

Rules of inference:-

we have already proven facts which are true.
and using those facts we have to infer new facts.

1. Modus ponens

$$(P \wedge P \rightarrow Q) \rightarrow Q$$

or

P

$P \rightarrow Q$

$$\therefore Q$$

2. Modus Tollens

$$(\sim Q \wedge P \rightarrow Q) \rightarrow \sim P$$

or

$\sim Q$

$P \rightarrow Q$

$$\therefore \sim P$$

$$P \wedge Q \equiv P$$

$$P \wedge Q \equiv Q$$

Hypothetical syllogism

$$P \rightarrow Q$$

$$Q \rightarrow R$$

$$\therefore P \rightarrow R$$

Simplification

Disjunctive Syllogism

$$\frac{\sim P \quad P \vee Q}{\therefore Q}$$

$$\sim P \wedge (P \vee Q) \rightarrow Q$$

$$(\sim P \wedge P) \vee (\sim P \wedge Q)$$

$$F \vee (\sim P \wedge Q)$$

$$(\sim P \wedge Q) \rightarrow Q$$

$$\sim (\sim P \wedge Q) \vee Q$$

$$(P \vee \sim Q) \vee Q$$

$$P \vee T = P$$

21/12/2020

Counting Techniques:- (Permutation & Combination)

- Q. How many passwords can be created of length 8 that contains digits (0-9) and alphabets (a-zA-Z) only?
- or How many number & can be generated product codes
- How many IP addresses can be generated of 46 length?

Permutation 5

{a, b, c}

1. a b c

2. a c b

3. b a c

4. b c a

5. c a b

6. c b a

6 different passwords can
be generated if no object
is repeated i.e. a a a
is ~~not~~ counted here.

Counting by
problem in listing down

what happened if
there are 5 or more digits
a, b, c, d, e?

Permutation problems

Counting objects through change of the positions
IN PERMUTATION \therefore arrangements of objects
matters.

password of length = 3 using symbols a, b, c
repetition is not allowed.

(a, b, c) (b, c)

3 choices 2 choices 1 choice

$$\begin{aligned}\text{no of password} &= 3 \times 2 \times 1 = 6 \\ &= 3!\end{aligned}$$

Suppose I have three pencils (a, b, c)

cost of ^{any} one pencil is 2 Rs

what will be the cost of 3 pencils?

