

MATB42: Assignment #7

1. (a) Find an equation of the tangent plane to the surface S defined parametrically by $\Phi(u, v) = (u^2 + v, v, u + v^2)$ at the point $(9, 0, 3)$.

$$v = 0 \qquad u + v^2 = 3 \implies u = 3$$

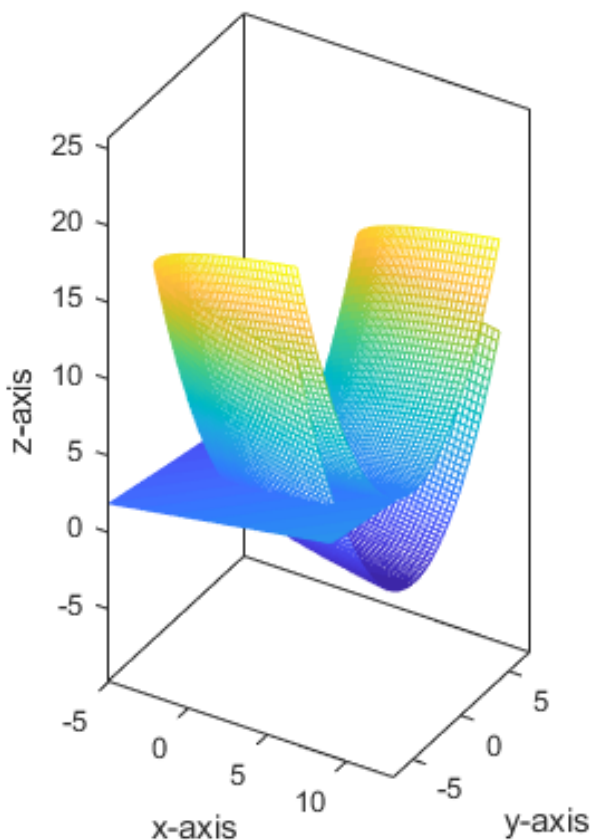
$$\phi_u = (2(3), 0, 1)$$

$$\phi_v = (1, 1, 2(0))$$

$$\phi_u \times \phi_v = (-1, 1 - 4uv, 2u)$$

$$\|\phi_u \times \phi_v\| = \sqrt{1 + (1 - 4uv)^2 + 4u^2}$$

- (b) Use symbolic algebra software to sketch the surface S and its tangent plane from part (a).



2. Use a surface integral to find the area of the triangle in \mathbb{R}^3 with vertices $(1, 1, 0)$, $(1, 2, 1)$ and $(3, 3, 2)$.
3. Calculate the surface area of the piece of the cone $x^2 + y^2 - z^2 = 0$ which lies outside the cylinder $x^2 + y^2 = 4$.
4. (a) Find the area of the portion of the unit sphere that is cut out by the cone $z = \sqrt{x^2 + y^2}$.
(cf. page 391, #10)
- (b) Find the area of the portion of the cone $z = \sqrt{x^2 + y^2}$ that is cut out by the unit sphere.
5. Let $\Phi : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a parametrization of a 2-dim surface S in \mathbb{R}^3 .

(a) Set

$$E = \|\phi_u\|^2,$$

$$F = \phi_u \cdot \phi_v,$$

$$G = \|\phi_v\|^2,$$

Show that the surface area of S is

$$A(S) = \iint_D \sqrt{EG - F^2} \, dA$$

- (b) What does the formula for $A(S)$ become if the vectors ϕ_u and ϕ_v are orthogonal?
- (c) Use parts (a) and (b) to compute the surface area of a sphere of radius a .
(cf. Marsden & Tromba, page 399, # 23.)
6. For each of the following surfaces S , sketch S (using symbolic software) and evaluate the surface integral $\int_S f \, dS$, where $f(x, y, z) = x$.
- (a) S is that part of the surface $y = 4 - x^2$ between $z = 0$ and $z = 1$, with $y \geq 0$.
- (b) S is the upper half of the unit sphere centered at the origin.
- (c) S is that part of the surface $x = \sin y$ with $0 \leq y \leq \pi$ and $0 \leq z \leq 2$.
7. Find the mass of the metallic surface S given by $z = 1 - \frac{x^2 + y^2}{2}$ with $0 \leq x \leq 1$, $0 \leq y \leq 1$, if the mass density at $(x, y, z) \in S$ is given by $m(x, y, z) = xy$.