Differential Calculus with Applications to Life Sciences

Math 102:105

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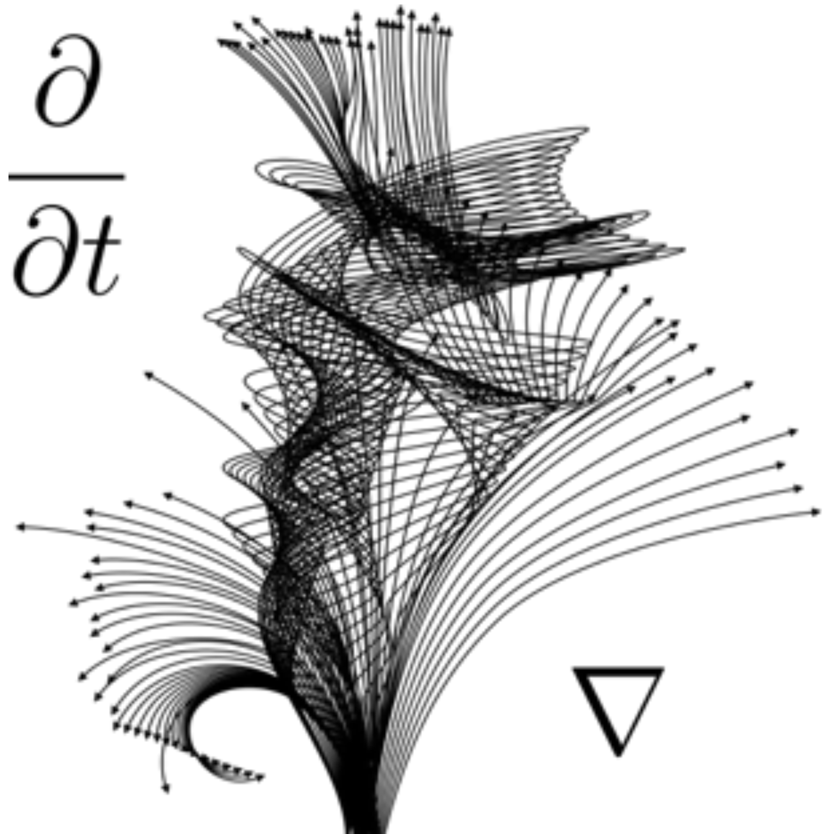
Agenda for today:

- Tangent line (ex)
- Linear approximation (ex)
- Newton’s method (ex)
- Tests of first derivative



Question

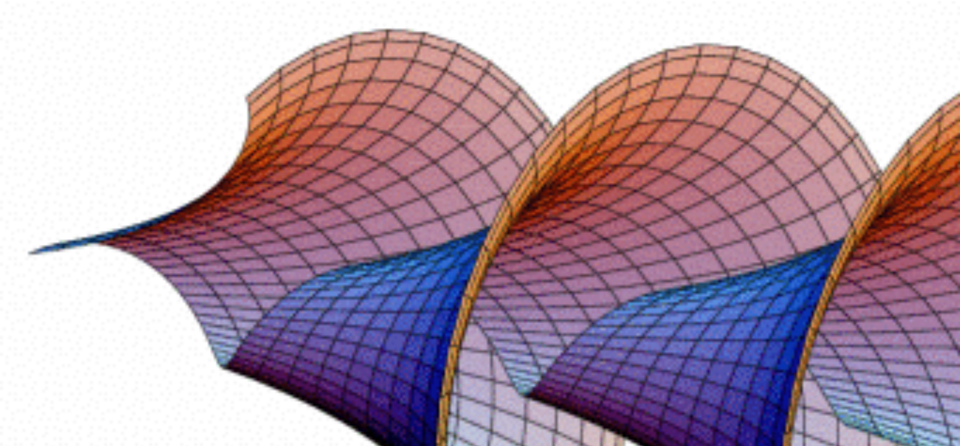
Find tangent line to $f(x) = x^2$ that goes through (1, -1). The point of tangency for this line will be at:

