

MATB42: Assignment #7

1. (a) Find an equation of the tangent plane to the surface  $S$  defined parametrically by  $\Phi(u, v) = (u^2 + v, v, u + v^2)$  at the point  $(9, 0, 3)$ .

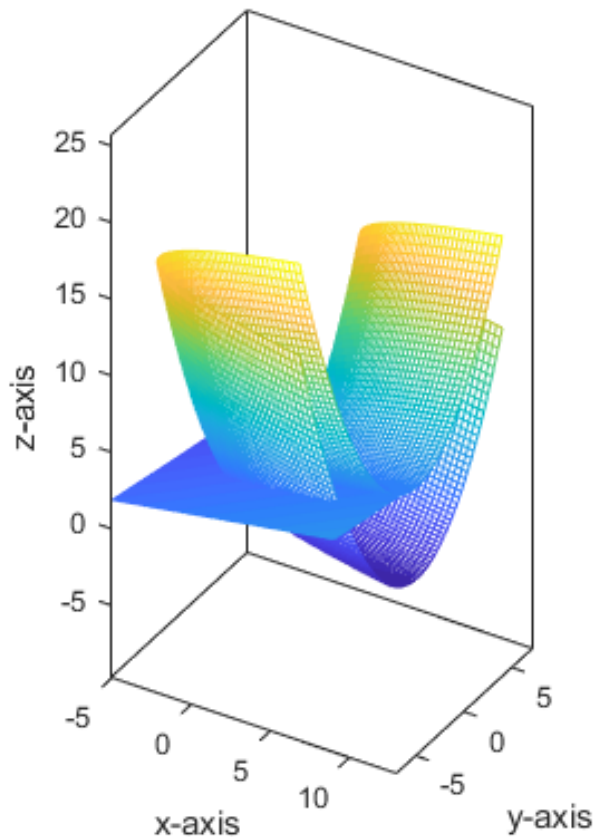
$$v = 0 \qquad u + v^2 = 3 \implies u = 3$$

$$\begin{aligned}\phi_u &= (2(3), 0, 1) \\ \phi_v &= (1, 1, 2(0)) \\ \phi_u \times \phi_v &= (-1, 1, 6)\end{aligned}$$

So the tangent plane can be given by

$$\begin{aligned}0 &= ((x - 9, y, z - 3) \cdot (-1, 1, 6)) \\ 0 &= (9 - x + y + 6z - 18) \\ 9 &= -x + y + 6z\end{aligned}$$

- (b) Use symbolic algebra software to sketch the surface  $S$  and its tangent plane from part (a).



2. Use a surface integral to find the area of the triangle in  $\mathbb{R}^3$  with vertices  $(1, 1, 0)$ ,  $(1, 2, 1)$  and  $(3, 3, 2)$ .
3. Calculate the surface area of the piece of the cone  $x^2 + y^2 - z^2 = 0$  which lies outside the cylinder  $x^2 + y^2 = 4$ .

4. (a) Find the area of the portion of the unit sphere that is cut out by the cone  $z = \sqrt{x^2 + y^2}$ .  
(cf. page 391, #10)

(b) Find the area of the portion of the cone  $z = \sqrt{x^2 + y^2}$  that is cut out by the unit sphere.

5. Let  $\Phi : D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a parametrization of a 2-dim surface  $S$  in  $\mathbb{R}^3$ .

(a) Set

$$E = \|\phi_u\|^2, \quad F = \phi_u \cdot \phi_v, \quad G = \|\phi_v\|^2,$$

Show that the surface area of  $S$  is

$$A(S) = \iint_D \sqrt{EG - F^2} dA$$

$$\iint_D \sqrt{EG - F^2} dA = \iint_D \sqrt{\|\phi_u\|^2 \|\phi_v\|^2 - (\phi_u \cdot \phi_v)^2}$$

(b) What does the formula for  $A(S)$  become if the vectors  $\phi_u$  and  $\phi_v$  are orthogonal?

(c) Use parts (a) and (b) to compute the surface area of a sphere of radius  $a$ .

(cf. Marsden & Tromba, page 399, # 23.)

6. For each of the following surfaces  $S$ , sketch  $S$  (using symbolic software) and evaluate the surface integral  $\int_S f dS$ , where  $f(x, y, z) = x$ .

(a)  $S$  is that part of the surface  $y = 4 - x^2$  between  $z = 0$  and  $z = 1$ , with  $y \geq 0$ .

(b)  $S$  is the upper half of the unit sphere centered at the origin.

(c)  $S$  is that part of the surface  $x = \sin y$  with  $0 \leq y \leq \pi$  and  $0 \leq z \leq 2$ .

7. Find the mass of the metallic surface  $S$  given by  $z = 1 - \frac{x^2 + y^2}{2}$  with  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ , if the mass density at  $(x, y, z) \in S$  is given by  $m(x, y, z) = xy$ .