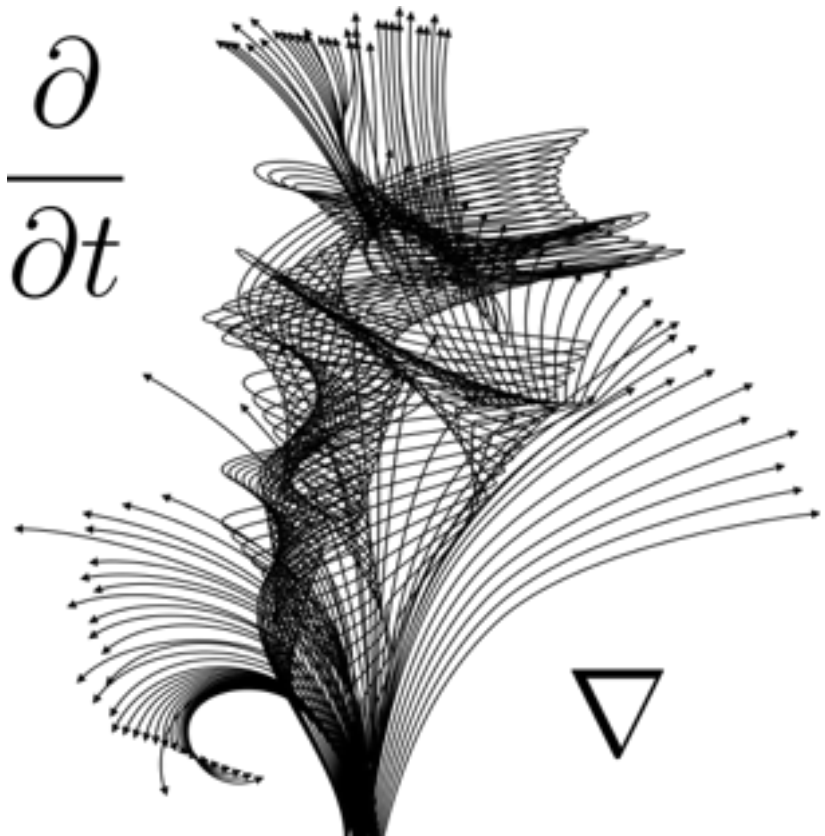
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

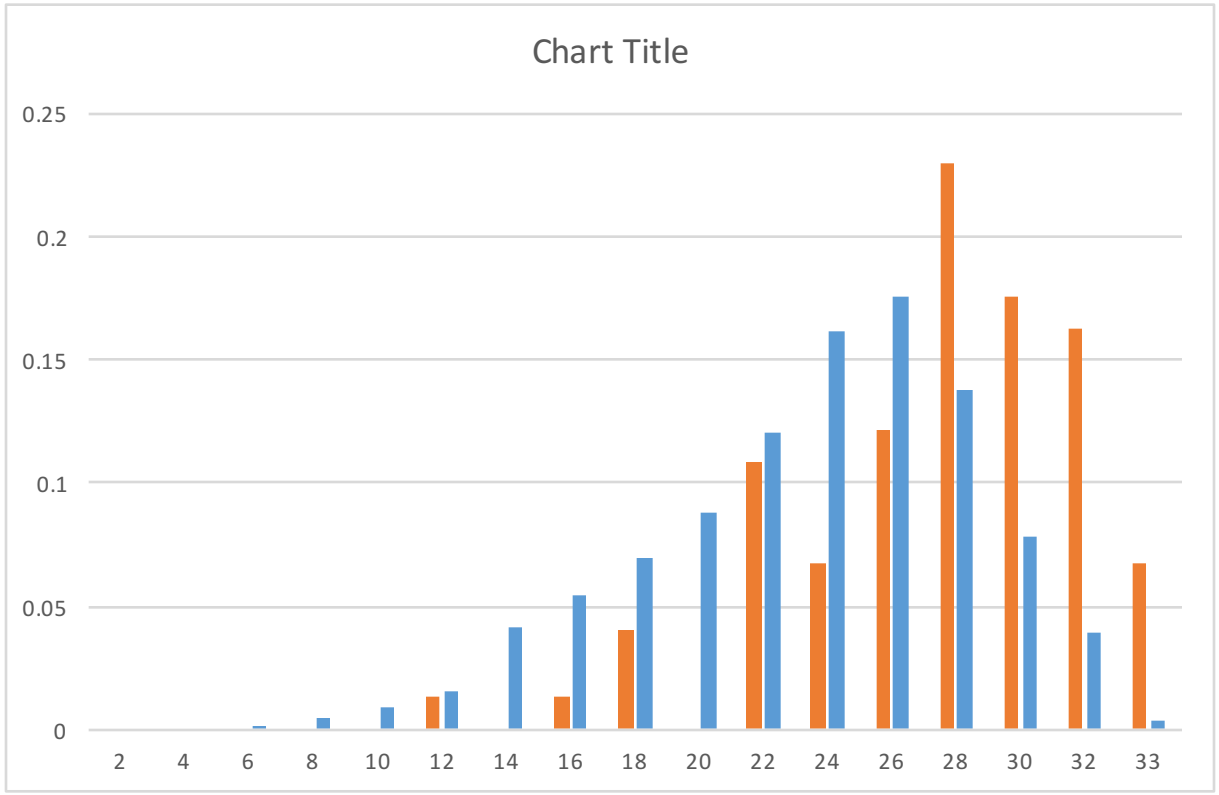
Agenda for today:

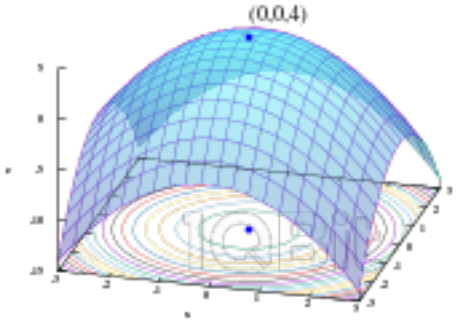
- Midterm
- SSR
- Related rates



Great job on the midterm =)

Will be handed to you next session unless you received it from Crowdmark.





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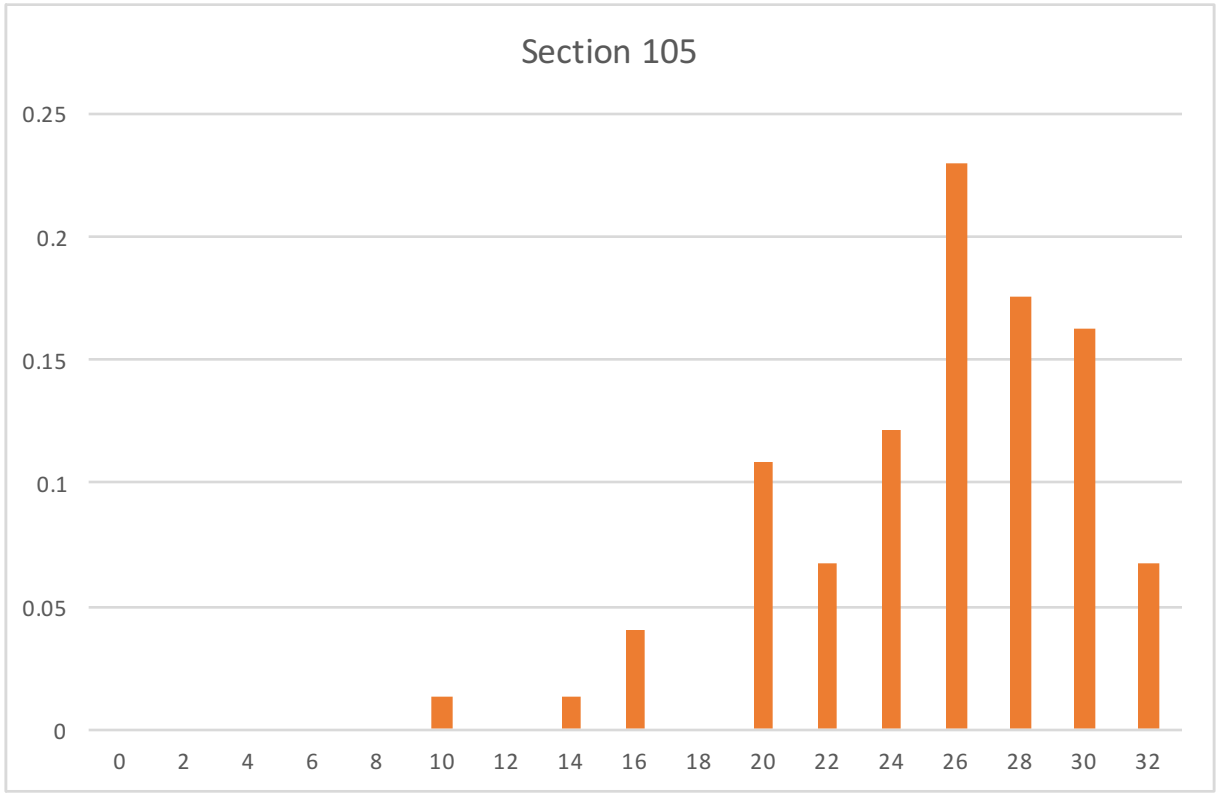