$(A) \quad (1+\sqrt{2}, 3-2\sqrt{2})$

$(B) \quad (1+\sqrt{2}, 3+2\sqrt{2})$

$(C) \quad (1, -1)$

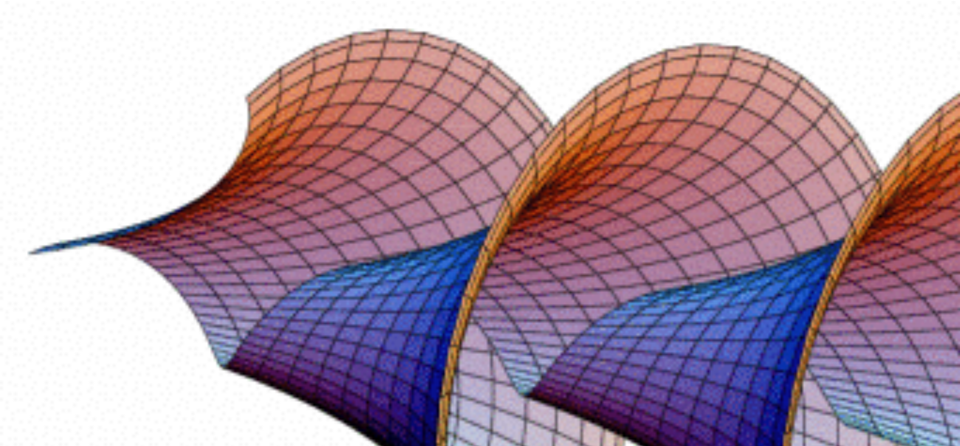
$(D) \quad (1-\sqrt{2}, 3-2\sqrt{2})$



Example

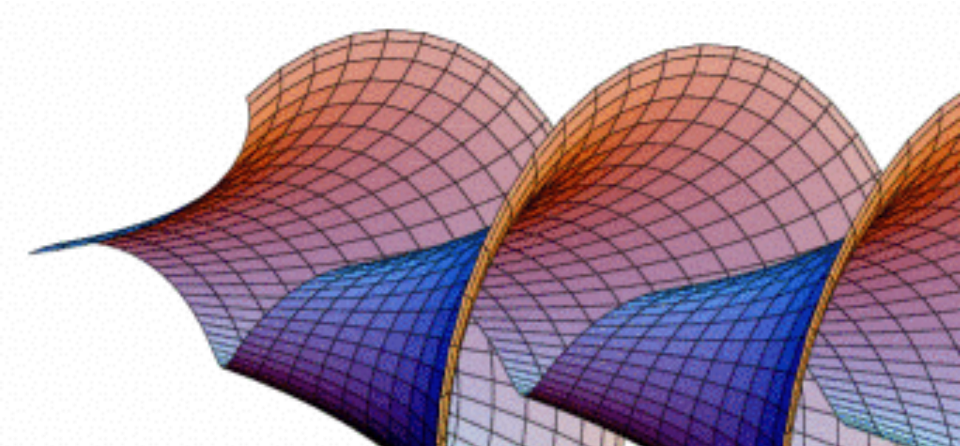
Estimate $\sqrt{3}$ using Newton's method.
Then using linear approximation.

Question: Which method is giving an overestimation/underestimation?
Question: Which one gives a better approximation?



