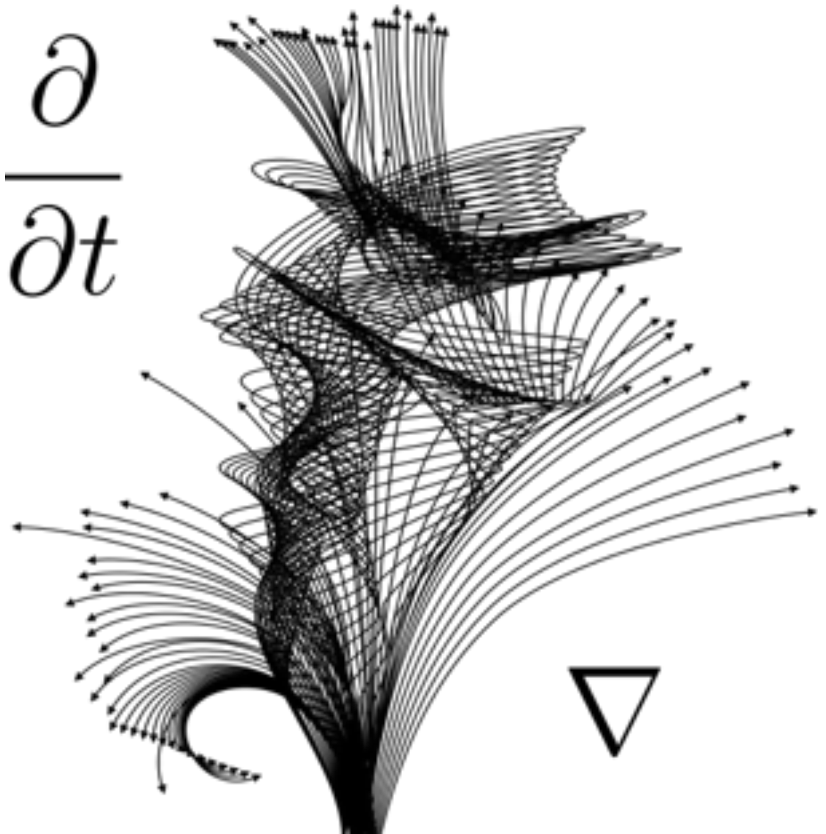
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

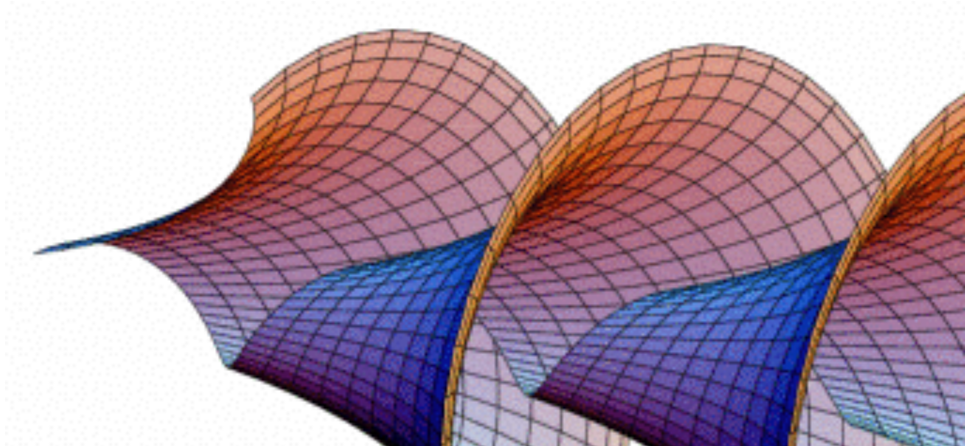
Agenda for today:

- A Hill function question
- Continue with Derivatives
- Introduce Continuity



New Office Hours :)

8:00AM - 9:30AM Thursdays (MATX 1118)
8:30AM - 9:30AM Mondays (MATX 1118)



