

MATB42: Assignment #8

1. (a) Let $f, g : \mathbb{R}^n \rightarrow \mathbb{R}$; $\mathbf{F}, \mathbf{G} : \mathbb{R}^n \rightarrow \mathbb{R}$; and define Δ , the *Laplacian*, by $\Delta f = \sum_{i=1}^n \frac{\partial^2 f}{\partial x_i^2}$.

Verify the following identities

- (i) $\operatorname{div}(\mathbf{F} + \mathbf{G}) = \operatorname{div} \mathbf{F} + \operatorname{div} \mathbf{G}$.
- (ii) $\operatorname{div}(f\mathbf{F}) = f \operatorname{div} \mathbf{F} + \mathbf{F} \cdot \operatorname{grad} f$.
- (iii) $\Delta(fg) = f\Delta g + g\Delta f + 2(\operatorname{grad} f) \cdot (\operatorname{grad} g)$.