STAT 503

Time Series Analysis

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Project:

Analysis and Modeling of Triplen Harmonic Current Time Series in Electrical Power Systems

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Submitted to:

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I. Introduction

The nature of loads supplied by electrical systems have substantially changed in recent years. Today, one can find different types of modern electrical appliances such as laptop, TV, compact fluorescent lamps in almost every home. Although being energy-efficient and satisfactory to technologic needs of modern human, widespread growth of such new loads is becoming quite a challenge to electrical power industry[1]-[3]. In a normal condition, all of the voltage and currents in a power system have the shape of nominal 60 Hz sinusoidal waveform. However, this type of new electric devices introduce distortions with higher frequencies which are commonly called 'harmonics'. Now, since all of the electrical components are initially designed to operate in a nominal condition, harmonics can potentially cause several problems such as breakdown of capacitors, interference with adjacent telephone lines, neutral wire overheating and etc. [4]-[5]

Therefore, it is quite essential for electrical engineers to estimate level of harmonic pollution in the system, and then, to plan for appropriate mitigation measures including installation of harmonic filters. Unfortunately, it is very difficult to predict harmonic currents/voltages in the system due to the very random nature of harmonic sources. There are several residential units in each system and their load trends vary significantly based on the usage pattern of each house residents. In this project, we target to analyze a sample time series obtained from triplen harmonic currents measured in an actual system (the so-called triplen harmonics with frequency of 180Hz are one of the major harmonic orders responsible for most of the issues). The time series will be also studied in a comparative manner to the fundamental standard currents that are much easier to predict based on total electrical power of feeders. The data is collected from an initial harmonic analysis on the result of measurements conducted in an electrical substation at Edmonton, Canada in March 2011. The objective is to establish a model to predict the harmonic current time series based on its own history and value of the fundamental current.

II. The Studied Time Series

Two time series are captured for the studies of this project: Fundamental and triplen harmonic currents associated with one day measurement at a substation. The raw electrical measurements have been processed in a harmonic analysis software to derive these two time series. The resolution of both time series are one minute with a total length reflecting the whole 24 hours (i.e. t=1~1440). Indeed, each data point in these time series refer to average of its measured value during a 1 minute period. In this report, the fundamental current and the triplen harmonic current are referred to as FC and THC, respectively. As discussed in the introduction section, the objective is to analyze and model THC_t based on FC_t. Figure 1 shows the sample data of the two times series used in this project.

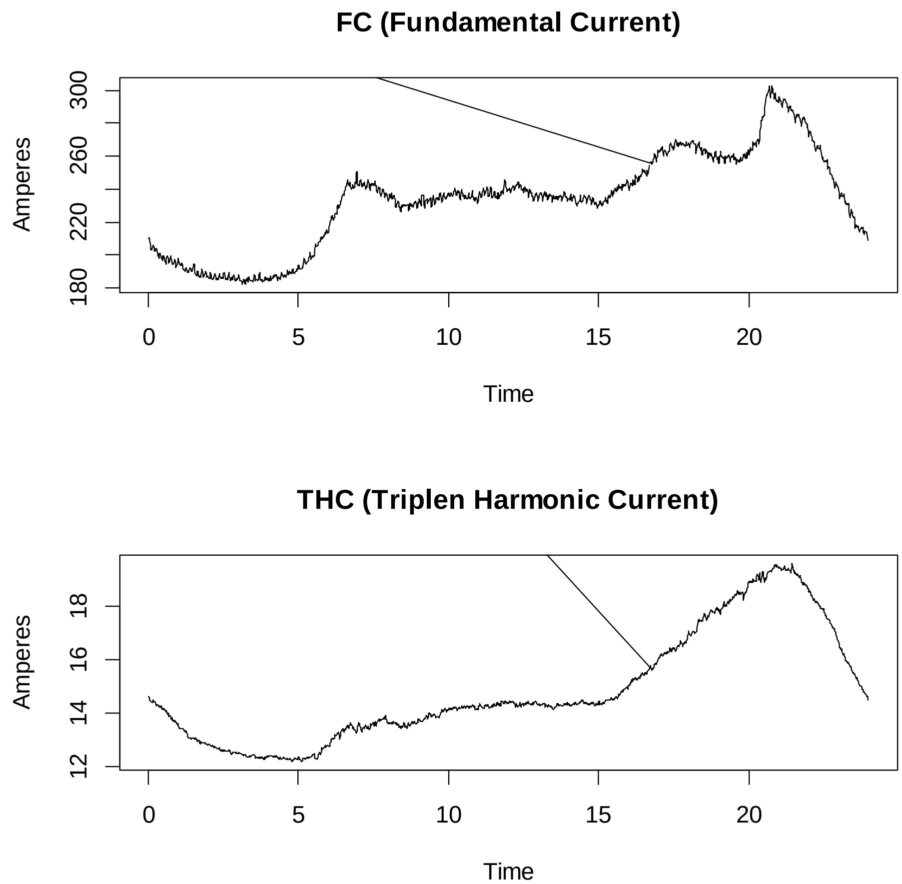


Figure 1 Original Time Series: Fundamental and Triplen Harmonic Current

III. Analysis and Modeling

Apparently, our time series of interest (THC) is not stationary based on Figure 1 where its mean varies significantly from time to time. This can be observed from its ACF plot too (Figure 2) as the ACF decreases in a very slow manner. Thus, the first step is to remove the trend of the THC time series to make it at least weakly stationary.

