Excercise 1.1

8

8.Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

- a) Smartphone B has the most RAM of these three smartphones.
- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

answer

- a) T
- b) T
- c) F
- d) F
- e) F

10

10. Let p and q be the propositions
    p: I bought a lottery ticket this week.
    q: I won the million dollar jackpot.
    Express each of these propositions as an English sentence.

- a) $\neg p$
- b) $p \vee q$
- c) $p \implies q$
- d) $p \wedge q$
- e) $p \iff q$
- f) $\neg p \implies \neg q$
- g) $\neg p \wedge \neg q$
- h) $\neg p \vee (p \wedge q)$

answer

- a) I didn’t bought a lottery ticket this week
- b) I bought a lottery ticket this week, or I won a the million dollar jackpot, or both.
- c) If I bought a lottery ticket this week, I would win the million dollar jackpot.
- d) I bought a lottery ticket this week and won the million dollar jackpot.
- e) I bought a lottery ticket this week if and only if I won the million dollar jackpot.
- f) If I didn’t buy a lottery ticket this week, then I didn’t win the million dollar jackpot.
- g) I didn't buy a lottery ticket this week and I didn't win the million dollar jackpot.
- h) I didn't buy a lottery ticket this week or I bought a lottery ticket this week and I win the million dollar jackpot.

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13. Let p and q be the propositions
    p: It is below freezing.
    q: It is snowing.
    Write these propositions using p and q and logical connectives (including negations).

- a) It is below freezing and snowing.

- b) It is below freezing but not snowing.
- c) It is not below freezing and it is not snowing.
- d) It is either snowing or below freezing (or both).
- e) If it is below freezing, it is also snowing.
- f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
- g) That it is below freezing is necessary and sufficient for it to be snowing.

answer

- a) $p \wedge q$
- b) $p \wedge \neg q$
- c) $\neg p \wedge \neg q$
- d) $p \vee q$
- e) $p \implies q$
- f) $(p \vee q) \wedge (p \implies \neg q)$
- g) $p \iff q$

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19. Determine whether each of these conditional statements is true or false.
    a) If 1 + 1 = 2, then 2 + 2 = 5.
    b) If 1 + 1 = 3, then 2 + 2 = 4.
    c) If 1 + 1 = 3, then 2 + 2 = 5.
    d) If monkeys can fly, then 1 + 1 = 3.

answer

- a) F
- b) T
- c) T
- d) T

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Write each of these statements in the form “if p, then q” in English. [Hint: Refer to the list of common ways to express conditional statements provided in this section.]

- a) I will remember to send you the address only if you send me an e-mail message.
- b) To be a citizen of this country, it is sufficient that you were born in the United States.
- c) If you keep your textbook, it will be a useful reference in your future courses.
- d) The Red Wings will win the Stanley Cup if their goalie plays well.
- e) That you get the job implies that you had the best credentials.
- f) The beach erodes whenever there is a storm.
- g) It is necessary to have a valid password to log on to the server.
- h) You will reach the summit unless you begin your climb too late.
- i) You will get a free ice cream cone, provided that you are among the first 100 customers tomorrow.

answer

- a) If you send me an e-mail message, then I will remember to send you the address.
- b) If you were born in the United States, then you will be a citizen of the United States.
- c) If you keep your textbook, then it will be a useful reference in your future courses.
- d) If the Red Wings goalie plays well, then the Red Wings will win the Stanley Cup.
- e) If you get the job, then you had the best credentials.
- f) If there is a storm, then the beach will erode.
- g) If you can log on to the server, then you have a valid password.
- h) If you begin your climb too late, then you will not reach the summit.
- i) If you are among the first 100 customers tomorrow, then you will get a free ice cream cone.

30

30. State the converse(逆命题), contrapositive(逆否命题), and inverse(否命题) of each of these conditional statements.

- a) If it snows tonight, then I will stay at home.
- b) I go to the beach whenever it is a sunny summer day.
- c) When I stay up late, it is necessary that I sleep until noon.

answer

Converse

- a) If I stay at home tonight, then it snows.
- b) If I go to the beach, then it is a sunny summer day.
- c) If I sleep until noon, then I stay up late.

