Binary Search

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Binary Search

→ This is done on Sorted Arrays.
→ There will be a sorted array given & then we need to perform Binary Search.

arr = [2, 4, 9, 10, 12, 14, 18, 19] → ascending

arr = [19, 18, 14, 12, 10, 8, -3, -16] → descending

} Sorted Arrays

Q: arr = [0:2, 1:4, 2:6, 3:9, 4:11, 5:12, 6:14, 7:20, 8:36, 9:48]

Target = 36

Binary Search Algorithm:

1. Find the middle element.
2. Check if target > middle ⇒ Search in the right side of the array

   else ⇒ Search in the left side of the array
3. If target == middle ⇒ Answer Found (10 (1):

Working:

s = 0, e = 9

arr = [0:2, 1:4, 2:6, 3:9, 4:11, 5:12, 6:14, 7:20, 8:36, 9:48]

target = 36

1: mid = $\frac{0+9}{2}$ = 4 (Integer value)

(middle element is at the 4th index)

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2: Check target > middle ; is 36 > 11 ?

YES!!

∴ arr's start & end will be updated like:

s (index 5), e (index 9)

arr = [2, 4, 6, 9, 11, 12, 14, 20, 36, 48]

(indices: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; index 7 (20) circled)

1: $mid = \frac{5+9}{2} = 7$

2: is 36 > 20 ? **YES!!**

s (index 8), e (index 9)

arr = [2, 4, 6, 9, 11, 12, 14, 20, 36, 48]

(indices: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9; index 8 (36) circled)

1: $mid = \frac{8+9}{2} = 8$

2: Here, target == middle

∴ 36 = 36 ∴ **Element Found !!**

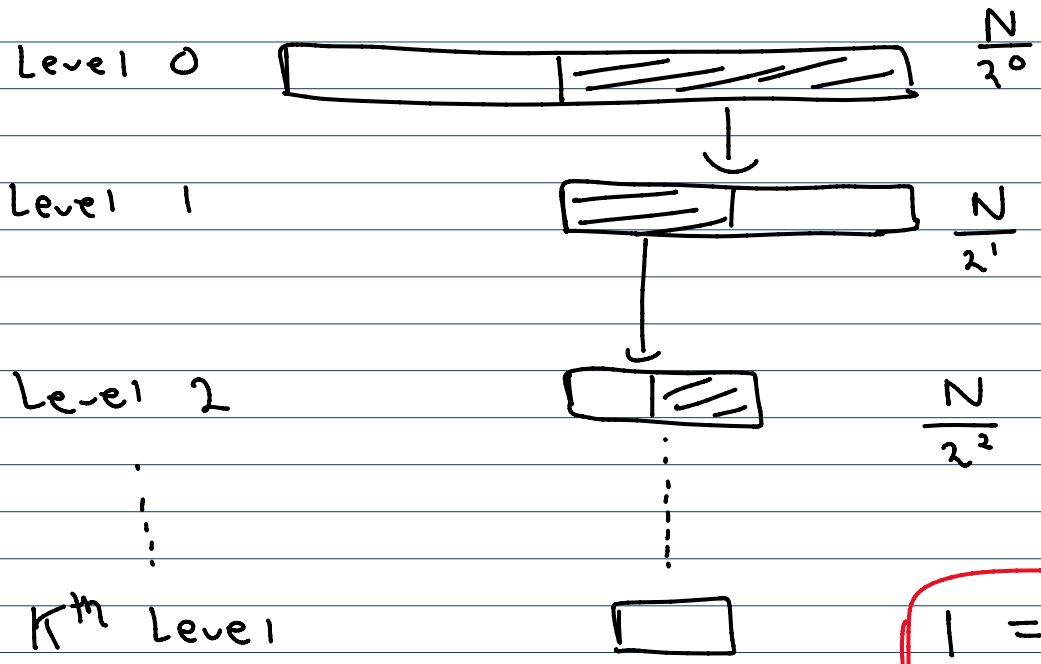
at index '8'

if 's' > 'e' : Element not found in the array.

Time Complexity

Best Case: $O(1)$ [element which we are searching is in the middle only]

Worst Case: $O(\log N)$



$$N = 2^K$$

$$\log N = \log(2^K)$$

$$\log N = K \log(2)$$

$$K = \frac{\log N}{\log 2} \Rightarrow \boxed{K = \log_2 N}$$

Total Comparisions in Worst Case → $\log N$

Suppose, Searching 1,000,000 elements (Considering worst case in both)

na a h

1,000,000 comparisions	$\log_2 1{,}000{,}000 \simeq 20$

Better way to find "mid"

$mid = \frac{start + end}{2}$ -> This may exceed the range

$\therefore mid = start + \frac{(end - start)}{2}$

$= \frac{2s + e - s}{2}$

$= \boxed{\frac{s+e}{2}}$

∴ Formula remains the same but only the way of writing is different. Thus, in the 2nd way, the range won't exceed.