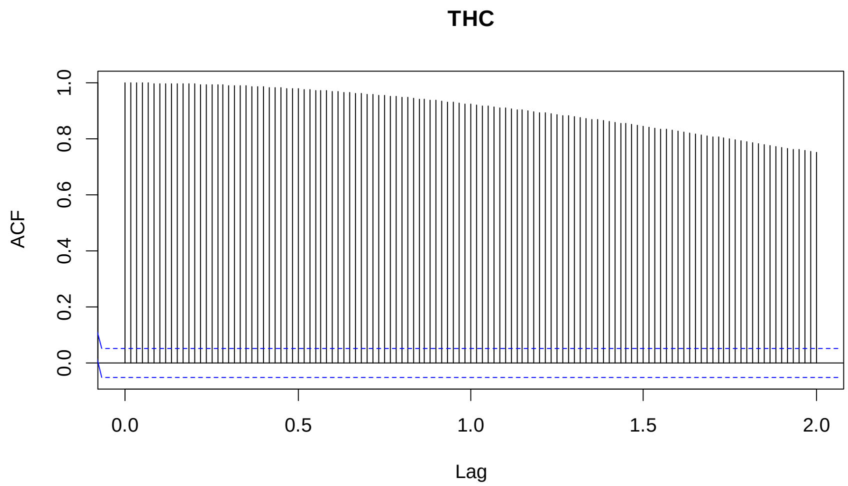
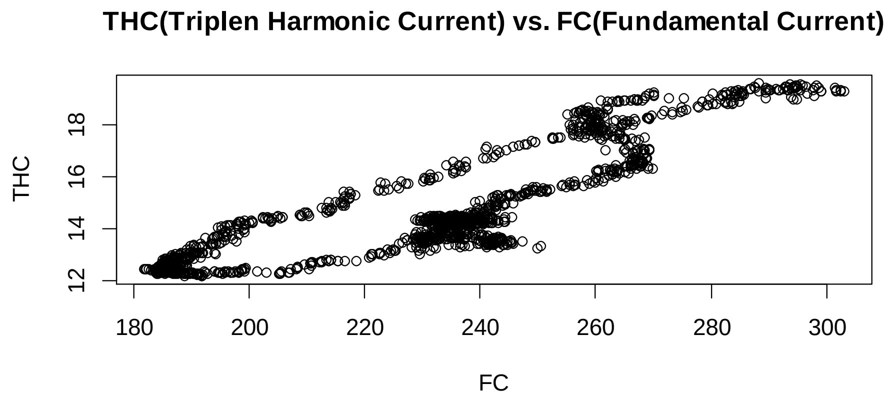


Figure 2 ACF of triplen harmonic current time series

a) Studying the relationship of two time series

A possible approach for detrending the THC_t time series is to investigate its relationship with the FC_t time series. As observed in Figure 1, there are considerable similarities between the general trend of the two time series. For example, both have higher values during evening time and lower values in the early morning. However, they are not quite following the exact same trends. This is intuitively expected as well since higher loads generally lead to higher fundamental current and consequently higher triplen harmonic current, but, the ratio is not constant since the loads type might also differ from time to time.

Figure 3 analyzes the relationship between FC_t and THC_t by a scatter plot as well as CCF. The scatter plot reveals an approximate linear relationship. Such linear relationship could be further discovered by a regression technique. The CCF plot, unfortunately, does not give any information as at which lag the two time series are the most correlated. However, from the physical point of view, we already know that dependency of THC on a lagged value of FC is somewhat meaningless. Since electrically-speaking, the harmonic current at each time is expected to be only dependent of the load amount (proportional to fundamental current) at that time regardless of the previous passed times. Therefore, in next section, we will only use a non-lagged term of fundamental current in our regression.



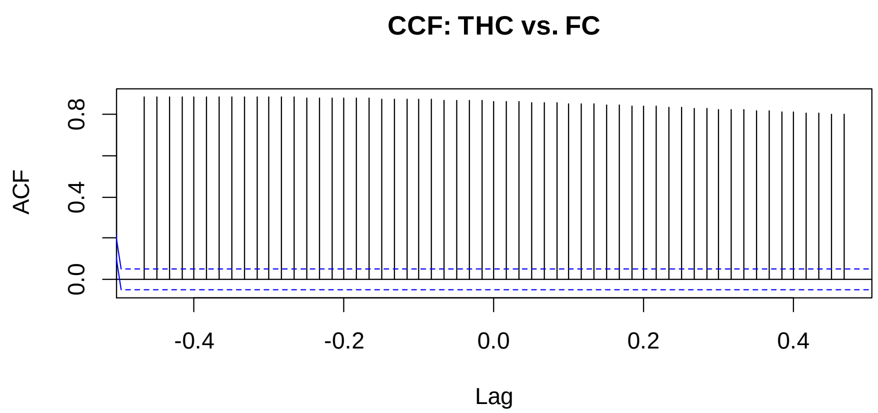


Figure 3 Relationship between fundamental and triplen harmonic current: Scatter plot and CCF