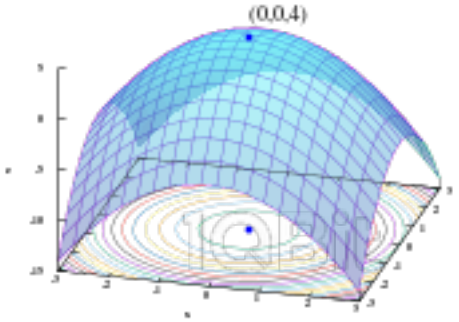

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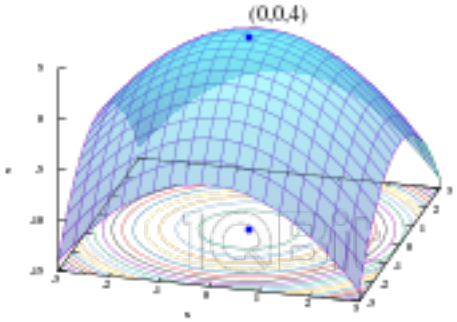
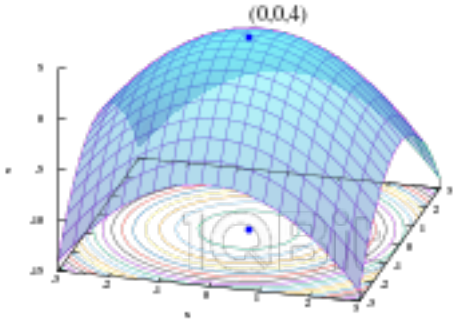
	Page 2	Page 3	Page 4	Page 5	Total
Section	5.973	3.405	9.608	6.000	24.986
Overall	5.318	2.745	9.046	5.854	22.963