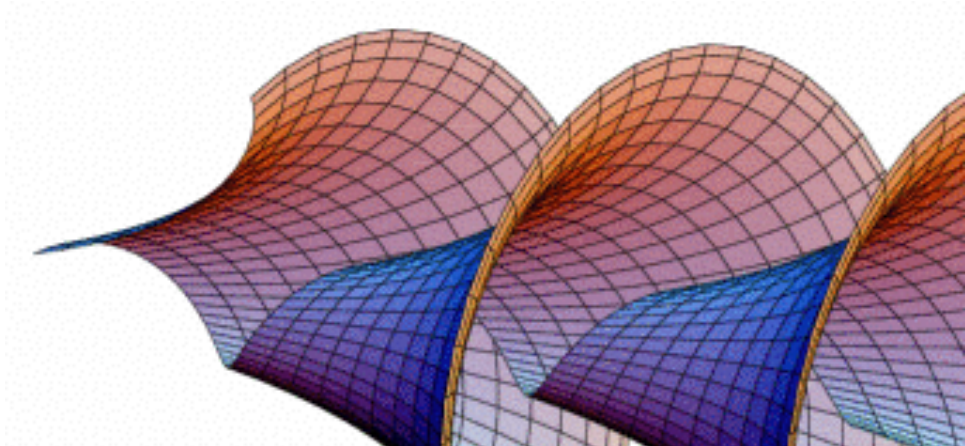
Upcoming due dates

OSH1: Friday Sept 16

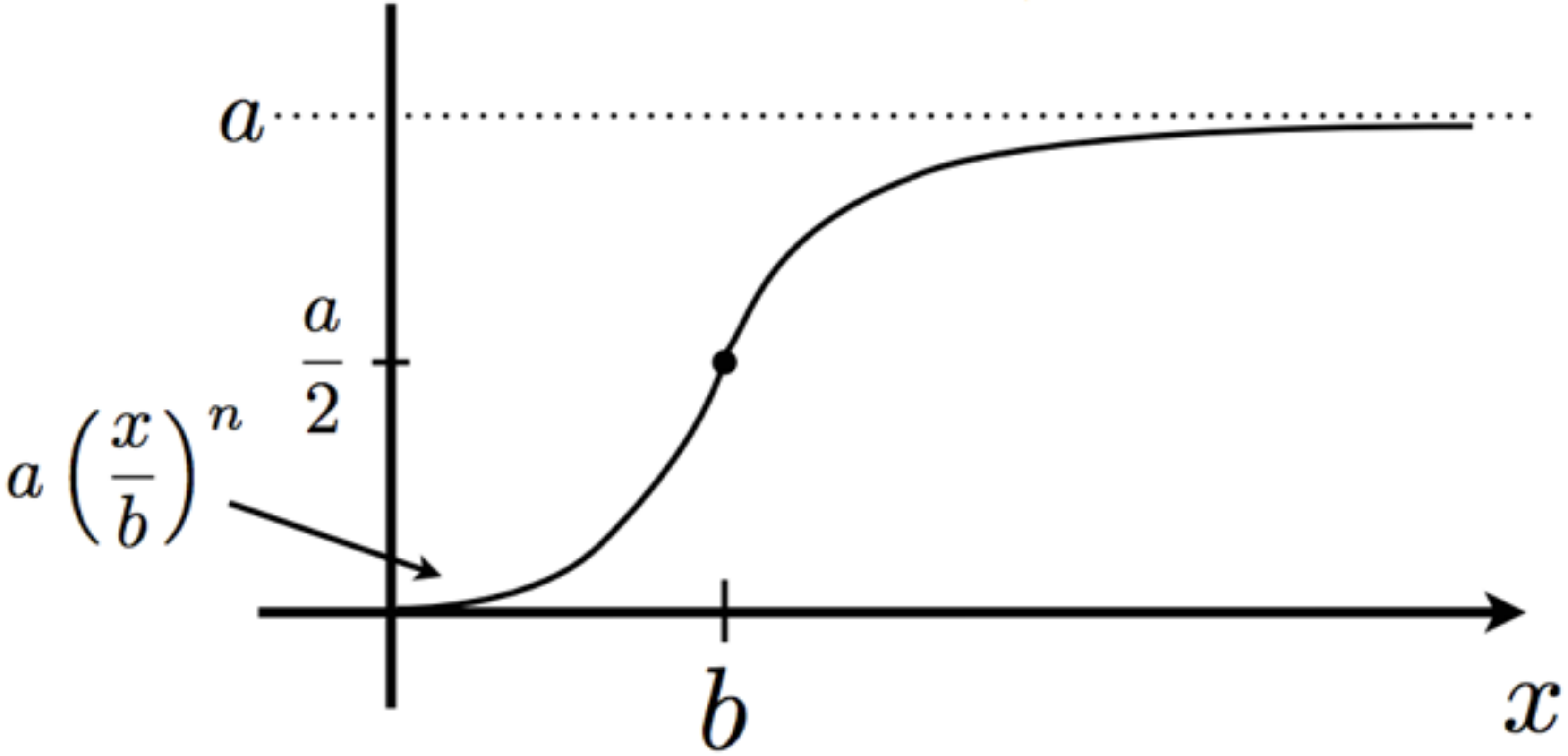
Course Logistics (on WeBWorK): Friday Sept 16

WeBWork Diagnostic Test (WW DT): Sunday Sept 18

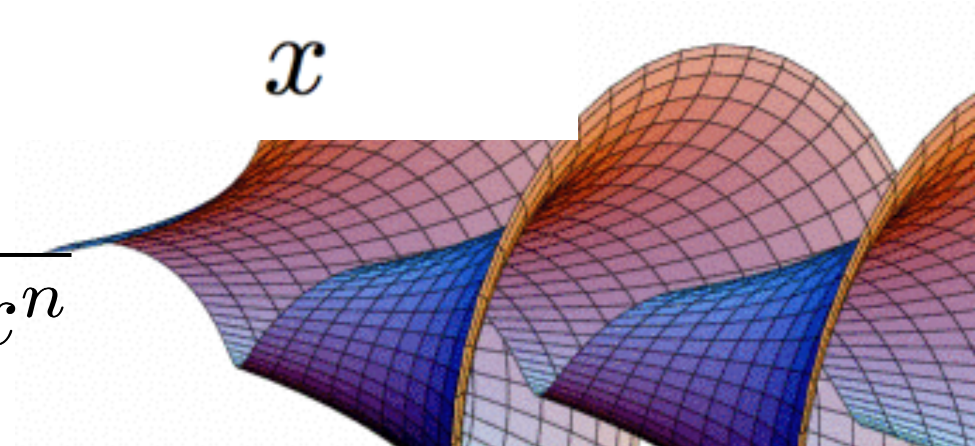
Pre-Lecture 3.1: Monday Sept 19



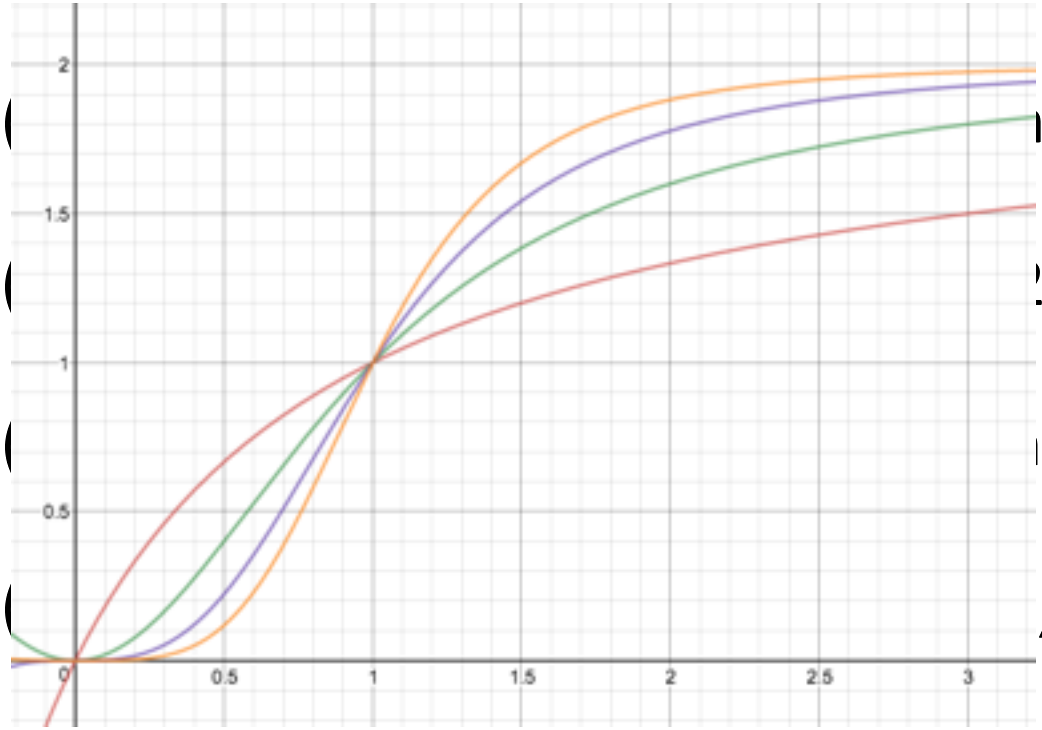
Last time: Sketching the Hill functions



$$f(x) = \frac{ax^n}{b^n + x^n}$$



Hill functions for different values of n

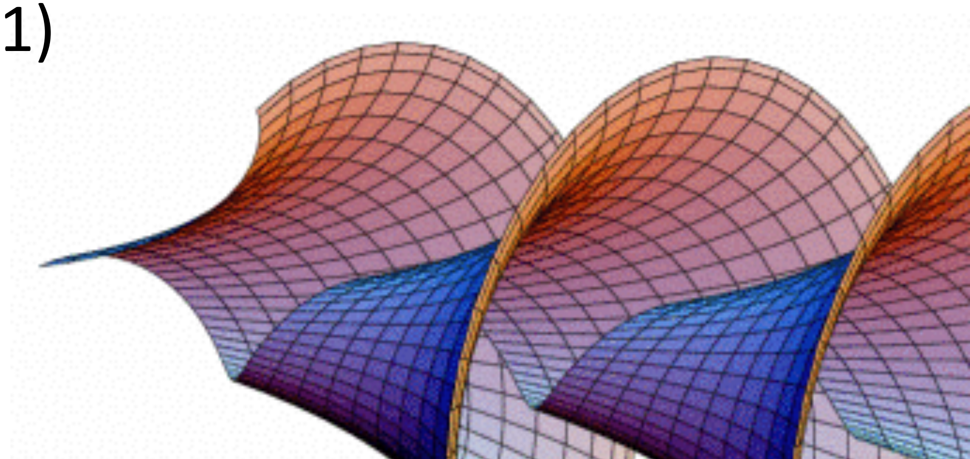


(=2), red (n=1)

