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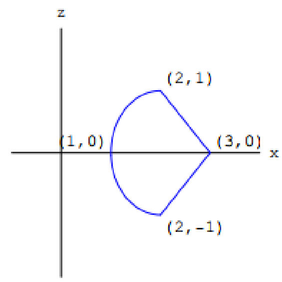
MATB42: Assignment #7 Mar 20th 2018

1. A surface S is obtained by rotation the given figure in the xy-plane about the z-axis. (The arc is part of a circle of radius 1 centered at (2,0).)



   (a) Paratemetrize S (in pieces) and compute the surface area.

   (b) Use a computer algebra system to sketch S.

2. Let S be the cone with vertex (2,3,3) and base the circle $x^2 + y^2 = 1$ in the xy-plane.

   (a) Paratemetrize S

   (b) Use a computer algebra system to sketch S.

   (c) Write down the integral that would give the surface area of S. (You are not expected to evaluate the integral.)

3. Let S be the self-intersecting rectangle in $\mathbb{R}^3$ given by the implicit equation $x^2 - y^2 z = 0$.

   (a) Give a parametrization of S and use a computer algebra system to provide a sketch.

   (b) Is your parametrization one-to-one? Explain.

   (c) Find the equation of the tangent plane to S at $\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$.

4. Let S be the surface defined by $x^2 + y^2 = 1$ for $0 \le z \le 1$ and by $x^2 + y^2 = z^2$ for $1 \le z \le 2$.

   (a) Use symbolic algebra software to sketch S.

   (b) Evaluate $\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$ where $\boldsymbol{F}(x, y, z) = (-y, x, z)$ and S is oriented by outward pointing normals.

   (a) Evaluate the (vector) surface integral $\int_S \boldsymbol{F} \cdot d\boldsymbol{S}$ in each of the following cases.

      i. $\boldsymbol{F}(x, y, z) = (1, x, z)$, S is the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, with $\boldsymbol{n}$ pointing upward.

      ii. $\boldsymbol{F}(x, y, z) = (2, x, z + y)$, S is that part of the plane $x + y + z = 1$ which lies in the first octant and $\boldsymbol{n}$ points upward.

      iii. Marsden & Tromba, page 425, #22.

5. Let S be the portion of the plane $x - 2y + z = 1$ that is cut off by the coordinate planes and the plane $x + y = 1$. Let $\boldsymbol{V}$ be the velocity field $\boldsymbol{V}(x, y, z) = (y, z, x^2)$. Find the flow across S when $\boldsymbol{n}$ points upward. Explain your answer.

6. Let S be the closed surface that consists of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, and its base $x^2 + y^2 \le 1$, $z = 0$. let $\boldsymbol{E}$ be the electric field $\boldsymbol{E}(x, y, z) = (2x, 2y, 2z)$. Directly calculate the electric flux across S.