ME 599/699 Robot Modeling & Control
Force and Impedance

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1 Introduction

So far, we cast robot motion control as position control or trajectory tracking: $q(t) \to q_d(t)$. This view comes from the historical focus of moving the end-effector in free space to complete tasks. What happens when the end-effector comes in contact with objects? This interaction due to contact is not handled well by position control An alternative to controlling the configuration during interaction is to control the force. We will look at force control in the next section. Finally, some challenges with force control will lead us to the general framework of impedance control.

2 Force Control

In some applications, we want the robotic manipulator to apply a force on an object, for example to push objects along a surface, or turn a valve. We may achieve this in two ways:

1. Directly through measurement of the applied force and error-based feedback
2. Indirectly through changes in static configuration.

2.1 Force-based

Consider the usual equations $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + J^T F_{tip}(t)$. If we can measure $F_{tip}(t)$ through some sensor, and have a desired force F_d at the end-effector, we can construct an error $F_e = F_d - F_{tip}(t)$. Let's choose the control

$$\tau = G(q) - J^T F_d - J^T \left(K_p F_e + K_i \int F_e(s) ds \right) \tag{1}$$

The closed-loop equations become

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + J^T F_{tip}(t) \tag{2}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = G(q) - J^T F_d - J^T \left(K_p F_e + K_i \int F_e(s) ds \right) + J^T F_{tip}(t) \tag{3}$$

In general, this system is hard to analyze. For quasi-static motions, where $\ddot{q} \approx 0$ and $\dot{q} \approx 0$, we get

$$(\approx 0) + (\approx 0) + G(q) = G(q) - J^T F_d - J^T \left(K_p F_e + K_i \int F_e(s) ds \right) + J^T F_{tip}(t) \tag{4}$$

$$\implies 0 = J^T \left((I + K_p) F_e + K_i \int F_e(s) ds \right) \tag{5}$$

$$\implies 0 = (I + K_p) \dot{F}_e + K_i F_e \quad \text{(if } J(q) \text{ is non-singular)} \tag{6}$$

So, if $\ddot{q}(t) \approx 0$, $\dot{q}(t) \approx 0$, $J(q)$ is non-singular, $K_p \geq 0$ and $K_d > 0$, then $F_e \to 0$. As you might guess, direct force control is difficult to achieve in practice.
Problems: There's a potential contradiction where we apply a time-varying torque $\tau(t) \neq 0$ at the robot's joints but need $\ddot{q}, \dot{q} = 0$. This situation might exist when the end-effector is in contact with something that doesn't move much. By contrast, what happens when the end-effector loses contact? The measured force drops to zero, and the end-effector accelerates due to F_e! Unexpected changes in contact turn out to be disastrous for force controllers intended to work on a particular contact configuration. These issues make force control unpopular except for very structured situations, requiring advanced methods.
Partial Solution: Add damping $-K_d \dot{q}$ to achieve slow motion

2.2 Configuration-based

Consider the usual equations $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + J^T F_{tip}(t)$. When the robot is stationary, and the end-effector is in contact with a surface, we obtain

$$G(q) = \tau + J^T(q) F_{tip}(t) \tag{7}$$

The idea is to then solve this equation, say for q^* and τ^*, when $F_{tip}(t) = F_d$, a desired force, and then design a controller τ that stabilizes the configuration and torque at these values. That is, $q(t) \to q^*$ and $\tau(t) \to \tau^*$. For example,

$$\tau = \underbrace{G(q^*) - J^T(q^*) F_d}_{\tau^*} + \underbrace{K_p(q - q^*) + K_d(\dot{q})}_{\text{stabilization}}.$$

Task: Analyze this control law.
Note that this approach does not monitor F_{tip}, but focuses on configuration q.

3 Impedance

When two mechanical systems come in contact, they impart forces on each other while constraining the motion of each other. However, it is not necessarily true that one system gets to dictate the force and motion at the contact. Both mechanical systems are influencing and reacting to the forces and motion, thereby this contact represents a true *interaction*.

References