( ), green (n=1)

(=2), red (n=1)

(, orange (n=1)



Derivatives (Cont’d)

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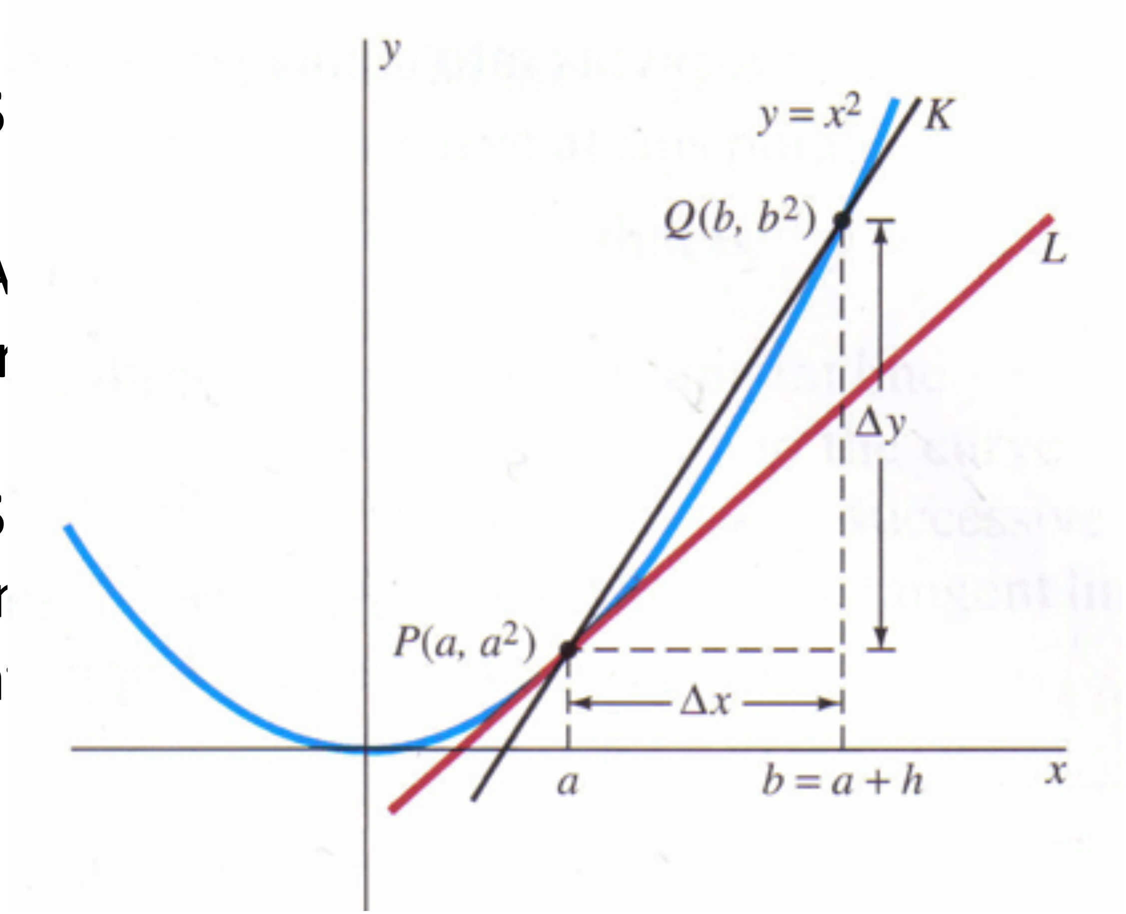
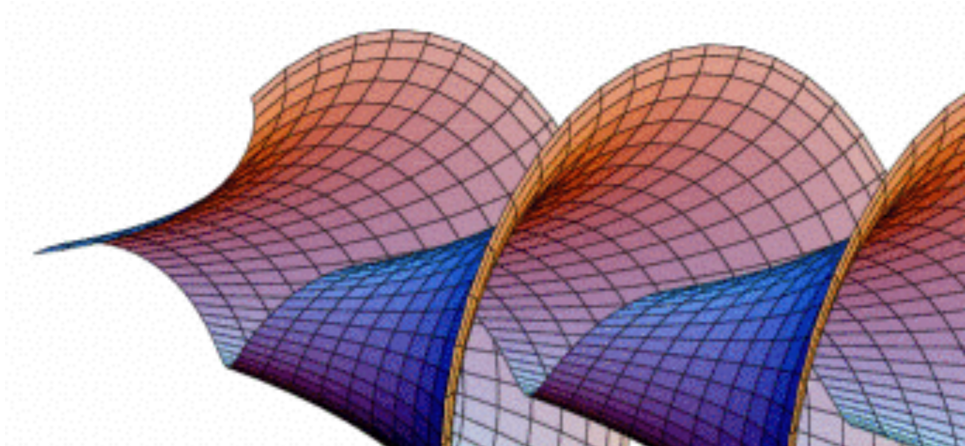
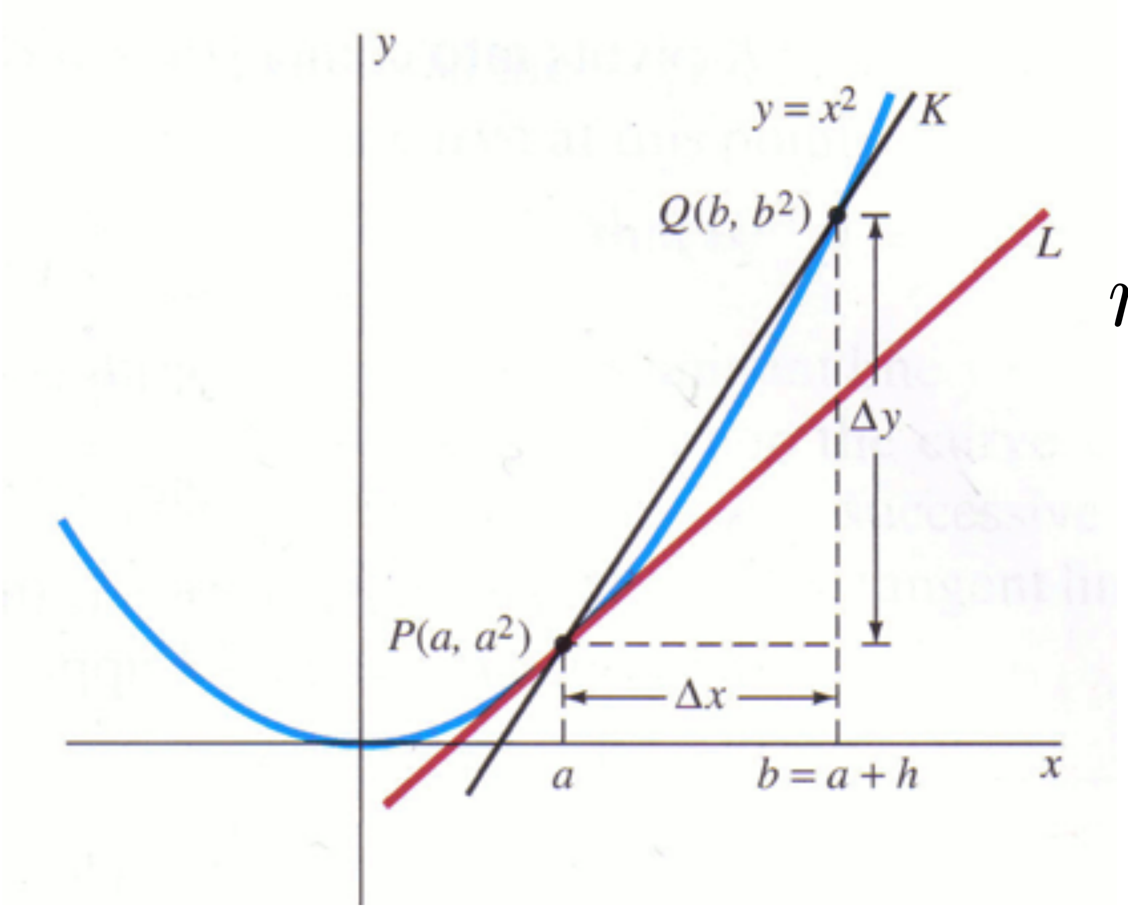
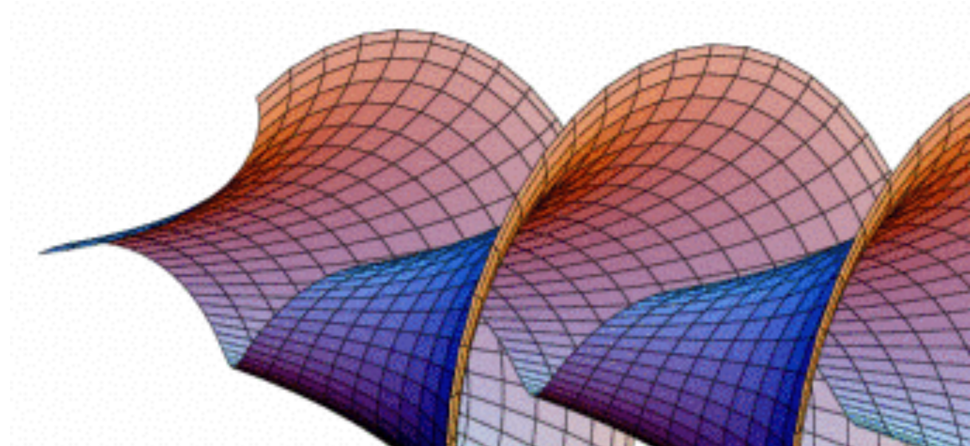




