Descriptive Statistics With R Software

Association of Variables
::
Correlation Coefficient

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Association of Two Variables

Example: Contineous variables

- Number of hours of study affect the marks obtained is an examination.
- Electricity/power consumption increases when the weather temperature increases.
- Weight of infants and small children increases as their height increases under normal circumstances.

Two variables are associated and the nature of variables is contineous.

Association of Two Variables

Example: **Discrete and counting variables**

- **We want to know whether male students prefer mathematics over female students.**
- **We want to know if the vaccine given to diseased persons was effective or not.**

These observations are based on counting and the two variables are discrete and counting.

Association of Two Variables

Example: Ranked observations

- **Two judges give ranks to a fashion model.**
- **Two persons give ranks to food prepared or their scores are ranked.**

These observations are the ranks of two variables (two judges).

Correlation Coefficient

X, Y **: Variables measured on contenuous scale.**

X **and** ***Y*** **are linearly related.**

$Y = a + bX$ **where** ***a*** **and** ***b*** **are unknown constant values.**

Correlation is a statistical tool to study the linear relationship between two variables.

Correlation Coefficient

Two variables are said to be correlated if the change in one variable results in a corresponding change in the other variable.

If two variables deviate in the same direction, i.e., the increase (or decrease) in one variable results in a corresponding increase (or decrease) in the other, the correlation is said to be positive or variables are said to be positively correlated.

Correlation Coefficient

If two variables deviate in the opposite direction, i.e., as one variable increases, the other decreases and vice versa, the correlation is said to be negative or the variables are said to be negatively correlated.

If one variable changes and the other variable remains constant on average or there is no change in the other variable, the variables are said to be independent or they have no correlation.

Correlation Coefficient

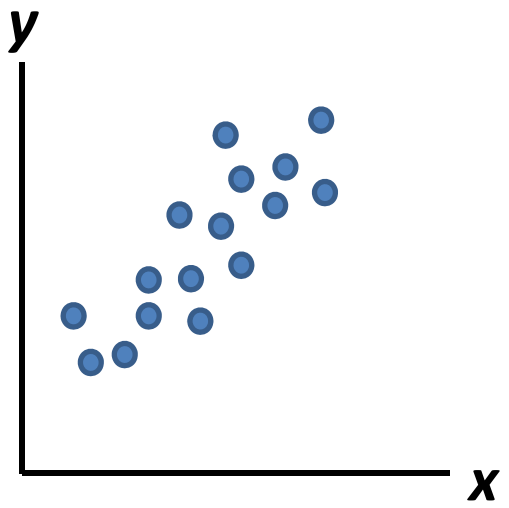


Fig. 1 : Positive correlation

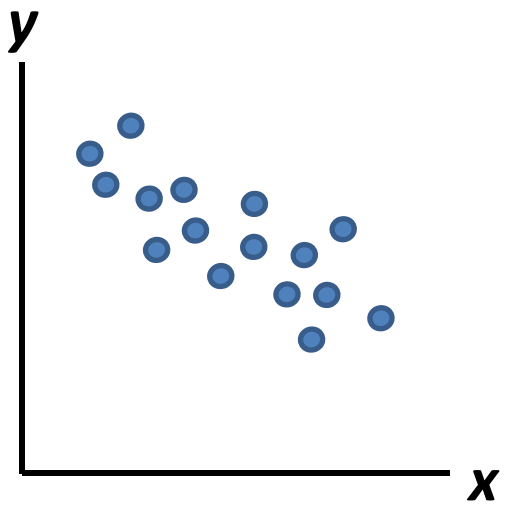


Fig. 2 : Negative correlation

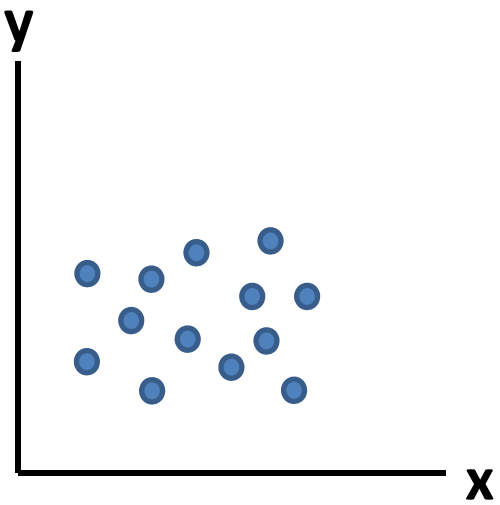


Fig. 3 : No correlation

Correlation Coefficient

Consider following two plots:

