Basic probability

Probability is a branch of mathematics associated with the analysis of random phenomena. In Computer Science, probability can arise in several ways: as part of the mathematics involved in analyzing a random process or as part of a computational approach to solving a particular problem. For this class, our main application of probability will be in analyzing and building programs associated with simple games. However, don't view this application as limiting. Probabilistic methods arise in applications like scientific computation, cryptography, and robotics.

Since many of you may have had limited exposure to probability, we review some basic terminology associated with probability theory to begin. A *trial* (or an experiment) is any procedure that can be infinitely repeated and has a well-defined set of possible *outcomes*, known as the *sample space*. If the sample space is finite, each outcome can be assigned a number between zero and one that corresponds to the likelihood of that particular outcome occurring. This number is the *probability* associated with the outcome. Since every trial always results in exactly one outcome from the sample space, the sum of the probabilities associated with the outcomes is always exactly one.

A simple example of a trial is a single roll of a fair six-sided die. The outcomes of this trial are the values on six sides of the die $\{1, 2, 3, 4, 5, 6\}$. Since the die is fair, the probabilities associated with these six outcomes are all equal so the probability of each outcome is exactly $\frac{1}{6}$. More generally, the probabilities associated with outcomes of a trial are said to be *uniformly distributed* if these probabilities all have equal value.

An *event* is a set of outcomes of a trial (a subset of the sample space). For a single six-sided die, a simple example of an event would be that the resulting roll is even. The probability of an event is the sum of the probabilities associated with its set of outcomes. For example, rolling an even number with a six-sided die corresponds to an event with three outcomes $\{2, 4, 6\}$, each with probability $\frac{1}{6}$. Therefore, the probability of that event is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$.