Differential Calculus with Applications to Life Sciences

Math 102:105

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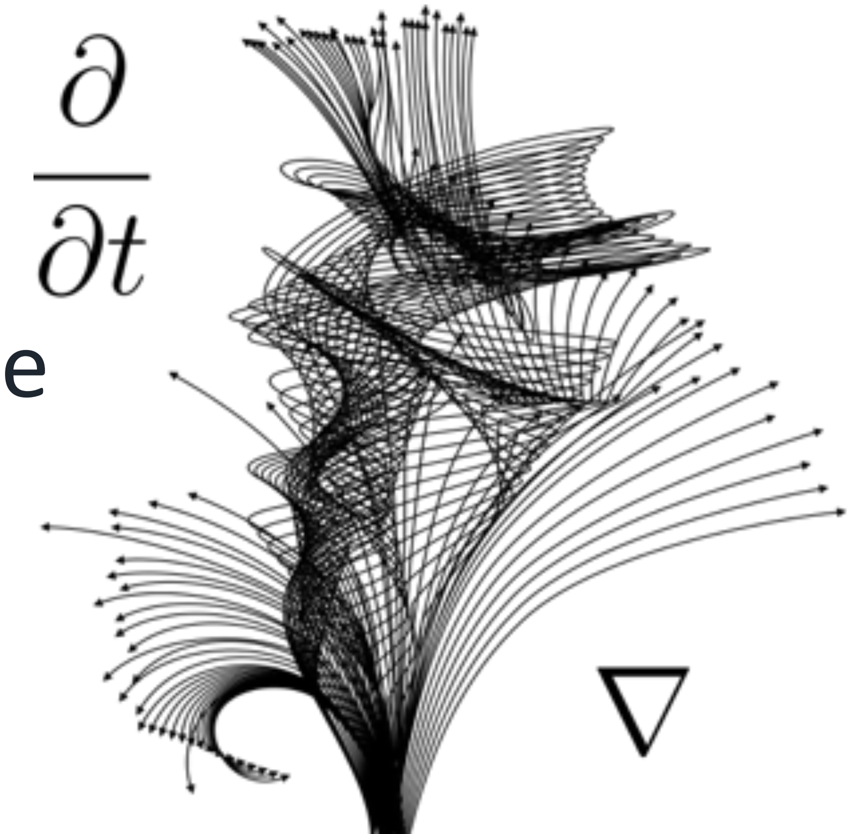
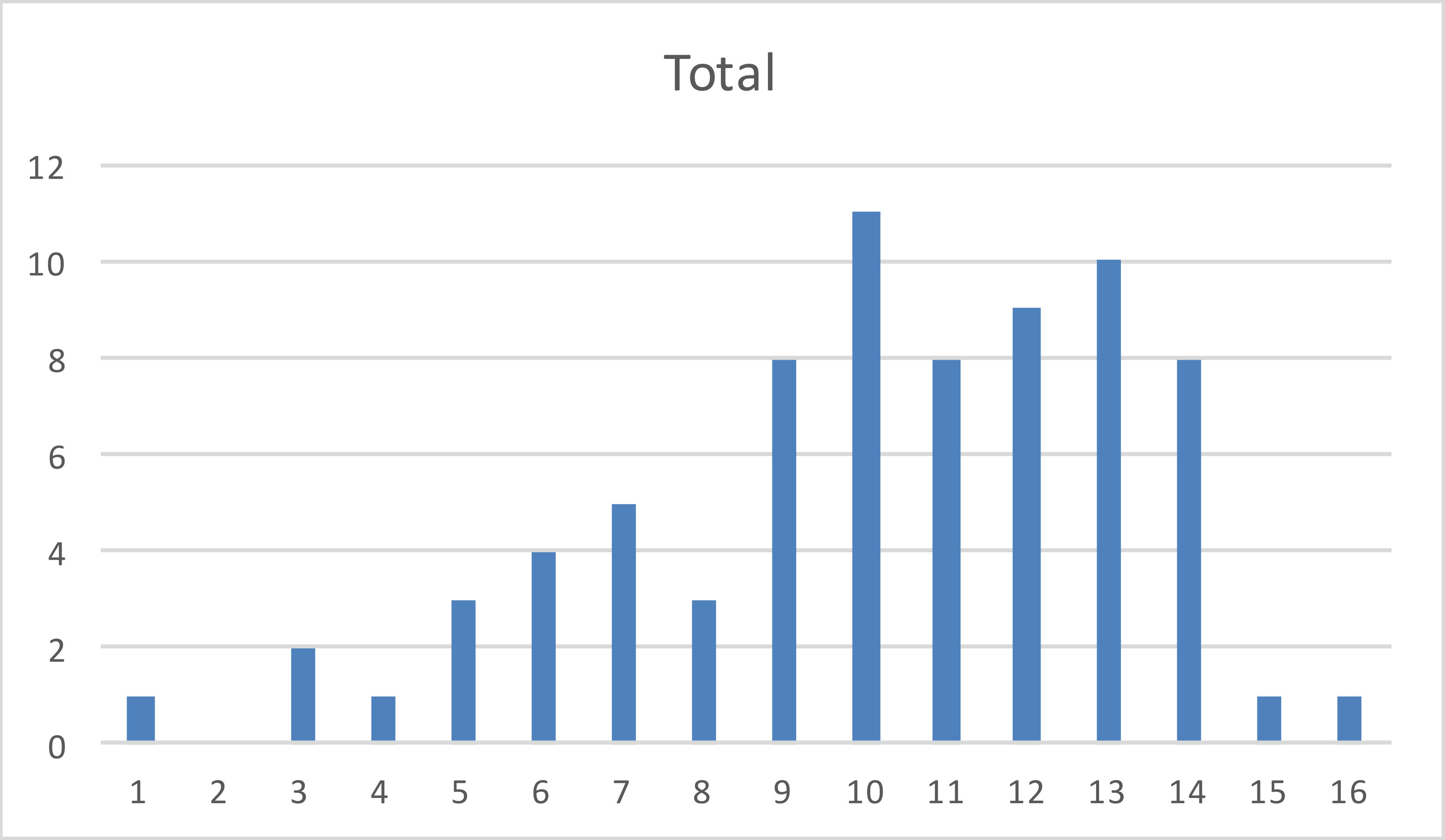
Agenda for today:

- Sketching graphs using the sign table
- Optimization

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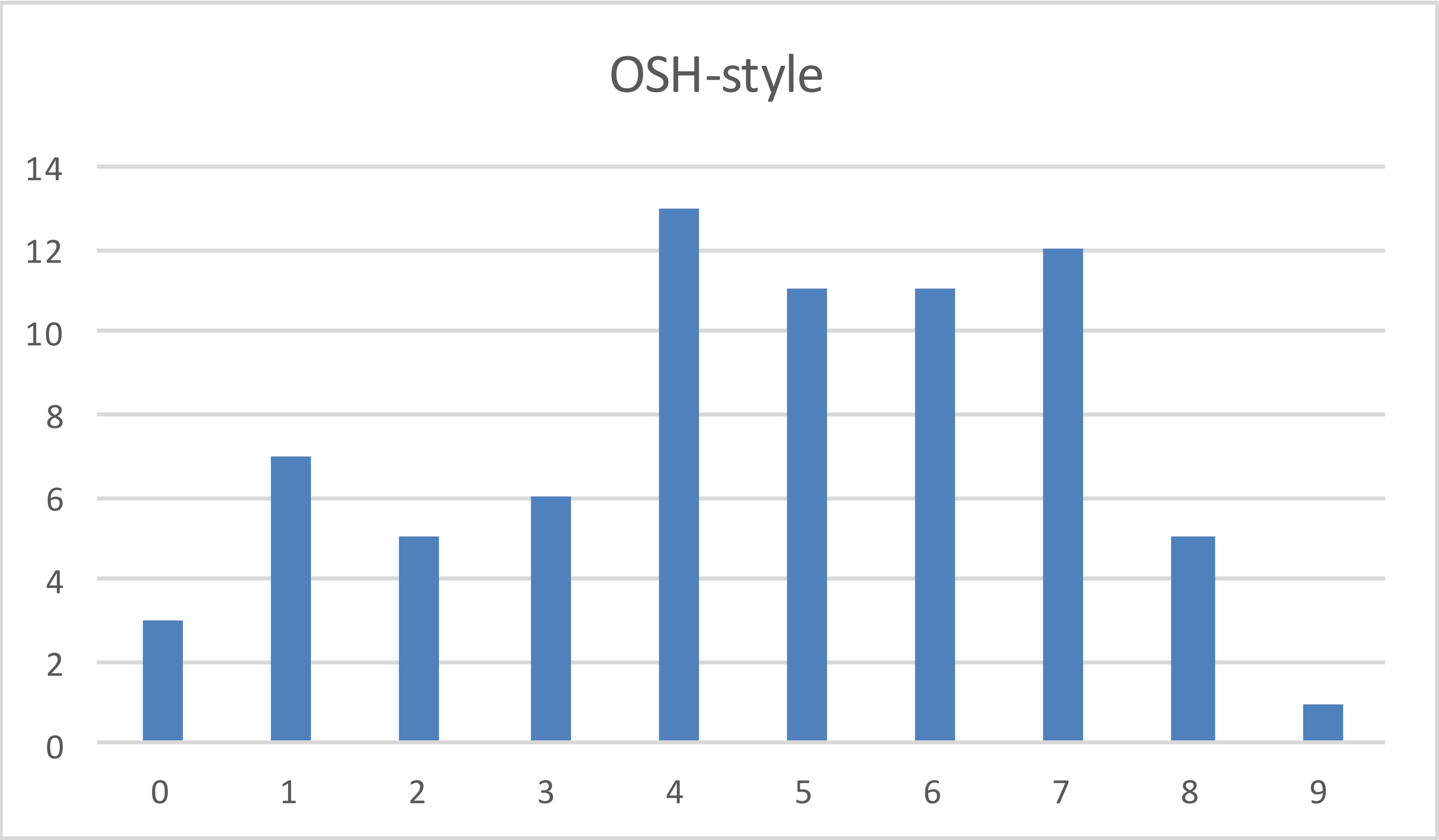