b) Linear Regression

At this step, THC is regressed on FC. The following is the result in R:

```
> fit = dynlm(THC ~ FC)
> summary(fit)

Time series regression with "ts" data:
Start = 0(2), End = 24(1)

Call:
dynlm(formula = THC ~ FC)

Residuals:
    Min      1Q  Median      3Q     Max
-2.6505 -0.8207 -0.3154  0.9468  2.3925

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.3970410  0.2253501   1.762   0.0783 .
FC          0.0619922  0.0009583  64.692   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 1.06 on 1438 degrees of freedom Multiple R-squared: 0.7443, Adjusted R-squared: 0.7441 F-statistic: 4185 on 1 and 1438 DF, p-value: < 2.2e-16

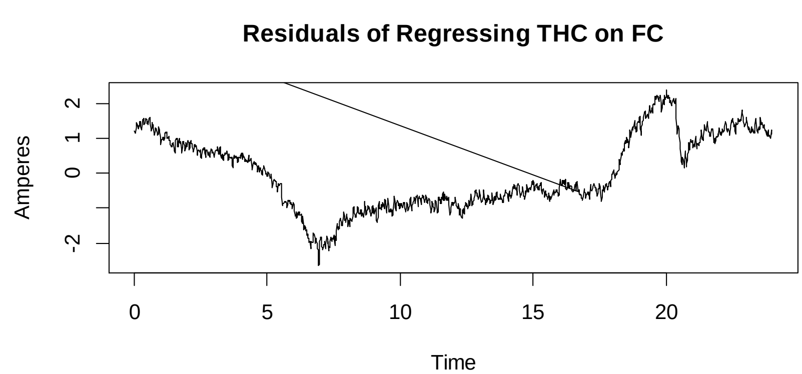
Based on the above result, THC can be modelled as follows:

$$THC_t = 0.062 \times FC_t + 0.397 + \varepsilon_t$$

Where ε_t represents the residuals from this regression.

c) Analysis of Residuals

Once we establish an appropriate model for the residuals, the behaviour of THC series can be thoroughly understood. The series of residuals are plotted on Figure 4. Compared to the original THC series, they are much closer to a stationary condition. This is also observable by comparing ACF of residuals to ACF of original THC. However, it is not yet purely stationary and there is still some trend in it. Figure 5 examines the relationship of these residuals with FC. To an appropriate extent, no further obvious correlation is observed between residuals and FC. Therefore, we have now to seek other methods to make the residuals stationary.



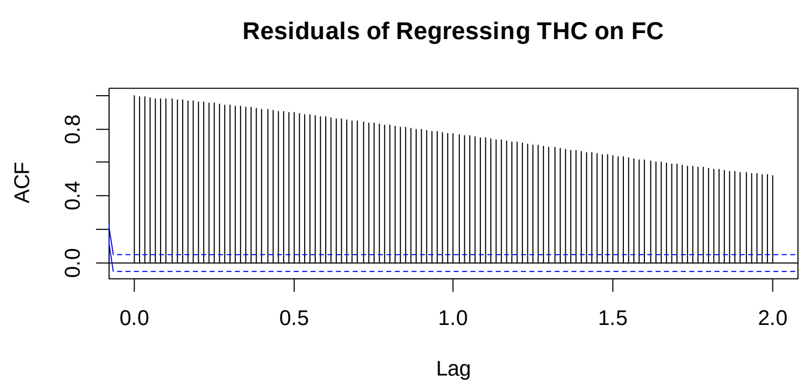


