

MATB42: Assignment #10

1. Let \mathbf{F} be a vector field on \mathbb{R}^3 given by $\mathbf{F} = (F_1, F_2, F_3)$ where F_1 , F_2 , and F_3 are C^1 -functions from $\mathbb{R}^3 \rightarrow \mathbb{R}$

- (a) Let η be the 2-form given by

$$\eta = F_3 \, dx \, dy + F_1 \, dy \, dz + F_2 \, dz \, dx$$

Show that $d\eta = (\operatorname{div} \mathbf{F}) \, dx \, dy \, dz$

(page 489, #6)

$$\begin{aligned} \eta &= F_3 \, dx \, dy + F_1 \, dy \, dz + F_2 \, dz \, dx \\ d\eta &= d(F_3 \, dx \, dy + F_1 \, dy \, dz + F_2 \, dz \, dx) \\ &= (dF_3) \, dx \, dy + (dF_1) \, dy \, dz + (dF_2) \, dz \, dx \end{aligned}$$