Reflective Number Theory: Structural Verification within Classical Mathematics

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1 Introduction

This document presents a rigorous analysis of **Reflective Number Theory** ($\mathbb{Z}_R$) fully confined within classical mathematics. No new axioms or external frameworks are introduced. We revisit historical prime definitions, restoring 1 as a prime, and demonstrate that the classical Riemann Hypothesis mechanism cannot be constructed under this historically faithful system.

2 Classical Foundations

Definition 2.1 (Prime Number, Classical). *An integer $p > 0$ is prime if it is divisible only by 1 and itself. Historically, 1 satisfies this condition and must be included. Exclusion of 1 is a later convention, not a requirement of mathematics itself.*

Remark 2.1. *All subsequent reasoning adheres strictly to classical arithmetic and factorization principles.*

3 Reflective Mapping and Structural Implications

Definition 3.1 (Reflection Mapping). *For any integer $a \in \mathbb{Z}$ and $x \in \mathbb{Z}$:*

$$R_a(x) = 2a - x.$$

This is a deterministic algebraic mapping requiring no additional assumptions.

Lemma 3.1 (Fixed Point Lemma)**.**

$$R_1(1) = 1 \quad \textit{and is the unique fixed point in } \mathbb{Z}^* = \mathbb{Z} \setminus \{0\}.$$

All other integers transform predictably under R_1, with $R_1(2) = 0 \notin \mathbb{Z}^*$.

4 Reflective Primes and Factorization

Definition 4.1 (Reflective Prime Set)**.**

$\mathbb{P}_R = \mathbb{Z}^* \setminus \{2\},$ *including* $1,$ *where each* $p \in \mathbb{P}_R$ *is divisible only by* 1 *and* p*.*

Remark 4.1. *Including* 1 *restores the historical and classical completeness of prime factorization. This does not violate any classical principle; it corrects a post-hoc convention.*

Corollary 4.1 (Factorization Consequence)**.** *For integers such as* 6*:*

$$6 = 2 \cdot 3 = 1 \cdot 2 \cdot 3 = 1^2 \cdot 2 \cdot 3 = \dots$$

This multiplicity is fully mechanical and reproducible, not philosophical.

5 Euler Product and RH Implications

Theorem 5.1 (Structural Failure of Classical Euler Product)**.** *The classical Euler product over* $\mathbb{P}_{classical}$*:*

$$\zeta(s) = \prod_{p \in \mathbb{P}_{classical}} \frac{1}{1 - p^{-s}}$$

cannot be mechanically constructed under $\mathbb{P}_R$ *because:*

1. *Inclusion of* 1 *introduces a singularity:* $\frac{1}{1-1^{-s}} = \frac{1}{0}$.
2. *Exclusion of* 2 *prevents representation of all integers.*

Corollary 5.1 (Vacuity of RH Mechanism)**.** *Under* $\mathbb{P}_R$*, the classical analytic continuation required to define non-trivial zeros of* $\zeta(s)$ *does not exist. Therefore, RH is structurally inapplicable, not as a new mathematics, but as a direct consequence of historically faithful primes.*

6 Reproducible Verification

- Symbolic computation with `Sympy`, `Sage`, or `mpmath` confirms factorization multiplicity and incomplete series reconstruction.
- All results are deterministic, reproducible, and fully classical.

7 Conclusion

By restoring 1 as prime and analyzing consequences mechanically:

1. No new axioms or frameworks are introduced.
2. All reasoning is strictly classical and verifiable.
3. RH mechanism is structurally inapplicable under historically accurate primes.

This document constitutes a fully verifiable, classical, and irreproachable demonstration.