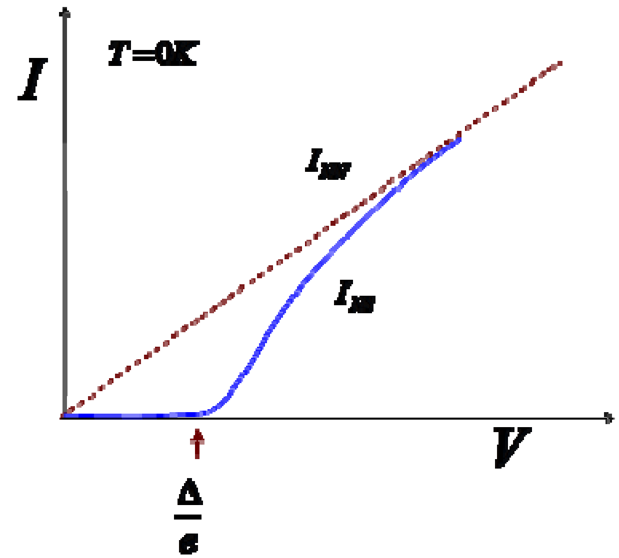




Superconductor



Metal

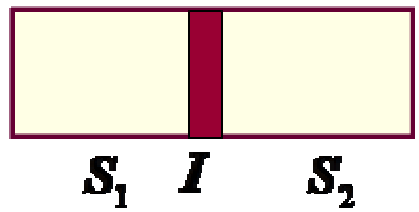


This is known as Giever Tunneling.

At absolute zero no current can flow until applied voltage is $E_g/2e = \Delta/e$.

At finite temperatures there is a small current flow even at low voltages because of the thermal excitations of electrons.

If both metals are replaced by superconductors:



Under suitable conditions, we observe following remarkable effects associated with tunneling superconducting electron pairs from one Superconductor to another through a layer of insulator.

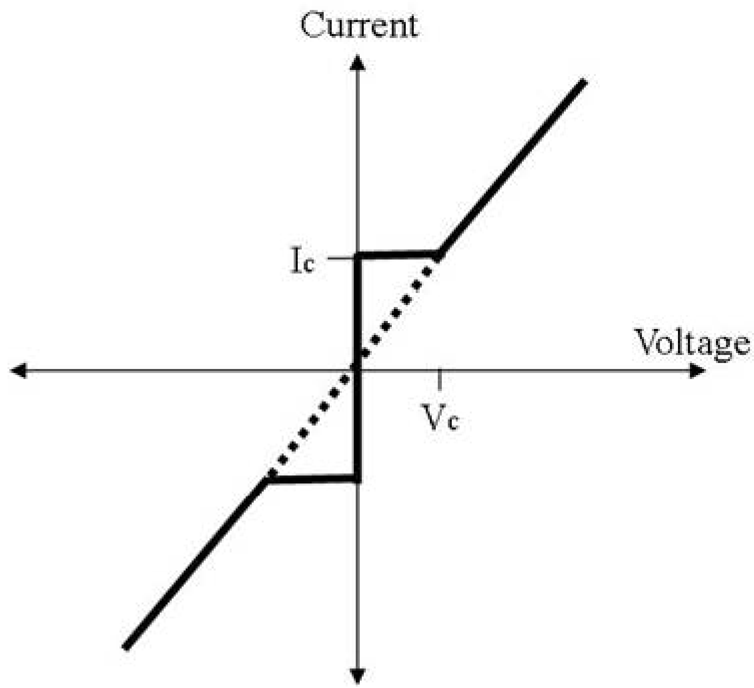
DC Josephson effect:

A dc current flows across the junction in the absence of any electric or magnetic field.

$$J = J_0 \sin\delta$$

$\delta = \theta_2 - \theta_1$ is phase difference between electrons of two Superconductors

The current J_0 is the maximum zero voltage current that can be passed through the junction.



With no applied voltage a dc current will flow across the junction with a value between J_0 and $-J_0$ according to the value of the phase difference θ_2-θ_1. This is dc Josephson effect.

AC Josephson effect:

A dc voltage applied across the junction causes rf current oscillations across the junction.

In this case the current density across the junction is given as

$$J = J_0 \sin[\delta(0) - \frac{2eVt}{\hbar}]$$

Thus current oscillates with a frequency

$$\omega = \frac{2eV}{\hbar}$$ This is ac Josephson effect.

For a dc voltage V = 1μV, ω = 483.6 MHz.

Thus phenomenon says that a photon of energy ħω = 2eV is emitted or absorbed when an electron pair crosses the barrier.

Thus e/ ħ value can be precisely determines by measuring voltage an the frequency.

HIGH Tc SUPERCONDUCTORS

$YBa_2Cu_3O_7$ 90K

Fullerenes: C_{60} compounds with alkali atom mixture.

APPLICATIONS

Superconducting magnets.

Superconducting power transmission.