Figure 4 The residual of triplen harmonic current (THC) time series after regressing on fundamental current

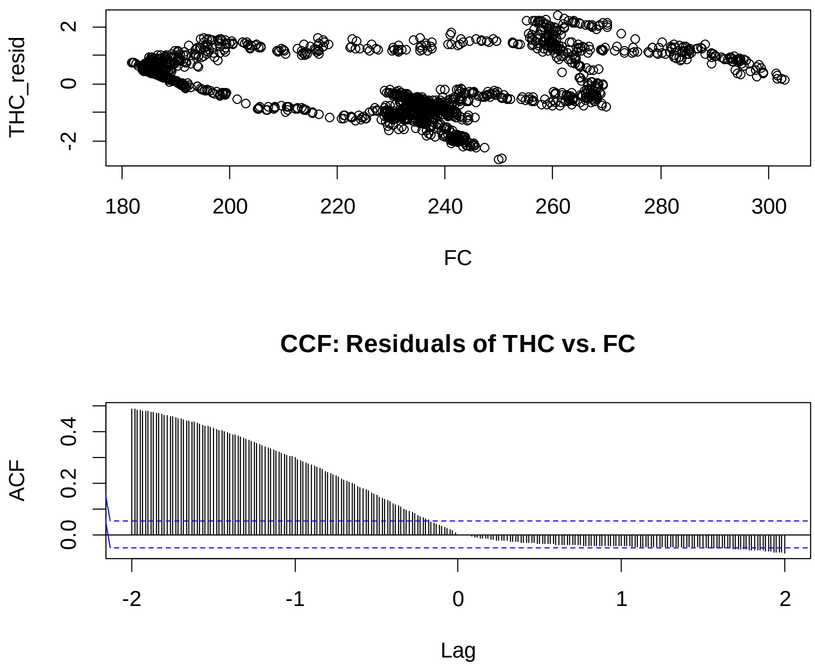


Figure 5 Relationship of residuals and the fundamental current

We try to take the first difference for this purpose. From the electrical engineering perspective, we should not expect the original time series to be stationary, since there might be significant differences in the type and amount of loads during different times of a day. However, intuitively, we can imagine that the changes in the type and amount of the loads might follow a similar trend throughout the day. As observed in Figure 6, after taking the first difference, the series now look very stationary. The differentiated series approximately exhibits a constant mean and variance throughout the time and the ACF gets small very fast too. Therefore, we can start to model residuals in an ARIMA framework.

d) ARIMA modeling

ACF and PACF are computed for the first difference of residuals as shown in Figure 7. Based on this plot, one can propose the following range of ARIMA models:

$$ARIMA(p = 0\sim6, d = 1, q = 0\sim4)$$

Some periodic spikes with approximate period of 8 are also observed in the ACF/PACF graphs (The physical meaning of a period equal to 8 might be the average time by which most of the home appliances change a status). Therefore, we also try to take a seasonal difference (S=8). The resulted ACF/PACF are shown on Figure 8. The seasonal values of PACF tail slowly, while, the seasonal ACF values becomes very small after one or two. Consequently, we propose the following seasonal ARIMA models to be investigated too:

$$ARIMA(p = 0{\sim}2, d = 1, q = 0{\sim}2) \times (P = 0{\sim}2, D = 1, Q = 0{\sim}1)_{S=8}$$

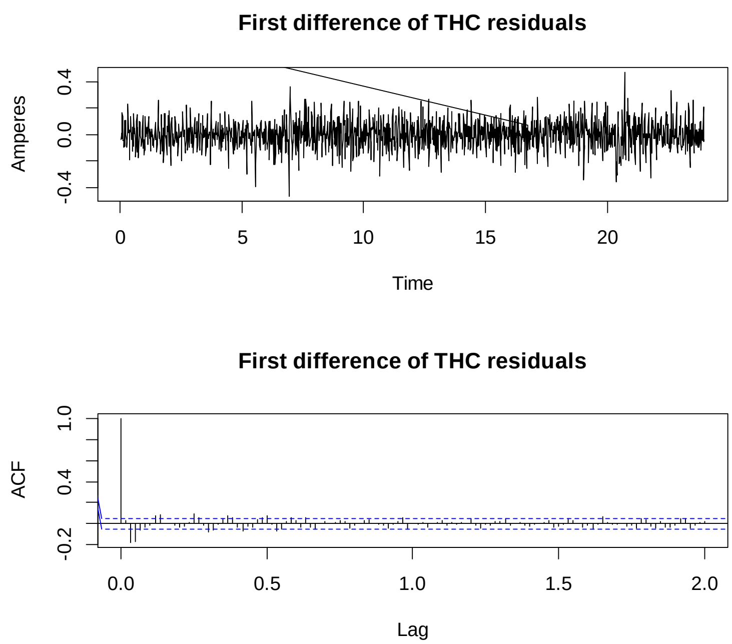


Figure 6 First difference of the residuals of THC (time series and ACF)

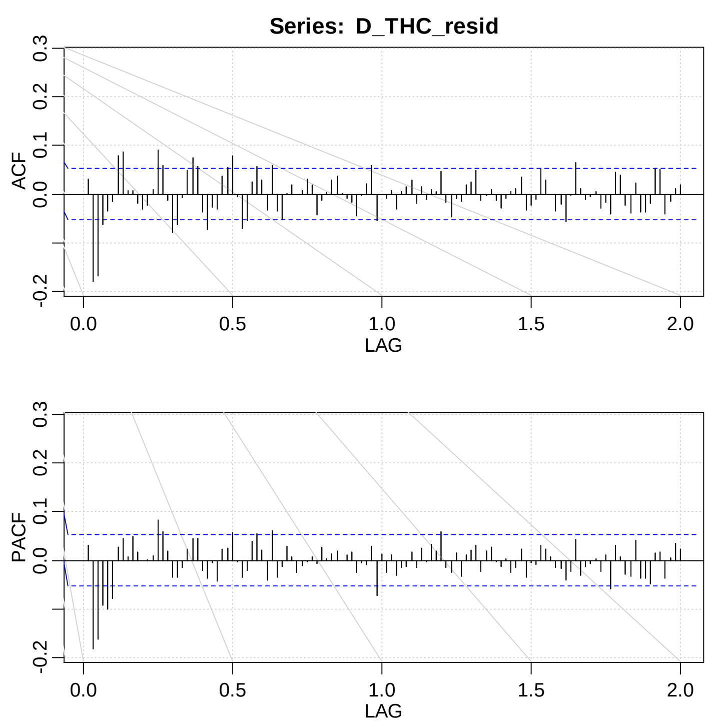


Figure 7 ACF and PACF computed after taking first difference of residuals

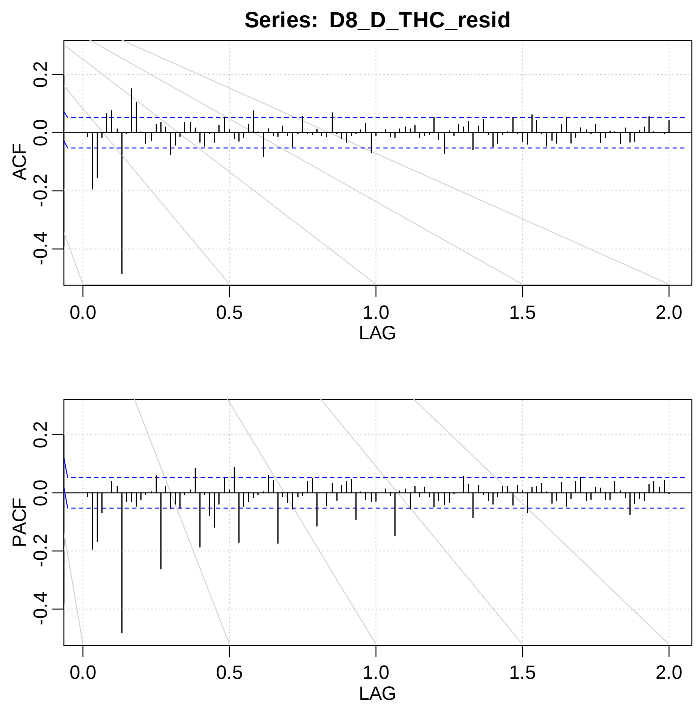


Figure 8 ACF and PACF of residuals after applying both seasonal (S=8) and first difference

All of the proposed possible seasonal and non-seasonal ARIMA models are investigated in R. Different model parameters with their corresponding Information Criteria indices are tabulated in Table I. In general, non-seasonal models have appeared to be better models based on these indices. Two non-seasonal models are chosen as the best candidates, based on minimum BIC, AIC & AICc. These two models are investigated separately in R. Figures 9 and 10 show the results for the models selected based on minimum BIC and AIC/AICc, respectively. Both models somehow perform at the same level according to the normality test of residuals. Hence, based on principle of parsimony, we chose the model highlighted by minimum BIC, since it has one less parameter:

$$\varepsilon_t : ARIMA(p = 5, d = 1, q = 2)$$

Table I. Information Criteria Indices obtained for different ARIMA models

No.	p	d	q	P	D	Q	S	AIC	AICc	BIC
1	0	1	0	-	-	-	-	-3.513481	-3.512086	-4.50982
2	0	1	1	-	-	-	-	-3.513613	-3.512212	-4.50629
3	0	1	2	-	-	-	-	-3.552696	-3.551288	-4.541712
4	0	1	3	-	-	-	-	-3.582675	-3.581257	-4.56803
5	0	1	4	-	-	-	-	-3.582064	-3.580634	-4.563757
