ME 599/699 Robot Modeling & Control
Feedback Control

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The task of motion control is to achieve that change

Ideal Behavior

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Ideally, $q_d(t) = q(t)$ at all times

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If we can apply large enough forces, we may assume that we can practically instantaneously set velocities. Eg: low inertia wheels.

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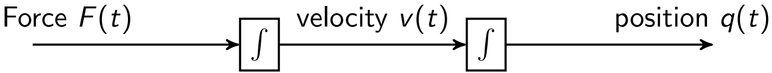
In other words, $q(t) = \int_0^t \int_0^t F(t) dv \ dq$, where $v(t) = \dot{q}(t)$.



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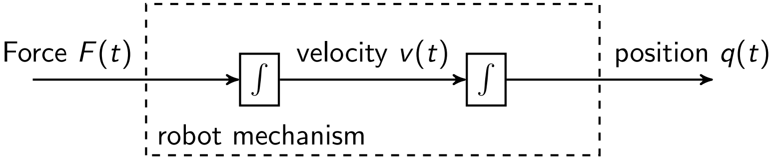

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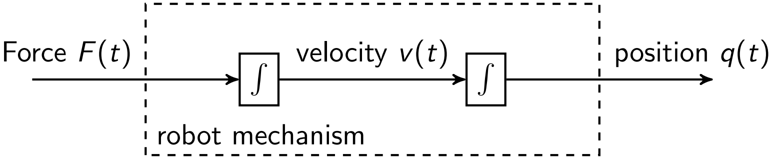

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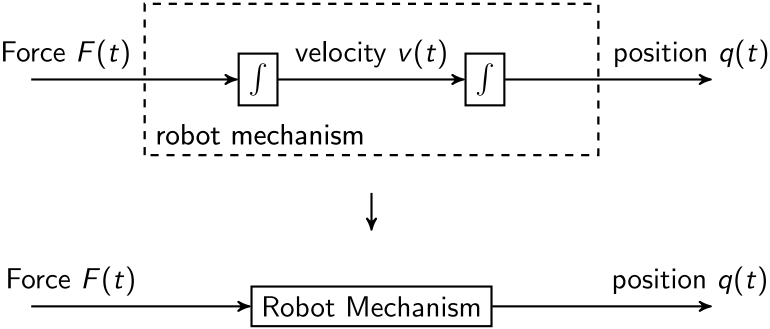

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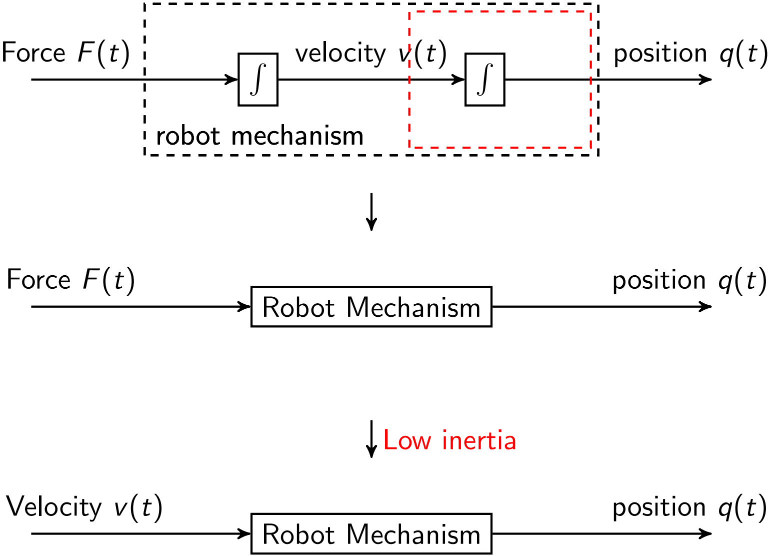

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$$q_d(t) \longrightarrow \boxed{\frac{d}{dt}} \longrightarrow \boxed{\frac{d}{dt}} \xrightarrow{\ddot{q}_d(t) = F(t)} \boxed{\int} \xrightarrow{v(t)} \boxed{\int} \xrightarrow{q(t)}$$

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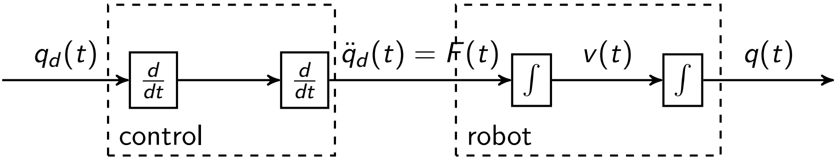

