Rules of inference :-

we have Already proven facts which are true.
and using those facts we have to infer new facts.

1. Modus ponen

$$(p \wedge p \rightarrow q) \rightarrow q$$

or

p
$p \rightarrow q$

$\therefore q$

2. Modus Tollen

$$(\sim q \wedge p \rightarrow q) \rightarrow \sim p$$

or

$\sim q$
$p \rightarrow q$

$\therefore \sim p$

$p \wedge q \equiv p$

$p \wedge q \equiv q$

Simplification

Hypothetical syllogism

$p \rightarrow q$

$q \rightarrow r$

$\therefore p \rightarrow r$

Disjunctive syllogism

$\sim p$

$p \vee q$

$\therefore q$

$\sim p \wedge (p \vee q) \rightarrow q$

$(\sim p \wedge p) \vee (\sim p \wedge q)$

$F \vee (\sim p \wedge q)$

$(\sim p \wedge q) \rightarrow q$

$\sim (\sim p \wedge q) \vee q$

$(p \vee \sim q) \vee q$

$p \vee T = \overline{p}$

21/12/2020

Counting Techniques:- (Permutation & Combination

Q. How many passwords can be created of length 8 that contains digits (0-9) and alphabets (a-z A-Z) only?

or How many number of product codes can be generated.

How many IP addresses can be generated of 16 length?

Permutations

{a, b, c}

1. a b c
2. a c b
3. b a c
4. b c a
5. c a b
6. c b a

6 different passwords can be generated if no object is repeated ie a a b is ~~not~~ counted here.

problem in listing down by counting

What happened if there are 5 or more digits a, b, c, d, e?

Permutation problems

counting objects through change of the positions

IN PERMUTATION :: arrangements of objects matters.

password of length = 3 using symbols a, b, c

Repetition is not allowed.

(a, b, c) (b, c)

___ ___ ___

3 choices 2 choices 1 choice

no of password = $3 \times 2 \times 1 = 6$

$= 3!$

Suppose I have three pencils (a, b, c)

cost of any one pencil is 2 Rs

what will be the cost of 3 pencils?

pencil a = 2
pencil b = 2
pencil c = 2

$2+2+2 = 6$

or $3 \times 2 = 6$

Pencils 2Rs

$2 \times 3 = 6$

Pencil Rs

$a \begin{Bmatrix} b & c \\ c & b \end{Bmatrix}$ 2 ways
2 RS

$b \begin{Bmatrix} a & c \\ c & a \end{Bmatrix}$ 2 ways

$c \begin{Bmatrix} a & b \\ b & a \end{Bmatrix}$ 2 ways

$3! = \underline{3 \times 2} \times 1$
$6 \times 1 = 6$

$$2 + 2 + 2 = 6$$

$= 3 \times 2$

1. $a \; b$ — $\underline{c}$ 1 RS 1 way
2. $a \; c$ — $\underline{b}$ 1 way
3. $b \; a$ — $\underline{c}$ 1 way
4. $b \; c$ — $\underline{a}$ 1 way
5. $c \; a$ — $\underline{b}$ 1 way
6. $c \; b$ — $\underline{a}$ 1 way

1 object $\longrightarrow$ 2 ways

3 objects $\longrightarrow$ $3 \times 2 = 6$ ways

objects ways

$3 \times 2 \times 1 = 3!$

obj way = 6 objects $\times$ 1

6×1

objects way = 6 ways.

4 Different objects (a, b, c, d)

4 objects

a [b c d] 3!

b [a c d] 3!

c [_ _ _] 3!

d _ _ _

4!

d [_ _ _] 3!

3!
+3!
+3!
+3!

$3! + 3! + 3! + 3!$

$6 + 6 + 6 + 6 = 24$

$4 \times 6 = 24$

$4 \times 3!$

objects ways

$= 4!$

$$4! = 4 \times \underbrace{3 \times 2 \times 1}$$

$$= 4 \times 3!$$

$$4! = \underset{\text{object}}{4} \left[\underset{\text{objects}}{\_} \left(\underset{\text{ways}}{\_ \quad \_} \right) \right]$$

$$
1.\ a \quad
\begin{array}{ccc}
3 & 2 & 1 \\
b & c & d \\
b & d & c \\
c & b & d \\
c & d & b \\
d & c & b \\
d & b & c
\end{array}
\quad 3!
$$

2\. b. [ ]

3\. c [ ]

4\. d [ ]

Permutation

Technique 1:- If we have a "n" different objects then they can be arranged themselves in $n!$ ways.

