Coordinates And Mobile Robots

Spring 2020

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Robot Configurations

Robot	Configuration Manifold	Coordinates
Point mass	$\mathbb{R}^3$	(x, y, z)
Pan-Tilt Camera	$\mathcal{S}^2$	(θ, ϕ)
Differential-Drive Robot	$\mathbb{R}^2 \times S^1 \quad [SE(2)]$	(x, y, θ)
Elbow Manipulator	$\mathcal{S}^1 \times \mathcal{S}^1 \quad$ [Torus]	(q_1, q_2)
Quadrotor	$\mathbb{R}^3 \times SO(3) \quad [SE(3)]$	(d, R)
Serial-Link Robot Arm	$\left(\mathcal{S}^1\right)^n$	$(q_1, q_2, \ldots, q_n)$

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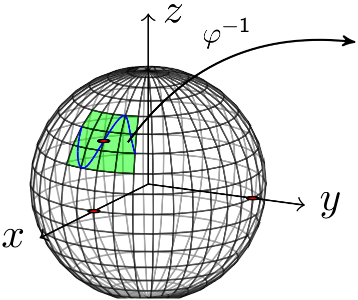
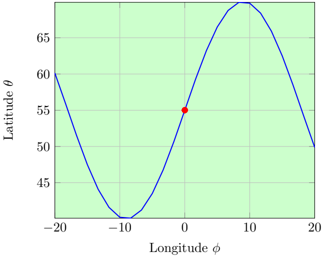
Example: Consider a circle drawn on the Sphere $\mathcal{S}^2$.

The Map Is Not The Territory

The green square in $\mathbb{R}^2$ becomes a patch on the sphere.





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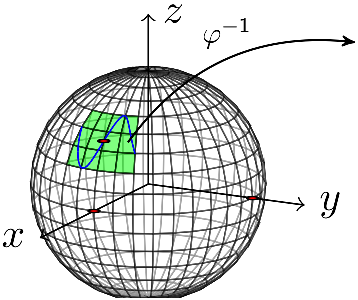

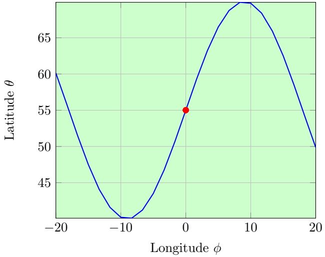

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Example: What happens as the square moves upwards in $\mathbb{R}^2$?

We compute using coordinates in $\mathbb{R}^n$. It's our responsibility to handle the manifold structure underlying our robots' configurations.

Over to Youtube

- DARPA Grand Challenge (ca. 2004)
- DARPA Urban Challenge (ca. 2007)
- DARPA Robotics Challenge (ca. 2015)

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1. Solves path-planning problems given initial and goal poses
2. Implements state-estimation algorithms such as the Extended Kalman Filter

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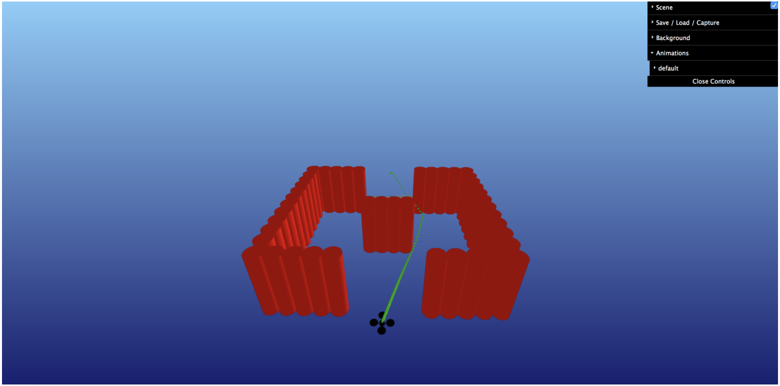

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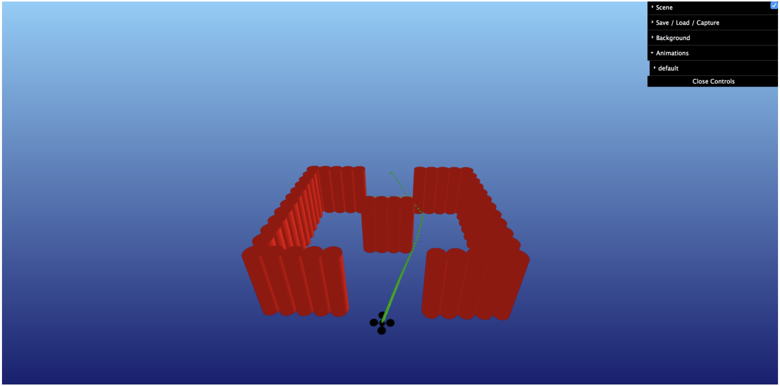

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(This code depicts what I want, but is not a valid solution)

Information & Resources

This is a group assignment: group sizes between 3 and 4 only.

Use `github`, `bitbucket`, or other `git`-compatible online repositories to collaborate and eventually ‘submit’ the assignment.

- Learn version control (`git`) and unix skills online.
- Some code I played with. (Uploaded to Canvas)
- Original papers.

Relevant Planning Topics

1. Graph Search: DFS, BFS, $A^\star$, Djikstra's Algorithm
2. Probabilistic Road Maps
3. Rapidly Exploring Random Trees

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The path/trajectory planning problems can be cast as an optimization problem.

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This version of the problem doesn't worry about control.
Out pops $q^*(t)$ and we try and use path following or trajectory tracking controllers.

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   (ii) Probabilistic Road Maps
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2. Polynomial/Parabolic Blends