HW2

Exercise 1.2

6

6.You can upgrade your operating system **if and only if** you have a 32-bit processor running at 1GHz or faster, at least 1GB of memory, and 16GB of available hard disk space, or a 64-bit processor running at 2GHz or faster, at least 2GB of memory, and at least 32GB of available hard disk space. Express your answer using the following propositions:

u: "You can upgrade your operating system"
b_{32}: "You have a 32-bit processor"
b_{64}: "You have a 64-bit processor"
g_1: "Your processor runs at 1GHz or faster"
g_2: "Your processor runs at 2GHz or faster"
r_1: "Your processor has at least 1GB of memory"
r_2: "Your processor has at least 2GB of memory"
h_{16}: "You have at least 16GB of available hard disk space"
h_{32}: "You have at least 32GB of available hard disk space"

answer

$(b_{32} \wedge g_1 \wedge r_1 \wedge h_{16}) \vee (b_{64} \wedge g_2 \wedge r_2 \wedge h_{32}) \iff u$

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8.Express these system specifications using the proposi-
tions
p: "The user enters a valid password,"
q: "Access
is granted,"
r: "The user has paid the subscription fee"
and logical connectives (including negations).

a) "The user has paid the subscription fee, but does not enter a valid password."
b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
c) "Access is denied if the user has not paid the subscription fee."
d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

answer

- a) $r \land \neg p$
- b) $(r \land p) \implies q$
- c) $\neg r \implies \neg q$
- d) $(r \land \neg p) \implies q$

Exercise 1.3

10

10.Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

answer

we assume that

- p: system software is being upgraded.
- q: user can access the file system.
- r: user can save new files.

we know that:

- $p \implies \neg q$
- $q \implies r$

- $\neg r \implies \neg p$

then:

- $((p \implies \neg q) \wedge (q \implies r) \wedge (\neg r \implies \neg p)) \implies (p \implies \neg p)$

then:
$\neg p \wedge p$
then:
the system is not consistent.

14

14.What Boolean search would you use to look for Web pages about hiking in West Virginia? What if you wanted to find Web pages about hiking in Virginia, but not in West Virginia?

answer

- HIKING AND WEST VIRGINIA
- HIKING AND VIRGINIA AND NOT WEST VIRGINIA

21

21.When three professors are seated in a restaurant, the hostess asks them: "Does everyone want coffee?" The first professor says "I do not know." The second professor then says "I do not know." Finally, the third professor says "No, not everyone wants coffee." The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

answer

- we assume that p_i is the i professor wants the coffee. u is everyone want coffee.
- $((\neg p_1 \vee \neg p_2 \vee \neg p_3) \implies \neg u)$

- $(p_1 \land p_2 \land p_3) \implies u$
- For the first professor, if p_1 is false, then he will know that u is not true, but he don't know, so p_1 is true.
- For the second professor, he knows p_1 is true. if p_2 is false, then he will know that u is not true, but he don't know, so p_2 is true.
- For the last professor, he knows p_1 and p_2 is true. Then the two proposition will be:
  - $\neg p_3 \implies \neg u$
  - $p_3 \implies u$
- So $p_3 \iff u$
- Because u is false, p_3 is false.
- so the first and second want coffee, the last professor doesn't want coffee.

37

37.Steve would like to determine the relative salaries of three coworkers using two facts.

- First, he knows that if Fred is not the highest paid of the three, then Janice is.
- Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most.

Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

answer

- we assume that
  - p_1 : Fred is the highest paid one.
  - p_2 : Janice is the highest paid one.
  - p_3 : Maggie is the highest paid one.
  - q_1 : Fred is the lowest paid one.
  - q_2 : Janice is the lowest paid one.
  - q_3 : Maggie is the lowest paid one.
- we know that
  - $axiom_1 : \neg p_1 \implies p_2$
  - $axiom_2 : \neg q_2 \implies p_3$
  - $axiom_3 : \neg(p_1 \land q_1) \land \neg(p_2 \land q_2) \land \neg(p_3 \land q_3)$
  - $axiom_4 : (p_1 \lor p_2 \lor p_3) \land \neg(p_1 \land p_2) \land \neg(p_2 \land p_3) \land \neg(p_1 \land p_3)$

- $axiom_5$: $(q_1 \vee q_2 \vee q_3) \wedge \neg(q_1 \wedge q_2) \wedge \neg(q_2 \wedge q_3) \wedge \neg(q_1 \wedge q_3)$

