Two-wire transmission lines

Outline:

- What is the purpose of this study?
- What and how was done in the study?
- What are possible future directions for the study?

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Why study it?

- Two-wire transmission lines are most common today
- Have a simple structure
- Therefore there are not so many dependencies for the parameters
- However, due to the first point, this topic is of great interest and is not extremely complex as a result

What we get from this study

- Understanding of how parameters of the line affect it's characteristics
- Can we use analytical formula in case parameters of the line are equal to certain value?
- Basic skills of using simulation software

  (COMSOL, for instance), although that was not the main point of the study

Parallel plate transmission line, theory

Assume $e^{-j\beta z}$ variation, no variation with x

TM modes: $\nabla_t^2 e_z + k_c^2 e_z = 0 \quad , \quad k_c^2 = \begin{cases} k_0^2 - \beta^2 & \text{in the air region} \\ k^2 - \beta^2 & \text{in the dielectric region} \end{cases}$

Let $k_c = \begin{cases} p & \text{for air region} \\ \ell & \text{for the dielectric region} \end{cases}$

$\ell^2 - p^2 = (\varepsilon_r - 1) k_0^2$

$$h_x(y)=\begin{cases}\dfrac{j\varepsilon_r k_0 Y_0}{\ell^2}\dfrac{\partial e_z}{\partial y} & \text{For dielectric region}\\ \dfrac{jk_0 Y}{p^2}\dfrac{\partial e_z}{\partial y} & \text{for air region}\end{cases}$$

$$e_z(y)=C_1\sin \ell y \qquad 0\le y\le a$$

$$e_z(y)=C_2\sin p(b-y) \qquad a\le y\le b$$

$$C_1\sin \ell a=C_2\sin pc$$

$$\frac{\varepsilon_r}{\ell}C_1\cos \ell a=-\frac{1}{p}C_2\cos pc$$

$\ell\tan \ell a=-\varepsilon_r\, p\tan pc$ Transcedental equation must be solved simultaneously with $\ell^2-p^2=(\varepsilon_r-1)k_0^2 \quad \Rightarrow \ell, p$

$$\beta = \sqrt{\mathrm{k}_0^2 - p^2} = \sqrt{k^2 - l^2}$$

Most of the modes will be nonpropagating if β is imaginary

The variation is $e^{-|\beta| z}$ and the field decays exponentially.

The value of β between k_0 and k can occur if $p = jp_0$

Let ℓ_0 to be the corresponding value of ℓ then :

$$\ell_0 \tan \ell_0 a = \varepsilon_r p_0 \tan p_0 c$$

$$\ell_0^2 + p_0^2 = (\varepsilon_r - 1) k_0^2$$

Low Frequency Solution

When the frequency is low,

k_0^2 is very small number , ℓ_0 and p_0 are very small $\Rightarrow$

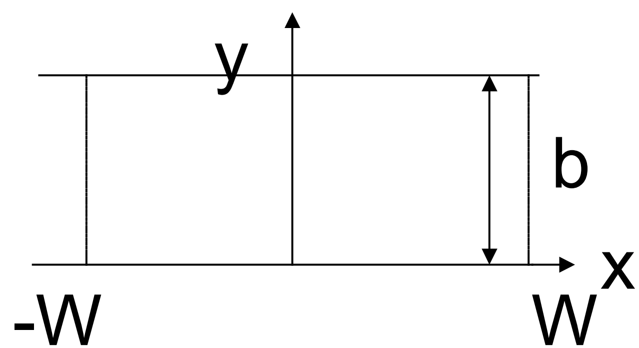
$$\ell_0{}^2 a = \varepsilon_r p_0^2 c$$

$$(\varepsilon_r - 1)\mathrm{k}_0^2 - p_0^2 = \frac{\varepsilon_r p_0^2 c}{a}$$

or $p_0^2 = \dfrac{(\varepsilon_r - 1)\mathrm{k}_0^2 a}{a + \varepsilon_r c}$ The solution for β is

$$\beta = \sqrt{\mathrm{k}_0^2 + p_0{}^2} = \sqrt{\frac{\varepsilon_r b}{a + \varepsilon_r c}}\, k_0 = \sqrt{\varepsilon_e}\, k_0$$

ε_e is the effective dielectric constant



$\varepsilon_e = \omega\sqrt{LC}$ $\quad L, C$ are the static inductance and capacitance per meter.