$$\text{pencil a} = 2$$

$$\text{pencil b} = 2$$

$$\text{pencil c} = 2$$

$$2 + 2 + 2 = 6$$

$$\text{or } 3 \times 2 = 6$$

Pencils

2 Rs

$$2 \times 3 = 6$$

← Pencil

↘ Rs

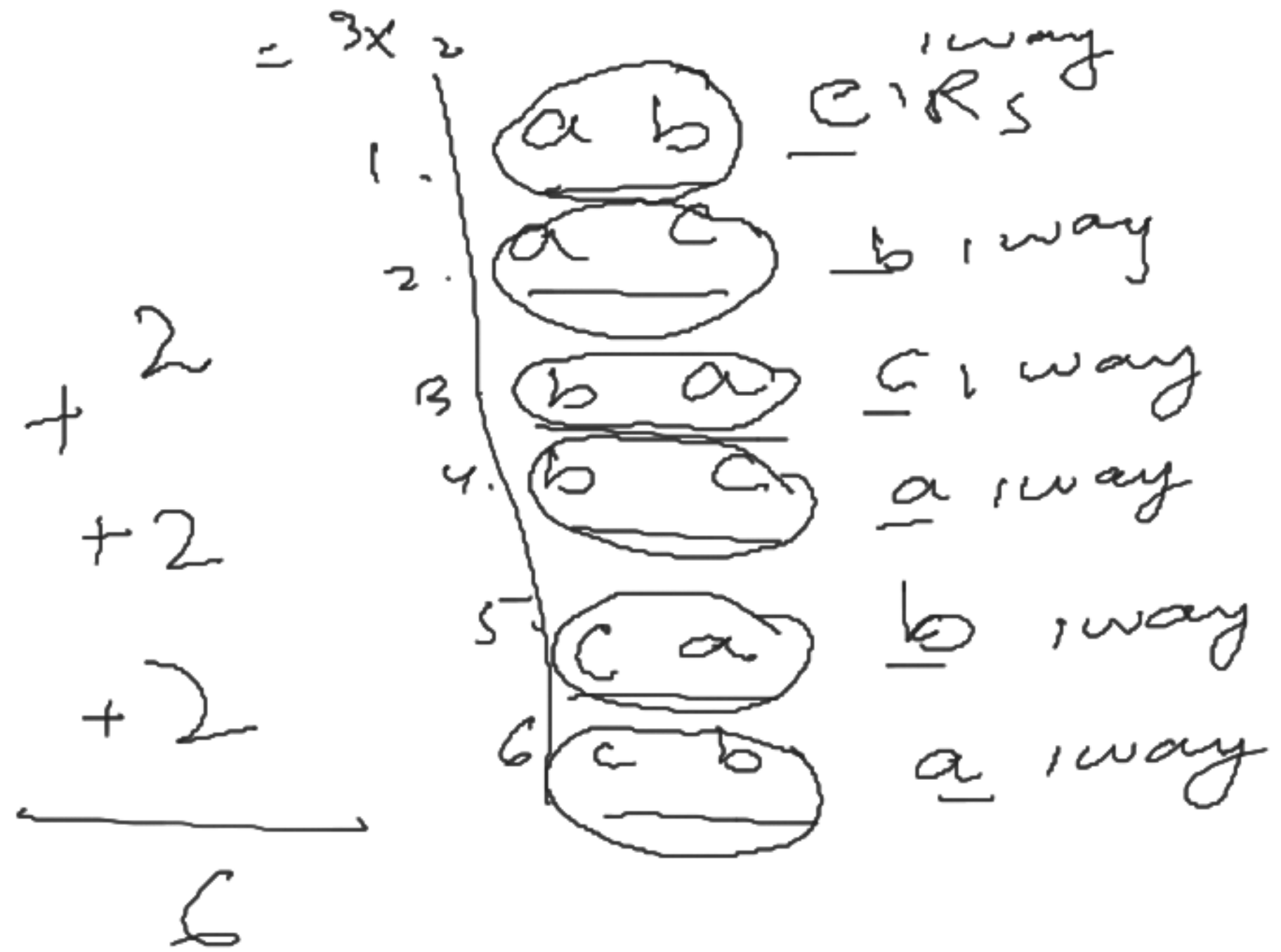
a { b c } 2 ways
 { c b } 2 ans

b { a c } 2 ways
 { c a }

c { a b } 2 ways
 { b a }

$$3! = \underline{3 \times 2 \times 1}$$

$$6 \times 1 = 6$$



1 object \rightarrow 2 ways
 3 objects $\rightarrow 3 \times 2 = 6$ ways
 objects \swarrow ways

