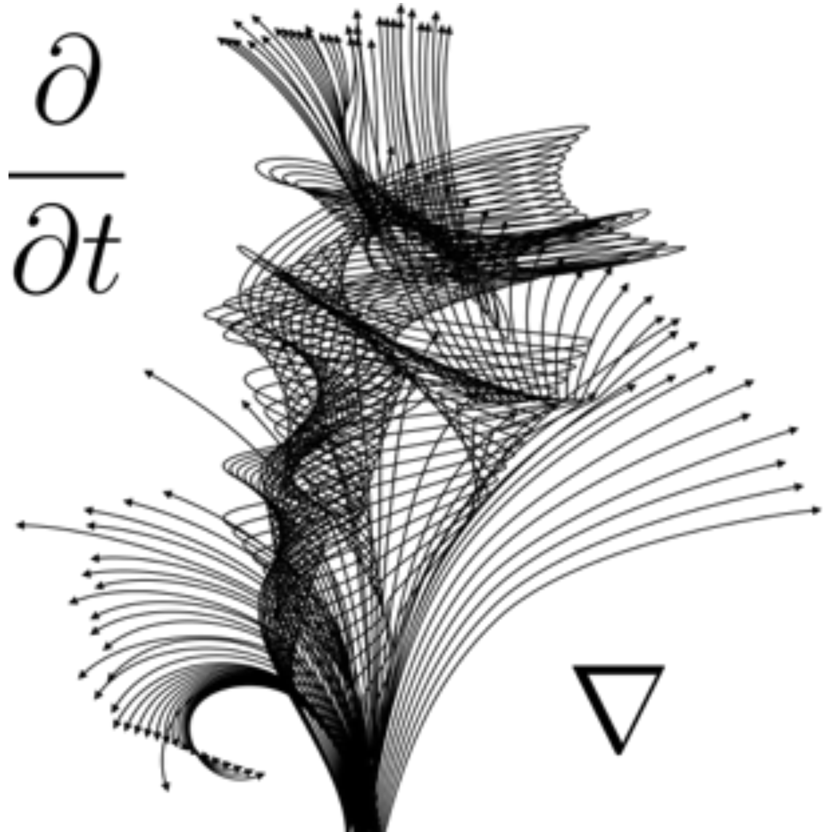
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

Agenda for today:

- Linear differential equations



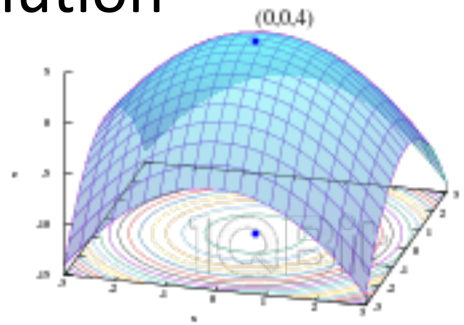
Slope field of the Logistic equation

Question: Is it possible to see from the slope field that the solution to the DE is blowing up to positive or negative infinity?

Answer: No! But we will use the information of the slope of a solution later on to predict this when we discuss Euler's method.

Question: No two solutions were crossing each other. Is this behaviour always true?

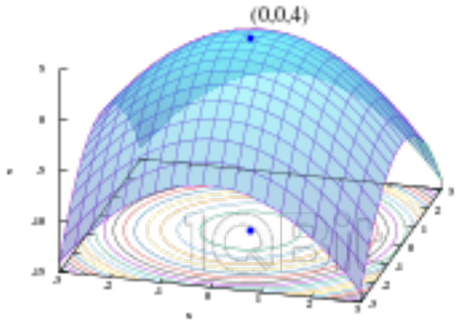
Answer: For DE y' = f(x, y) if f and df/dy are continuous then the solution passing any initial condition is unique.



More on uniqueness

Question: What about $(y')^2 - y' - 2 = 0$?

Answer: This is still not of the form y’ = f(y) even after solving for y’.



More on uniqueness

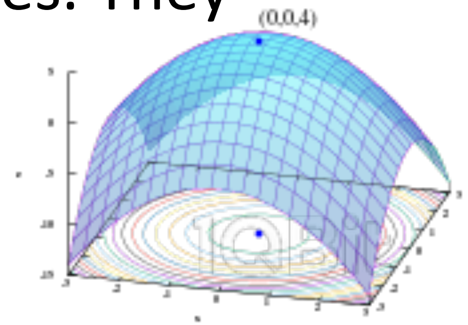
Question: What about $(y')^2 - y' - 2 = 0$?

Answer: This is still not of the form y' = f(y) even after solving for y'.

Question: What about $y' = 3y^{2/3}$?