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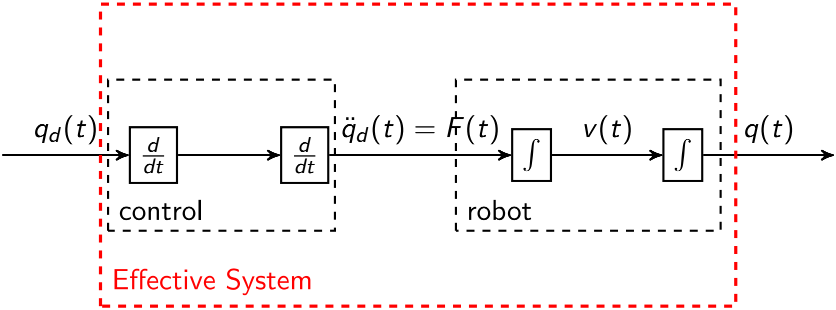
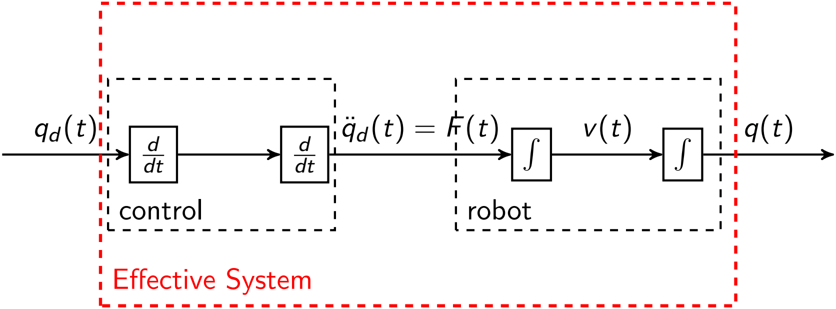


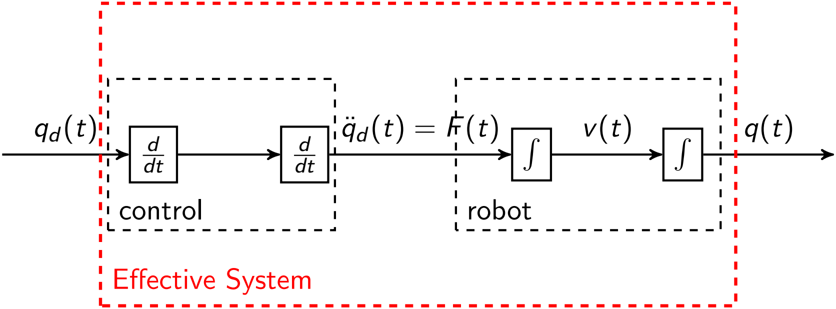
Model Inversion: Open-Loop



We've managed to make our robot react to desired position instantly, by

- Setting the initial condition
- Knowing the 'input' $q_d(t)$ perfectly

Model Inversion: Open-Loop



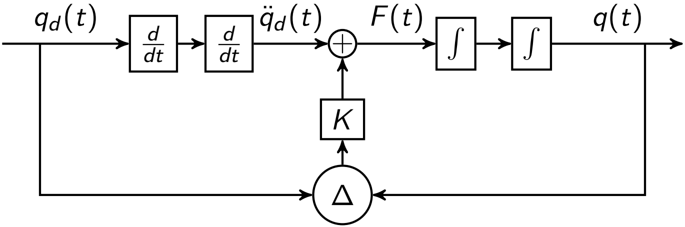
Issues:

- What if desired position is not known ahead of time?
- What if a disturbance force $f_d(t)$ acts, so that input is $F(t) + f_d(t) \neq \ddot{q}_d(t)$?

Closed-Loop Control

One way to account for these issues is to also react to errors in position:

$$F(t) = \ddot{q}_d(t) + K\left(q_d(t) - q(t)\right)$$



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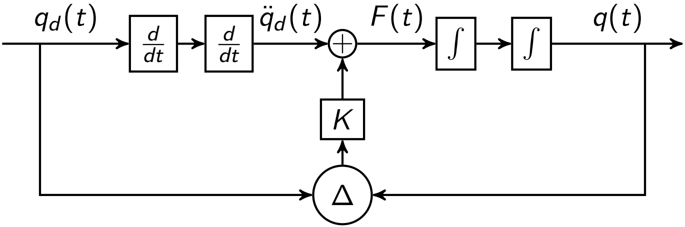

