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MATB42: Assignment #6 Mar 6th 2018

1. Let $\omega = \dfrac{-y}{x^2+y^2}\ dx + \dfrac{x}{x^2+y^2}\ dy$. Calculate $\displaystyle\int_{\boldsymbol{\gamma}} \omega$ where

   (a) $\boldsymbol{\gamma}$ is the boundary of the triangle with vertices (in order) (0,1), (2,3) and (2,1).

   (b) $\boldsymbol{\gamma}$ is the boundary curve of the region $\left\{(x,y) \in \mathbb{R}^2 \middle| \dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{4} \leq 1\right\}$ oriented in a counter clockwise direction

   (c) $\boldsymbol{\gamma}$ is the graph of the polar equation $r = 3 + 2\sin\theta$ oriented in the clockwise direction.

2. Let $\omega = (y^2 + z\ \ \ln 3)\ dx + (2xy + \sin z)\ dy + (y\cos z + (x+1)\ln 3)\ dz$. Determine if $\omega$ is exact. If it is, use the algorithm given in class to find the potential function $g$.

3. Evaluate the double integral $\displaystyle\int_{-1} 1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x^2y^2 dy\ dx$, by first finding an equivalent line integral.

4. Let $R$ be a region in $\mathbb{R}^2$ and let $\boldsymbol{\gamma}$ be a counterclockwise parametrization of $\partial R$. Let $\boldsymbol{F} = (F_1, F-2)$ be a $C^1$ vector field defined throughout $R$ and on $\partial R$ and let $\boldsymbol{n}$ be the outward pointing unit normal ector to $\boldsymbol{\gamma}$. Use Green's theorem to give a double integral over $R$ which is equivalent to $\displaystyle\int_{\boldsymbol{\gamma}} \boldsymbol{F}\cdot\boldsymbol{n}\ ds$.

5. Give a parametrization for each of the following surfaces, use a computer algebra sustem to plot the surface and find a unit vector normal to the surface.

   (a) The piece of the cylinder $y^2 + z^2 = 1$ between $x = -1 and x = 3$.

   (b) The piece of the plane $z = x + y + 5$ which lies over the uni dik $x^2 + y^2 \leq 1$.

   (c) The piece of the sphere $x^2 + y^2 + z^2 = 4$ which lies above the plane $z = 1$.

   (d) The piece of the plane $x + y + z = 1$ which lies above the parallelogram: $0 \leq y - x \leq 1, 0 \leq y + x2 \leq 1$.

6. Let $S$ be the surface given parameterically by $\boldsymbol{\Phi}(u,v) = (u^2, 3v, u^2+v)$ where $(u,v) \in D$, the interior of a triangle with vertices (0,0), (3,0) and (3,3).

   (a) Find the surface area of $S$.

   (b) Find the equation of the tangent plane to $S$ at the point (4,9,7).

7. Suppose the surface $S$ is the graph of a function $f : \mathbb{R}^2 \to \mathbb{R}$. Give a natural parametrization of $S$ (in terms of $f$) and derive the formula $\|\boldsymbol{\phi}_u \times \boldsymbol{\phi}_v\| = \sqrt{1 + \|\text{grad } f\|^2}$