Characteristic impedance of strip transmission line

Konstantin Pandakov, konstantin.pandakov@student.tut.fi, Sergio Moreschini, sergio.moreschini@student.tut.fi

***Abstract*—The main aim of this paper is to compare calculated value of characteristic impedance of a microstrip transmission line according to expressions given in literature with simulation results (with help of COMSOL Multiphysics®). Our study is done for a strip transmission line in two different cases: in the first one we discuss about outer strip transmission line, in the second about inner strip transmission Line. This last case is studied for three different situations: one line, two coupled coplanar and two coupled broadside transmission lines.**

***Index Terms*—microstrip, transmission line, coupled lines, coplanar, broadside, characteristic impedance.**

I. Introduction

THIS document has been prepared in order to provide essential information about strip transmission Line and the parameter involved in the study of this physical object. A strip Line is a planar type of transmission line, its geometry is characterized by a thin conducting strip and a big region mode of substrate. It can support a TEM (transverse electromagnetic) wave, but can also support TM (transverse magnetic) and TE (transverse electric) modes (avoided in practice). The target of this paper is discuss about the approximations for calculation of characteristic impedance Z_c given in literature and determine how close they are at our calculations done with COMSOL Multiphysics®.

II. Modeling of strip transmission lines

The following transmission lines can be divided in two main groups: outer and inner strip transmission line. For the inner transmission line it is taken into account three different cases: in the first one we consider only one strip line, in the second it is performed a coupled coplanar strip, and in third it is shown coupled broadside strip.

A. *Outer strip transmission lines*

Outer strip transmission lines is performed as flat conductor placed on surface of a substrate (non-conducting) having relatively small thickness (μm). The lower side of the substrate is considered as ground plane (with zero potential).

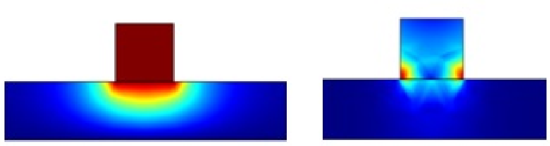

Fig. 1. Electric potential (left figure) and magnetic flux density (right figure) in the outer strip system.

In the first experiment we had deals with the following parameters:

$H = T = 0.5\ mm$, $\epsilon_r = 9.7$,

and width of line was being changed.

Here H - thickness of a substrate, T - thickness of a strip, ϵ_r - dielectric constant of alumina (material of the substrate). Aluminium was chosen as material of transmission line.

Fig. 1 performs simulation results - electric potential and magnetic flux density of the system under consideration. For electric potential blue color is zero potential and for magnetic flux density is zero magnetic potential.

With help of experimental data we calculated characteristic impedance using the following expressions:

$$Z_c = \sqrt{\frac{L}{C}} \tag{1}$$

where capacitance per meter is determined through density of electric field energy W_e and potential of a microstrip line V as

$$C = \frac{2W_e}{V^2} \tag{2}$$

and inductance per meter:

$$C = \frac{2W_m}{I^2} \tag{3}$$

where W_m - density of magnetic field energy and I is current of a microstrip line. In all cases we chose $V = 25\ V$ and $I = 1\ A$.

In order to determine characteristic impedance through design approach we used the following expressions:

Capacitance per meter of an outer strip of width W at a hight H above a ground plane:

$$C_a = \frac{2\pi\epsilon_0}{\ln(\frac{8H}{W} + \frac{W}{4H})}, \qquad \frac{W}{H} \leq 1 \tag{4}$$

$$C_a = \epsilon_0\Big[\frac{W}{H} + 1.393 + 0.667\ln(\frac{W}{H} + 1.444)\Big], \frac{W}{H} > 1 \tag{5}$$

where $\epsilon_0 = 8.85 \cdot 10^{-12}$ - electric constant.

The effective dielectric constant for an outer microstrip line is given by:

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\Big(1 + \frac{12H}{W}\Big)^{-0.5} + F(\epsilon_r, H) - \\ -0.217(\epsilon_r - 1)\frac{T}{\sqrt{WH}} \tag{6}$$

where T - thickness of a strip, ϵ_r - electric permittivity of a substrate and

$$F(\epsilon_r, H) = 0.02(\epsilon_r - 1)\Big(1 - \frac{W}{H}\Big)^2, \frac{W}{H} \leq 1 \tag{7}$$

$$F(\epsilon_r, H) = 0, \qquad \frac{W}{H} > 1 \tag{8}$$

Characteristic impedance

$$Z_c = \sqrt{\frac{\mu_0 \epsilon_0}{\epsilon_e}} \frac{1}{C_a} \tag{9}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ - magnetic constant.