Example

Find the derivative of $(x^4 + 3x^7)^9$



Example

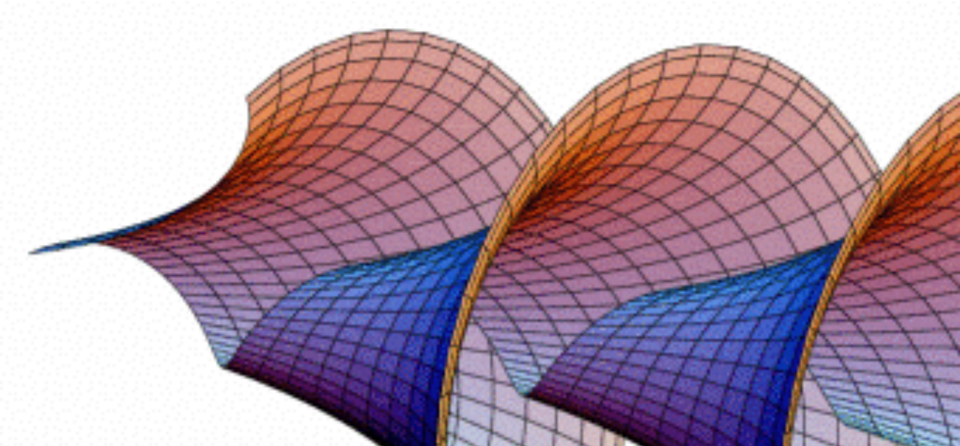
Find the derivative of $\sin(\sqrt{x})$

$(A) \quad \dfrac{\sqrt{x}}{2x}\cos(x)$

$(B) \quad \dfrac{1}{2}\sqrt{x}\cos(\sqrt{x})$

$(D) \quad \cos(\sqrt{x})$

$(D) \quad \dfrac{\sqrt{x}}{2x}\cos(\sqrt{x})$



Increasing/decreasing functions

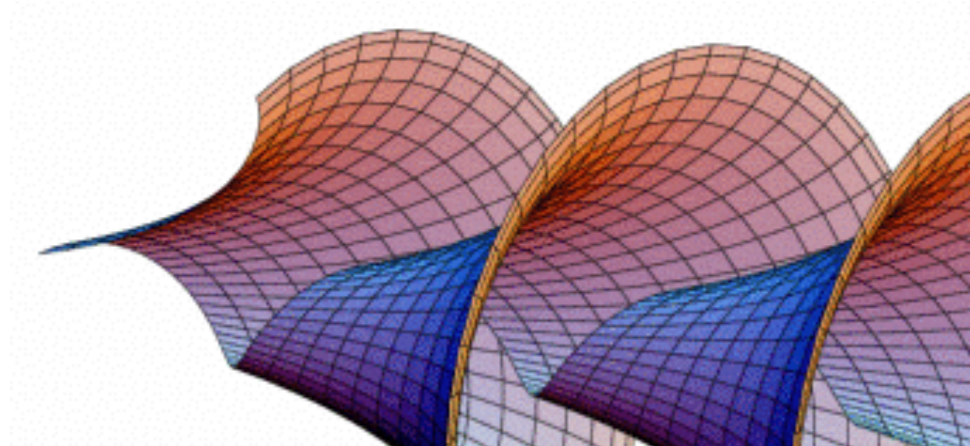
Given a function defined on an interval D :

