State-Space Models of Dynamical Systems

Spring 2020

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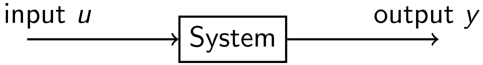
Example:
A bouncing ball is a system that interacts with the rest of the universe through gravitational forces, and the reaction forces with the ground.
The height of the ball above the ground is one possible output.

Dynamical Systems

From the view of the environment, the system accepts inputs from the environment and produces outputs.



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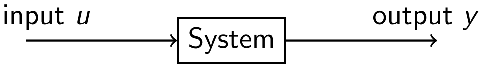

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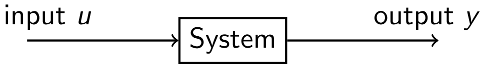

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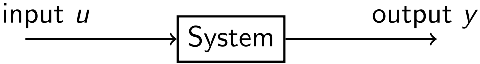

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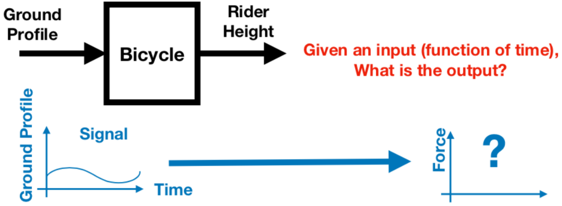
This process occurs in 'real-time'. We predict the transformation using models of the dynamical system.

State: Motivation

For example, consider a bicycle as the system, with the ground's shape as input and the seat height as output.



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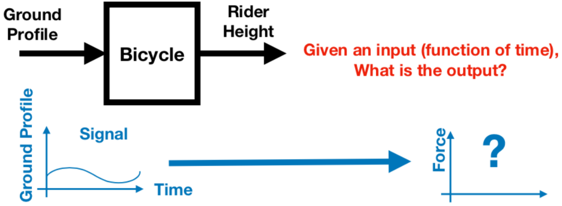

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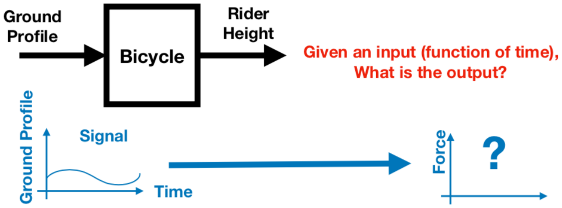

Question: Why is the output not just a multiple of the input at each time?

Answer: The system has an internal state that acts as a memory of the past inputs.

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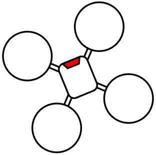
For physical robots, state consists of configuration q and velocity, or

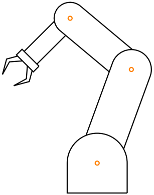
$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

Dynamics

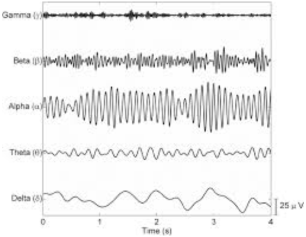
- The system has a state ***x***
- The dynamics are captured by how ***x*** changes with time



Quadrotor

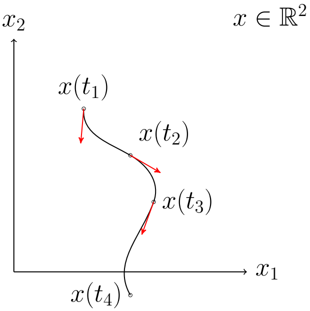


Robot Arm

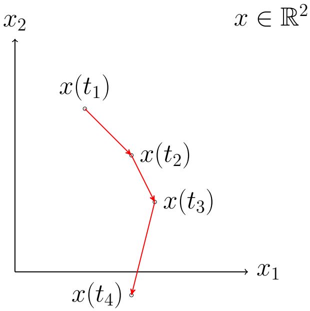


EEG

Dynamics



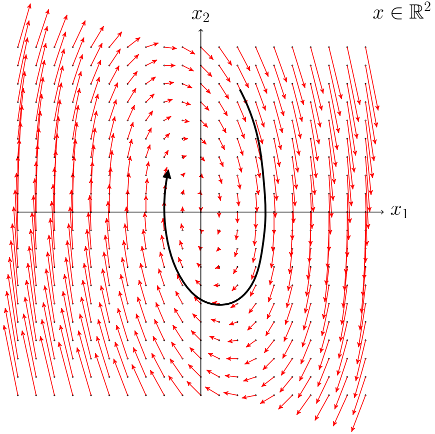
Continuous-time

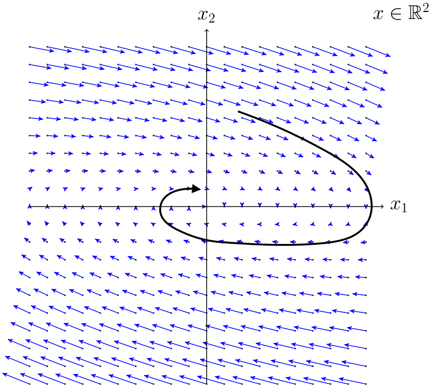


Discrete-time

Think of vectors assigned to each point *x*
(A vector field)

Dynamics





Two continuous vector fields

Dynamical Systems

For continuous-time systems, the vector field is the output of a map f, so that the instantaneous change in x is

$$\frac{d}{dt}x(t) = \dot{x}(t) = f(x(t), u(t), t).$$

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For continuous-time dynamical system, this model consists of nonlinear ordinary differential equations (ODEs).

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Once we have $x(t)$ for $t \geq t_0$, we can calculate $y(t) = h(x(t), u(t), t)$ for each t.