6	1	1	0	-	-	-	-	-3.513087	-3.511686	-4.505764
7	1	1	1	-	-	-	-	-3.516831	-3.515423	-4.505847
8	1	1	2	-	-	-	-	-3.575828	-3.57441	-4.561182
9	1	1	3	-	-	-	-	-3.581849	-3.580419	-4.563542
10	1	1	4	-	-	-	-	-3.581201	-3.579758	-4.559233
11	2	1	0	-	-	-	-	-3.545104	-3.543696	-4.53412
12	2	1	1	-	-	-	-	-3.584251	-3.582833	-4.569605
13	2	1	2	-	-	-	-	-3.588962	-3.587533	-4.570655
14	2	1	3	-	-	-	-	-3.590539	-3.589095	-4.56857
15	2	1	4	-	-	-	-	-3.579802	-3.578343	-4.554172
16	3	1	0	-	-	-	-	-3.569904	-3.568486	-4.555258
17	3	1	1	-	-	-	-	-3.585848	-3.584419	-4.567541
18	3	1	2	-	-	-	-	-3.592263	-3.59082	-4.570295
19	3	1	3	-	-	-	-	-3.591288	-3.589829	-4.565658
20	3	1	4	-	-	-	-	-3.58902	-3.587543	-4.559729
21	4	1	0	-	-	-	-	-3.577078	-3.575648	-4.558771
22	4	1	1	-	-	-	-	-3.585002	-3.583559	-4.563034
23	4	1	2	-	-	-	-	-3.591236	-3.589777	-4.565606
24	4	1	3	-	-	-	-	-3.589927	-3.588451	-4.560636
25	4	1	4	-	-	-	-	-3.59225	-3.590755	-4.559298
26	5	1	0	-	-	-	-	-3.585586	-3.584143	-4.563617
27	5	1	1	-	-	-	-	-3.587193	-3.585734	-4.561563
28	5	1	2	-	-	-	-	-3.601468	-3.599991	-4.572176
29	5	1	3	-	-	-	-	-3.603609	-3.602113	-4.570656
30	5	1	4	-	-	-	-	-3.589873	-3.588355	-4.553259
31	6	1	0	-	-	-	-	-3.590249	-3.58879	-4.564619
32	6	1	1	-	-	-	-	-3.589188	-3.587711	-4.559897
33	6	1	2	-	-	-	-	-3.589051	-3.587556	-4.556099
34	6	1	3	-	-	-	-	-3.602316	-3.600799	-4.565702
35	6	1	4	-	-	-	-	-3.600843	-3.599302	-4.560568
36	0	1	0	0	1	0	8	-2.91177	-2.910379	-3.91177
37	0	1	0	0	1	1	8	-3.507207	-3.505812	-4.503545
38	0	1	0	1	1	0	8	-3.178905	-3.17751	-4.175243
39	0	1	0	1	1	1	8	-3.517053	-3.515653	-4.50973
40	0	1	0	2	1	0	8	-3.246477	-3.245076	-4.239154