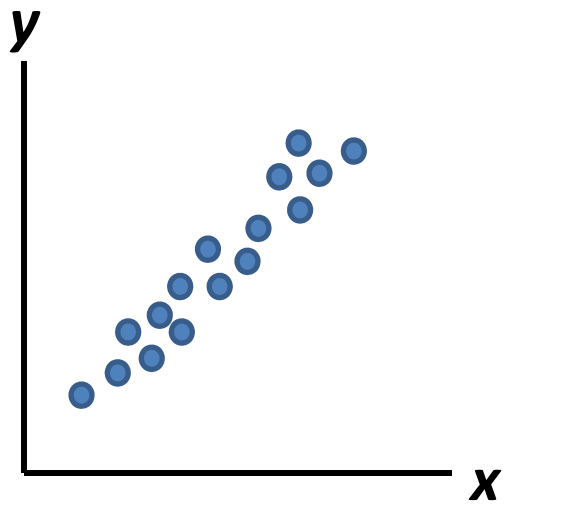


Fig A: strong positive correlation

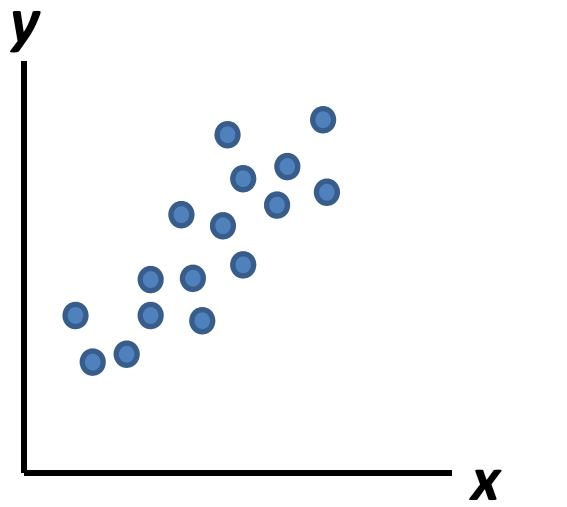


Fig. B: Moderate positive correlation

Correlation Coefficient

How to quantitatively measure the degree of linear relationship?

Use correlation coefficient.

It is based on the concepts of covariance and variance.

What is covariance?

Correlation Coefficient

What is covariance?

Recall variance.

When there is only one variable, variation exists.

When there are two variables, beside their individual variations, their co-variation also exists, provided they affect each other.

Covariance

***X*, *Y* : Two variables**

***n* pairs of observations are available as (x_1,y_1), (x_2,y_2),…, (x_n,y_n)**

The covariance between the variables *X* and *Y* is defined as

$$\operatorname{cov}(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

where

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Similar definition is available for grouped data in frequency table.

Covariance

R command:

`x,y` : Two data vectors

`cov(x,y)`: covariance between x and y.

Command `cov(x,y)` calculates the covariance with divisor (*n* – 1)

$$\mathrm{cov}(x, y) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

Coefficient of Correlation

Also called as Karl Pearson Coefficient of Correlation

$$r \equiv r(x,y) = \frac{\mathrm{cov}(x,y)}{\sqrt{\mathrm{var}(x).\mathrm{var}(y)}}$$

$$= \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

$$= \frac{\sum_{i=1}^{n}x_i y_i - n\,\bar{x}\,\bar{y}}{\sqrt{\left(\sum_{i=1}^{n}x_i^{\,2}-n\bar{x}^2\right)\left(\sum_{i=1}^{n}y_i^{\,2}-n\bar{y}^2\right)}}$$

Coefficient of Correlation

r **measures the degree of linear relationship**

Also called as *Bravis-Pearson Correlation Coefficient* or *Product Moment Correlation Coefficient*

Prof. Karl Pearson (1857 – 1936) presented first regorous treatment of correlation and acknowledged Prof. Auguste Bravis (1811 – 1863) for his initial mathematical formula for correlation coefficient

Coefficient of Correlation

Limits of r : $-1 \leq r \leq 1$

$r > 0$: Indicates positive association between X and Y

$\Rightarrow$ X and Y are positively correlated.

$r < 0$: Indicates negative association between X and Y

$\Rightarrow$ X and Y are negatively correlated.

$r = 0$: Indicates no association between X and Y

$\Rightarrow$ X and Y are uncorrelated.

Coefficient of Correlation

Value of *r* has two components – sign and magnitude.

Sign of *r* indicates the nature of association.

+ sign of *r* indicates positive correlation. As one variable increases (or decreases), other variable also increases (or decreases).

– sign of *r* indicates negative correlation. As one variable increases (or decreases), other variable decreases (or increases).

Coefficient of Correlation

Magnitude of *r* indicates the degree of linear relationship.