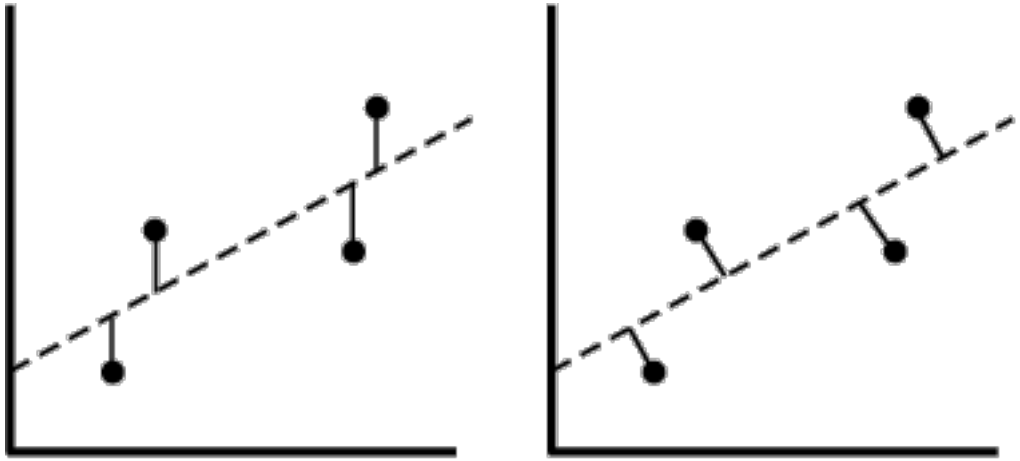
Another optimization problem

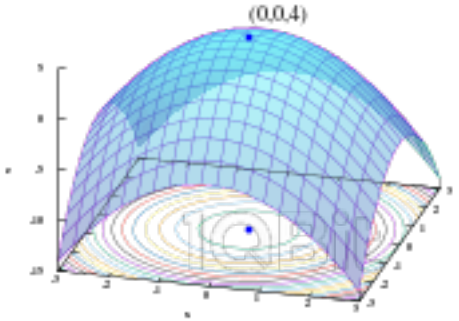
A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. At what time are the two boats closest to each other?

(A) 2:00PM (B) 2:30PM (C) 3:00PM (D) 3:30PM



Least square model fitting





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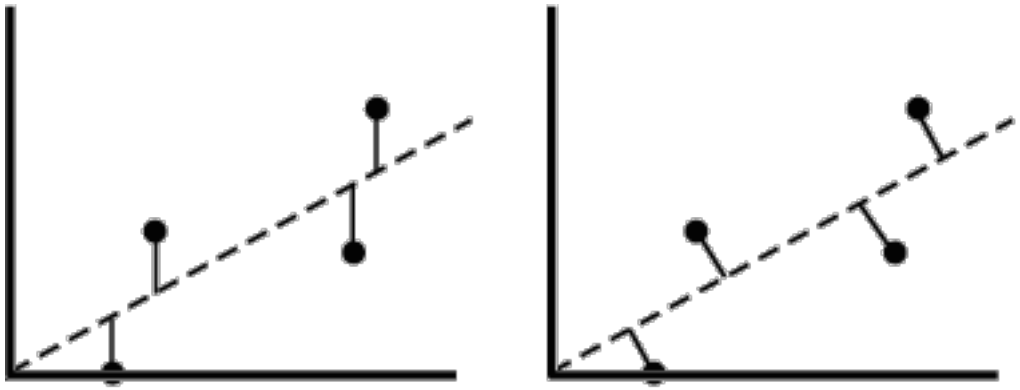
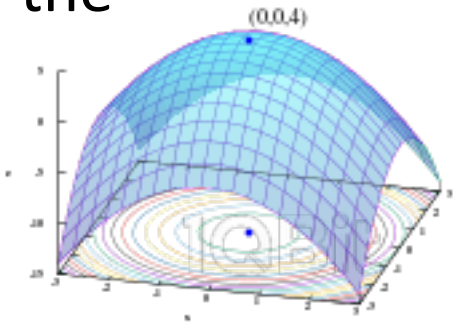


Which one gives a better fitting?

Bonus: what is the objective function to be optimized for SSR when the residual is the perpendicular offset from y = ax?



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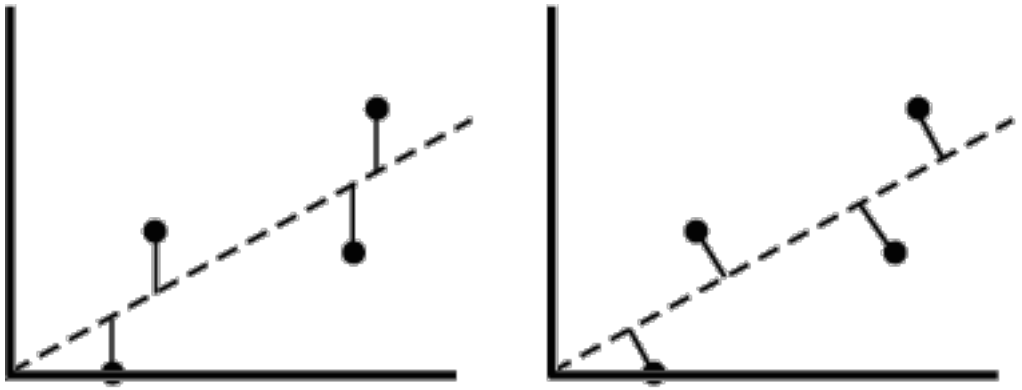
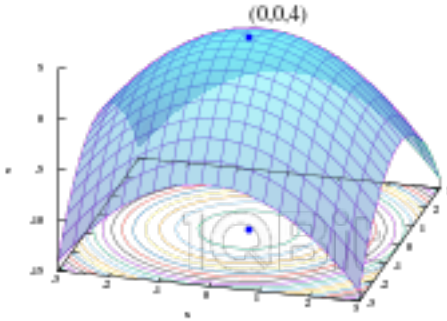


