

ALTERNATE STANDARD NORMAL DISTRIBUTION TABLE: AREA FROM 0 TO z

Table 4—Areas of a Standard Normal Distribution (Appendix I)—provides the areas under the standard normal distribution that are to the left of a specified z value. Such areas are equivalent to the cumulative probability $P(z < z_0)$, where z is a standard normal variable and z_0 is a fixed value. Section 7.2 shows how to use Table 4.

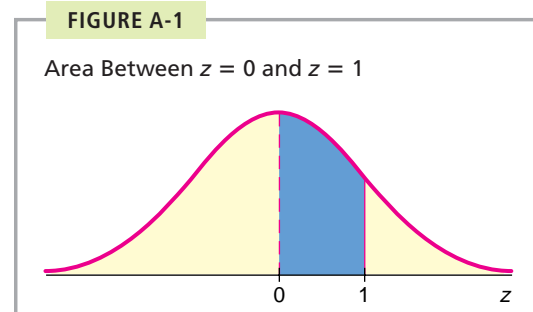
An alternate standard normal distribution table provides areas under the standard normal distribution that are between 0 and a specified positive z value. Table A on the preceding page is such a table.

EXAMPLE 1 Area between 0 and z

Find the area under the standard normal curve between $z = 0$ and $z = 1$. This area is shown in Figure A-1.

SOLUTION: In the upper-left corner of the table we see the letter z . The column under z gives us the units value and tenths for z . The other column headings indicate the hundredths value of z . The table entries give the areas under the normal curve from the mean $z = 0$ to a specified value of z . To find the area from $z = 0$ to $z = 1$, we observe that if $z = 1$, then the units value of z is 1 and the tenths value is 0. So we look in the column labeled z for 1.0. The area from $z = 0$ to $z = 1$ is given in the corresponding row of the column with heading 0.00 because $z = 1$ is the same as $z = 1.00$. The area we read from the table for $z = 1.00$ is 0.3413. ♦

Table A gives areas under the normal curve for regions *beginning* at $z = 0$ and extending to a specified positive z value. However, because the normal curve is symmetrical, we also can use the table directly to find areas beginning with a negative z value and extending to $z = 0$. Example 2 shows this process.



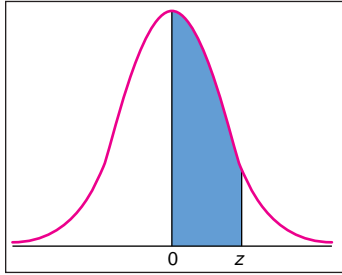


TABLE A Areas of a Standard Normal Distribution (Alternate Version of Appendix I Table 4)

The table entries represent the area under the standard normal curve from 0 to the specified value of z .										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999

For values of z greater than or equal to 3.70, use 0.4999 to approximate the shaded area under the standard normal curve.

EXAMPLE 2

*Area between 0 and
negative z value*

Find the area under the standard normal curve from $z = -2.34$ to 0.

SOLUTION: The area from $z = -2.34$ to 0 is the same as the area from $z = 0$ to 2.34. (See Figure A-2.) By Table A, the area from 0 to 2.34 is 0.4904. Therefore, the area from $z = -2.34$ to 0 is also 0.4904. ♦

To find areas other than those between a given z value and $z = 0$, we use Table A together with addition or subtraction of areas we find in Table A. Figure A-3 on the next page shows how to combine areas. As you study the figure, notice that

1. For areas extending from one side of the mean $z = 0$ to the other side, we *add* areas found in Table A.
2. For areas completely on one side of the mean $z = 0$ (but not bordering $z = 0$), we *subtract* areas found in Table A.
3. The area extending from $z = 0$ and including the entire right half of the graph is 0.5000. Likewise, the area extending from $z = 0$ and including the entire left half of the graph is 0.5000.