- From $axiom_1$, we know that: $p_1 \vee p_2$ $(collary_1)$
- From $axiom_2$, we know that: $q_2 \vee p_3$ $(collary_2)$
- From $axiom_3$, we know that: $\neg(p_2 \wedge q_2)$ $(collary_3)$

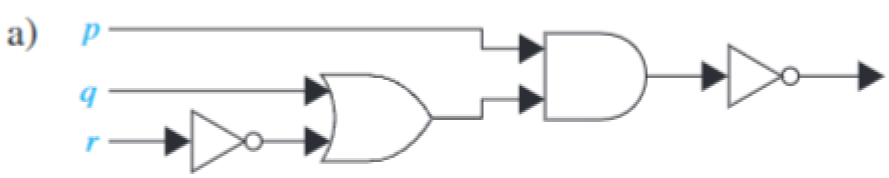
Case1: p_2 is true.
then from $collary_3$, q_2 is false.
then from $collary_2$, p_3 is true.
then from $axiom_4$, p_1 and p_2 are false.
We can't know who is the least paid one.

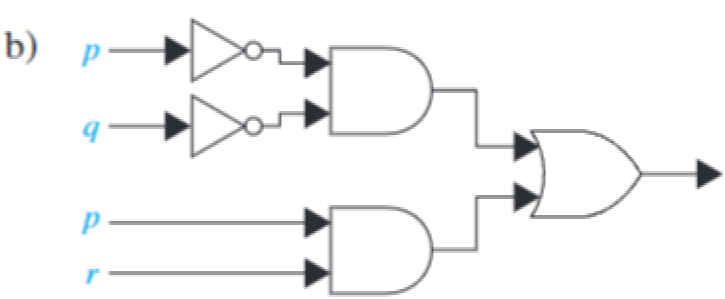
Case2: q_2 is true.
then from $collary_3$, p_2 is false.
then from $collary_2$, p_1 is true.
then from $axiom_4$ and $axiom_5$, p_2 p_3 q_1 q_3 are false.
Done.

Case3: q_2 and p_2 are both false.
then from $collary_1$ and $collary_2$, p_1 and p_3 are true.
violate the $axiom_4$

45

45.Find the output of each of these combinatorial circuits.





- a)
  $\neg(p \wedge (q \vee \neg r))$
- b)
  $(\neg p \wedge \neg q) \vee (p \wedge r)$

Exercises 1.3

4.Use truth tables to verify the associative laws
a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$.
b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

answer

- a)

p	q	$p \vee q$	r	$(p \vee q) \vee r$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	T	T
T	T	T	F	T
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

p	q	r	$q \vee r$	$p \vee (q \vee r)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	T	T	T
T	T	F	T	T

p	q	r	$q \vee r$	$p \vee (q \vee r)$
T	F	F	F	T
F	T	F	T	T
F	F	F	F	F

- b)

p	q	$p \wedge q$	r	$(p \wedge q) \wedge r$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F
T	T	T	F	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	F	T	F	F
F	T	T	T	F
F	F	T	F	F
T	T	F	F	F
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

12.Show that each of these conditional statements is a tautology by using truth tables.
a) $[\neg p \wedge (p \vee q)] \implies q$
b) $[(p \implies q) \wedge (q \implies r)] \implies (p \implies r)$
c) $[p \wedge (p \implies q)] \implies q$
d) $[(p \vee q) \wedge (p \implies r) \wedge (q \implies r)] \implies r$

- a)

p	q	$\neg p$	$p \vee q$	$[\neg p \wedge (p \vee q)]$	$[\neg p \wedge (p \vee q)] \implies q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

so $[\neg p \wedge (p \vee q)] \implies q$ is a tautology.

- b)

p	q	r	$p \implies q$	$q \implies r$	$p \implies r$	$(p \implies q) \wedge (q \implies r)$	$[(p \implies q) \wedge (q \implies r) \implies p] \implies r$
T	T	T	T	T	T	T	T
T	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	F	T	T	F	F	T
F	F	F	T	T	T	T	T

so $[(p \implies q) \wedge (q \implies r)] \implies (p \implies r)$ is a tautology.

- c)

p	q	$p \implies q$	$p \wedge (p \implies q)$	$[p \wedge (p \implies q)] \implies q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

so $[p \wedge (p \implies q)] \implies q$ is a tautology.

