

Homework 2: Theory (9 pts)  
 Programming Languages (CSCI 3300)  
 Due: Thursday, Sept. 12 by 12:45pm

The following defines the Iffy Language:

Syntax:

(Bool)  $b ::= 1 \mid 0 \mid b \wedge b \mid b \vee b \mid \text{if } b \text{ then } b \text{ else } b$

Single-step reduction:

$$\begin{array}{c}
 \frac{}{(1 \wedge 1) \rightsquigarrow 1} \text{ ANDTRUE} \quad \frac{}{(0 \wedge 0) \rightsquigarrow 0} \text{ ANDFALSE} \quad \frac{}{(0 \wedge 1) \rightsquigarrow 0} \text{ ANDFALSE1} \\
 \frac{}{(1 \wedge 0) \rightsquigarrow 0} \text{ ANDFALSE2} \quad \frac{b_1 \rightsquigarrow b'_1}{(b_1 \wedge b_2) \rightsquigarrow (b'_1 \wedge b_2)} \text{ AND1} \quad \frac{b_2 \rightsquigarrow b'_2}{(b_1 \wedge b_2) \rightsquigarrow (b_1 \wedge b'_2)} \text{ AND2} \\
 \frac{}{(1 \vee 1) \rightsquigarrow 1} \text{ ORTRUE} \quad \frac{}{(1 \vee 0) \rightsquigarrow 1} \text{ ORTRUE1} \quad \frac{}{(0 \vee 1) \rightsquigarrow 1} \text{ ORTRUE2} \quad \frac{}{(0 \vee 0) \rightsquigarrow 0} \text{ ORFALSE} \\
 \frac{b_1 \rightsquigarrow b'_1}{(b_1 \vee b_2) \rightsquigarrow (b'_1 \vee b_2)} \text{ OR1} \quad \frac{b_2 \rightsquigarrow b'_2}{(b_1 \vee b_2) \rightsquigarrow (b_1 \vee b'_2)} \text{ OR2} \quad \frac{}{\text{if } 1 \text{ then } b_1 \text{ else } b_2 \rightsquigarrow b_1} \text{ IFTRUE} \\
 \frac{}{\text{if } 0 \text{ then } b_1 \text{ else } b_2 \rightsquigarrow b_2} \text{ IFFALSE} \quad \frac{b \rightsquigarrow b'}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b' \text{ then } b_1 \text{ else } b_2} \text{ IF1} \\
 \frac{b_1 \rightsquigarrow b'_1}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b \text{ then } b'_1 \text{ else } b_2} \text{ IF2} \quad \frac{b_2 \rightsquigarrow b'_2}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b \text{ then } b_1 \text{ else } b'_2} \text{ IF3}
 \end{array}$$

1. Show that the following is derivable:

$$(0 \wedge (1 \vee 1)) \rightsquigarrow 0 \wedge 1$$

2. Show that the following is derivable:

$$\text{if } (0 \vee 1 \vee (1 \wedge 0)) \text{ then } 0 \wedge (\text{if } 0 \text{ then } 1 \vee 1 \text{ else } 0) \text{ else } (0 \wedge 1) \rightsquigarrow \text{if } (0 \vee 1 \vee (1 \wedge 0)) \text{ then } 0 \wedge (\text{if } 0 \text{ then } 1 \text{ else } 0) \text{ else } (0 \wedge 1)$$