FIGURE A-2

Area from $z = -2.34$ to 0 Equals Area from $z = 0$ to 2.34

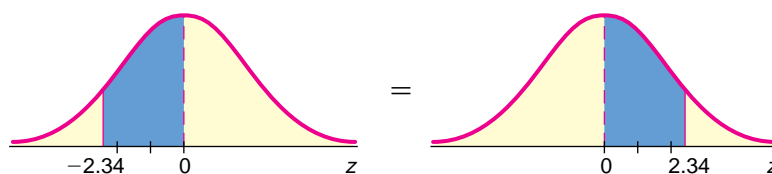
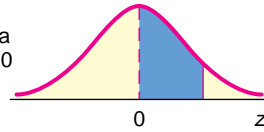


FIGURE A-3

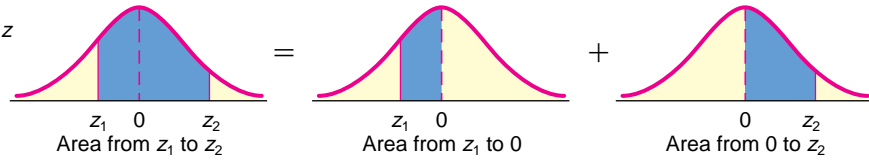
Patterns for Finding Areas Under the Standard Normal Curve

(a) Area between a given z value and 0

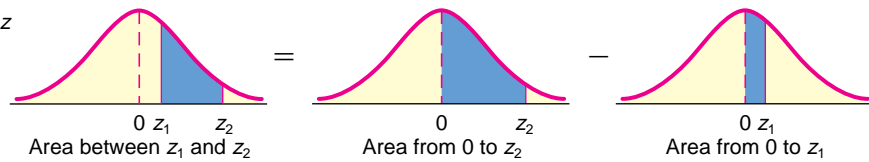


Use Table A in Appendix I directly.

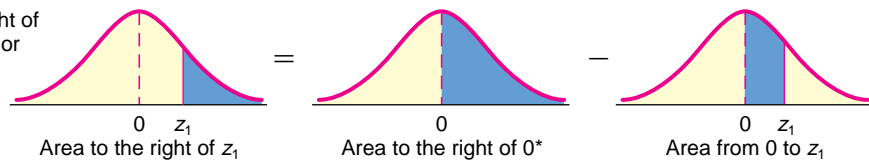
(b) Area between z values on either side of 0



(c) Area between z values on same side of 0

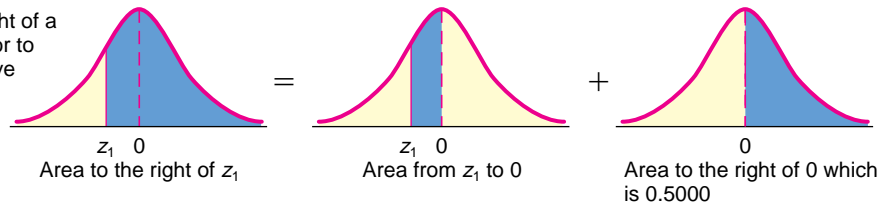


(d) Area to the right of a positive z value or to the left of a negative z value



*This area = 0.5000 since the area under the entire curve is 1 and the area to the right of 0 is half the area under the entire curve.

(e) Area to the right of a negative z value or to the left of a positive z value

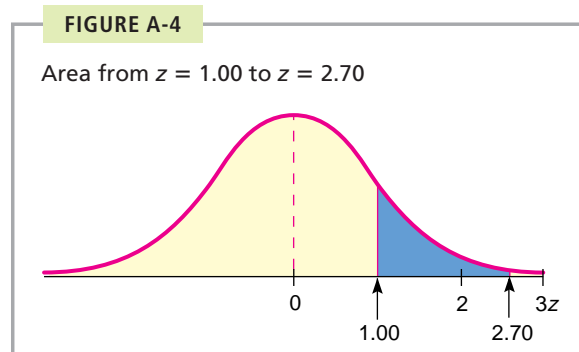


EXAMPLE 3
Area between two
positive z values

Find the area under the standard normal curve in Figure A-4 from $z = 1.00$ to $z = 2.70$.

SOLUTION: The area we are trying to find lies entirely to the right of $z = 0$ and does not border $z = 0$. Therefore, we need to subtract component areas.

$$\begin{aligned} \left(\begin{array}{c} \text{Area from} \\ 1.00 \text{ to } 2.70 \end{array} \right) &= \left(\begin{array}{c} \text{Area from} \\ 0 \text{ to } 2.70 \end{array} \right) - \left(\begin{array}{c} \text{Area from} \\ 0 \text{ to } 1.00 \end{array} \right) \\ &= 0.4965 - 0.3413 = 0.1552 \end{aligned}$$



EXAMPLE 4
Area to the left of a
negative z value

Find the area under the standard normal curve to the left of $z = -0.94$.