- d)

p	q	r	$p \vee q$	$p \implies r$	$q \implies r$	$(p \vee q) \wedge (p \implies r) \wedge (q \implies r)$	$[(p \vee q) \wedge (p \implies r) \wedge (q \implies r)] \implies r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

so $[(p \vee q) \wedge (p \implies r) \wedge (q \implies r)] \implies r$ is a tautology.

24.Show that $\neg(p \oplus q)$ and $p \iff q$ are logically equivalent

Pf: by truth table.

- $\neg(p \oplus q)$

p	q	$p \oplus q$	$\neg(p \oplus q)$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

- $p \iff q$

p	q	$p \implies q$	$q \implies p$	$p \iff q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

so $\neg(p \oplus q)$ and $p \iff q$ are logically equivalent.

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28.Show that $(p \implies q) \vee (p \implies r)$ and $p \implies (q \vee r)$ are logically equivalent.

answer

- $(p \implies q) \vee (p \implies r)$

$$(p \implies q) \vee (p \implies r) \equiv p \implies (q \vee r) \equiv (\neg p \vee q) \vee (\neg p \vee r) \equiv \neg p \vee q \vee r$$

- $p \implies (q \vee r)$

$$p \implies (q \vee r) \equiv \neg p \vee q \vee r$$

Consequently $(p \implies q) \vee (p \implies r)$ and $p \implies (q \vee r)$ are logically equivalent.

31

31.Show that $p \leftrightarrow q$ and $(p \implies q) \wedge (q \implies p)$ are logically equivalent.

$p \leftrightarrow q \equiv (p \implies q) \wedge (q \implies p)$

answer

Pf: by truth table

p	q	$p \implies q$	$q \implies p$	$p \leftrightarrow q$	$(p \implies q) \wedge (q \implies p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

so $p \leftrightarrow q$ and $(p \implies q) \wedge (q \implies p)$ are logically equivalent.

36

36.Show that $(p \wedge q) \implies r$ and $(p \implies r) \wedge (q \implies r)$ are not logically equivalent.

$(p \wedge q) \implies r \not\equiv (p \implies r) \wedge (q \implies r)$

answer

Notice that when p is true, q is false and r is false.

- $(p \wedge q) \implies r$ is true.
- $(p \implies r) \wedge (q \implies r)$ is false.

So $(p \wedge q) \implies r \not\equiv (p \implies r) \wedge (q \implies r)$.

38

38.Find the dual of each of these compound propositions.

- a) $p \vee \neg q$
- b) $p \wedge (q \vee (r \wedge T))$
- c) $(p \wedge \neg q) \vee (q \wedge F)$

answer

- a) $p \wedge \neg q$
- b) $p \vee (q \wedge (r \vee F))$
- c) $(p \vee \neg q) \wedge (q \vee T)$

66

66.Determine whether each of these compound propositions is satisfiable.

- a) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s) \wedge (p \vee q \vee \neg s)$
- b) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg s) \wedge (\neg p \vee \neg r \vee \neg s) \wedge (p \vee q \vee \neg r) \wedge (p \vee \neg r \vee \neg s)$
- c) $(p \vee q \vee r) \wedge (p \vee \neg q \vee \neg s) \wedge (q \vee \neg r \vee s) \wedge (\neg p \vee r \vee s) \wedge (\neg p \vee q \vee \neg s) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee s) \wedge (\neg p \vee \neg r \vee \neg s)$

answer

- a) when p is true, q is false and r is false, the proposition is true. So the propostion is satisfiable.
- b) when p is false, q is true, r is true and s is false, the proposition is true. So the propostion is satisfiable.
- c) when p is true, q is true, r is false and s is true, the proposition is true. So the propostion is satisfiable.

71

71.Explain the steps in the construction of the compound proposition given in the text that asserts that every column of a 9 × 9 Sudoku puzzle contains every number.

answer

- define a variant $x_{r,c,n}$

- $x_{r,c,n}$ is **true** if the number n is in row r and column c.
- $x_{r,c,n}$ is **false** if the number n is **not** in row r and column c.

- for each column, any number n_i must appear.
  - $$\forall n \in 1, 2, ..., 9, \forall c \in 1, 2, ..., 9, \\ \bigvee_{r=1}^{9} x_{r,c,n}$$
- for each column, any number n_i don't appear more than once.
  - $$\forall n \in 1, 2, ..., 9, \forall c \in 1, 2, ..., 9, \forall r_1, r_2 \in 1, 2, ..., 9, r_1 \neq r_2, \\ \neg(x_{r_1,c,n} \wedge x_{r_2,c,n})$$