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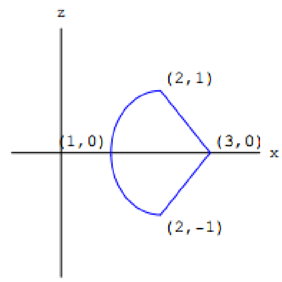
MATB42: Assignment #8 Mar 20th 2018

1. A surface S is obtained by rotation the given figure in the xy-plane about the z-axis. (The arc is part of a circle of radius 1 centered at (2,0).)



   (a) Paratemetrize S (in pieces) and compute the surface area.

   We have that the upper line when rotated, can be parametrized by a restricted cone and similarly for the bottom. The top and bottom respectively can be written as

$$\mathbf{\Phi}(u,\theta) = ((1-u)3\cos\theta, (1-u)3\sin\theta, 3u),\ 0 \le u \le 1,\ 0 \le \theta \le 2\pi$$
$$\mathbf{\Phi}(u,\theta) = ((1+u)3\cos\theta, (1+u)3\sin\theta, -3u),\ -1 \le u \le 0,\ 0 \le \theta \le 2\pi$$

For the circular portion to the left, when rotated around, it will be the inner half of a torus, so the equation will be

(b) Use a computer algebra system to sketch S.

2. Let S be the cone with vertex (2,3,3) and base the circle $x^2 + y^2 = 1$ in the xy-plane.

(a) Paratemetrize S Starting with a base of a circle, we get $(\cos\theta, \sin\theta, 1)$ with $0 \le \theta \le 2\pi$. To change into a cone multiply x and y by $(1-u)$ with $0 \le u \le 1$ and finally to shift the vertex, add $(2u, 3u, 2u)$ where $z = 2u$ since the base equation already has a 1, so $1 + ku <= 3 \implies k \le 2$.

$$\implies \mathbf{\Phi}(u,\theta) = ((1-u)\cos\theta + 2u, (1-u)\sin\theta + 3u, 1 + 2u)$$

(b) Use a computer algebra system to sketch S.

(c) Write down the integral that would give the surface area of S. (You are not expected to evaluate the integral.)

$$\begin{aligned}
\boldsymbol{\phi}_\theta &= (-(1-u)\sin\theta,\ (1-u)\cos\theta,\ 0) \\
\boldsymbol{\phi}_u &= (-\cos\theta + 2,\ -\sin\theta + 3,\ 2) \\
\boldsymbol{\phi}_\theta \times \boldsymbol{\phi}_u &= ((2(1-u)\cos\theta), (2(1-u)\sin\theta), \\
&\quad (-(1-u)\sin\theta)(-\sin\theta + 3) - ((1-u)\cos\theta)(-\cos\theta + 2)) \\
&= ((2-2u)\cos\theta, (2-2u)\sin\theta, (1-u)\sin^2\theta - (3-3u)\sin\theta + (1-u)\cos^2\theta - (2-2u)\cos\theta) \\
&= ((2-2u)\cos\theta, (2-2u)\sin\theta, (1-u) - (3-3u)\sin\theta - (2-2u)\cos\theta) \\
\|\boldsymbol{\phi}_\theta \times \boldsymbol{\phi}_u\| &= \sqrt{(2-2u)^2\cos^2\theta + (2-2u)^2\sin^2\theta + ((1-u) - (3-3u)\sin\theta - (2-2u)\cos\theta)^2} \\
&= \sqrt{(2-2u) + ((1-u) - (3-3u)\sin\theta - (2-2u)\cos\theta)^2} \\
\implies \mathcal{A}(S) &= \int_0^1 \int_0^{2\pi} \sqrt{(2-2u) + ((1-u) - (3-3u)\sin\theta - (2-2u)\cos\theta)^2}
\end{aligned}$$

3. Let S be the self-intersecting rectangle in $\mathbb{R}^3$ given by the implicit equation $x^2 - y^2 z = 0$.

   (a) Give a parametrization of S and use a computer algebra system to provide a sketch.

   (b) Is your parametrization one-to-one? Explain.

   (c) Find the equation of the tangent plane to S at $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$.

4. Let S be the surface defined by $x^2 + y^2 = 1$ for $0 \leq z \leq 1$ and by $x^2 + y^2 = z^2$ for $1 \leq z \leq 2$.

   (a) Use symbolic algebra software to sketch S.

   (b) Evaluate $\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$ where $\boldsymbol{F}(x, y, z) = (-y, x, z)$ and S is oriented by outward pointing normals.

   (a) Evaluate the (vector) surface integral $\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$ in each of the following cases.

      i. $\boldsymbol{F}(x, y, z) = (1, x, z)$, S is the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, with $\boldsymbol{n}$ pointing upward.

      ii. $\boldsymbol{F}(x, y, z) = (2, x, z+y)$, S is that part of the plane $x+y+z = 1$ which lies in the first octant and $\boldsymbol{n}$ points upward.

      iii. Marsden & Tromba, page 425, #22.

5. Let S be the portion of the plane $x - 2y + z = 1$ that is cut off by the coordinate planes and the plane $x + y = 1$. Let $\boldsymbol{V}$ be the velocity field $\boldsymbol{V}(x, y, z) = (y, z, x^2)$. Find the flow across S when $\boldsymbol{n}$ points upward. Explain your answer.

6. Let S be the closed surface that consists of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$, and its base $x^2 + y^2 \leq 1$, $z = 0$. let $\boldsymbol{E}$ be the electric field $\boldsymbol{E}(x, y, z) = (2x, 2y, 2z)$. Directly calculate the electric flux across S.