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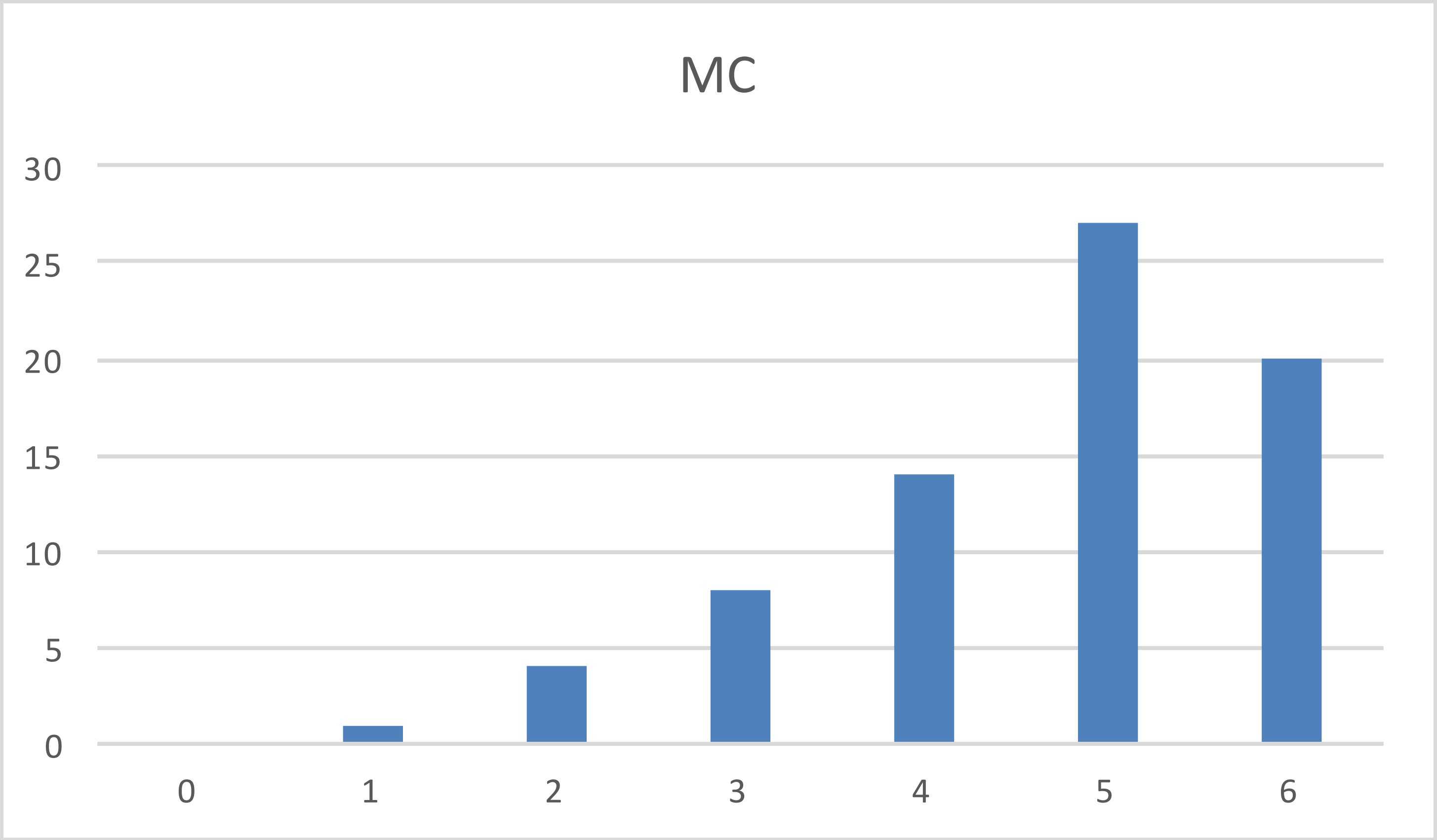
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Last time

