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MATB42: Assignment #6 Mar 6th 2018

1. Let $\omega = \dfrac{-y}{x^2+y^2}\ dx + \dfrac{x}{x^2+y^2}\ dy$. Calculate $\displaystyle\int_{\gamma} \omega$ where
   (a) γ is the boundary of the triangle with vertices (in order) (0,1), (2,3) and (2,1).
   (b) γ is the boundary curve of the region $\left\{ (x,y) \in \mathbb{R}^2 \middle| \dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{4} \le 1 \right\}$ oriented in a counter clockwise direction
   (c) γ is the graph of the polar equation $r = 3 + 2\sin\theta$ oriented in the clockwise direction.
2. Let $\omega = (y^2 + z\ \ln 3)\ dx + (2xy + \sin z)\ dy + (y\cos z + (x+1)\ln 3)\ dz$. Determine if ω is exact. If it is, use the algorithm given in class to find the potential function g.
3. Evaluate the double integral $\displaystyle\int_{-1} 1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x^2y^2 dy\ dx$, by first finding an equivalent line integral.
4. Let R be a region in $\mathbb{R}^2$ and let $\boldsymbol{\gamma}$ be a counterclockwise parametrization of ∂R. Let $\boldsymbol{F} = (F_1, F-2)$ be a C^1 vector field defined throughout R and on ∂R and let $\boldsymbol{n}$ be the outward pointing unit normal ector to $\boldsymbol{\gamma}$. Use Green's theorem to give a double integral over R which is equivalent to $\displaystyle\int_{\gamma} \boldsymbol{F}\cdot\boldsymbol{n}\ ds$.
5. Give a parametrization for each of the following surfaces, use a computer algebra sustem to plot the surface and find a unit vector normal to the surface.
   (a) The piece of the cylinder $y^2 + z^2 = 1$ between $x = -1 and x = 3$.
   (b) The piece of the plane $z = x + y + 5$ which lies over the uni dik $x^2 + y^2 \le 1$.
   (c) The piece of the sphere $x^2 + y^2 + z^2 = 4$ which lies above the plane $z = 1$.
   (d) The piece of the plane $x + y + z = 1$ which lies above the parallelogram: $0 \le y - x \le 1, 0 \le y + x2 \le 1$.