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Mar 6th 2018

MATB42: Assignment #6

1. Let $\omega = \dfrac{-y}{x^2 + y^2}\,dx + \dfrac{x}{x^2 + y^2}\,dy$. Calculate $\displaystyle\int_\gamma \omega$ where

   (a) γ is the boundary of the triangle with vertices (in order) (0,1), (2,3) and (2,1).

   (b) γ is the boundary curve of the region $\left\{ (x,y) \in \mathbb{R}^2 \,\middle|\, \dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{4} \le 1 \right\}$ oriented in a counter
       clockwise direction

   (c) γ is the graph of the polar equation $r = 3 + 2\sin\theta$ oriented in the clockwise direction.

2. Let $\omega = (y^2 + z\ \ln 3)\ dx + (2xy + \sin z)\ dy + (y\cos z + (x+1)\ln 3)\ dz$. Determine if ω is exact. If it is, use the algorithm given in class to find the potential function g.

3. Evaluate the double integral $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x^2 y^2 dy\ dx$, by first finding an equivalent line integral.

4. Let R be a region in $\mathbb{R}^2$ and let γ be a counterclockwise parametrization of ∂R. Let $\boldsymbol{F} = (F_1, F-2)$ be a C^1 vector field defined throughout R and on ∂R and let $\boldsymbol{n}$ be the outward pointing unit normal ector to γ. Use Green's theorem to give a double integral over R which is equivalent to $\displaystyle\int_\gamma \boldsymbol{F} \cdot \boldsymbol{n}\ ds$.

5. Give a parametrization for each of the following surfaces, use a computer algebra sustem to plot the surface and find a unit vector normal to the surface.

   (a) The piece of the cylinder $y^2 + z^2 = 1$ between $x = -1 and x = 3$.

   (b) The piece of the plane $z = x + y + 5$ which lies over the uni dik $x^2 + y^2 \le 1$.

   (c) The piece of the sphere $x^2 + y^2 + z^2 = 4$ which lies above the plane $z = 1$.

   (d) The piece of the plane $x + y + z = 1$ which lies above the parallelogram: $0 \le y - x \le 1, 0 \le y + x2 \le 1$.

6. Let S be the surface given parameterically by $\boldsymbol{\Phi}(u,v) = (u^2, 3v, u^2 + v)$ where $(u,v) \in D$, the interior of a triangle with vertices (0,0), (3,0) and (3,3).

   (a) Find the surface area of S.

   (b) Find the equation of the tangent plane to S at the point (4,9,7).

7. Suppose the surface S is the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$. Give a natural parametrization of S (in terms of f) and derive the formula $\|\boldsymbol{\phi}_u \times \boldsymbol{\phi}_v\| = \sqrt{1 + \|\operatorname{grad}\ f\|^2}$

8. A paraboloid of revolution S is parameterized by $\boldsymbol{\phi}(u,v) = u\cos v, u\sin v, u^2,\ 0 \le u \le 2, 0 \le v \le 2\pi$.

   (a) Find an equation in x, y and z describing the surface.

   (b) What are the geometric meanings of the parameters u and v?

   (c) Find a unit vector orthogonal to the surface of $\boldsymbol{\phi}(u,v)$.

   (d) Find the equation for the tangent plane at $\boldsymbol{\phi}(u_0, v_0) = (1,1,2)$ and express your answer in the following two ways:

      i. parameterized by u and v; and
      ii. in terms of x, y and z.

   (e) Find the area of S.