$$3 \times 2 \times 1 = 3!$$

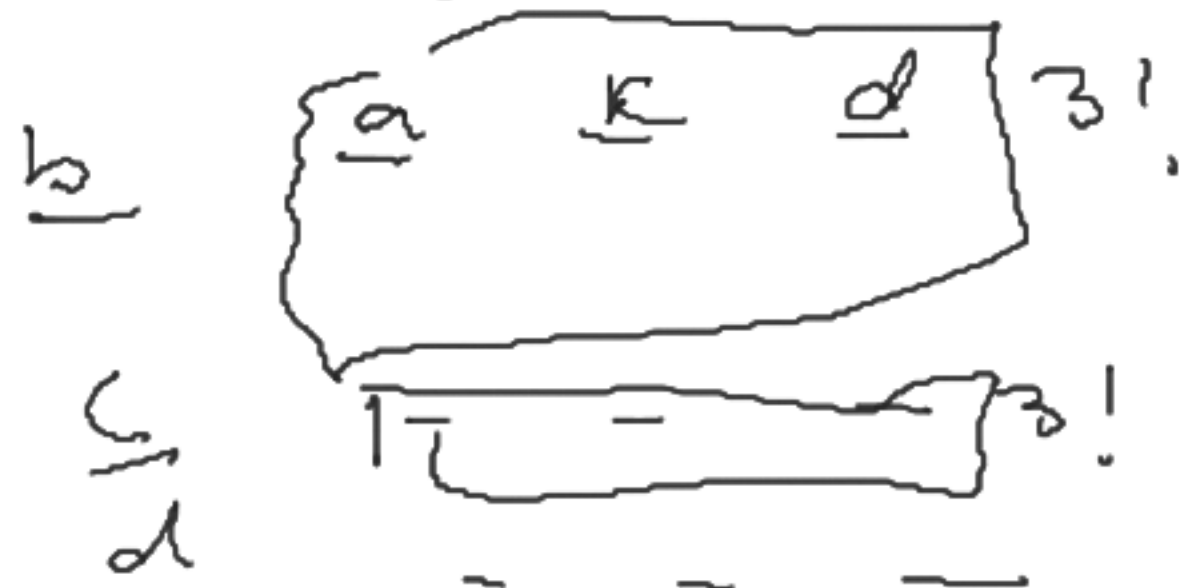
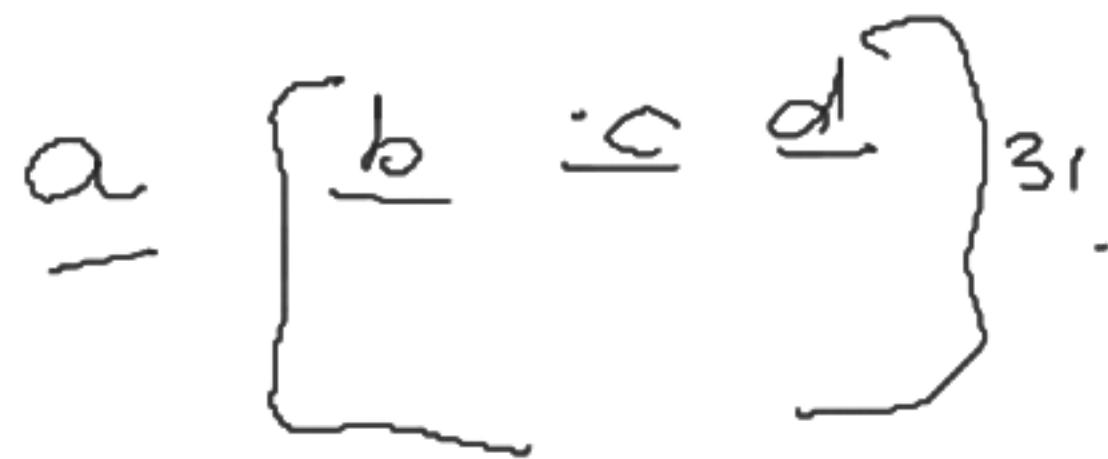
obj way = 6 objects \times 1

$$6 \times 1$$

objects way = 6 ways.

4 Different objects (a, b, c, d)

4 objects



3!



$$\begin{array}{r}
 3! \\
 + 3! \\
 + 3! \\
 + 3! \\
 \hline
 4 \times 3! \\
 \text{ways} \\
 = 4!
 \end{array}$$