Table I shows us comparison of experimental (”Exp.” in tables) and designed values of characteristic impedance for to cases.

TABLE I
EXPERIMENTAL AND DESIGN DATA FOR OUTER STRIP SYSTEM

	Z_c, Ω			
	W, mm	Exp.	Design	RE, %
$\frac{W}{H} \leq 1$	0.5	58.18	58.57	0.67
$\frac{W}{H} > 1$	1	39.56	37.55	5

As we can see relative error (RE) does not exceed 5%.

B. Inner strip transmission lines

This type of transmission line is completely surrounded by dielectric material. The strip is situated between two ground planes.

In this experiment we had deals with the following parameters:

$H = 5\ mm$, $T = 20\ \mu m$, $\epsilon_r = 2.2$,

and Fig. 2 presents simulation results for electric potential (ϕ) and magnetic flux density (B) in the considering system.

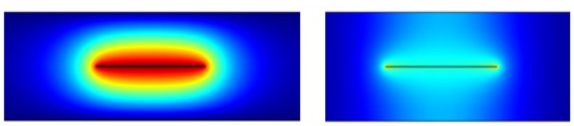

Fig. 2. ϕ and B in the inner strip system.

In order to determine design parameters we applied such formulas:

$$Z_c = \frac{120\pi K}{4\sqrt{\epsilon_r} K'} \tag{10}$$

where

$$\frac{K}{K'} = \frac{1}{\pi} \ln\left(2\frac{1+\sqrt{k}}{1-\sqrt{k}}\right), \qquad 0.7 \leq k < 1 \tag{11}$$

$$\frac{K}{K'} = \left[\frac{1}{\pi} \ln\left(2\frac{1+\sqrt{k'}}{1-\sqrt{k'}}\right)\right]^{-1}, \quad 0 < k \leq 1 \tag{12}$$

and

$$k = \frac{1}{\cosh(\frac{\pi W}{4H})} \tag{13}$$

$$k' = \tanh(\frac{\pi W}{4H}) \tag{14}$$

where W was variable parameter.

In this case, after comparison results (Table II), relative error does not exceed 3%.

TABLE II
EXPERIMENTAL AND DESIGN DATA FOR INNER STRIP SYSTEM

	Z_c, Ω			
	W, mm	Exp.	Design	RE, %
$k \leq 0.7$	10	45.3	44.1	2.64
$k < 1$	5	69.25	67.76	2.16

1) Coupled coplanar strip transmission lines: The system performs two strip lines separated by length S. For this type of simulation there are two different modes: even (potentials are equal and currents are codirectional vectors) and odd (one strip has $+V$ potential and another $-V$, currents are differently directed).

Figure 3 shows modeled results for coplanar system where we used the following parameters:

$W = 10\ mm$, $T = 20\ \mu m$, $\epsilon_r = 2.2$, $S = 2.5\ mm$.

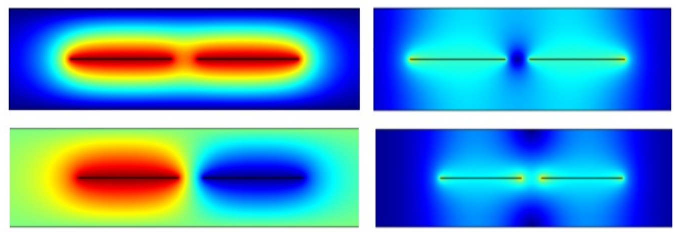

Fig. 3. ϕ and B for even (upper figures) and odd (lower figures) modes in the coplanar system.