Q1. How many passwords of length 8 can be generated using alphabets (a-z) and digits (0-9) only if repetition is not allowed?

(10 digits 0-9)

a-z = 26

26+10 = 36 objects

$\underset{36}{\_}\ \underset{35}{\_}\ \underset{34}{\_}\ \underset{33}{\_}\ \underset{32}{\_}\ \underset{31}{\_}\ \underset{30}{\_}\ \underset{29}{\_}$

choices

$36 \times 35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29$

Q1 ii) How many password of length 8 if they start with 3 Digits digits must followed by alphabets (a – z)

D	D	D	A	A	A	A	A
10	9	8	26	25	24	23	22

$10 \times 9 \times 8 \times 26 \times 25 \times 24 \times 23 \times 22$

= no of passwords.

Q1. 1

iii) Any two specified digits must come together at any position in the password.

for example.

ab12cdef

b12acdef

6712abcd

67abcd12

count two specified digits as 1 object.

Hence total digits (0 − 9) = 10 − 1 = 9 objects

{0, 12, 3, 4, 5, 6, 7, 8, 9}

9 digits objects + 26 alphabets

$= \overset{a-z}{26} + 9 = 35$ objects are to be arrange in 8 positions

$\frac{\quad}{35}$ $\frac{\quad}{34}$ $\frac{\quad}{33}$ $\frac{\quad}{32}$ $\frac{\quad}{31}$ $\frac{\quad}{30}$ $\frac{\quad}{29}$ $\frac{\quad}{28}$

$= {}^{35}P_{8}$ ~~35~~ $\times 34 \times 33 \times 32 \times 31 \times 30 \times 29 \times 28 =$ ~~Ans~~ ${}^{35}P_{8}$

shape 1

= no of passwords in which 1, 2 ~~occur~~

a 2, 1

shape 2

for shape 1 1, 2 there will be

x no of passwords

for shape 2 2, 1 x no of passwords

total no of password = $x + x = 2x$

total password = $(35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29 \times 28) \times 2$

Q2 How many natural numbers of three different digits can be formed using digits (0, 1, 2, 3, 4, 5) that lies b/w 320 — 445 ?

<u>Solution</u>-

H T U

for example 322, ~~322~~ ~~132~~ ~~323~~ 324

~~673~~ ~~4574~~

~~441~~

(0, 1, 2, 3, 4, 5)

320 — 445

H T U

1. Count all those number which Start with 32 and > 320

H	T	U
3 fix	2 fix	3 choices
1 choice	1 Choice	(1, 4, 5)

$1 \times 1 \times 3 = 3$ naturals numbers that start with 32.

2) Count natural number 3 ≥ 2 start with 3

H T U

3 fix

1 choice

2 choices

(4, 5)

(4 choices)

$= 1 \times 2 \times 4$

$= 8$ natural numbers

3 4 0
3 4 1
3 4 2
3 4 5

3 5 0
3 5 1
3 5 2

3 5 4

3) Count all the natural numbers that start with 4.

$$320 < 405 < 445$$

H	T	U
fix 4		
1 choice	4 choices (0, 1, 2, 3)	4 choices

Natural numbers $= 1 \times 4 \times 4$
$= 16$

445

(0, 1, 2, 3)

40: 1, 2, 3, 5

41: 0, 2, 3

42: 0, 1, 3, 5

43: 0, 1, 2

Total natural number $> 320 < x < 445$

$= 3 + 8 + 16 = 27$

$320 < x < 445$

$(0, 1, 2, 3, 4, 5)$

fix

$\underline{3}$ $\underline{2}$ fix ___

1 choice 1 choices 3 choices

$1 \times 1 \times 3 = 3$ natural numbers

$\underline{3}$ $\underline{2}$ $\underline{1}$

3 2 4

3 2 5

3 4 $\underline{0}$

3 4 $\underline{1}$

3 4 2

3 4 5

4 choices

32

~~321~~

~~312~~

320

340

32

$\underline{34}$

35