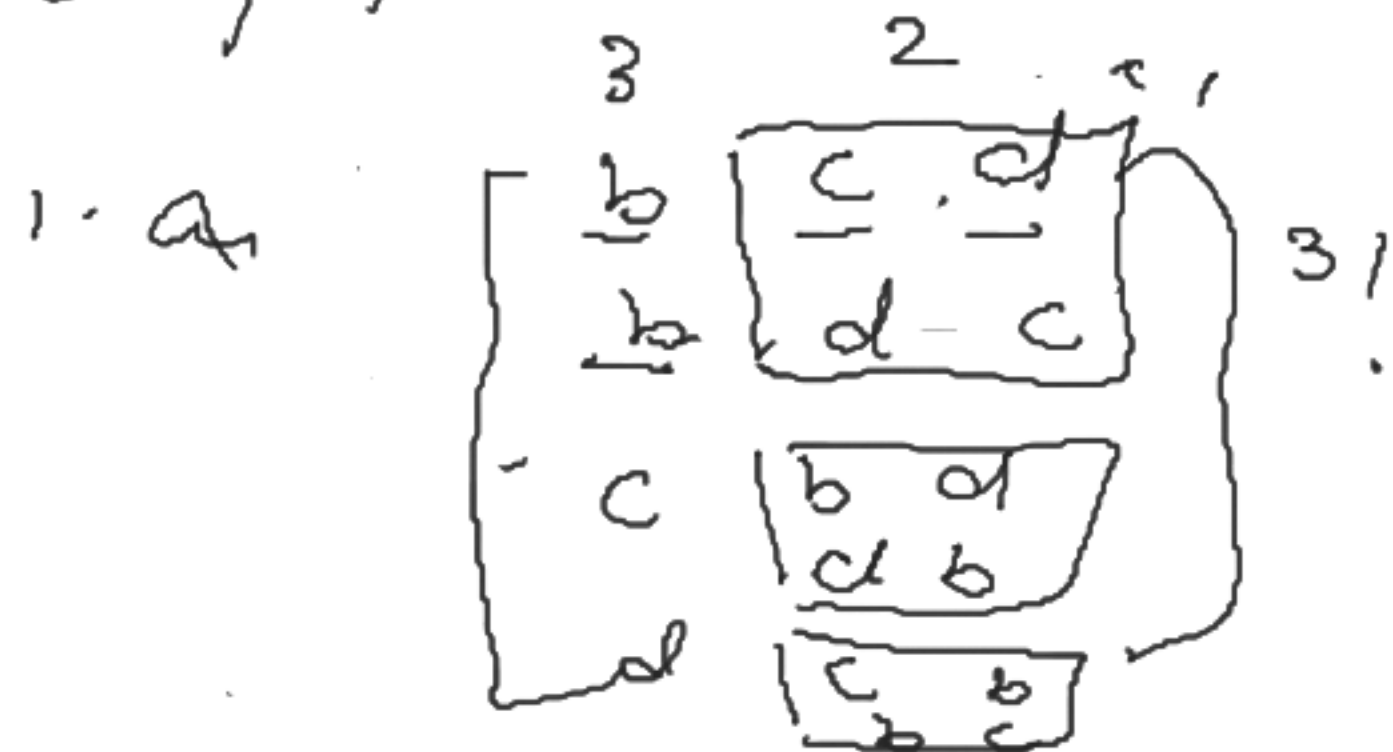
$$\begin{array}{r}
 3! + 3! + 3! + 3! \\
 6 + 6 + 6 + 6 = 24 \\
 4! \times 6 = 24
 \end{array}$$

$$4! = 4 \times 3 \times 2 \times 1$$

$$= 4 \times 3!$$

$$4! = 4 \left[\begin{array}{c} \text{ways} \\ \text{objects} \end{array} \right]$$

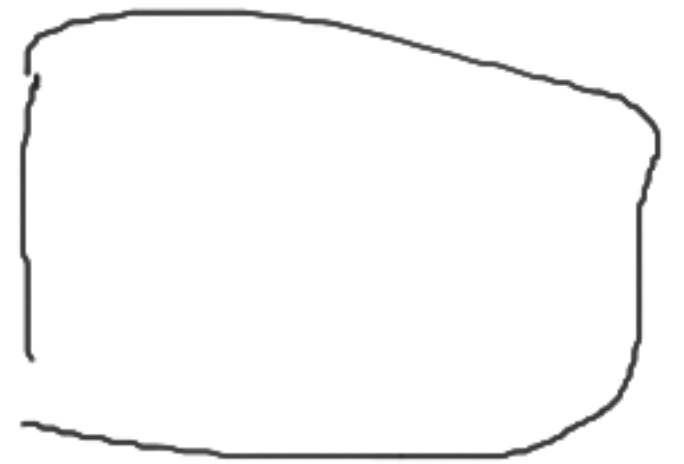
object



2. b

$$[\quad]$$

3. c



4. d



Permutation

Technique 1:- If we have " n " different objects then they can be arranged themselves in $n!$ ways.

Q1. How many passwords of length 8 can be generated using alphabets $(a-z)$ and digits only if repetition is not allowed?

No digits
 $0-9$

$a-z = 26$

$26 + 10$
 $= 36$ objects

$\begin{array}{cccccccc} _ & _ & _ & _ & _ & _ & _ & _ \\ 36 & 35 & 34 & 33 & 32 & 31 & 30 & 29 \end{array}$
choices

$36 \times 35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29$

Q1 ii) How many password of length 8 if they start with 3 ^{digits} ~~digits~~ must followed by alphabets (a-z)

<u>D</u>	<u>D</u>	<u>D</u>	<u>A</u>	<u>A</u>	<u>A</u>	<u>A</u>	<u>A</u>
10	9	8	26	25	24	23	22

$$10 \times 9 \times 8 \times 26 \times 25 \times 24 \times 23 \times 22$$

= no. of passwords.

