Descriptive Statistics With R Software

Moments

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Raw and Central Moments

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Moments

Moments are used to describe different characteristics and features of a frequency distribution, viz., central tendency, disperson, symmetry and peakedness (hump) of frequency curve.

Notations for Ungrouped (Discrete) Data

Observations on a variable *X* are obtained as $x_1, x_2, \ldots, x_n$.

Notations for Grouped (Continuous) data

Observations on a variable X are obtained and tabulated in K class intervals in a frequency table as follows. The mid points of the intervals are denoted by $x_1, x_2, \ldots, x_k$ which occur with frequencies $f_1, f_2, \ldots, f_K$ respectively and $n = f_1 + f_2 + \ldots + f_K$.

Class intervals	Mid point (x_i)	Absolute frequency (f_i)
$e_1 - e_2$	$x_1 = (e_1 + e_2)/2$	f_1
$e_2 - e_3$	$x_2 = (e_2 + e_3)/2$	f_2
…	…	…
$e_{K-1} - e_K$	$x_K = (e_{K-1} + e_K)/2$	f_K

Moments about Arbitrary Point *A*

The r^{th} moment of a variable X about any arbitrary point A based on observations $x_1, x_2, \ldots, x_n$ is defined as

- **For ungrouped (discrete) data**

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^r$$

- **For grouped (continuous) data**

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{K} f_i (x_i - A)^r$$

where $n = \sum_{i=1}^{K} f_i$

Raw Moments

The r^{th} (sample) moment around origin A = 0 is called as raw moment and is defined as follows:

- **For ungrouped (discrete) data**

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{n} x_i^r$$

- **For grouped (continuous) data**

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{K} f_i x_i^r$$

where $n = \sum_{i=1}^{K} f_i$

Raw Moments

The first and second raw moments are obtained by substituting

***r* = 1 and *r* = 2 respectively as follows:**

For ungrouped (discrete) data	For grouped (continuous) data
$\mu_1' = \frac{1}{n}\sum_{i=1}^{n} x_i$ **Arithmetic mean**	$\mu_1' = \frac{1}{n}\sum_{i=1}^{K} f_i x_i$ **Arithmetic mean**
$\mu_2' = \frac{1}{n}\sum_{i=1}^{n} x_i^2$	$\mu_2' = \frac{1}{n}\sum_{i=1}^{K} f_i x_i^2$ where $n = \sum_{i=1}^{K} f_i$

Note that when *r* = 0, $\mu_0 = 1$ for ungrouped and grouped data both.

Central Moments

The moments of a variable X about the arithmetic mean $\overline{x}$ are called central moments.

The r^{th} (sample) central moment based on observations $x_1, x_2, \ldots, x_n$ is defined as follows:

❖ **For ungrouped (discrete) data**

$$\mu_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^r$$

❖ **For grouped (continuous) data**

$$\mu_r = \frac{1}{n}\sum_{i=1}^{K} f_i (x_i - \overline{x})^r$$

where $n = \sum_{i=1}^{K} f_i, \quad \overline{x} = \frac{1}{n}\sum_{i=1}^{K} f_i x_i$

Central Moments

The first and second central moments are obtained by substituting

***r* = 1 and *r* = 2 respectively as follows:**

❖ **For ungrouped (discrete) data**

$$\mu_1 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x}) = 0$$

$$\mu_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2 \quad \text{: Sample variance}$$

Central Moments

The first and second central moments are obtained by substituting $r = 1$ and $r = 2$ respectively as follows:

❖ **For grouped (continuous) data**

$$\mu_1 = \frac{1}{n}\sum_{i=1}^{K} f_i (x_i - \overline{x}) = 0$$

$$\mu_2 = \frac{1}{n}\sum_{i=1}^{K} f_i (x_i - \overline{x})^2 \quad \text{: Sample variance}$$

where $n = \sum_{i=1}^{K} f_i, \quad \overline{x} = \frac{1}{n}\sum_{i=1}^{K} f_i x_i$

❖ **Note that when $r = 0$, $\mu_0 = 1$ for ungrouped and grouped data both.**

Relationship Between Central and Raw Moments

$$\mu_0 = \mu'_0 = 1$$

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2 - \mu_1'^2$$

$$\mu_3 = \mu'_3 - 3\mu'_1\mu'_2 + 2\mu_1'^3$$

$$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu_1'^2 - 3\mu_1'^4$$