41	0	1	0	2	1	1	8	-3.518132	-3.516724	-4.507148

No.	p	d	q	P	D	Q	S	AIC	AICc	BIC
42	0	1	1	0	1	0	8	-2.910611	-2.909217	-3.90695
43	0	1	1	0	1	1	8	-3.507847	-3.506447	-4.500525
44	0	1	1	1	1	0	8	-3.177669	-3.176269	-4.170346
45	0	1	1	1	1	1	8	-3.516702	-3.515294	-4.505718
46	0	1	1	2	1	0	8	-3.245215	-3.243807	-4.234231
47	0	1	1	2	1	1	8	-3.517537	-3.516119	-4.502891
48	0	1	2	0	1	0	8	-2.959292	-2.957892	-3.951969
49	0	1	2	0	1	1	8	-3.546196	-3.544788	-4.535212
50	0	1	2	1	1	0	8	-3.215406	-3.213998	-4.204422
51	0	1	2	1	1	1	8	-3.55729	-3.555872	-4.542645
52	0	1	2	2	1	0	8	-3.288274	-3.286856	-4.273629
53	0	1	2	2	1	1	8	-3.557498	-3.556068	-4.539191

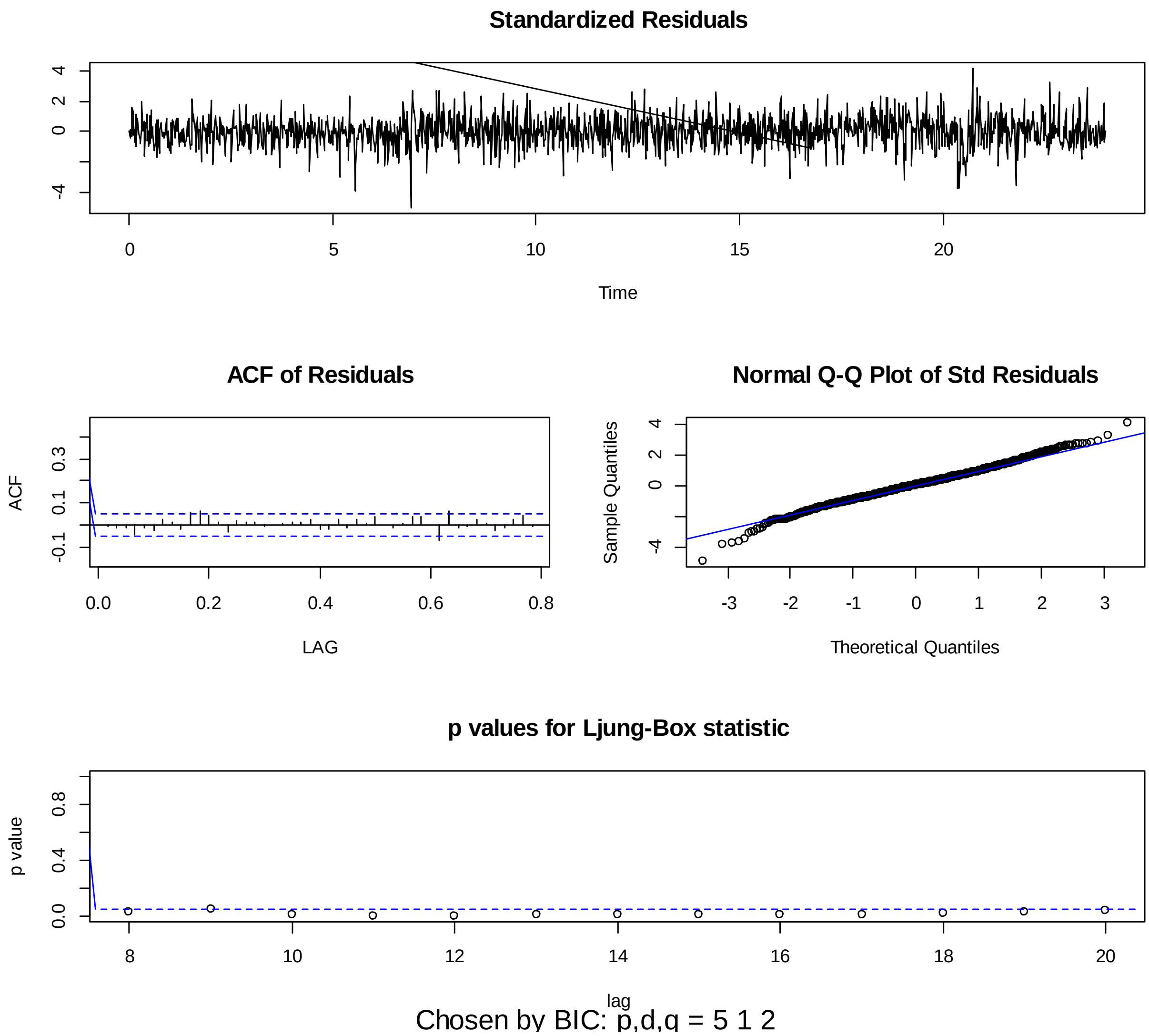


Figure 9 The chosen ARIMA model by BIC: (p=5,d=1,q=2)

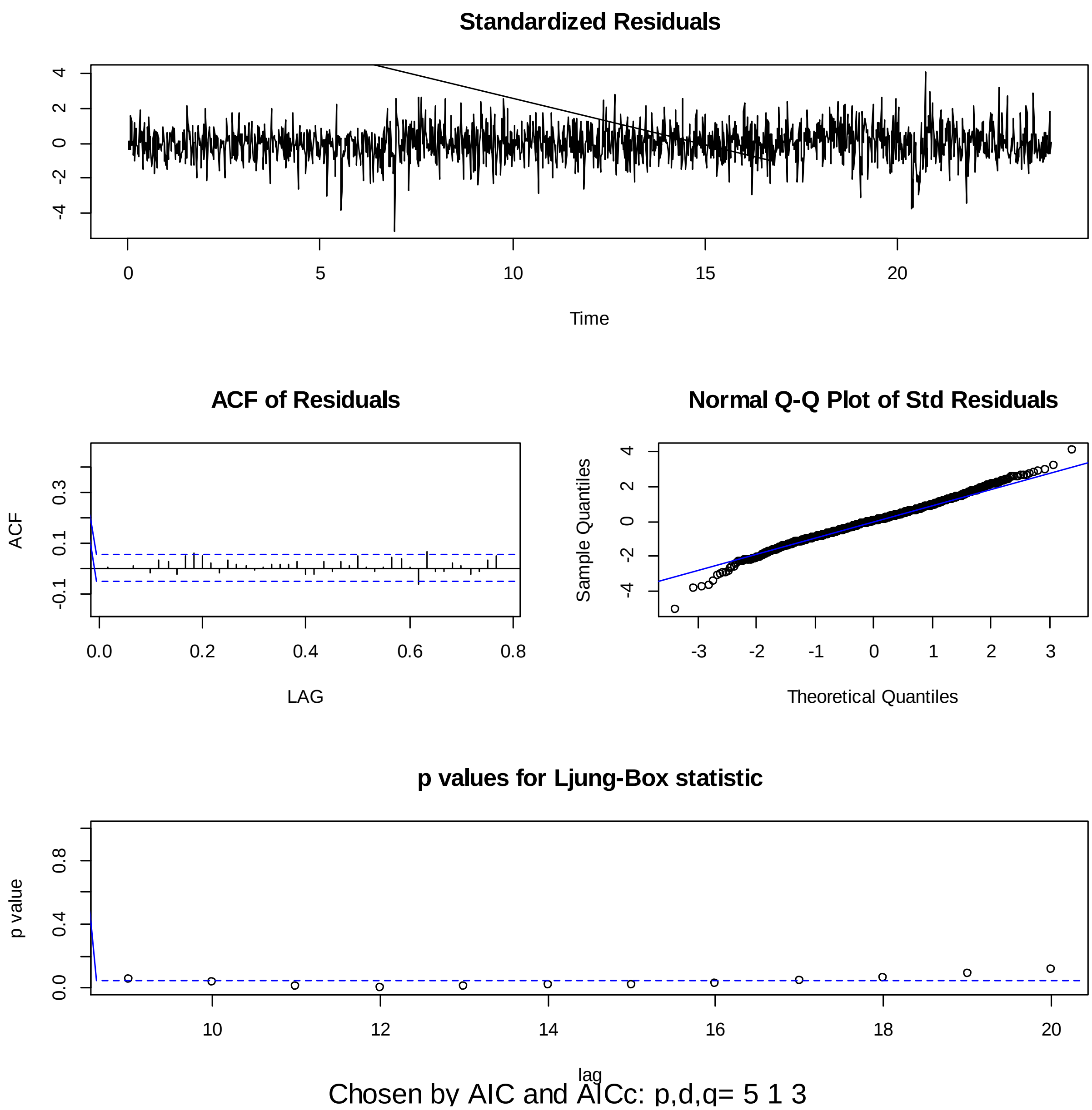


Figure 10 The chosen ARIMA model by AIC & AICc: (p=5,d=1,q=3)

e) The Overall Model

Based on the works of previous sections, the following is the overall model for THC time series:

$$THC_t = 0.062 \times FC_t + 0.397 + \varepsilon_t$$

Where ε_t itself is an ARIMA process as follows:

$$(-.162B^5 - .023B^4 + .043B^3 - 1.073B^2 + 1.141B + 1)(1 - B)\varepsilon_t = (.91B^2 - 1.164B + 1)\omega_t$$

Where ω_t is white noise with variance of $\sigma_\omega^2 = 0.001$.

IV. Model Evaluation by Forecasting

In this section, the accuracy of developed model is evaluated by applying the forecast method on our data. It is assumed that the last hour of THC time series is not available and this period is forecasted by the FC time series and the model we have established so far. Then, the predicted values are compared