Q1. 1
iii) Any two specified digits must come together at any position in the password.

for example.

a b (12) c d e f

b (12) a c d e f

67 (12) a b c d

67 a b c d (12)

count two specified digits as 1 object.

Hence total digits $(0-9) = 10 - 1 = 9$ objects

$\{0, \textcircled{12}, 3, 4, 5, 6, 7, 8, 9\}$

9 digits objects + 26 alphabets

$= 26 + 9 = 35$ objects are to be

arrange in 8 positions

$$= \overline{35} \times \overline{34} \times \overline{33} \times \overline{32} \times \overline{31} \times \overline{30} \times \overline{29} \times \overline{28} = \text{not}$$

= no of passwords in which ^{shape 1} 1, 2 ~~can~~

a 2, 1
shape 2

for shape ① 1, 2 there will be

* no of passwords

for shape ② 2, 1 * no of passwords

total no of password = * + * = 2 *

total password = $(35 \times 34 \times 33 \times 32 \times 31 \times 30 \times 29 \times 28) \times 2$

Q2 How many natural numbers of three different digits can be formed using digits (0, 1, 2, 3, 4, 5) that lies b/w 320 — 445?

Solution.

H T U

for example 322, ~~322~~ ~~132~~ ~~323~~ 324
~~613~~ ~~454~~
441

(0, 1, 2, 3, 4, 5)

320 — 445

H T U

— — —

1. Count all those number which
start with 32 and > 320

$\begin{array}{c} H \\ \text{3 fix} \\ \text{1 choice} \end{array}$ $\begin{array}{c} T \\ \text{2 fix} \\ \text{1 choice} \end{array}$ $\begin{array}{c} U \\ \text{3 choices} \\ (1, 4, 5) \end{array}$

$1 \times 1 \times 3 = 3$ natural numbers that start
with 32.

2) Count natural number ^{start with 3} $3 \geq 2$

$$\begin{array}{c}
 H \\
 \underline{3 \text{ fix}} \\
 1 \text{ choice}
 \end{array}
 \begin{array}{c}
 T \\
 \underline{2 \text{ choices}} \\
 (4, 5)
 \end{array}
 \begin{array}{c}
 U \\
 \underline{(4 \text{ choices})}
 \end{array}
 = 1 \times 2 \times 4 = 8 \text{ natural numbers}$$

3 4 0
 3 4 1
 3 4 2
 3 4 5

3 5 0
 3 5 1
 3 5 2

3 5 4

3) Count all the natural numbers that start with 4.

$$320 < \underline{405} < 445$$

H T U
fix 4 4 choices 4 choices
1 choice (0, 1, 2, 3) 4 choices

$$\text{natural numbers} = 1 \times 4 \times 4 \\ = 16$$

445

(0, 1, 2, 3)

