

Models of Robotic Manipulators

Spring 2020

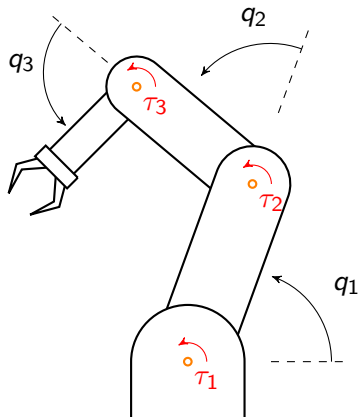
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Robot Configurations: Joint Variables



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Dynamics

The configuration q changes with time according to the second-order nonlinear ordinary differential equation

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + \tau_{friction} + \tau_e, \quad (1)$$

where

- ▶ $D(q)$: inertia matrix (some formulations use $M(q)$)
- ▶ $C(q, \dot{q})$: Coriolis terms
- ▶ $G(q)$: the conservative forces (gradient of potential energy)
- ▶ τ is the force or torque vector generated by actuators at these joints
- ▶ $\tau_{friction}$ represents dissipative forces
- ▶ τ_e represents externally applied forces

State Space Models

How do these dynamics compare to the usual state-space system?

$$\dot{x} = f(x, u)$$

$$y = h(x, u)$$

The configuration $q \in \mathbb{R}^n$ defines a state

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}.$$

We may then use formulate a state space model

$$\dot{x} = f(x) + g(x)u$$

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The output y above corresponds to the task space coordinate of the robot.

WARNING: Roboticians do not use symbols x and y as above.

Goals

For the remainder of this course, we will

- ▶ Understand joint coordinates q
- ▶ Understand properties of the dynamics model (1)
- ▶ Understand the relationship $x = FK(q)$, where x is the task coordinates (output) and FK is the forward kinematics map
- ▶ Derive controllers that make $q(t) \rightarrow q_d(t)$
- ▶ Derive controllers that make $x(t) \rightarrow x_d(t)$
- ▶ Derive controllers that enforce relationships between external forces τ_e and joint velocity \dot{q} (Energy-based control)

Goals



At the end of this half of the course, you should

- ▶ understand how to derive equations and controllers in principle;
- ▶ know how to leverage software libraries to perform calculations involving robot models and controllers.