On Economic Scarcity

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1 Introduction

The question of economic scarcity is often left vaguely defined and relatively neglected. In this essay a more concise definition of the problem of scarcity is presented based on set theoretical terms. In doing so we find a novel approach to the foundations of economics, with potentially beneficial uses both for theory and practice.

2 Body

We often find the economic problem to be defined somewhere along the following lines:

> The world's resources are finite. Human needs are infinite.
> Therefore, we will always have scarcity.
> Money is our mechanism to achieve the best allocation of those limited resources.

Let us mark the set of resources R, and the set of human needs H. It seems to be assumed that:

$$|R| < \aleph_0$$

as well that

$$|H| \geq \aleph_0$$

.

We see that the conclusion says that any relation between R and H, will not be "onto" H, rather only "onto" a proper subset $H' \subset H$, which we call:

$$money \subseteq R \times H'$$

.

Definition. An relation between R and H is an *Economy*.

Examples of an *Economy*: Dictatorship, Free Market System, Money-Based Economies, Resource-Based-Economy, Communism, Socialism and more.

Definition. Given two members of H, denoted h_1 and h_2, we say

$$h_1 \beta h_2$$

if both h_1 and h_2 belong to the same person (or group of people).

Proposition. β is an equivalence relation.

Proof. Reflexivity: let $h \in H$. Clearly, $h \beta h$.
Symmetry: let $h_1, h_2 \in H$. It is easy to see that if $h_1 \beta h_2$ then $h_2 \beta h_1$.
Transitivity: Given $h_1, h_2, h_3 \in H$, if h_1 and h_2 belong to the same person, and so do h_2 and h_3, it follows that h_1 and h_3 also belong to the same person, meaning $h_1 \beta h_3$.

From this we see that β divides H to equivalence classes, $[h]_\beta$, which represent the needs of a given person, or group of people, denoted $\hbar$[1].

Coming back to our examples of Economy. Now we can better define the difference between a dictatorship and a democracy.

Definition. We define economic equality, based on the number of $\hbar$ equivalence classes in the codomain of *Economy E*, divided by the number of those in H.

For example,

$$|H/\beta| = 100$$

$$|\{\hbar | \forall h \in \hbar \exists r \in R. < r, h > \in E\}| = 4$$

is an example of 4 an *Economy* where 4 people, have their needs met by the resource allocation chosen, and the other 96 do not.

This measure of inequality helps define dictatorship, as well as egalitarian regimes. Other measure can rely of choosing a specific need (member of H), such as medical care or food, and asking about the cardinality of

$$\{\hbar | food \in \hbar\}$$

compared to

$$\{\hbar | \exists r \in R. < r, food > \in E\}$$

Using the same measures we can asses the effectivity of other economies, such as monetary based democracy.

Question. Is a money-based *Economy* a function? Let us take $r \in R$ such as $r =$ painkillers. r can be used both to satisfy ease in pain denoted h_1 ($\in H$), or sold to satisfy a need for food denoted h_2 ($\in H$). So we see such an *Economy* consists both of $< r, h_1 >$ and $< r, h_2 >$, namely, not a function.

Productivity Traditionally, the productivity of a firm or an economy overall is the efficiency by with the firm's capital and labor are transformed to products.

[1] Original symbols consist of "curly b" instead of β and "curly h" instead of $\hbar$

The more a firm produces from the same labor and capital, it has been said its productivity is higher.

We can look at the set of resources for a given Economy E, and define the way these resources are combined together for production.

Definition. For two members of R, r_1, r_2 we define two different operations. The first operation is "Addition", which is the same thing you do when you put oranges and apples in the fruit bowl. So if r_1 =fork and r_2 = spoon, $r_1 \bigoplus r_2$ would be a new resource: cutlery.

The second operation is combining, and is denoted $\bigotimes .Soforthesame\mathrm{r}_1 =$ fork, and r_2 = spoon, the combination $r_1 \bigoplus r_2$ = a spork (a tool with both characteristics of a spoon and a fork).

Consider resource "Addition", and notice that: $r_1 \bigoplus r_2 = r_2 \bigoplus r_1$. This means we can take the spoon first or the fork first, we still get the same cutlery. We call this "Symmetry of resource addition".

Consider now that instead of adding something to the spoon, we chose to add "nothing" to the spoon. This is obviously only a theoretical idea, but if we try it we can say that adding "nothing" (zero) to a resource, gives us back just that same resource. In denotation: $\forall r \in R.r \bigoplus 0 = r$ We call this the "zero" of r.

The next two attributes of R, along with the definitions above will provide for us a full notion of the Vector Space of resources. If we choose a number (real number) lets say 2, we can observe the following, quite obvious thing, that taking 2 of an apple, or anything, and adding that with 2 of anything else, for example an orange, gives us twice the sum of of a single orange and a single apple. It can be written more visually so (notice the brackets): $(2 * r_1) + (2 * r_2) = 2 * (orange + apple)$ If instead of 2, we choose any number this still is true: $c * (r_1 + r_2) = c * r_1 + c * r_2$

Second attributes, which is again as obvious as it is useful, says: Take any two real numbers $c1, c2$ $(c1 + c2) * r = c1 * r + c2 * r$

To complete the notion, saying that that order in which 3 members of R are added doesn't matter (under the name "associativity"), will mean (again look at the brackets): $r1 \bigoplus (r2 \bigoplus r3) = (r1 \bigoplus r2) \bigoplus r3$

These seemingly "empty" attributes of adding together resources, results in that Resources consist of a Vector Space, and this allows us to better understand the way in which "non-productive" economies and "productive" economies can be defined.

Definition. Vectors mean a list of numbers (short, long or even with "infinite" length). When two vectors are combined using the symbol $\bigotimes, anewvectorisproduced, whichhasinitbothlistsoft$
$2 * wheels, abicycleskeleton.Combiningthemtogether, with \bigoplus cangiveoutabicycle.$

But this is not always the case. We still miss the skills required, the expertise of bike shop, and the degree to which putting the parts together is able to generate bike according which is of quality. This notion is captured by a special function v, defined as such:

$v : P(R) \to R$

v is a valuation function, the ability of some person to take some resources (a subset of R) and give out another resource.

The set of all possible v functions is the possible productivity of all people for a given set of resources.

References