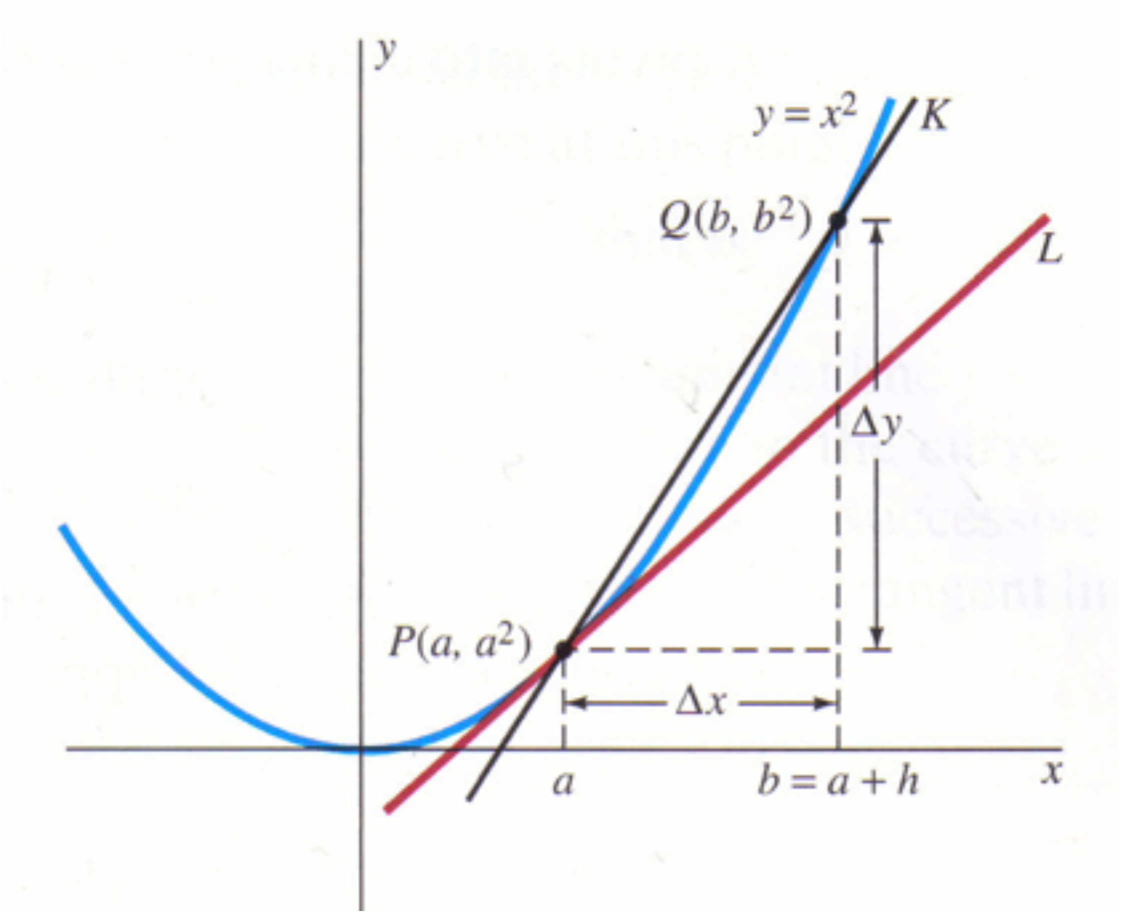
Average rate of change



$$m = \frac{f(b) - f(a)}{b - a}$$
$$= \frac{f(a+h) - f(a)}{h}$$

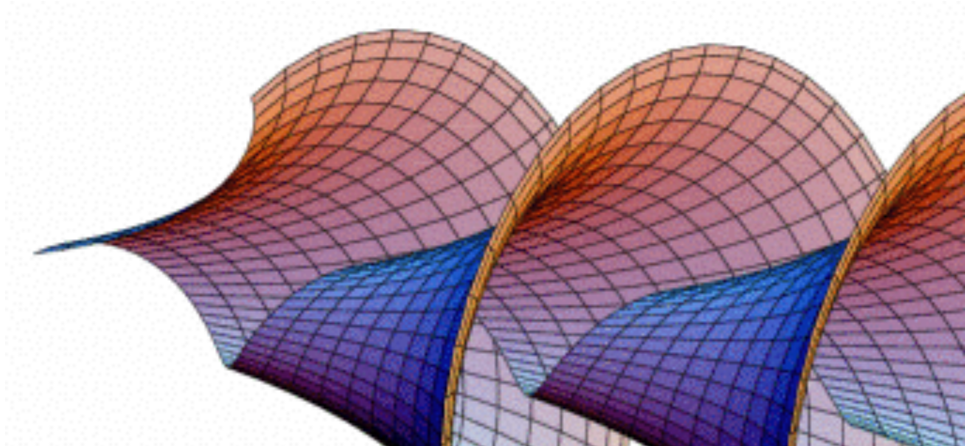


Derivative = Instantaneous rate of change



$$\lim_{b \to a} \frac{f(b) - f(a)}{b - a}$$

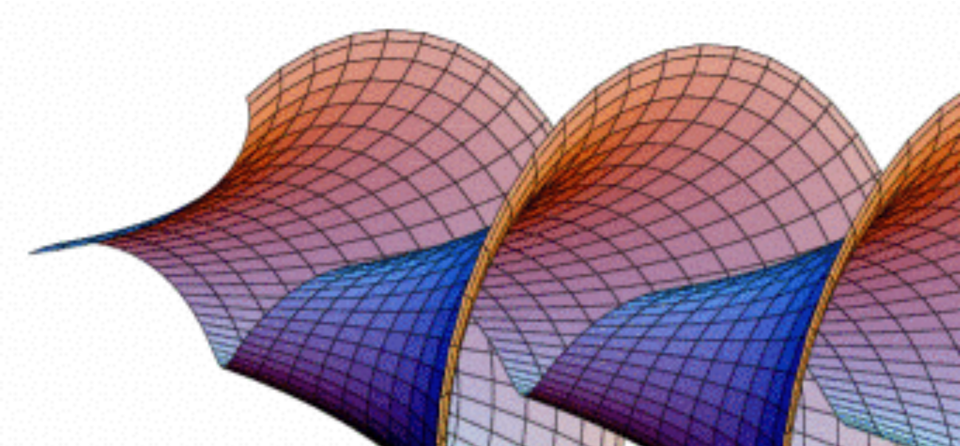
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Notation of the derivate

$$f'(a) = \lim_{b \to a} \frac{f(b) - f(a)}{b - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\left. \frac{d}{dx} f \right|_a = \left. \frac{df}{dx} \right|_a = f'(a)$$



