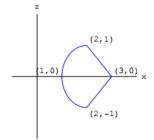
MATB42: Assignment #8

- 1. A surface S is obtained by rotation the given figure in the xy-plane about the z-axis. (The arc is part of a circle of radius 1 centered at (2,0).)
 - (a) Paratemetrize S (in pieces) and compute the surface area. We have that the upper line when rotated, can be parametrized by a restricted cone and similarly for the bottom. The top and bottom respectively can be written as



$$\Phi(u,\theta) = ((1-u)3\cos\theta, (1-u)3\sin\theta, 3u)$$
$$0 \le u \le 1, \ 0 \le \theta \le 2\pi$$
$$\Phi(u,\theta) = ((1+u)3\cos\theta, (1+u)3\sin\theta, -3u)$$
$$-1 \le u \le 0, \ 0 \le \theta \le 2\pi$$

2. Use a computer algebra system to sketch S. For the circular portion to the left, when rotated around, it will be the inner half of a torus, so the equation will be

- 3. Let S be the cone with vertex (2,3,3) and base the circle $x^2 + y^2 = 1$ in the xy-plane.
 - (a) Paratemetrize S Starting with a base of a circle, we get $(\cos \theta, \sin \theta, 1)$. To change into a cone multiply x and y by (1-u) and finally to shift the vertex, add (2u, 3u, 2u) where z=2u since the base equation already has a 1, so it needs to be one less.

$$\implies \Phi(u,\theta) = ((1-u)\cos\theta + 2u, (1-u)\sin\theta + 3u, 1+2u)$$

- (b) Use a computer algebra system to sketch S.
- (c) Write down the integral that would give the surface area of S. (You are not expected to evaluate the integral.)

- 4. Let S be the self-intersecting rectangle in \mathbb{R}^3 given by the implicit equation $x^2 y^2z = 0$.
 - (a) Give a parametrization of S and use a computer algebra system to provide a sketch.
 - (b) Is your parametrization one-to-one? Explain.
 - (c) Find the equation of the tangent plane to S at $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$.
- 5. Let S be the surface defined by $x^2 + y^2 = 1$ for $0 \le z \le 1$ and by $x^2 + y^2 = z^2$ for $1 \le z \le 2$.
 - (a) Use symbolic algebra software to sketch S.
 - (b) Evaluate $\int_{S} \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = (-y, x, z)$ and S is oriented by outward pointing normals.
 - (a) Evaluate the (vector) surface integral $\int_S {m F} \cdot d{m S}$ in each of the following cases.
 - i. F(x, y, z) = (1, x, z), S is the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, with n pointing upward.
 - ii. F(x, y, z) = (2, x, z + y), S is that part of the plane x + y + z = 1 which lies in the first octant and n points upward.
 - iii. Marsden & Tromba, page 425, #22.
- 6. Let S be the portion of the plane x 2y + z = 1 that is cut off by the coordinate planes and the plane x + y = 1. Let V be the velocity field $V(x, y, z) = (y, z, x^2)$. Find the flow across S when n points upward. Explain your answer.
- 7. Let S be the closed surface that consists of the hemisphere $x^2+y^2+z^2=1, z\geq 0$, and its base $x^2+y^2\leq 1, z=0$. let \boldsymbol{E} be the electric field $\boldsymbol{E}(x,y,z)=(2x,2y,2z)$. Directly calculate the electric flux across S.