$-1 \leq r \leq 1$

$r = 1 \Rightarrow$ Perfect linear relationship

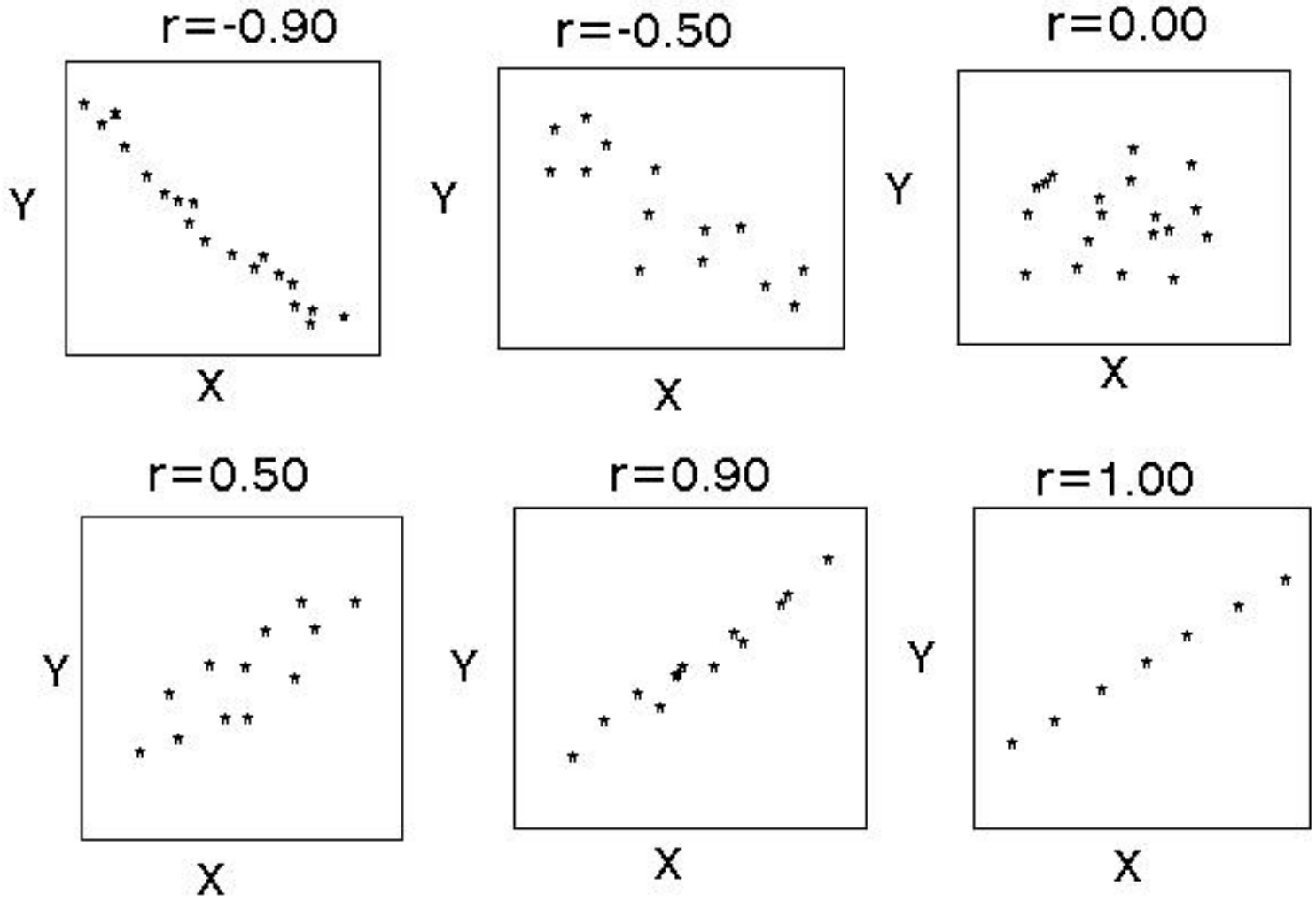
$r = 0 \Rightarrow$ No correlation

Any other value of *r* between 0 and 1 indicates the degree of linear relationship.

$r = +1 \Rightarrow$ Perfect linear and increasing relationship between *X* and *Y.*

$r = -1 \Rightarrow$ Perfect linear and decreasing relationship between *X* and *Y.*

Coefficient of Correlation



Coefficient of Correlation

Value of *r* close to zero indicates that

- **the variables are independent**

  or

- **the relationshop is nonlinear.**

If relationship between *X* and *Y* is nonlinear, then the degree of linear relationship may be low and *r* is then close to 0 even if the variables are clearly not independent.

So when *X* and *Y* are independent then *r*(*X*, *Y*) = 0 but not conversely true.

Coefficient of Correlation

Correlation coefficient is symmetric

$$r(X, Y) = r(Y, X)$$

Example:

Correlation coefficient between height and weight is the same as of the correlation between weight and height.

Coefficient of Correlation

Correlation coefficient is independent of units of measurement of *X* and *Y*.

Example:

- **One person measures height is meters and weight in kilograms. Finds correlation coefficient r_1**
- **Another person measures height in centimeters and weight in grams. Finds correlation coefficient r_2**

Then $r_1 = r_2$