Contrapositive

- a) If I do not stay at home tonight, then it does not snow.
- b) If I don't go to the beach, then it is not a sunny summer day.
- c) If I don't sleep until noon, then I don't stay up late.

Inverse

- a) If it does not snow tonight, then I will not stay at home.
- b) If it is not a sunny summer day, then I will not go to the beach.
- c) If I do not stay up late, then I do not sleep until noon.

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32. How many rows appear in a truth table for each of these compound propositions?

- a) $(q \implies \neg p) \vee (\neg p \implies \neg q)$
- b) $(p \vee \neg t) \wedge (p \vee \neg s)$
- c) $(p \implies r) \vee (\neg s \implies \neg t) \vee (\neg u \implies v)$
- d) $(p \wedge r \wedge s) \vee (q \wedge t) \vee (r \wedge \neg t)$

answer

- a) 4
- b) 8
- c) 64
- d) 32

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37. Construct a truth table for each of these compound propositions.

- a) $p \implies \neg q$
- b) $\neg p \iff q$
- c) $(p \implies q) \vee (\neg p \implies q)$
- d) $(p \implies q) \wedge (\neg p \implies q)$
- e) $(p \iff q) \vee (\neg p \iff q)$
- f) $(\neg p \iff \neg q) \iff (p \iff q)$

answer

- a)

p	q	$\neg q$	$p \implies \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

- b)

p	q	$\neg p$	$\neg p \implies q$	$q \implies \neg p$	$\neg p \iff q$
T	T	F	T	F	F
T	F	F	T	T	T

p	q	$\neg p$	$\neg p \implies q$	$q \implies \neg p$	$\neg p \iff q$
F	T	T	T	T	T
F	F	T	F	T	F

- c)

p	q	$\neg p$	$p \implies q$	$\neg p \implies q$	$(p \implies q) \vee (\neg p \implies q)$
T	T	F	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	F	T

- d)

p	q	$\neg p$	$p \implies q$	$\neg p \implies q$	$(p \implies q) \wedge (\neg p \implies q)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

- e)
  单表过长，分两张列。

p	q	$\neg p$	$p \implies q$	$q \implies p$	$p \iff q$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

p	q	$\neg p$	$\neg p \implies q$	$q \implies \neg p$	$\neg p \iff q$
T	T	F	T	F	F
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	T	F

所以

p	q	$(p \iff q) \vee (\neg p \iff q)$
T	T	T
T	F	T
F	T	T
F	F	T

- f)
  这下更长了

p	q	$p \implies q$	$q \implies p$	$p \iff q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

p	q	$\neg p$	$\neg q$	$\neg p \iff \neg q$	$p \iff q$	$(\neg p \iff \neg q) \iff (p \iff q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

44

44.If $p_1, p_2, ..., p_n$ are n propositions, explain why

$$\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$$

is true if and only if at most one of $p_1, p_2, ..., p_n$ is true.

answer

Pf:
case 1: for any p_i, p_i is false,
then $\neg p_i$ is true,
then $\neg p_i \vee \neg p_j$ is true for any $j \neq i$,
then $\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$ is true.

case 2: assume that p_{i_0} is true, for all $i \neq i_0$, p_i is false.
then $\neg p_{i_0}$ is false, for all $i \neq i_0$, $\neg p_i$ is true.
then for any pair of p_i and p_j, $i \neq j$, $\neg p_i \vee \neg p_j$ is true.
then $\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$ is true.

case 3: assume that more than one of $p_1, p_2, ..., p_n$ is true, say p_{i_0} and p_{i_1} are true.
then $\neg p_{i_0}$ is false, $\neg p_{i_1}$ is false,
then $\neg p_{i_0} \vee \neg p_{i_1}$ is false,
then $\bigwedge_{i=1}^{n-1} \bigwedge_{j=i+1}^{n} (\neg p_i \vee \neg p_j)$ is false.

Done