The time average stored magnetic energy is W_m

$$W_m = \frac{\mu_0}{4}\int_0^b \int_{-w}^{w} H_x^2\, dxdy = \frac{\mu_0}{2} WbJ_z^2 = \frac{1}{4} LI_z^2$$

$$I_z = 2WJ_z \qquad \Rightarrow L = \frac{\mu_0 b}{2\mathrm{W}}$$

$$C_2 = C_1 \sin \ell_0 a / j \sinh p_0 c \approx -jC_1 \beta \ell_0 a / p_0 c$$

$$0 \le y \le a$$

$$e_z = C_1 \ell_0 y$$

$$e_y = -\frac{j\beta}{\ell_0} C_1 = -jC_1 \sqrt{\frac{b}{(\varepsilon_r - 1)c}}$$

$$h_x = \frac{j\varepsilon_r k_0 Y_0}{\ell_0} C_1 = jY_0 C_1 \sqrt{\frac{(\varepsilon_r c + a)\varepsilon_r}{(\varepsilon_r - 1)c}}$$

The capacitance $C = \frac{C_a C_d}{C_a + C_d}$, $C_a = \frac{\varepsilon_0 2\mathrm{W}}{c}$, $C_d = \frac{\varepsilon_0 \varepsilon_r 2\mathrm{W}}{a}$

$$C = \frac{\varepsilon_0 \varepsilon_r 2\mathrm{W}}{\varepsilon_r c + a}$$

$$LC = \frac{\varepsilon_0 \varepsilon_r \mu_0 b}{\varepsilon_r c + a} \qquad \beta = \omega \sqrt{LC}$$

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Field expressions:

$$C_2 = C_1 \sin \ell_0 a / j \sinh p_0 c \approx -jC_1 \beta \ell_0 a / p_0 c$$

$$0 \le y \le a$$

$$e_z = C_1 \ell_0 y$$

$$e_y = -\frac{j\beta}{\ell_0} C_1 = -jC_1 \sqrt{\frac{b}{(\varepsilon_r - 1)c}}$$

$$h_x = \frac{j\varepsilon_r k_0 Y_0}{\ell_0} C_1 = jY_0 C_1 \sqrt{\frac{(\varepsilon_r c + a)\varepsilon_r}{(\varepsilon_r - 1)c}}$$

In the air region:

$$e_z = C_1 \ell_0 a \frac{(b-y)}{c}$$

$$e_y = -\frac{j\beta\ell_0 a}{p_0{}^2 c} C_1 = -jC_1 \varepsilon_r \sqrt{\frac{b}{(\varepsilon_r - 1)c}}$$

$$h_x = \frac{jk_0 Y_0}{\ell_0} C_1 = jY_0 C_1 \sqrt{\frac{(\varepsilon_r c + a)\varepsilon_r}{(\varepsilon_r - 1)c}}$$

$$V = -\int_0^b e_y \, dy = jC_1 \sqrt{\frac{b}{(\varepsilon_r - 1)c}} (a + \varepsilon_r c)$$

$$I_z = 2WJ_z = 2WH_x$$

The characteristic impedance is :

$$Z_c = \frac{V}{I} = \frac{Z_0}{2W}\sqrt{\frac{(a+\varepsilon_r c)b}{\varepsilon_r}} = \sqrt{\frac{L}{C}}$$

In the low frequency limit, the dominant mode of propagation becomed a TEM mode（quasi-TEM mode）
At hight frequency the mode of propagation is an E mod

Example:
Two plate transmission line

- Problems:
  - Plates are infinitely thin
  - Simulation mesh might get very big
- Solutions:
  - Define boundaries of inner square (or rectangle) as conductors
  - Pick appropriate size by trial and error, so that the simualtion results do not change drastically, compared with previous values

Calculating impedance

- We need L and C of the line
  - They are the part of the energy equations
  - Calculate them using these equations:

$$L=\frac{2W_m}{I^2}$$

$$U=\frac{2W_e}{U^2}$$

- Optimization: set 1 for current and voltage in the simulation parameters

Calculating impedance...

- The only step that is left now is:

$$Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{2W_m}{2W_e}} = \sqrt{\frac{W_m}{W_e}}$$

- Then we can calculate the analytical function and compare the plots of two solutions

- In the formula above we consider I = 1A and U = 1V

Two parallel plates

- Consider following:
  - We have two infinitely thin plates of width *w*
  - They are separated by *b* meters of insulator
  - Let insulator be air
  - How impedance of such structure can be calculated?
  - Analytic formula:

$$Z \approx \eta \frac{b}{w}$$

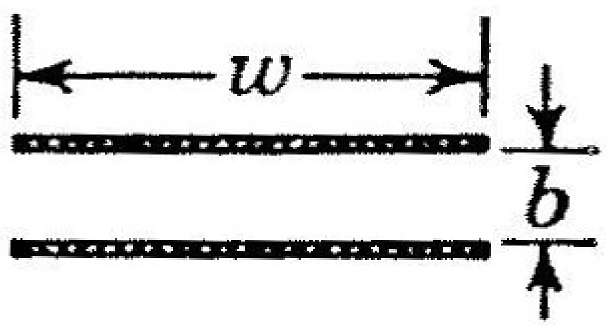
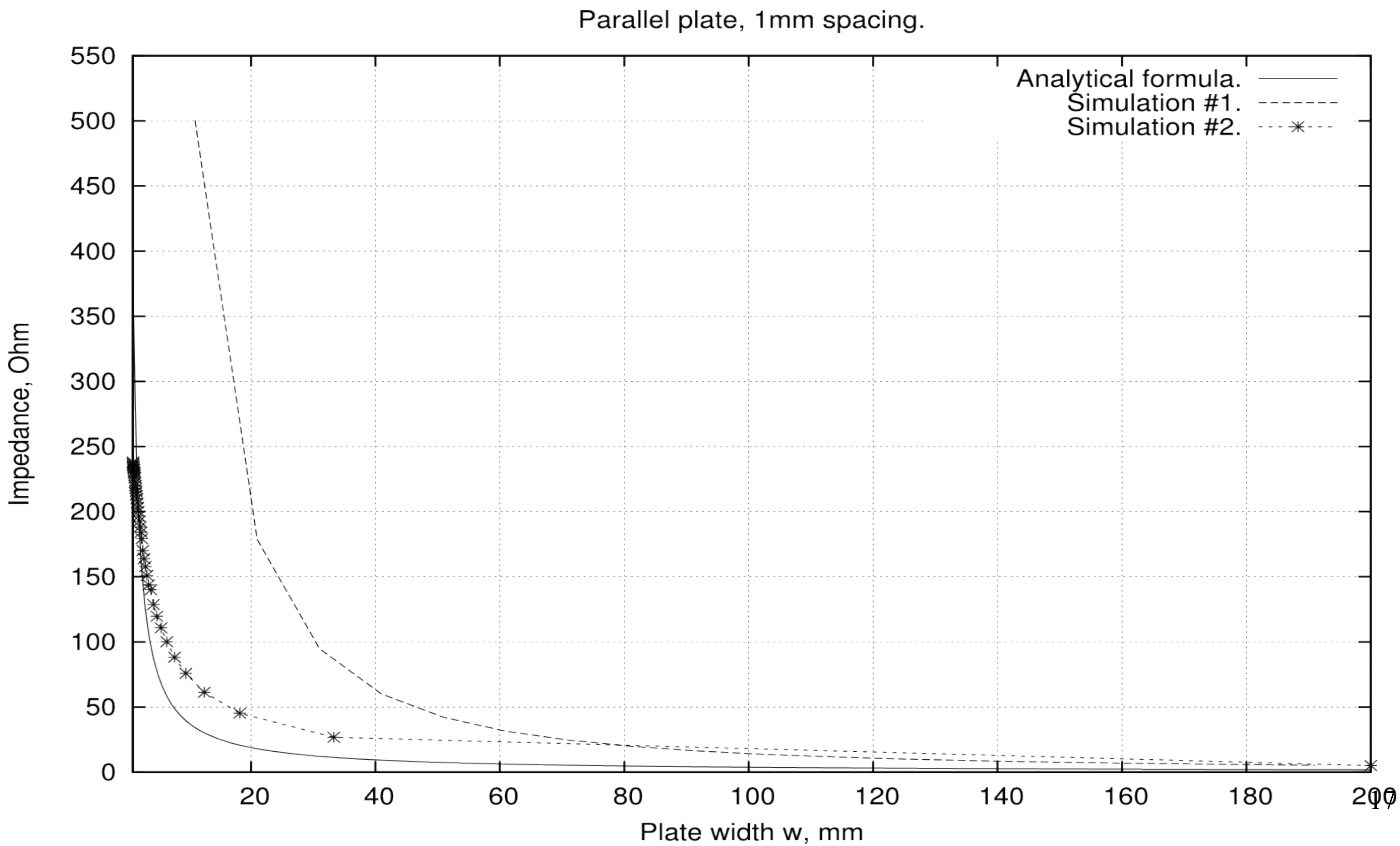