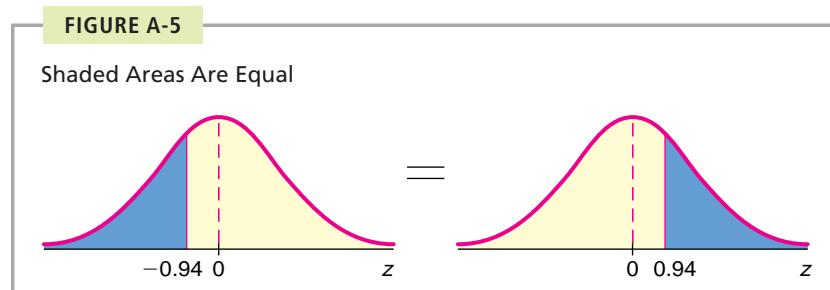
SOLUTION: We sketch the area and notice that the area to the left of -0.94 is the same as the area to the right of 0.94 (see Figure A-5).

To find the area to the right of 0.94 , we observe

$$\begin{aligned} \left(\begin{array}{c} \text{Area to the} \\ \text{right of } 0.94 \end{array} \right) &= \left(\begin{array}{c} \text{Area to the} \\ \text{right of } 0 \end{array} \right) - \left(\begin{array}{c} \text{Area from} \\ 0 \text{ to } 0.94 \end{array} \right) \\ &= 0.5000 - 0.3264 = 0.1736 \end{aligned}$$



We have practiced the skill of finding areas under the standard normal curve for various intervals along the z axis. This skill is important, since *the probability that z lies in an interval is given by the area under the standard normal curve above that interval.*



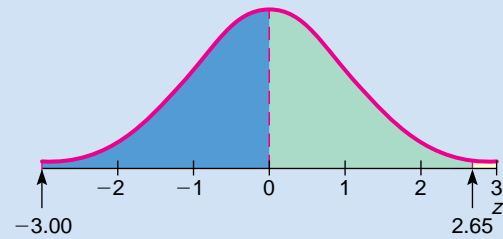
GUIDED EXERCISE 1

Area between a negative z value and a positive z value

Find the area from $z = -3.00$ to $z = 2.65$. First, we *draw the picture* (see Figure A-6) and observe the location of the requested area. Next, we find component areas from Table A and combine them appropriately.

- Look at Figure A-6. Should we add or subtract component areas?
- Find the area under the standard normal curve between $z = 0$ and $z = 2.65$.
- Find the area under the standard normal curve between $z = -3.00$ and $z = 0$.
- Use parts (b) and (c) to find the area under the standard normal curve between $z = -3.00$ and $z = 2.65$.

FIGURE A-6 Area Between $z = -3.00$ and $z = 2.65$



➡ Since the area extends from the left side of $z = 0$ to the right side, we add the component areas.

➡ We look under the z column of Table A until we find 2.6; then we stay in this row and move to the right until we are in the column headed by 0.05. The area from $z = 0$ to $z = 2.65$ is given by the entry 0.4960.

➡ Since the area from $z = -3.00$ to $z = 0$ is the same as that from $z = 0$ to $z = 3.00$, we look down the z column until we find 3.0. Then we move to the right in this row until we are in the column headed by 0.00. This entry is 0.4987, which is the area from $z = -3.00$ to $z = 0$.

$$\begin{aligned}
 \left(\begin{array}{c} \text{Area from} \\ -3.00 \text{ to } 2.65 \end{array} \right) &= \left(\begin{array}{c} \text{Area from} \\ -3.00 \text{ to } 0 \end{array} \right) + \left(\begin{array}{c} \text{Area from} \\ 0 \text{ to } 2.65 \end{array} \right) \\
 &\quad \downarrow \qquad \qquad \downarrow \\
 &= 0.4987 + 0.4960
 \end{aligned}$$

The desired area is 0.9947.