40
1
2
3

41
0
1
2
3

42
0
1
3

43
0
1
2

Total natural number $> 320 < x < 445$

$$= 3 + 8 + 16 = 27$$

$$320 < x < 445$$

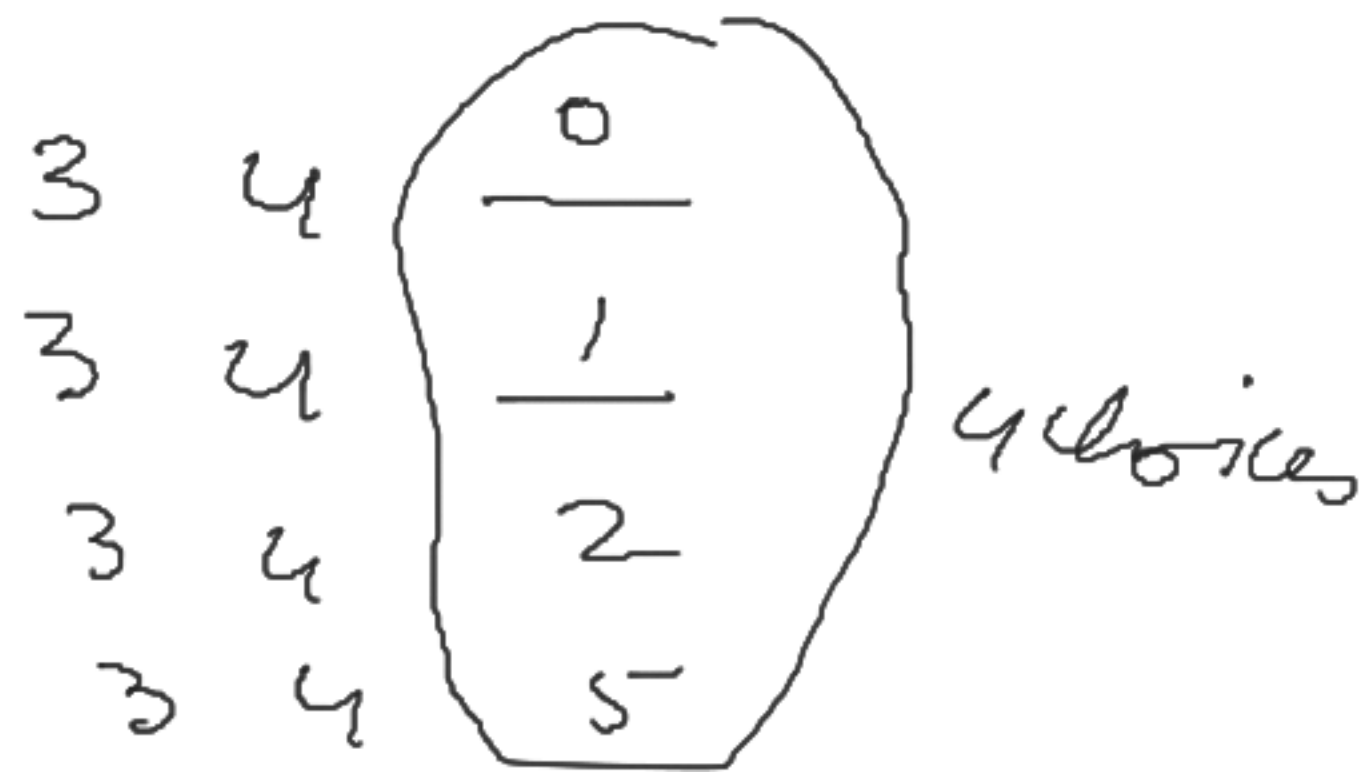
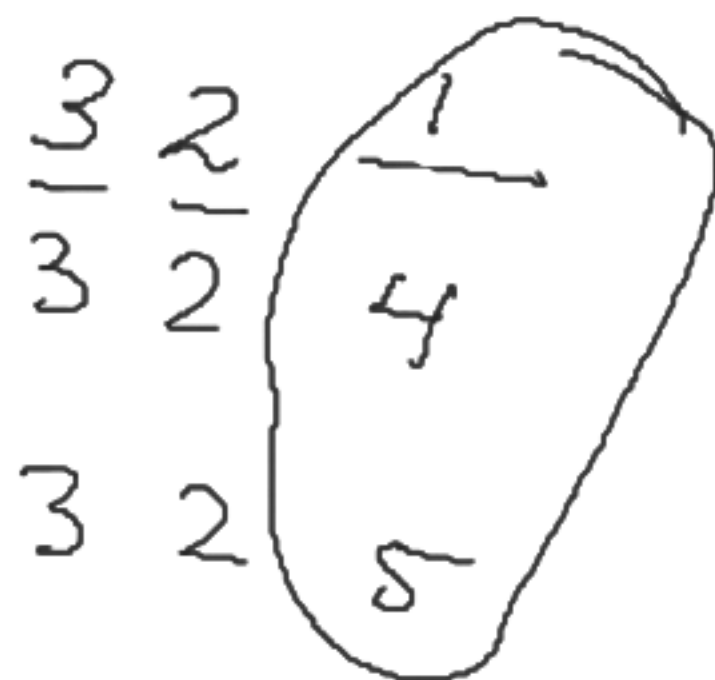
(0, 1, 2, 3, 4, 5)

fix

3 2 fix

1 choice 1 choice 3 choices

1X1X3 = 3 natural numbers



32

~~32~~

~~32~~

320
X

340

32

3, 4

35