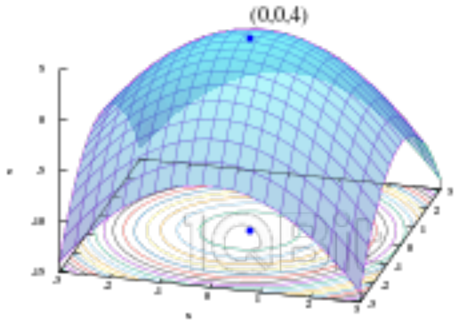
Answer: This is of the form y'= f(y) but df/dy is not continuous!

Note: Even in this case the solutions don't cross with different slopes. They become tangent to each other.



What you should be able to do

- Identify steady states for a DE.
- Draw/interpret the phase line for a DE.
- Draw/interpret a slope field for a DE.
- Determine stability of steady states and long-term behaviour of solutions.
- Sketch the graphs of solutions using phase line and/or slope fields (slopes, concavity, IPs, horizontal asymptotes).

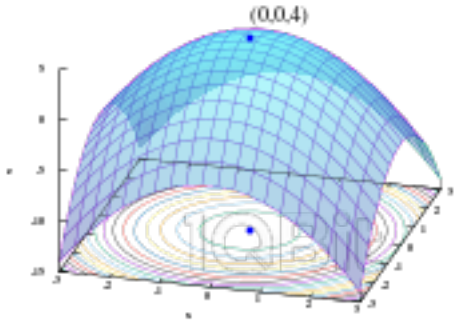


Example

Consider the DE $y' = \sin(y)$

A solution satisfying the initial condition $y(0) = y_0$ will approach y^*as $x \to \infty$. Which one is correct?

$$(A) \quad y_0 = -3\pi/2, y^* = -2\pi.$$
$$(B) \quad y_0 = -\pi/2, y^* = -\pi/2.$$
$$(C) \quad y_0 = \pi/4, y^* = \pi/2.$$
$$(D) \quad y_0 = 3\pi/4, y^* = \pi.$$
$$(E) \quad y_0 = \pi, y^* = 0.$$