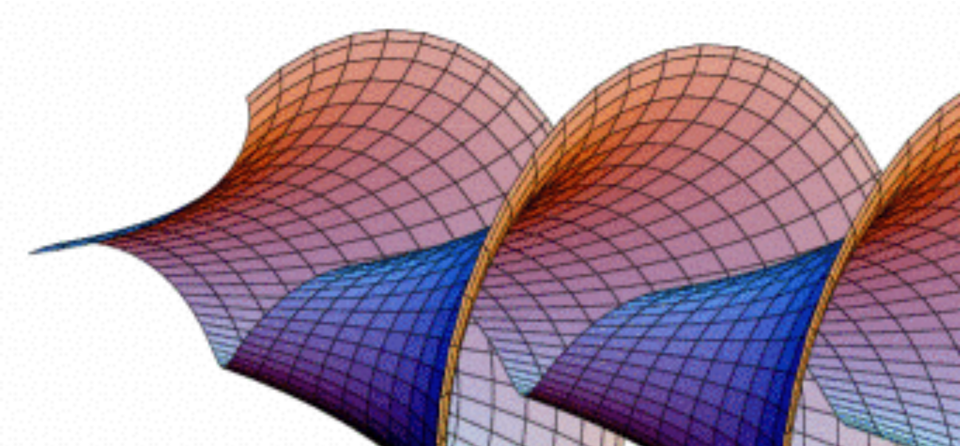
Examples

Last time: $f(x) = x^2$

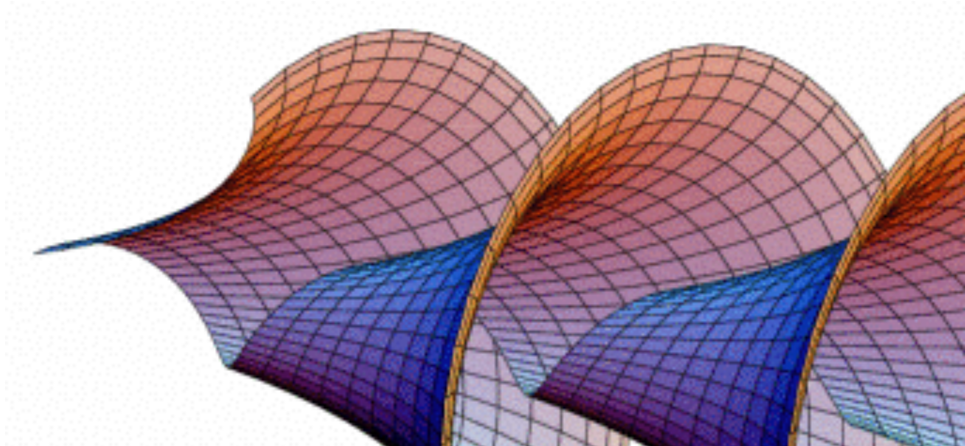
Now let's try: $f(x) = x^3$

What is the derivate of f at every point a?

What is the derivate of f at every point x?

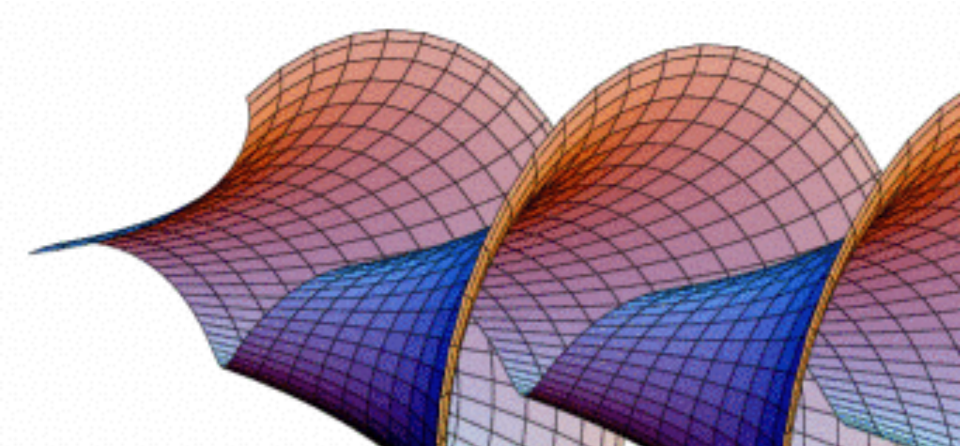


Derivative of a line



Derivative of a line

$$f(x) = mx + b$$

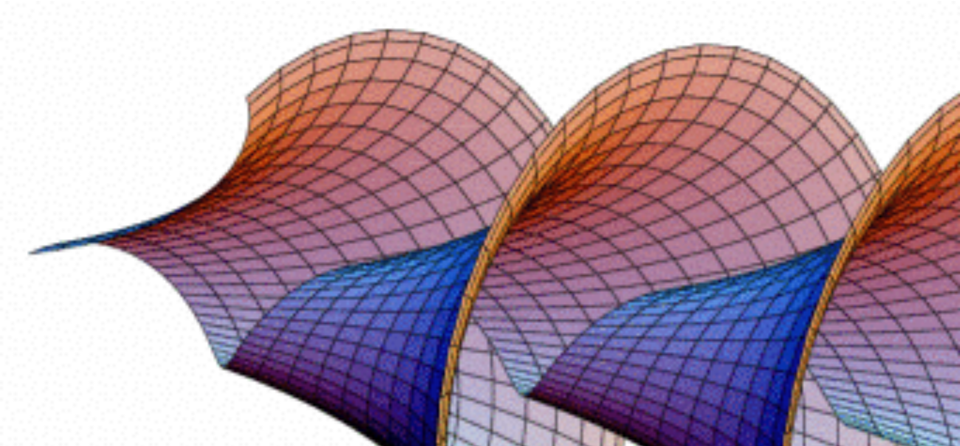


Derivative of a line

Slope of the **tangent line to a line** is the slope of the line!