Critical points

Test of the first derivative at a critical point

Convexity

Test of the second derivative at a critical point

First derivative test

A critical point x=a is a local extremum when f'(x) changes sign at x=a.

- If f'(x) goes from - to 0 to + then x=a is a min of f(x).
- If f'(x) goes from + to 0 to - then x=a is a max of f(x).
- Note: if the sign of f'(x) goes from - to 0 to - or from + to 0 to + then x=a is NOT a local extremum (eg: f(x)= x^3)

Convexity

We say a function is concave/convex up on some interval if f'(x) is increasing on that interval.

When f"(x) exists, same as f"(x)>0.

We say a function is concave/convex down on some interval if f'(x) is decreasing on that interval.

When f"(x) exists, same as f"(x)<0.

Second derivative test

If $f'(x)$ is differentiable at $x=a$, then the critical point $x=a$ is a local extremum when $f''(a) \neq 0$.

- If $f''(a) > 0$, then $f'(x)$ goes from - to 0 to + so $x=a$ is a min of $f(x)$.

- If $f''(a) < 0$, then $f'(x)$ goes from + to 0 to - so $x=a$ is a max of $f(x)$.

- Note: if $f''(x)=0$ at the critical point the test is inconclusive
- Example: $f(x)= x^3$ and $g(x) = x^4$

Question

$$f(x) = x^5 - x^3$$

(A) a maximum at x=0 and a minimum at x= $\sqrt{3/5}$

(B) a minimum at x=0 and a maximum at x= $\sqrt{3/5}$

(C) no extremum at x=0 and a minimum at x= $\sqrt{3/5}$

(D) a mystery point at x=0 and a minimum at x= $\sqrt{3/5}$

Inflection point

An inflection point of f(x) is a point at which the concavity changes from up to down or down to up.

In terms of derivatives:

A point a is an inflection point of a function f(x) provided that a is a local minimum or a local maximum of f’(x)

Inflection point

So to find an inflection point of y= f(x) we may perform:

(A) First derivative test on f(x)

(B) Second derivative test on f(x)

(C) First derivative test on f’(x)

(D) Second derivative test on f’(x)

Inflection point

So to find an inflection point of $y= f(x)$ we may perform:

(A) First derivative test on $f(x)$ [$f'(a) = 0$, and f' changes sign]

(B) Second derivative test on $f(x)$ [$f'(a) = 0$, $f''(a)$ nonzero]

(C) First derivative test on $f'(x)$ [$f''(a) = 0$, and f'' changes sign]

(D) Second derivative test on $f'(x)$ [$f''(a) = 0$, $f'''(a)$ nonzero]

Sketching a graph - sign table

$$f(x) = 3x^4 - 4x^3$$

Question

Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

See you on Thursday

Oct 13	WW 5
Oct 14	OSH 3