Order Agnostic Binary Search

-> This approach is used when we don't know if the array is sorted in ascending order or descending order

	0	1	2	3 (M)	4	5	6	7 (e)
arr =	90,	75,	18,	12,	6,	4,	3,	1

target = 75

IF target > middle => search in left OR else right

IF start > end => Descending

IF Start > end => Descending

IF end > Start => Ascending

Searching in Matrices

	0	1	2
0	18	9	12
1	36	-4	91
2	44	36	16

Ans = [1, 2]

Target = 91

```
For r=0; r<n, r++
    For c=0; c<n; c++;
        if arr[r][c] = tgt
            Ans
else return -1;
```

$N * N = N^2 = O(N^2)$

$O(N * m)$

IF N rows & m cols

Q. Matrix is sorted in row wise & col wise manner

	0	1	2	3
0	10	20	30	40
1	15	25	35	45
2	28	29	37	49
3	33	34	38	50

Lowest bound

upper bound

Target = 37

Case 1: if element == target

=> ans. is found

Case 2: If element < target

row ++

Case 3: If element > target

col --

For lowest bound, we consider the 1st element of 0th row (Here 10)

For upper bound, we consider the entire last column

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Col --

Explanation for Case 3:

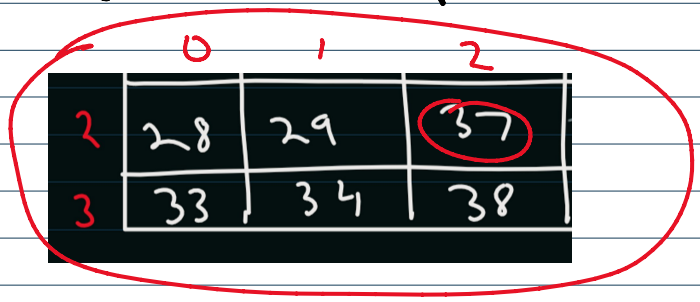
1] For column, we start with last column & take its 1st element (40). We observe that $40 > 37$. ∴ All elements in that column will be > target. (∵ the matrix is sorted). Thus we ignore the column. Thus, we do {Col --}

2] Then we come to the 2nd column & find out that $30 < 37$. That leads us to Case 2

Explanation for Case 2:

1] We start with element 30 (arr[0][2]) bcoz srch space is now reduced. We see the 30 < target (37). Thus all elements in that row will be < target. Hence, we ignore the row. Thus we do {rowtt}

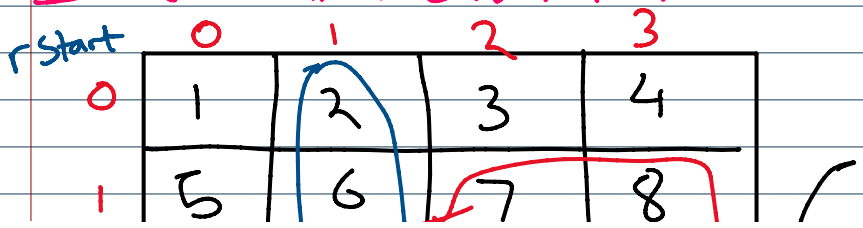
2] We also ignore the 2nd row for the same reason. Thus our srch space now will be:



Now we come in arr[2][2]. Here the element == target

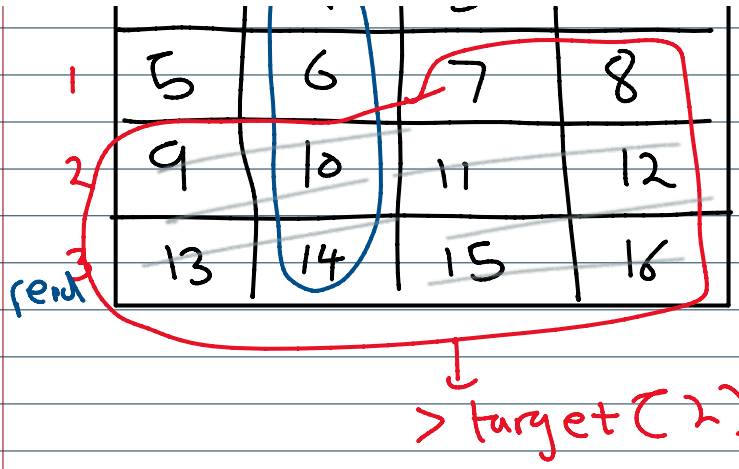
Thus, answer found

Q Search in a Sorted Matrix



Target = 2

Take middle column & perform BS on it



find middle column & perform BS on it

Upper & lower bounds are given (rstart, rend)

middle element = 6.
we see that 6 > 2.
Then obviously all the rows below it will be ignored

① IF element == target

// Ans found

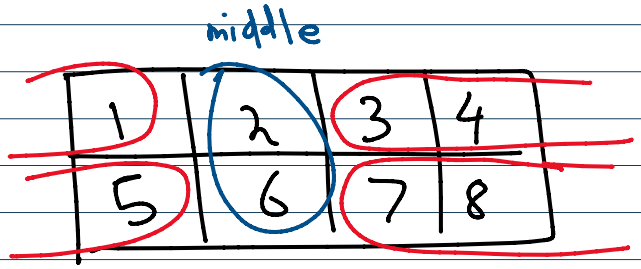
② IF element > target

// Ignore rows after it

③ IF element < target

// Ignore rows above it

In the end, only 2 rows will be left (consider above example)



If target = 3

① Check whether the mid column you are at contains the answer or not

② If not, then consider 4 parts.

③ Srch in all 4 parts till you get answer

Time Complexity: T.C = log(N) (BS har rahe hai)

We considered N rows, & we are taking M columns.

↓

Searching

↓
Searching
across 4 columns

(3) Search in all 4 parts till you get answer

4 log(m)
↳ can be ignored

$$T.C = O(\log(N) + \log(m))$$