$$f(x) = mx + b$$

$$f'(x) = m$$



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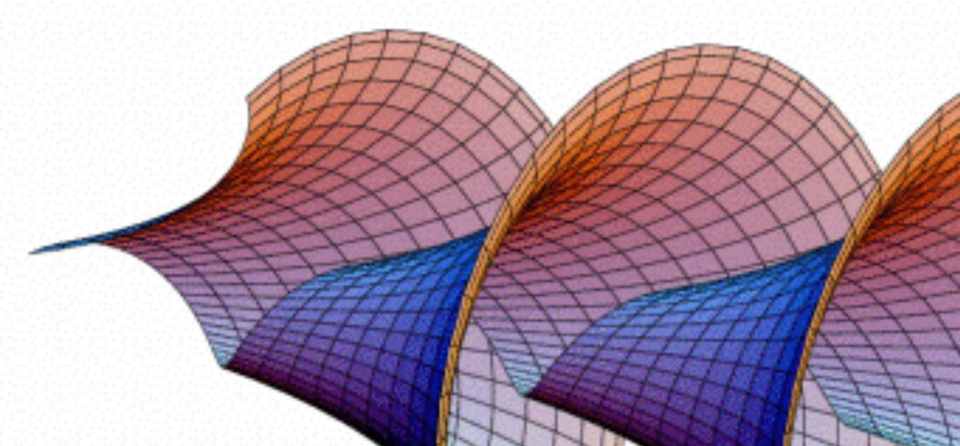
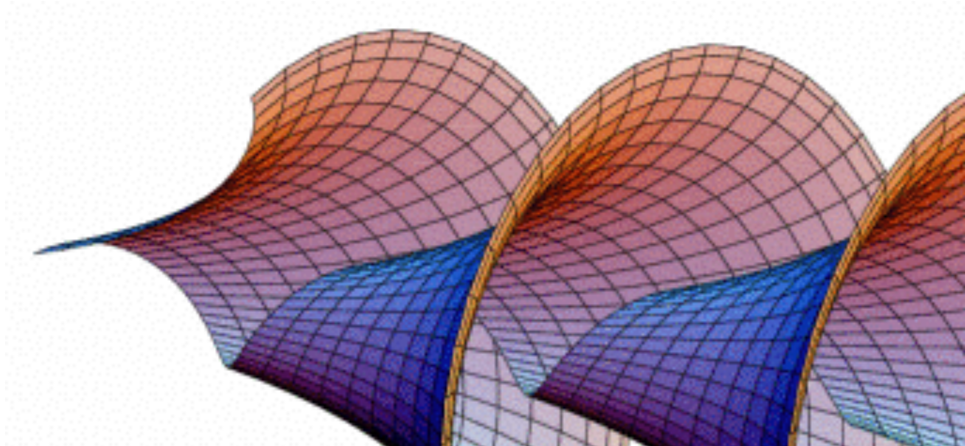


Question

True or False: The derivate of

$$f(x) = \frac{x^2}{x}$$

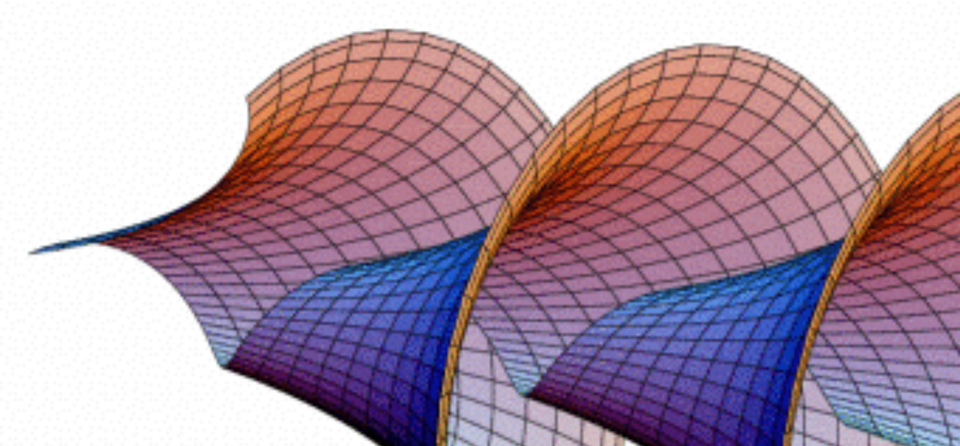
is 1 at $x = 0$?



Piecewise functions

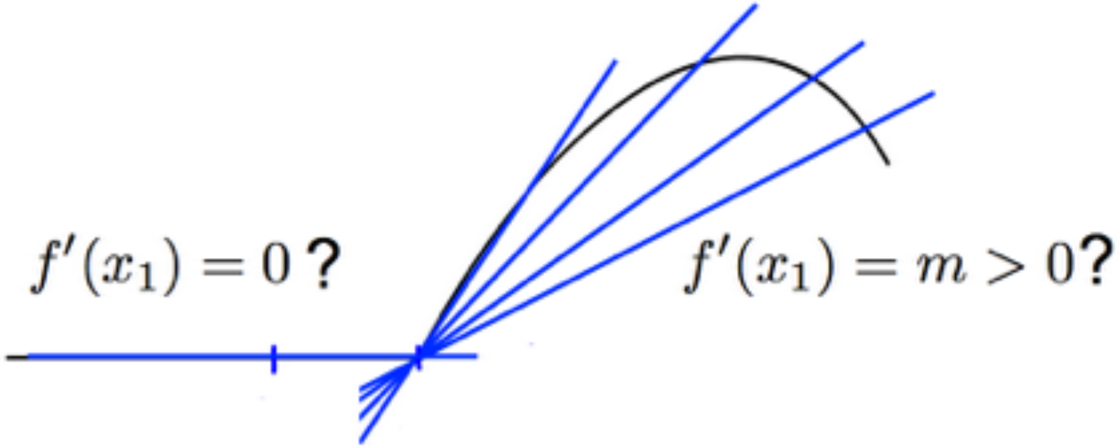
$$f(x) = \begin{cases} x^3 & x \geq 0 \\ -x & x < 0 \end{cases}$$

What is the derivate at x = 0 ?