For designing the following expression are suitable:

$$Z_c = \frac{120\pi(2H-T)}{4\sqrt{\epsilon_r}(W + (H/\pi)C_f A_i)} \tag{15}$$

where index $i = e,\ o$ denotes even or odd mode,

$$C_f = 2\ln(\frac{4H-T}{2H-T}) - \frac{T}{2H}\ln(\frac{T(4H-T)}{(2H-T)^2}) \tag{16}$$

$$A_e = 1 + \frac{\ln(1+\tanh(\frac{\pi S}{4H}))}{\ln(2)} \tag{17}$$

$$A_o = 1 + \frac{\ln(1+\coth(\frac{\pi S}{4H}))}{\ln(2)} \tag{18}$$

Characteristic impedance for the case with $W < 0.7H$ can be determined as:

$$Z_c^i = \frac{120\pi K'}{4\sqrt{\epsilon_r} K} \tag{19}$$

where $\frac{K}{K'}$ is obtained through (11) and (12) and

$$k_e = \tanh(\frac{\pi W}{4H})\tanh(\frac{\pi(W+S)}{4H}) \tag{20}$$

$$k_o = \tanh(\frac{\pi W}{4H})\coth(\frac{\pi(W+S)}{4H}) \tag{21}$$

and

$$k'_e = \sqrt{1-k_e^2} \tag{22}$$

$$k'_o = \sqrt{1-k_o^2} \tag{23}$$

TABLE III
DATA FOR COUPLED COPLANAR STRIP SYSTEM

			Z_c, Ω		
Mode		H, mm	Exp.	Design	RE, %
Even	$W \geq 0.7H$	10	49	47.87	2.36
	$W < 0.7H$	20	130.13	127.46	2
Odd	$W \geq 0.7H$	10	40.87	38.64	5.46
	$W < 0.7H$	20	58.45	53.86	7.86

Table III shows us comparison of simulated and designed results for different cases (parameter H was being changed). For odd mode there is quite big error (more than 5%).

2) Coupled broadside strip transmission lines: This type of lines is very similar to the previous but separation by S is performed in vertical axis.

Modeled results are presented in Fig. 4 for parameters:

$W = 10\ mm$, $H = 5\ mm$, $T = 20\ \mu m$, $\epsilon_r = 2.2$, $S = 2.5\ mm$.

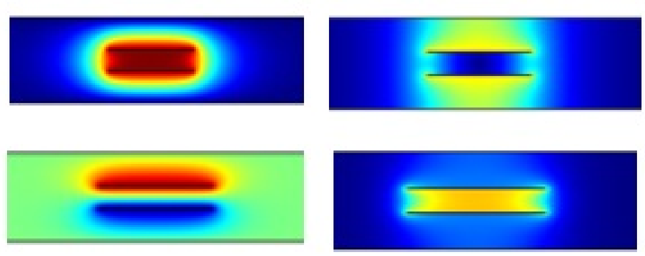

Fig. 4. ϕ and B for even (upper figures) and odd (lower figures) modes in the broadside system.

Design expressions:

Characteristic impedance for the case with $W \geq 0.35S$ and $W \geq 0.7H(1 - \frac{S}{2H})$

$$Z_c^e = \frac{120\pi}{2\sqrt{\epsilon_r}} \left[\frac{W}{2H - S - 2T} + 0.4413 + \frac{1}{\pi} \cdot \right. \tag{24}$$

$$\left. \cdot \Big(\ln(\frac{2H}{2H - S - 2T}) + \frac{S + 2T}{2H - S - 2T} \ln(\frac{2H}{S + 2T}) \Big) \right]^{-1}$$

$$Z_c^o = \frac{120\pi}{2\sqrt{\epsilon_r}} \left[\frac{W}{2H - S - 2T} + \frac{W}{S} + C_f^o + \frac{2}{\pi} \cdot \right. \tag{25}$$

$$\left. \cdot \Big((1 + \frac{T}{S}) \ln(1 + \frac{T}{S}) + \frac{T}{S} \ln(\frac{T}{S}) \Big) \right]^{-1}$$

where

$$C_f^o = \frac{2H - 2T}{\pi S} \left[\ln(\frac{2H - 2T}{2H - S - 2T}) + \right. \tag{26}$$

$$\left. + \frac{S}{2H - S - 2T} \ln(\frac{2H - 2T}{S}) \right]$$

Data for different modes are collected in Table IV. Relative error is less than 4%.

TABLE IV
DATA FOR COUPLED BROADSIDE STRIP SYSTEM

		Z_c, Ω	
Mode	Exp.	Design	RE, %
Even	65.28	62.8	3.8
Odd	20.48	20.1	1.86

III. CONCLUSION

The obtained results show us that in fact approximation is a really good way to proceed when we want to calculate the terms of the strip line. Small discrepancy can be explained as error of calculation and simulation. Moreover for modeling we used nonideal systems (finite geometrical and physical parameters).

As we can see, however, for odd mode in coplanar system there is significant error. It means that to be satisfied with only calculated results is not suitable approach for designing of strip transmission lines.

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