**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:\_ B 1-stats.norm.cdf(50,45,8)= 0.26598552904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans:- #employees greater than 44 years age

1-stats.norm.cdf(44,38,6)= 0.15865525393145707

#employees between age 38 and 44

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)= 0.3413447460685429

Obsevation-*FALSE. More employees are there between age 38 and 44*

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- #employees under 30 years of age

stats.norm.cdf(30,38,6)= 0.09121121972586788

#employees out of 400 expected to attend the training

400\*0.09121121972586788=36.484487890347154

1. *Ans. TRUE* A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.
2. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:- *The difference between  and  is .*

***Step-by-step explanation:***

*According to the****Central Limit Theorem****, any****large sum****of****independent****,****identically distributed(iid)****random variables is approximately****Normal****.*

*The****Normal distribution****is defined by two parameters, the****mean****, , and the****variance****,  and written as .*

*Given   are two independent identically distributed random variables.*

*From the properties of****normal random variables****,*

*if  and  are two independent identically distributed random variables then*

* *the****sum****of normal random variables is given by*

*,*

* *and the****difference****of normal random variables is given by*

**

* *When  , the****product****of X is given by*

**

* *When  , the****linear combination****of X and Y is given by*

**

*Given to find, *

*Thus, following the property of multiplication, we get*

**

*and following the property of addition,*

**

*And the difference between the two is given by*

**

*The mean of  and  is same but the var() of   is 2 times more than the variance of .*

*The difference between the two says that the two given variables are****identically****and****independently****distributed.*

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:- (C) stats.norm.interval(0.99,100,20)

= (48.48341392902199, 151.516586070978)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:- # Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

= Mean Profit is Rs 540 Million

# Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD = np.sqrt((9)+(16))

print('Standard Deviation is Rs', SD\*45, 'Million')

= Standard Deviation is Rs 225.0 Million

# A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

= Range is Rs (99.00810347848784, 980.9918965215122) in Millions

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans:- # To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

= 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:-

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)= 0.0477903522728147

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)= 0.040059156863817086