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MATB42: Assignment #8 Mar 20th 2018

1. (a) Let $f, g : \mathbb{R}^n \to \mathbb{R}$; $\boldsymbol{F}$, $\boldsymbol{G}$: $\mathbb{R}^n \to \mathbb{R}$; and define Δ, the *Laplacian*, by $\Delta f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$.

   Verify the following identities

   (i) $\text{div}(\boldsymbol{F} + \boldsymbol{G}) = \text{div } \boldsymbol{F} + \text{ div } \boldsymbol{G}$.

   (ii) $\text{div}(f\boldsymbol{F}) = f \text{ div } \boldsymbol{F} + \boldsymbol{F} \cdot \text{ grad} f$.

   (iii) $\Delta(fg) = f\Delta g + g\Delta f + 2(\text{grad} f) \cdot (\text{grad} g)$.

   (b) Let $f, g : D \subset \mathbb{R}^3 \to \mathbb{R}$ be of class C^1. If R is a solid region contained in D then

$$\iiint_R \nabla f \cdot \nabla g \, dV = \iint_{\partial R} f \nabla g \cdot \boldsymbol{n} \, dS - \iiint_R f \nabla^2 g \, dV$$

   ($\nabla^2 g = \text{div } (\nabla g)$).

2. Use the Divergence Theorem to verify your asnwer to question 7 on assignment 8.

3. Let $\boldsymbol{F}(x, y, z) = (x, y^2, e^{yz})$ and let R be a cube centered at the origin with sides of length 2. Evaluate $\int_S \text{div } \boldsymbol{F} \, dV$ directly and by using the Divergence Theorem.

4. Let B be the pyramid with top vertex (0,0,1) and base vertices (0,0,0), (1,0,0), (0,1,0) and (1,1,0). Let S be the 2-dim closed surface bounding B, oriented in the outward direction. Use Gauss' theorem to calculate $\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$, where $\boldsymbol{F}(x, y, z) = (x^2 y, 3y^2 z, 9z^2 x)$.