against the actual available data, in order to assess our model. First of all, the residuals portion in the model (i.e. ε_t) is forecasted by R codes for the last hour and plotted along actual value as shown on Figure 11. As observed, the actual values lie within the 95% confidence interval very well, indicating that the established ARIMA model has fitted the regression residuals of THC series very well.

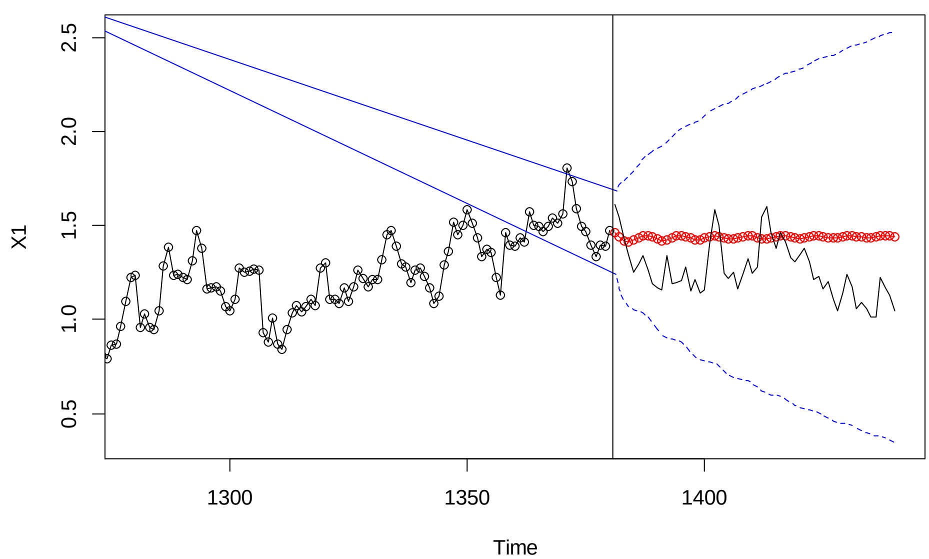


Figure 11 Forecasting the last hour of ε_t (the residual time series) based on the ARIMA model

The rest of the THC time series can be also forecasted by using the regression portion of the model. Figure 12 compares the predicted and actual values of the THC time series for the last hour. In overall, the results confirm accuracy of the established model.

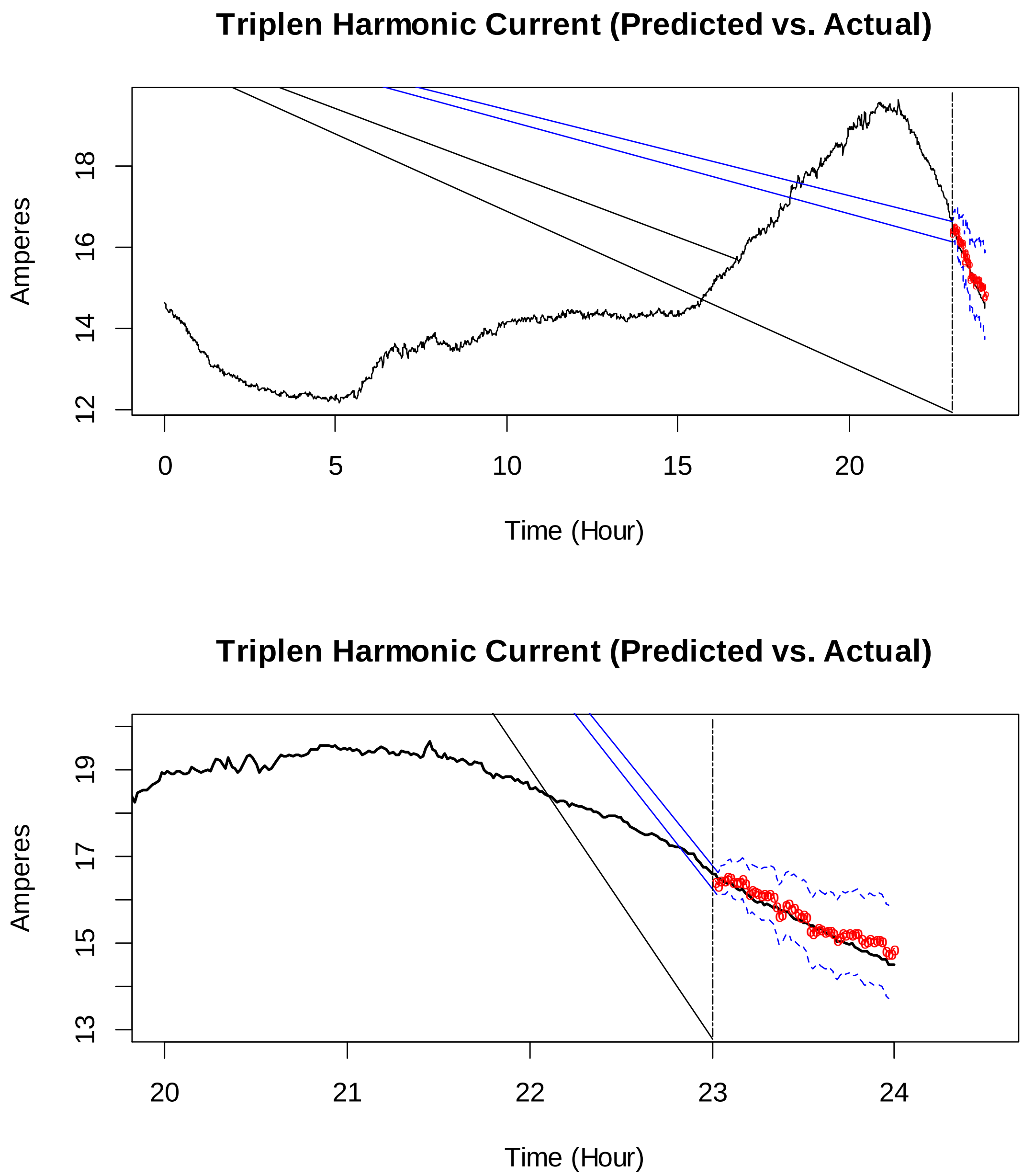


Figure 12 Comparison of predicted and actual value for the last hour of THC time series

V. SUMMARY

In this project, a model was developed to analyze and forecast triplen harmonic current based on its history and fundamental current. Two sample time series were captured from real measurement data to conduct this task. A model was developed by using regression techniques and the basics of ARIMA modeling. The accuracy of the model was also confirmed by comparing forecasted value of the last hour against the actual data. In conclusion, this work has shown the possibility of modeling the so-called triplen harmonic current as a linear function of fundamental current plus a random ARIMA term.

I. REFERENCES

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