Fig. 1: Cross section of the line

Calculation results



Simulation screens

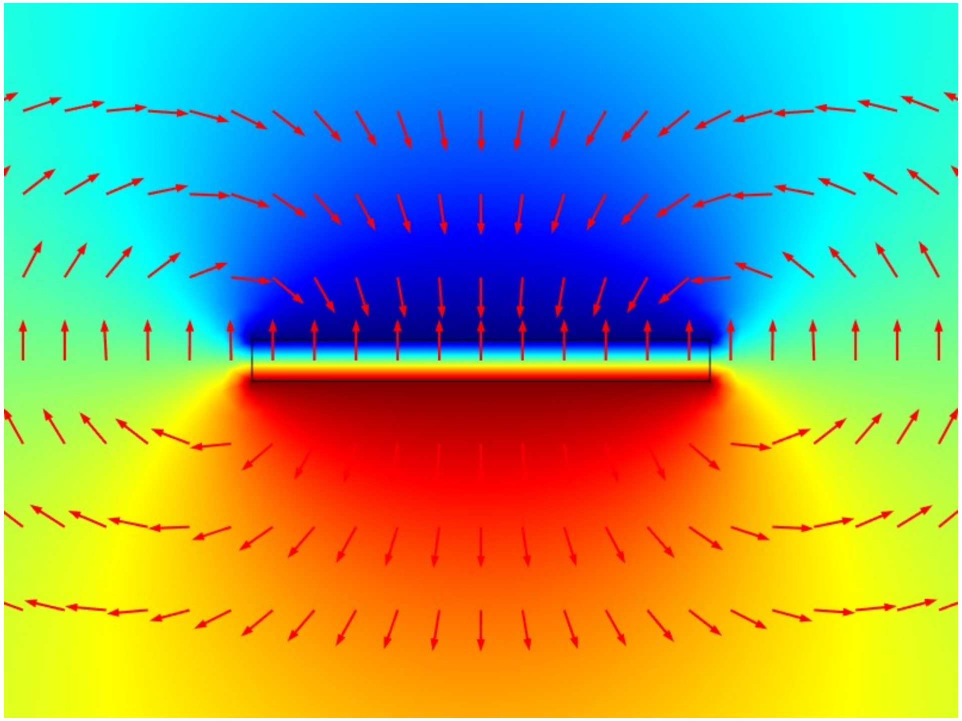


Fig. 2: Electric potential and its density in the two plate transmission line.

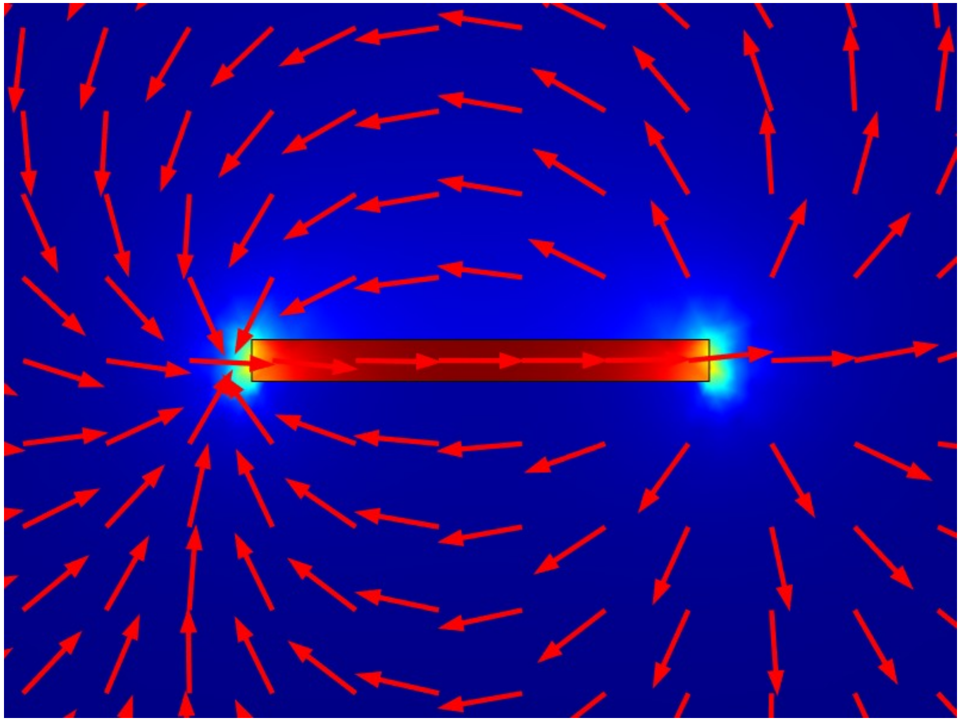


Fig. 3: Magnetic flux and its density in the two plate transmission line.

Some conclusions on case

- It can be seen from plot that the analytical formula is not always correct
- The bigger the plate width gets, the more accurate our estimation becomes
- The analytical formula approaches values of simulation, meaning that the error is decreasing and consequently getting closer to actual value

Future directions

- Fix simulations that were done improperly
- Make a proper report
- Reviev other cases (Shielded pair, for instance)
- Review different dielectrics (polyethylene, rubber)

COMSOL TIME

- Show some parts of the models in COMSOL
- Questions?