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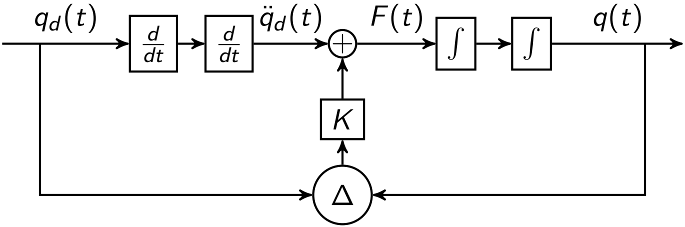

Issue: How do we choose K? Why will a good choice be possible?
Another issue: How do you know what $q(t)$ is?

Recap

We studied a simple point-mass system.

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We then related force to change in $q(t)$ using an ordinary differential equation:

$$\frac{d^2}{dt^2}q(t) = F(t).$$

This ODE is a forward dynamics model: how the state and input affect the change in state.

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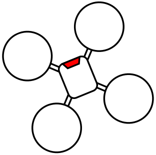
For point-mass:

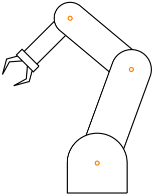
$$\dot{x} = \begin{bmatrix} 0 \\ F(t) \end{bmatrix} = f(x(t), F(t)).$$

Dynamics

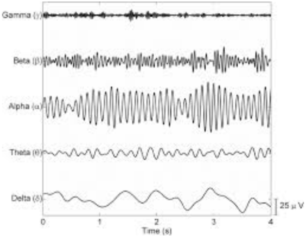
- The system has a state ***x***
- The dynamics are captured by how ***x*** changes with time



Quadrotor

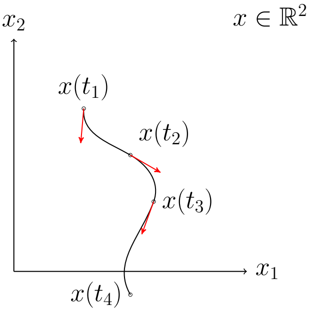


Robot Arm

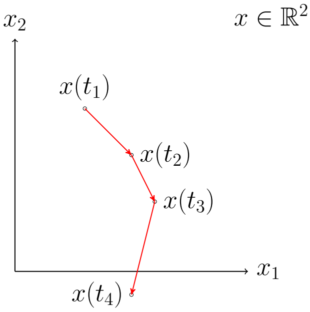


EEG

Dynamics



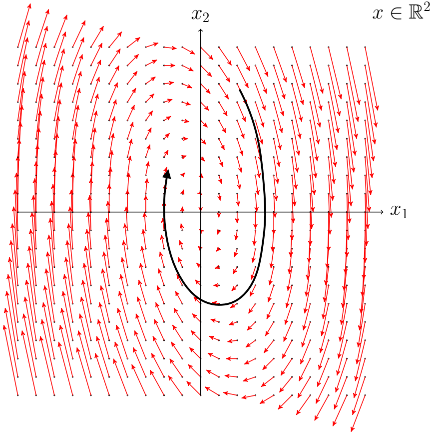
Continuous-time

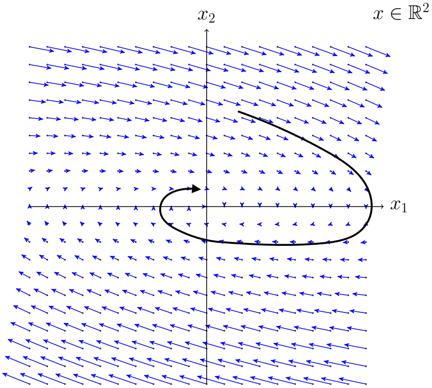


Discrete-time

Think of vectors assigned to each point *x*
(A vector field)

Dynamics





Two continuous vector fields

Forward Dynamics

Continuous Time:

$$\dot{x}(t) = f(x(t), u(t), t),$$

where the independent variable is time $t \in \mathbb{R}$
Discrete Time:

$$x_{t+1} = f_t(x_t, u_t),$$

where the independent variable is $t \in \mathbb{N}$

Inverse Dynamics

Idea: given some desired value $\dot{x}_d(t)$ for $\dot{x}(t)$, what $u(t)$ will achieve it?

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For example, for the point mass, we said that we want $\ddot{q}(t)$ to equal $\ddot{q}_d(t)$, so we choose

$$F(t) = \ddot{q}_d(t)$$

Sensor Model

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