

MATB42: Assignment #6

1. Let $\omega = \frac{-y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy$. Calculate $\int_{\gamma} \omega$ where
 - (a) γ is the boundary of the triangle with vertices (in order) (0,1), (2,3) and (2,1).
 - (b) γ is the boundary curve of the region $\left\{ (x,y) \in \mathbb{R}^2 \left| \frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} \leq 1 \right. \right\}$ oriented in a counter clockwise direction
 - (c) γ is the graph of the polar equation $r = 3 + 2 \sin \theta$ oriented in the clockwise direction.
2. Let $\omega = (y^2 + z \ln 3) dx + (2xy + \sin z) dy + (y \cos z + (x+1) \ln 3) dz$. Determine if ω is exact. If it is, use the algorithm given in class to find the potential function g .
3. Evaluate the double integral $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x^2 y^2 dy dx$, by first finding an equivalent line integral.
4. Let R be a region in \mathbb{R}^2 and let γ be a counterclockwise parametrization of ∂R . Let $\mathbf{F} = (F_1, F_2)$ be a C^1 vector field defined throughout R and on ∂R and let \mathbf{n} be the outward pointing unit normal vector to γ . Use Green's theorem to give a double integral over R which is equivalent to $\int_{\gamma} \mathbf{F} \cdot \mathbf{n} ds$.