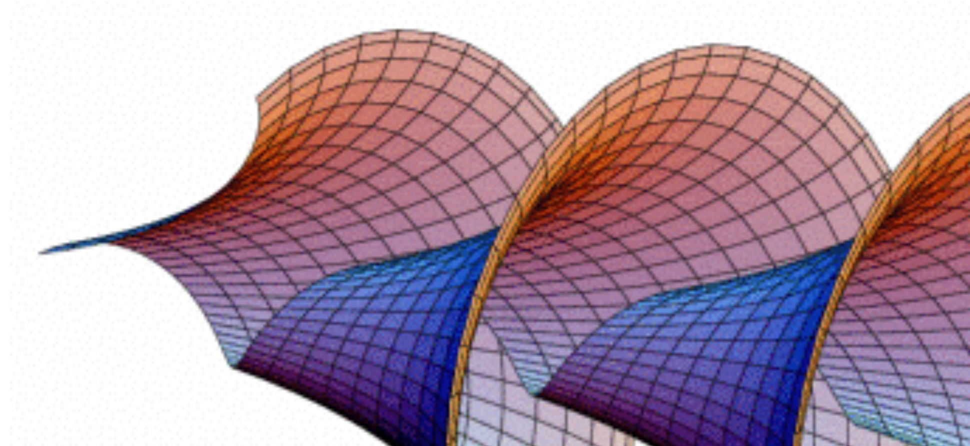


Piecewise functions

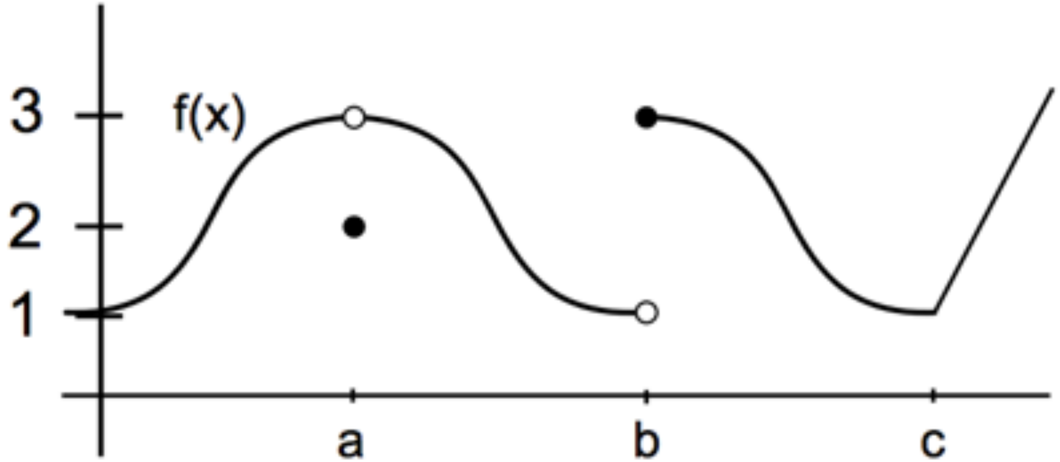
The derivate at x = 0:



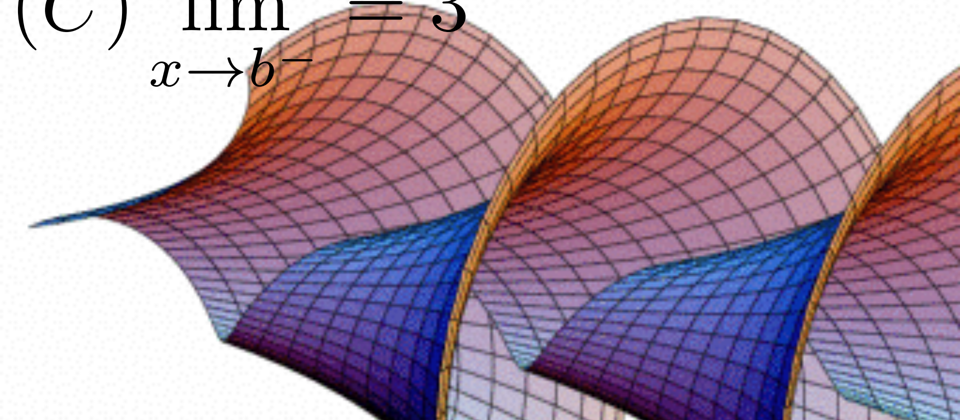
(D) is none?



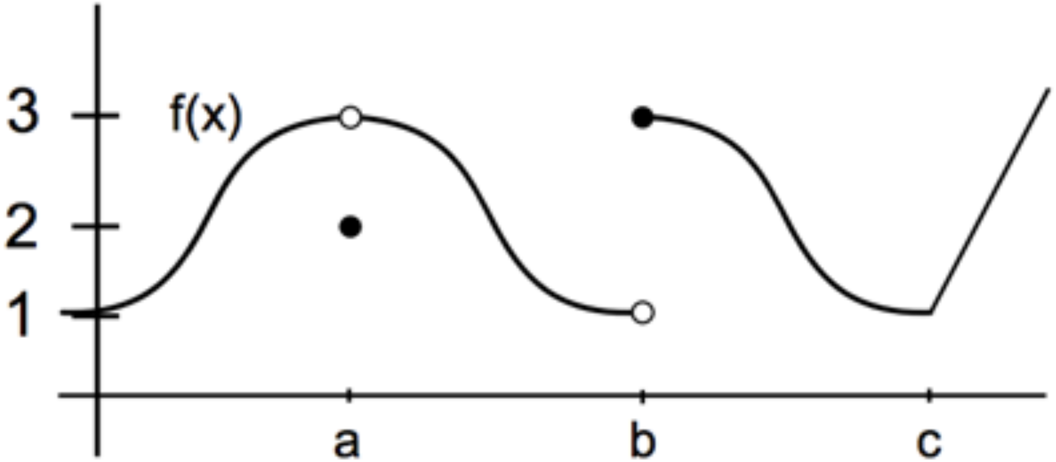
Right and left limits



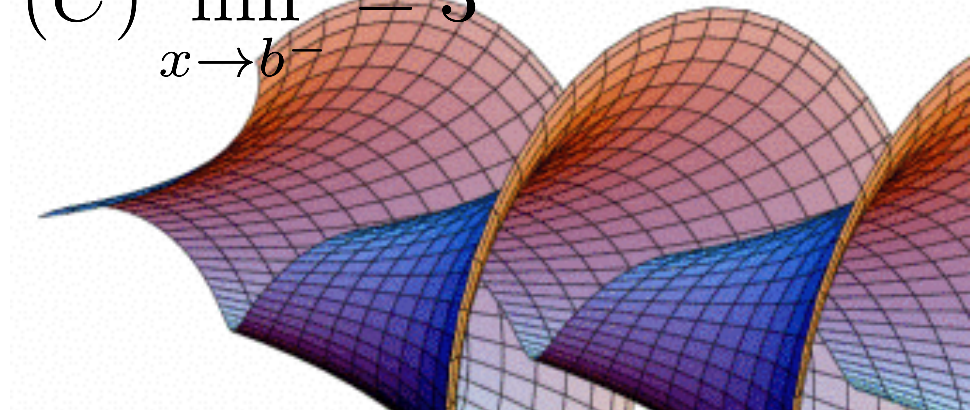
$$(A)\ \lim_{x\to a^+} = 3 \qquad (B)\ \lim_{x\to a^-} = 2 \qquad (C)\ \lim_{x\to b^-} = 3$$