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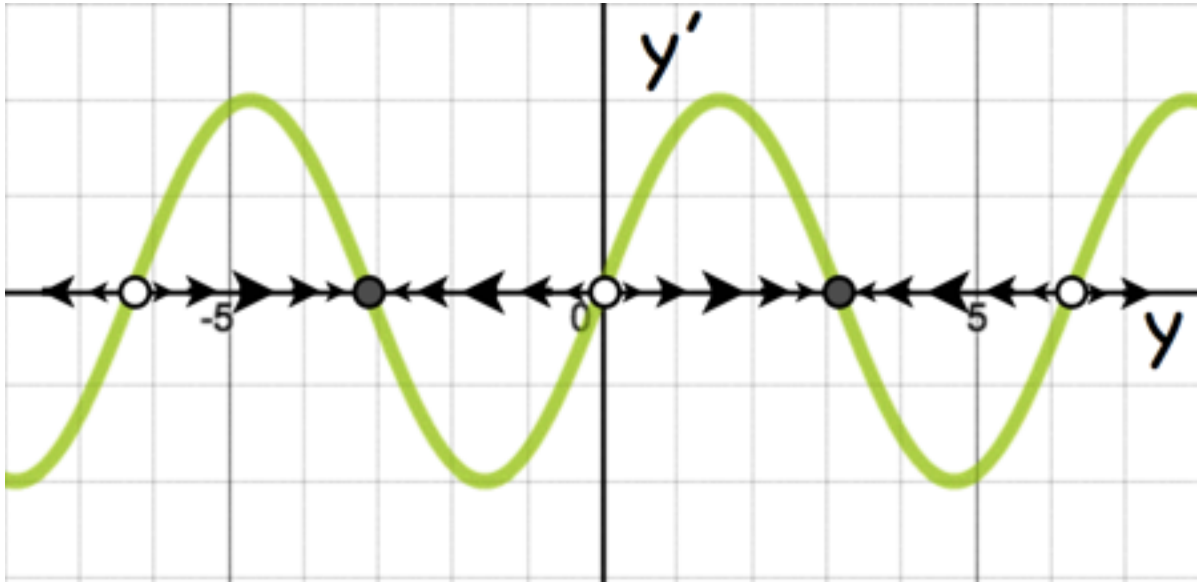
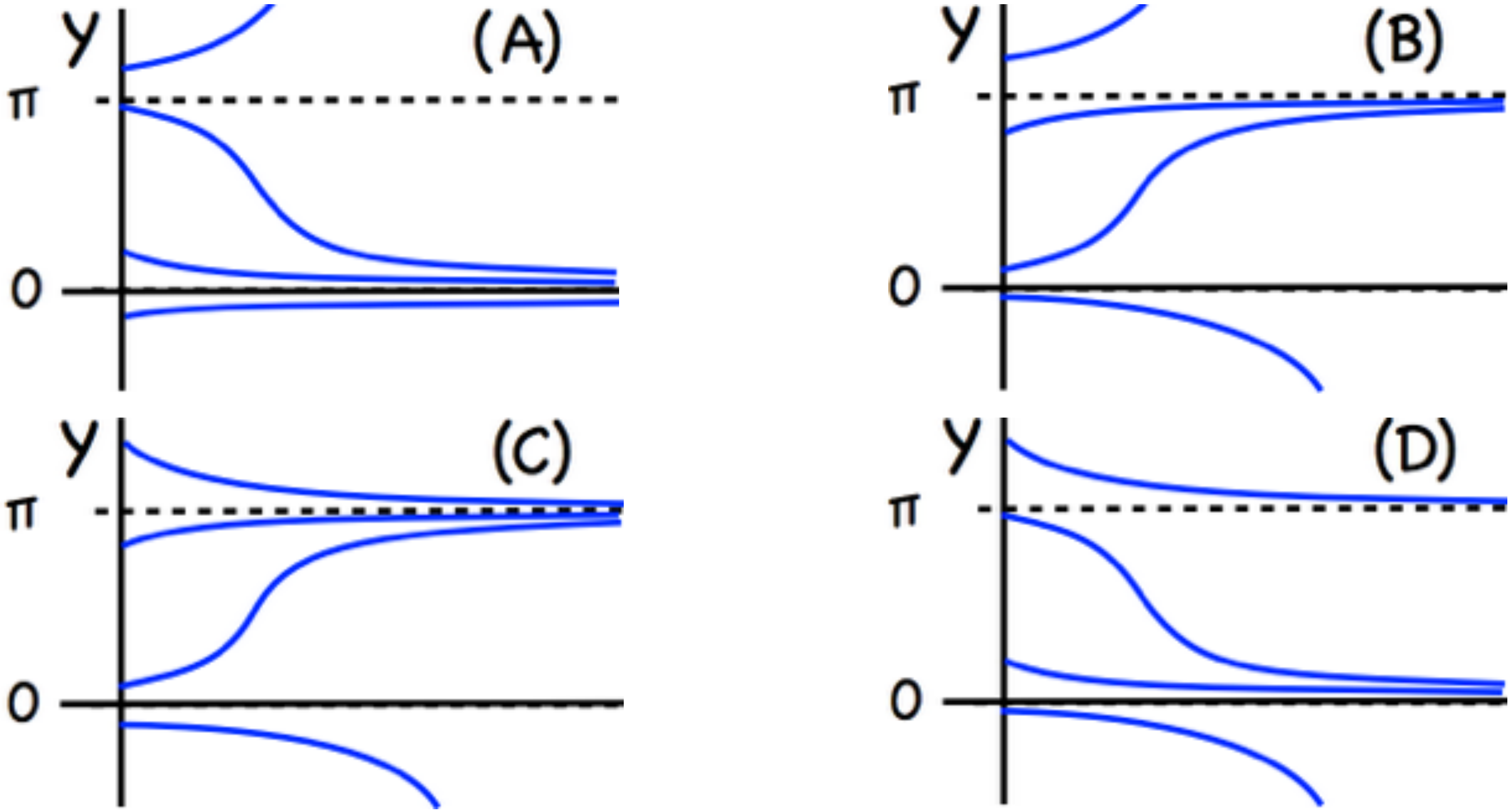
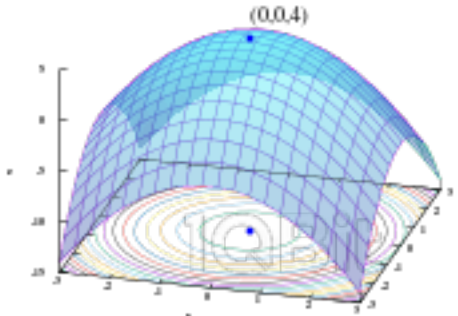


Example

Which one is a better sketch of some of the solutions to $y' = \sin(y)$?

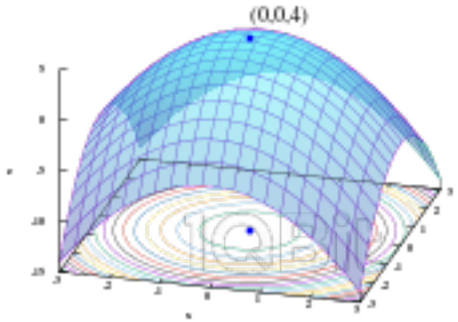




Quiz 3

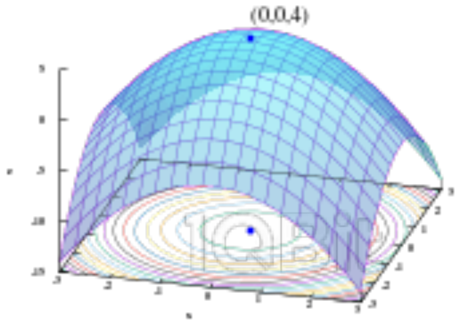
Everything up to here!

