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MATB42: Assignment #8 Mar 20th 2018

1. (a) Let $f, g : \mathbb{R}^n \to \mathbb{R}$; $\boldsymbol{F}$, $\boldsymbol{G}$: $\mathbb{R}^n \to \mathbb{R}$; and define Δ, the *Laplacian*, by $\Delta f = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$.

   Verify the following identities

   (i) $\operatorname{div}(\boldsymbol{F} + \boldsymbol{G}) = \operatorname{div} \boldsymbol{F} + \operatorname{div} \boldsymbol{G}$.

   (ii) $\operatorname{div}(f\boldsymbol{F}) = f \operatorname{div} \boldsymbol{F} + \boldsymbol{F} \cdot \operatorname{grad} f$.

   (iii) $\Delta(fg) = f\Delta g + g\Delta f + 2(\operatorname{grad} f) \cdot (\operatorname{grad} g)$.

   (b) Let $f, g : D \subset \mathbb{R}^3 \to \mathbb{R}$ be of class C^1. If R is a solid region contained in D then

$$\iiint_R \nabla f \cdot \nabla g \, dV = \iint_{\partial R} f \nabla g \cdot \boldsymbol{n} \, dS - \iiint_R f \nabla^2 g \, dV$$