Theory Of Automata & Formal Languages

LECTURE 3

Recursive Definition & Regular Expressions

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Recursive Definition

- The following three steps are used in recursive definition

1. Some basic words are specified in the language.
2. Rules for constructing more words are defined in the language.
3. No strings except those constructed in above, are allowed to be in the language.

TASK

- Defining the language PALINDROME, defined over $\Sigma = \{a,b\}$

Step 1:

Λ, a and b are in PALINDROME.

Step 2:

If x is palindrome, then axa and bxb will also be PALINDROME.

Step 3:

No strings except those constructed in above, are allowed to be in PALINDROME.

Practice Questions

Step 1:

aa and bb are in L

Step 2:

s(aa)s and s(bb)s are also in L, where s belongs to Σ^*

Step 3:

No strings except those constructed in above, are allowed to be in L

A Language L, defined over {a, b}, containing strings having at least two consecutive a's or two consecutive b's.

Practice Questions

Step 1:

aa is in L

Step 2:

s(aa)s is also in L, where s belongs to b^*

Step 3:

No strings except those constructed in above, are allowed to be in L

A Language L, defined over {a, b}, containing strings having exactly two consecutive a's and any number of b's.

Practice Question

- What is the Recursive Definition of Regular Expressions?

BUT WHAT ARE REGULAR EXPRESSIONS?

Regular Expressions

- a^* generates Λ, a, aa, aaa, …
  - So $L_1 = \{\Lambda, a, aa, aaa, \ldots\}$

- a^+ generates a, aa, aaa, aaaa, …
  - So $L_2 = \{a, aa, aaa, aaaa, \ldots\}$

- NOTE: a^* and a^+ are called the regular expressions (RE) for L_1 and L_2 respectively.

  Note: L_2 can also be generated by aa^* and a^*a.

Regular Expressions

- Compound Regular Expressions can be expressed as:

- R_1+R_2 for OR/UNION Operation

- $R_1.R_2$ for CONCATENATION Operation

- $R_1{}^*$ for KLEENE STAR Operation

- $R_1{}^+$ for PLUS Operation

Practice Question

- What is the Recursive Definition of Regular Expressions?

<u>Step 1:</u> Every letter of Σ including Λ is a regular expression.

<u>Step 2:</u> If r_1 and r2 are regular expressions then

1. (r_1)
2. $r_1\ r_2$
3. $r_1 + r_2$ and
4. r_1^*
5. r_1^+

are also regular expressions.

<u>Step 3:</u> Nothing else is a regular expression.

Types of Definitions

- Set-builder Definition
- Definition of Formal Languages
- Tabular Definition
- Descriptive Definition
- Recursive Definition
- Definition using Regular Expressions

Definition by Regular Expressions

- Consider the language $L=\{\Lambda, x, xx, xxx,\ldots\}$ of strings, defined over $\Sigma = \{x\}$.
  - We can write this language as the Kleene star closure of alphabet Σ or $L=\Sigma^*=\{x\}^*$
  - This language can also be expressed by the regular expression x^*.

Definition by Regular Expressions

- Similarly the language $L=\{x, xx, xxx,\ldots\}$, defined over $\Sigma = \{x\}$, can be expressed by the regular expression x^+.

Examples

- Consider another language L, consisting of all possible strings, defined over $\Sigma = \{a, b\}$.

- This language can also be expressed by the regular expression

$$(a + b)^*$$

Practice Question

- A language L, of strings having exactly double a, defined over Σ = {a, b}.

$$b^*aa\,b^*$$

Practice Question

- A language L, of strings having exactly two a's, defined over $\Sigma = \{a, b\}$.

$$b^*ab^*ab^*$$

Practice Question

- A language L, of strings of even length, defined over $\Sigma = \{a, b\}$.

$$((a+b)(a+b))^*$$

Practice Question

- A language L, of strings of odd length, defined over $\Sigma = \{a, b\}$.

$$(a+b)((a+b)(a+b))^*$$

or

$$((a+b)(a+b))^*(a+b)$$

TASK

- Consider the language, defined over $\Sigma=\{a, b\}$ of words ending in "b".

- Consider the language, defined over $\Sigma=\{a, b\}$ of words not ending in "a".

Equivalent Regular Expressions

- Two regular expressions are said to be equivalent if they generate the same language.

- Example:
  Consider the following regular expressions
  $r_1 = (a + b)^* (aa + bb)$
  $r_2 = (a + b)^* aa + (a + b)^* bb$ then
  both regular expressions define the language of strings ending in aa or bb.

Regular Languages

- Any Language associated with Regular Expression is called as a Regular Language.

- In other words, the language generated by any regular expression is called a Regular Language.

Regular Languages

- If r_1, r_2 are regular expressions, corresponding to the languages L_1 and L_2 then the languages generated by $r_1 + r_2$, r_1r_2, r_2r_1, r_1^* and r_2^* are also regular languages.

1. $r_1 + r_2$, is the language $L_1 + L_2$ or $L_1 \cup L_2$

2. r_1r_2 , is the language L_1L_2, of strings obtained by prefixing every string of L_1 with every string of L_2

3. r_1^*, is the language L_1^*, of strings obtained by concatenating the strings of L, including the null string.

Remarks

- All finite languages are regular languages. A language contains even thousand words, its RE may be expressed, placing ‘ + ’ between all the words.

Thanks

- End of Lecture
- Q/A