- Exponential functions and their derivatives
- Inverse functions, the log function and its derivative
- Applications using exponential functions
- Qualitative analysis of ODEs (slope fields, phase lines, steady states, stability)



Newton's law of cooling (revisited)

The rate of change of an object's temperature is proportional to the difference between the objects temperature and the surrounding environment. Find a differential equation that expresses this phenomena.



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My differential equation looks like

$$(A) \quad T'(t) = kT(t)$$

$$(B) \quad T'(t) = E - kT(t)$$

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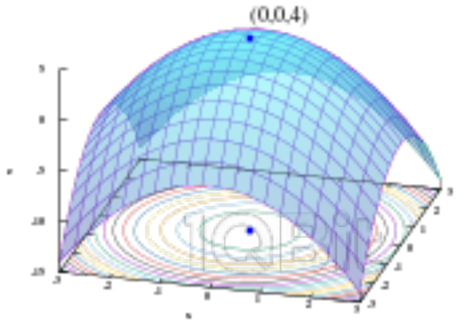

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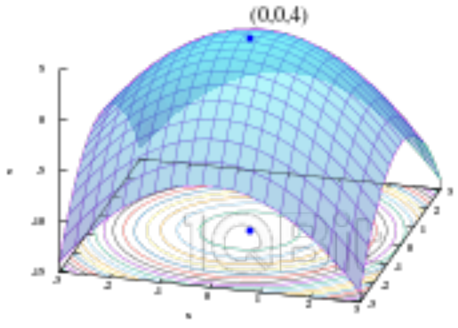

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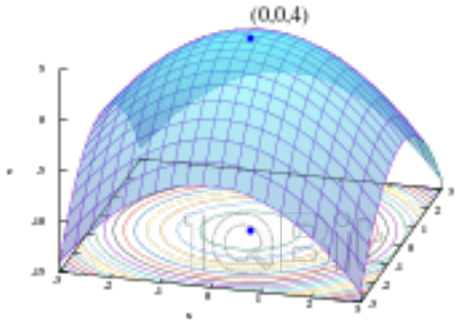
How to solve this? $$T'(t) = k(E - T(t))$$

First of all… What is the unit of k?



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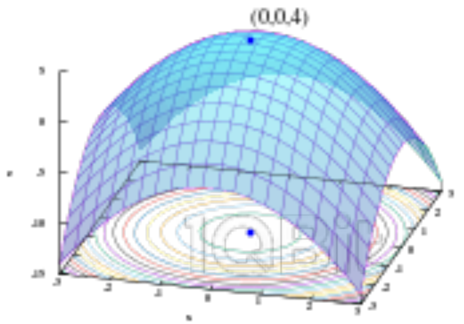
Let's assume the surrounding is water at temperature 273K. What is the DE expressing this same phenomena is temperature was measures in centigrades?



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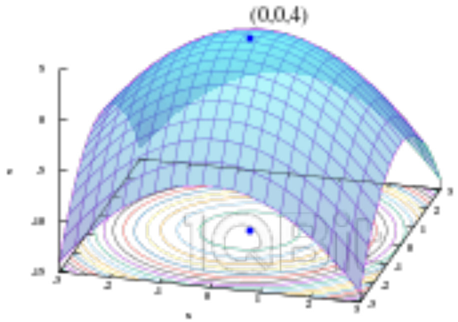

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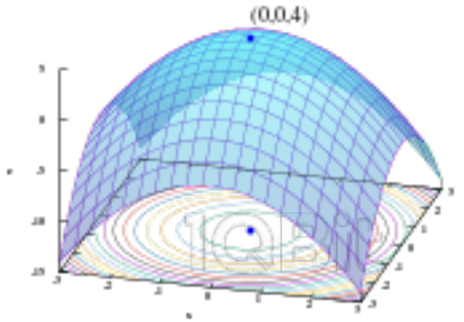

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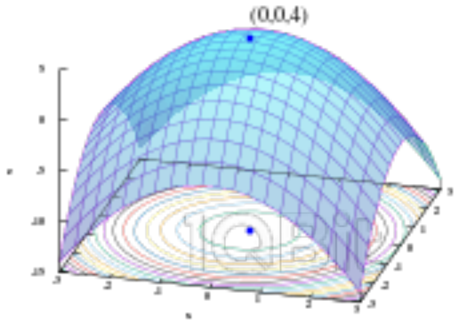
And we can convert back to Kelvin: $$T(t) = (T_0 - E)e^{-kt} + E$$



See you 9:30AM Thursday for QUIZ 3

Nov 16 PL11.2