$$\mathrm{SSR}(a) = \sum_{i=1}^{n} (y_i - a x_i)^2 \qquad \mathrm{SSR}(a) = \sum_{i=1}^{n} \frac{(y_i - a x_i)^2}{1 + a^2}$$



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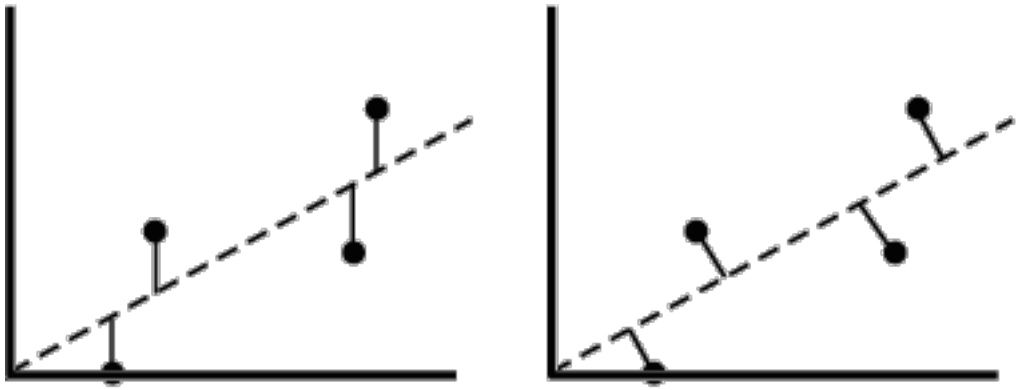
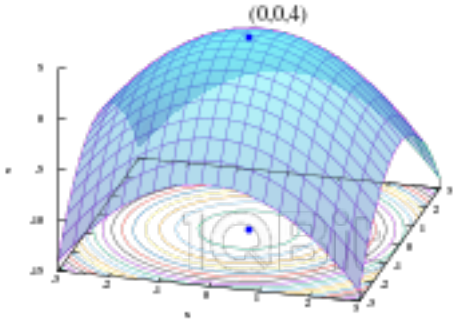


$$a = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$$



Chain rule revisited

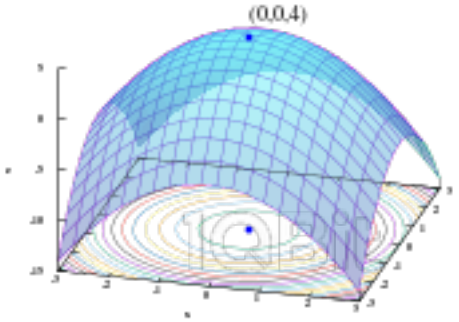
If $h(x) = (x^3 - 2x + 1)^6$ then the derivate is...

$(A) \quad 6(x^3 - 2x + 1)^5$

$(B) \quad (x^3 - 2x + 1)^6(3x^2 - 2)$

$(C) \quad 6(x^3 - 2x + 1)^5(3x^2 - 2)$

$(D) \quad 6(x^3 - 2x + 1)^5(x^3 - 2x + 1)$



Related rates: Intro

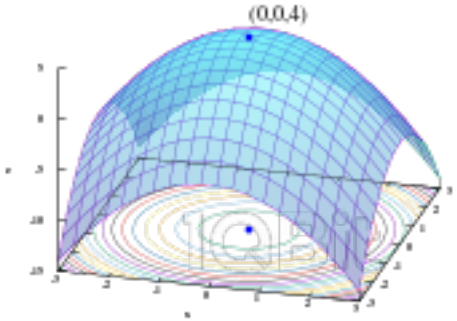
Gas costs $1.25/litre. Your car consumes 7litres/100 km. You've driven 130 km. How much does it cost to drive one more km?

