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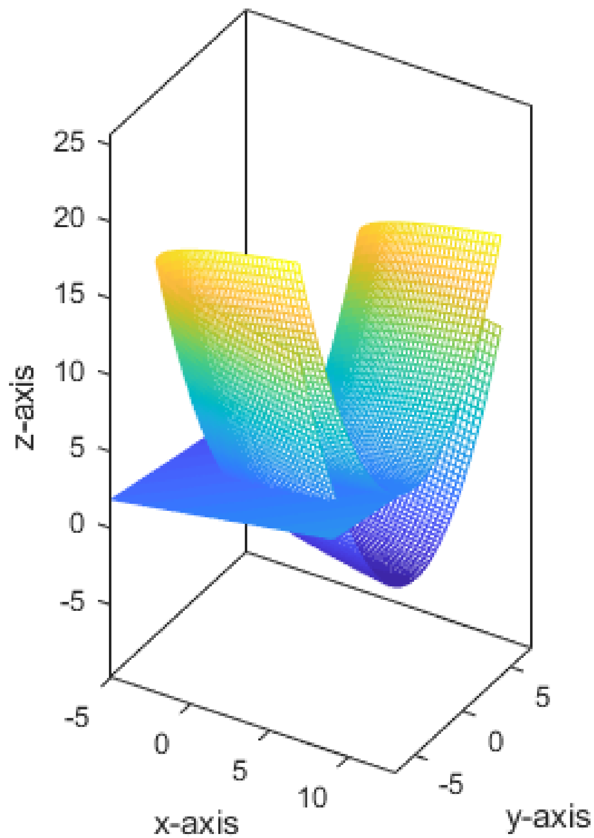
MATB42: Assignment #7 Mar 13th 2018

1. (a) Find an equation of the tangent plane to the surface S defined parametrically by $\mathbf{\Phi}(u,v) = (u^2+v,\ v,\ u+v^2)$ at the point (9,0,3).

$$
\begin{aligned}
\boldsymbol{\phi}_u &= (2u, 0, 1) \\
\boldsymbol{\phi}_v &= (1, 1, 2v) \\
\boldsymbol{\phi}_u \times \boldsymbol{\phi}_v &= (-1, 1-4uv, 2u) \|\boldsymbol{\phi}_u \times \boldsymbol{\phi}_v\| \qquad = \sqrt{1+(1-4uv)^2+4u^u}
\end{aligned}
$$

(b) Use symbolic algebra software to sketch the surface S and its tangent plane from part (a).



2. Use a surface integral to find the area of the triangle in $\mathbb{R}^3$ with vertices (1,1,0), (1,2,1) and (3,3,2).

3. Calculate the surface area of the piece of the cone $x^2+y^2-z^2=0$ which lies outside the cylinder $x^2+y^2=4$.

4. (a) Find the area of the portion of the unit sphere that is cut out by the cone $z=\sqrt{x^2+y^2}$. (cf. page 391, #10)

(b) Find the area of the portion of the cone $z=\sqrt{x^2+y^2}$ that is cut out by the unit sphere.

5. Let $\mathbf{\Phi} : D \subset \mathbb{R}^2 \to \mathbb{R}^3$ be a parametrization of a 2-dim surface S in $\mathbb{R}^3$.

(a) Set

$$E = \|\boldsymbol{\phi}_u\|^2, \qquad F = \boldsymbol{\phi}_u \cdot \boldsymbol{\phi}_v, \qquad G = \|\boldsymbol{\phi}_v\|^2,$$

Show that the surface area of S is

$$A(S) = \iint_D \sqrt{EG - F^2}\, dA$$

(b) What does the formula for $A(S)$ become if the vectors $\boldsymbol{\phi}_u$ and $\boldsymbol{\phi}_v$ are orthogonal?

(c) Use parts (a) and (b) to compute the surface area of a sphere of radius a.
(cf. Marsden & Tromba, page 399, # 23.)

6. For each of the following surfaces S, sketch S (using symbolic software) and evaluate the surface integral $\int_S f\, dS$, where $f(x, y, z) = x$.

   (a) S is that part of the surface $y = 4 - x^2$ between $z = 0$ and $z = 1$, with $y \geq 0$.

   (b) S is the upper half of the unit sphere centered at the origin.

   (c) S is that part of the surface $x = \sin y$ with $0 \leq y \leq \pi$ and $0 \leq z \leq 2$.

7. Find the mass of the metallic surface S given by $z = 1 - \dfrac{x^2 + y^2}{2}$ with $0 \leq x \leq 1$, $0 \leq y \leq 1$, if the mass density at $(x, y, z) \in S$ is given by $m(x, y, z) = xy$.