The function is increasing on D, if for every choice of a and b with $a < b$,

$$f(a) \leq f(b)$$

The function is decreasing on D, if for every choice of a and b with $a < b$,

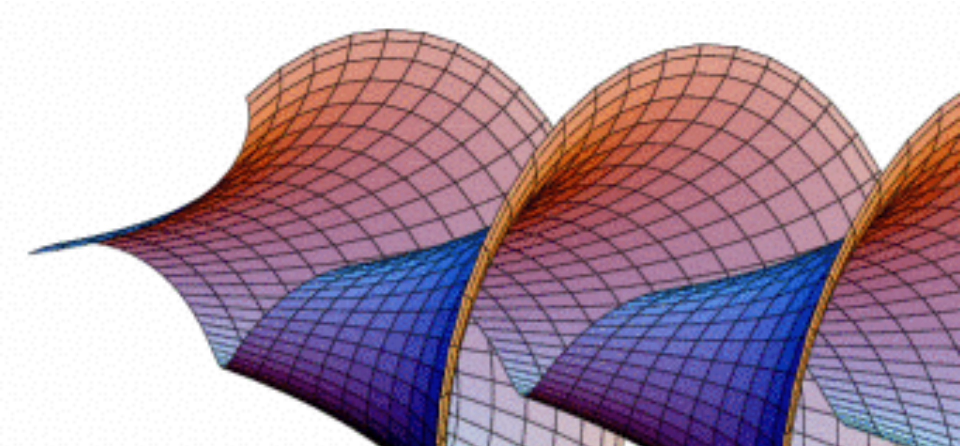
$$f(a) \geq f(b)$$



Increasing/decreasing functions

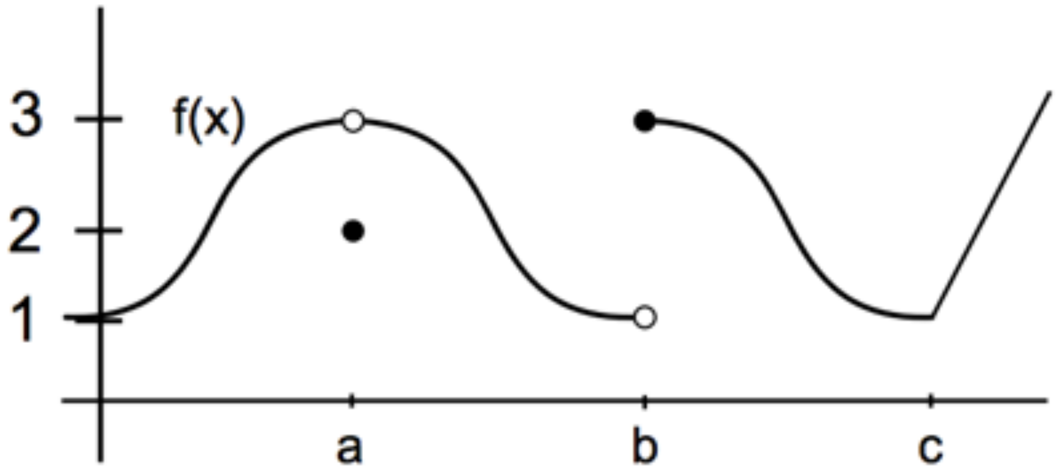
If $f'(x) \geq 0$ on D then f is **increasing** on D.

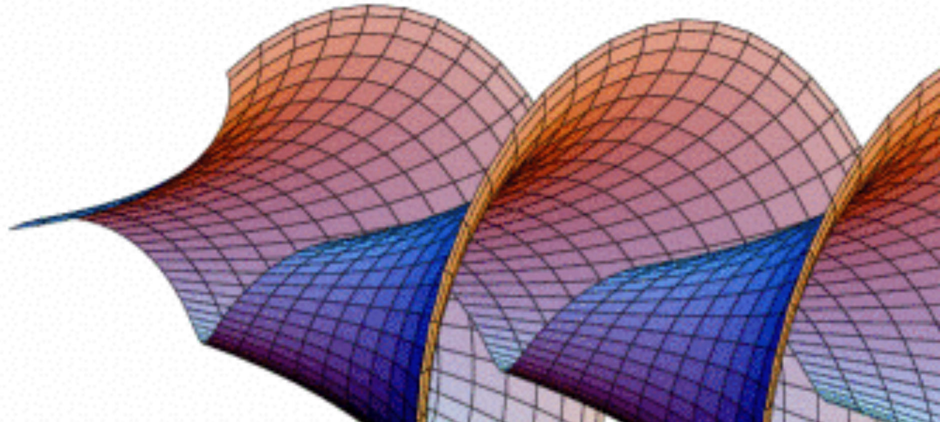
If $f'(x) \leq 0$ on D then f is **decreasing** on D.



Local min/max

Question: Which one is a local minimum?





Local min/max

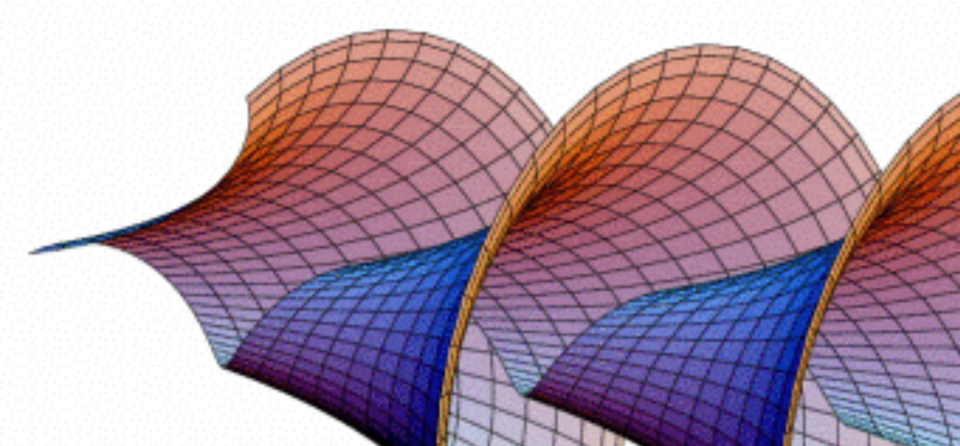
Given a function defined on an interval D and a is a point in D :