(A) $1.25 \cdot (7/100) \cdot 130$

(B) $1.25 \cdot (7/100)$

(C) $125/7$

(D) $7/125$



Related rates: Intro

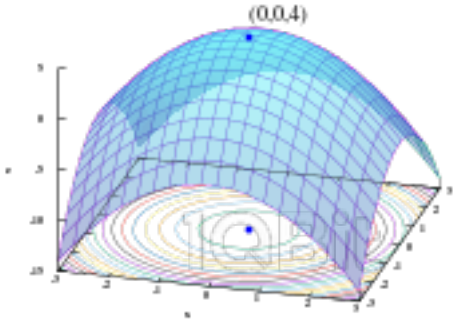
In terms of related rates and by using the chain rule:

Cost of gas is given by $c(\ell) = 1.25\ell$

Consumption of gas is given by $\ell(x) = 0.07x$

Total cost is given by $c(x) = c(\ell(x)) = 1.25(0.07x)$

The slope is given by chain rule according to $c'(x) = c'(\ell(x)).\ell'(x)$



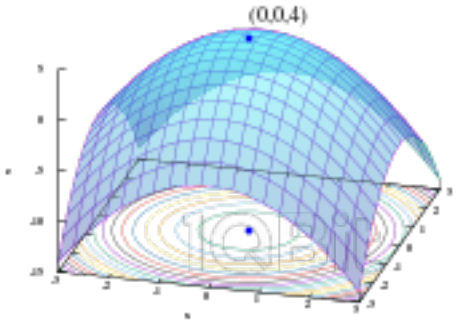
Related rates: Intro

Conclusion: **the fact that we had driven 130km was irrelevant!**

We could have solved the problem by only considering the related rates as follows:

Cost of gas is given by $c = 1.25\ell$

Rate of change of cost is given by $\frac{d}{dx}c = 1.25\frac{d}{dx}\ell = 1.25 \times 0.07$



Another example

The radius of a spherical tumour grows at a constant rate, k. Determine the rate of growth of the volume of the tumour when the radius is one centimetre.

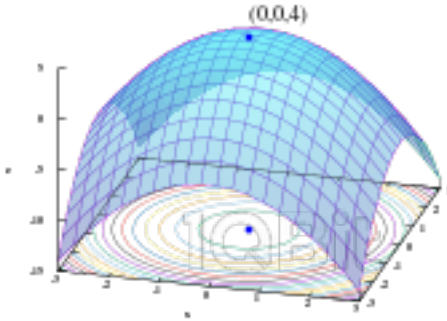
Which one is the useful relation to start from?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2\ k$

(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi$



Another example

The radius of a spherical tumour grows at a constant rate, k. Determine the rate of growth of the volume of the tumour when the radius is one centimetre.

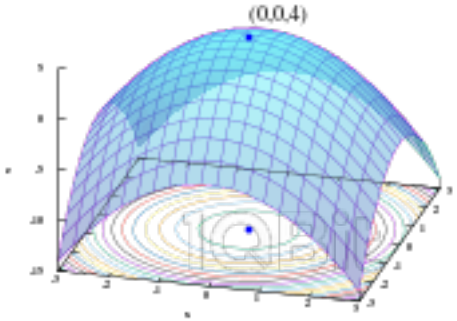
Which one is the correct relation between the related rates?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2\ k$

(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi\ k^3$



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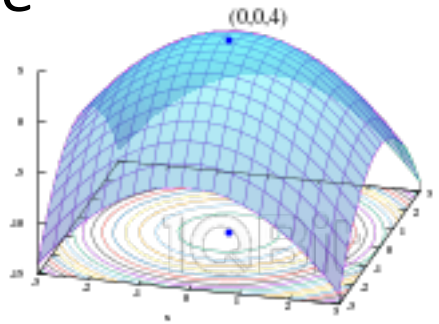
(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi\ k^3$

The fact that the radius of tumour is one was irrelevant in finding the related rates. But we can plug it in!



Good luck with the busy week…

Oct 25	WW 6
Oct 26	PL8.2
Oct 27	WW 7
Oct 28	OSH 4