ME 599/699 Robot Modeling & Control
Fall 2021

Dynamics

Hasan A. Poonawala

Department of Mechanical Engineering
University of Kentucky

Email: hasan.poonawala@uky.edu
Web: https://www.engr.uky.edu/~hap

Introduction

Two related problems for robotics:

- Forward Dynamics: given τ, calculate $\ddot{q}$ (simulation)
- Inverse Dynamics: given $\ddot{q}$, what τ produces it? (control)

Introduction

- Since our robot is a powered mechanism, we will obtain models for the motion that are generalizations of the Newton's Second Law $F = ma$
- a can be joint acceleration $\ddot{q}$ or task-variable accelerations $\ddot{x}$
- There are largely two approaches :
  - Apply Newton's law to every rigid body (need to know contact forces)
  - Use an energy-based formation, which can ignore contact forces

Recursive Newton-Euler Methods

- Apply Newton-Euler equations to a link in the frame attached to it's center of mass (easy to encode)
- Forward pass: Since the frames are not inertial, propagate the coriolis accelerations from the base to the end-effector
- Backward pass: propagate the torques that achieve the accelerations, and contact forces they imply, from end-effector frame to the base

Recursive Newton-Euler Methods

- Most simulators implement the RNE algorithm also for simulation
- $\mathcal{O}(n)$ complexity, which is fast
- Method generalizes to all kinematic trees
- Closed chains / parallel mechansisms can be handled by additional steps
- Screw-theory-based approaches may be better for parallel mechanisms

Euler-Lagrangian Models

- Derive's equations from the total energy of the system
- Avoids needing to account for internal joint forces
- Difficult to automate, at least so far
- Deep structural insights into robot dynamics

Euler-Lagrangian Models

- Define the coordinates q of the system
- Define the Lagrangian $\mathcal{L}(q, \dot{q})$ = Kinetic Energy - Potential Energy
- For each DoF q_i:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = \sum \text{ Generalized Forces}$$

- The robot equations are

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + \tau_{friction} + \tau_e,$$

Properties

- The matrix $\dot{D}(Q) - 2C$ is skew symmetric.
- Bounded Inertia: For a system with revolute joints, there exist λ_m and λ_M such that

$$\lambda_m I_{n\times n} \leq D(q) \leq \lambda_M I_{n\times n} < \infty \qquad (1)$$

- Linearity in Parameters: We can derive a function $Y(q, \dot{q}, \ddot{q})$ and parameter set θ such that

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \ddot{q})\theta \qquad (2)$$

Dynamics Including Actuators

- For torque-controlled robots, use

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \tau_{friction} + \tau_e,$$

- For voltage-controlled motors, we often use

$$\underbrace{M(q)}_{+\text{motor inertia}} \ddot{q} + C(q,\dot{q})\dot{q} + \underbrace{B\dot{q}}_{+\text{motor friction}} + G(q) = u + \tau_e,$$