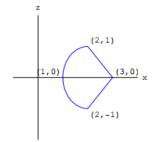
## MATB42: Assignment #8

- 1. A surface S is obtained by rotation the given figure in the xy-plane about the z-axis. (The arc is part of a circle of radius 1 centered at (2,0).)
  - (a) Paratemetrize S (in pieces) and compute the surface area.

We have that the upper line when rotated, can be parametrized by a restricted cone and similarly for the bottom. The top and bottom respectively can be written as



$$\Phi(u,\theta) = ((1-u)3\cos\theta, (1-u)3\sin\theta, 3u)$$
$$0 \le u \le 1, \ 0 \le \theta \le 2\pi$$
$$\Phi(u,\theta) = ((1+u)3\cos\theta, (1+u)3\sin\theta, -3u)$$
$$-1 \le u \le 0, \ 0 \le \theta \le 2\pi$$

(b) Use a computer algebra system to sketch S. For the circular portion to the left, when rotated around, it will be the inner half of a torus, so the equation will be

- 2. Let S be the cone with vertex (2,3,3) and base the circle  $x^2 + y^2 = 1$  in the xy-plane.
  - (a) Paratemetrize S Starting with a base of a circle, we get  $(\cos \theta, \sin \theta, 1)$ . To change into a cone multiply x and y by (1-u) and finally to shift the vertex, add (2u, 3u, 2u) where z=2u since the base equation already has a 1, so it needs to be one less.

$$\implies \Phi(u,\theta) = ((1-u)\cos\theta + 2u, (1-u)\sin\theta + 3u, 1+2u)$$

- (b) Use a computer algebra system to sketch S.
- (c) Write down the integral that would give the surface area of S. (You are not expected to evaluate the integral.)

- 3. Let S be the self-intersecting rectangle in  $\mathbb{R}^3$  given by the implicit equation  $x^2 y^2z = 0$ .
  - (a) Give a parametrization of S and use a computer algebra system to provide a sketch.
  - (b) Is your parametrization one-to-one? Explain.
  - (c) Find the equation of the tangent plane to S at  $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$ .
- 4. Let S be the surface defined by  $x^2 + y^2 = 1$  for  $0 \le z \le 1$  and by  $x^2 + y^2 = z^2$  for  $1 \le z \le 2$ .
  - (a) Use symbolic algebra software to sketch S.
  - (b) Evaluate  $\int_{S} \mathbf{F} \cdot d\mathbf{S}$  where  $\mathbf{F}(x, y, z) = (-y, x, z)$  and S is oriented by outward pointing normals.
  - (a) Evaluate the (vector) surface integral  $\int_S {m F} \cdot d{m S}$  in each of the following cases.
    - i. F(x, y, z) = (1, x, z), S is the upper hemisphere  $x^2 + y^2 + z^2 = 1$ ,  $z \ge 0$ , with n pointing upward.
    - ii. F(x, y, z) = (2, x, z + y), S is that part of the plane x + y + z = 1 which lies in the first octant and n points upward.
    - iii. Marsden & Tromba, page 425, #22.
- 5. Let S be the portion of the plane x 2y + z = 1 that is cut off by the coordinate planes and the plane x + y = 1. Let  $\mathbf{V}$  be the velocity field  $\mathbf{V}(x, y, z) = (y, z, x^2)$ . Find the flow across S when  $\mathbf{n}$  points upward. Explain your answer.
- 6. Let S be the closed surface that consists of the hemisphere  $x^2 + y^2 + z^2 = 1$ ,  $z \ge 0$ , and its base  $x^2 + y^2 \le 1$ , z = 0. let **E** be the electric field  $\mathbf{E}(x, y, z) = (2x, 2y, 2z)$ . Directly calculate the electric flux across S.