The function has a **local minimum** at a if for all x in D:

$$f(a) \leq f(x)$$

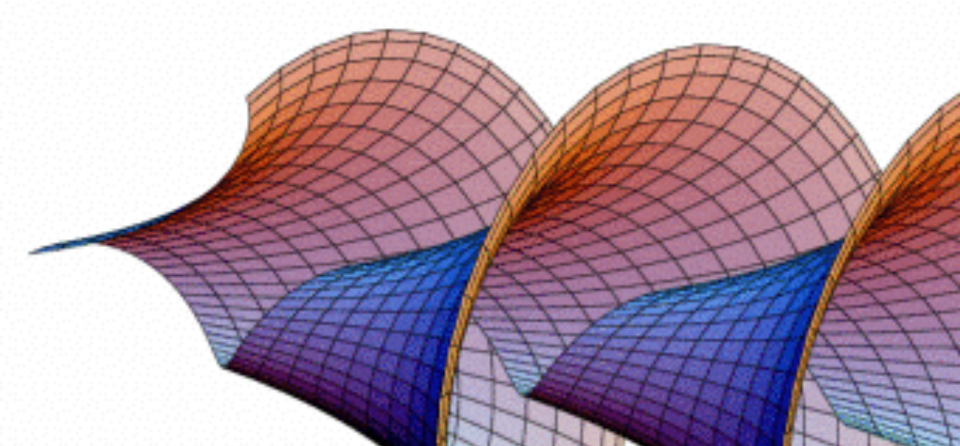
The function has a **local maximum** at a if for all x in D:

$$f(a) \geq f(x)$$



Local min/max

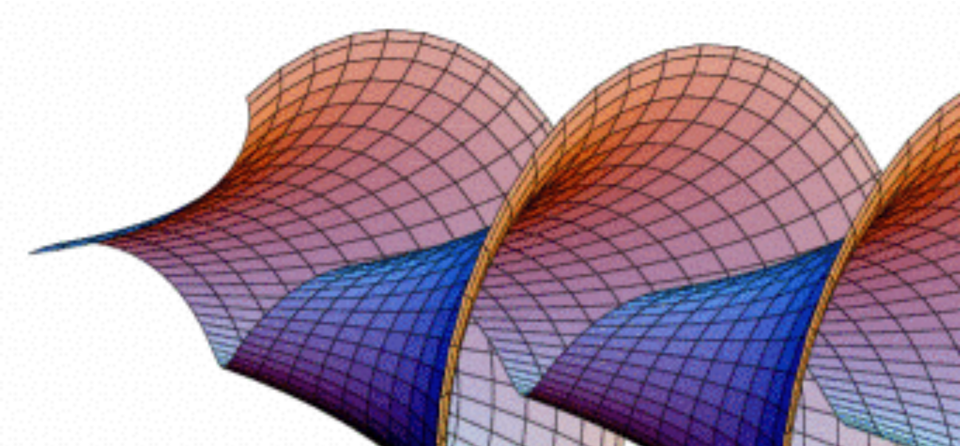
If f' changes sign at x= a then a is a local min/max (aka **local extremum**).



Critical points

If (1) f’(a) = 0 or (2) f’ is undefined at a even though f(a) is defined, a is called a critical point.

So: to find local max/min, **first** step is finding the critical points, and **second** step is determining the sign of f away from the critical points.



See you on Thursday!

IMPORTANT: Quiz will be in the beginning of the class so do not be late.

Oct 5	PL5.2
Oct 6	WW 4
Oct 6	Quiz 2