Right and left limits



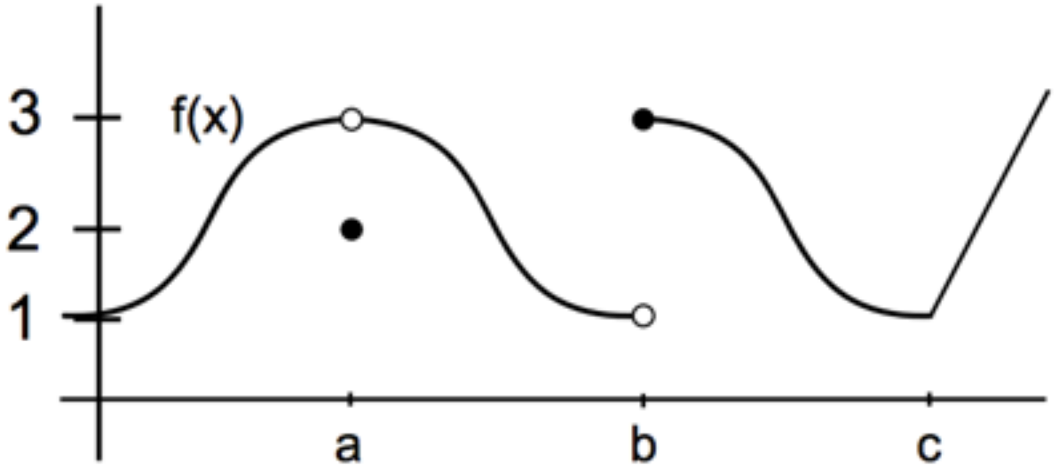
$$(A)\ \lim_{x\to a^+} = 3 \qquad (B)\ \lim_{x\to a^-} = 2 \qquad (C)\ \lim_{x\to b^-} = 3$$

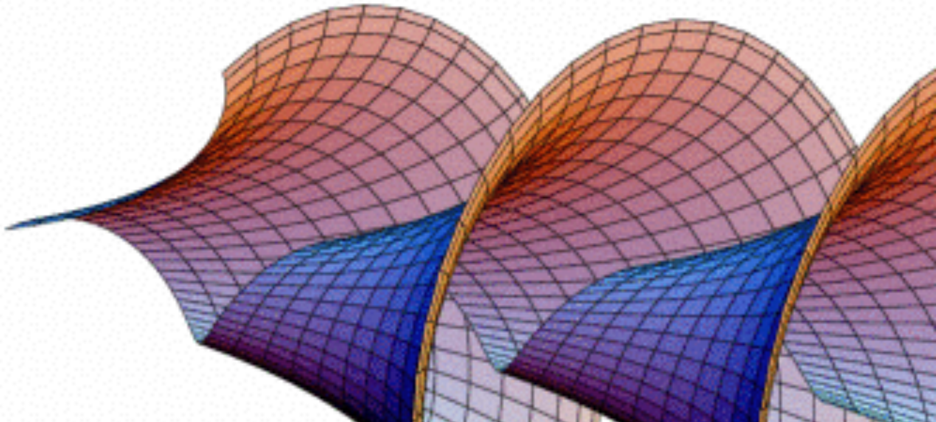


Limits

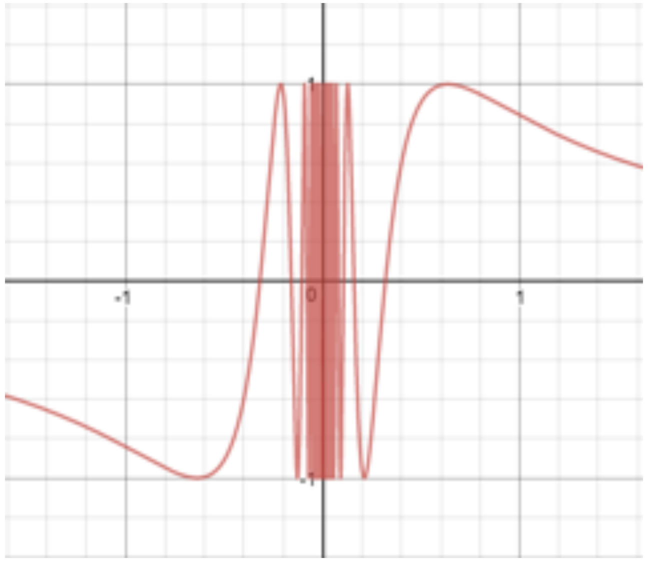
We say the limit of a function f(x) at the point x = a exists if the right and left limits at x = a **exist** and are **equal.**

TRUE/FALSE: Limit of the following function exists at x= a?

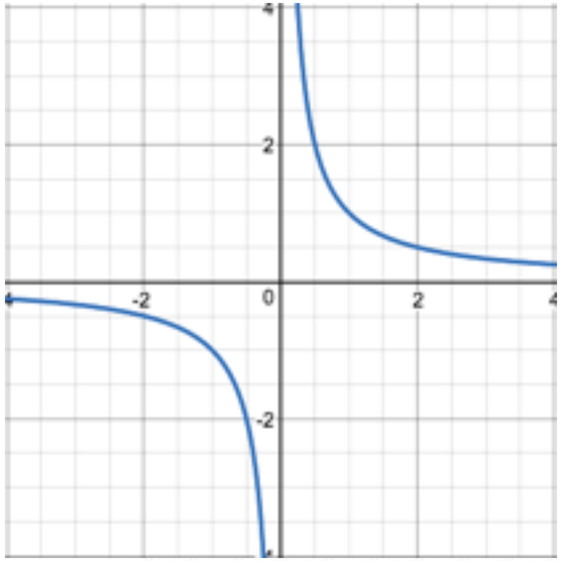




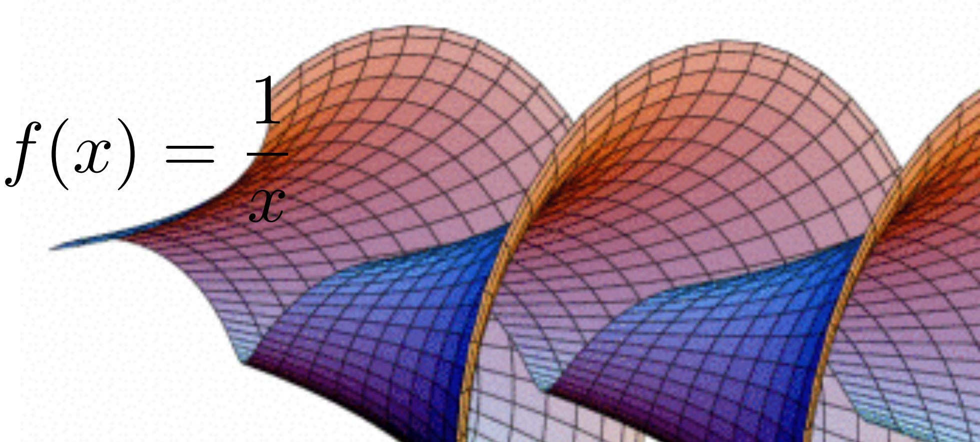
Examples of anomalies



$$f(x) = \sin\left(\frac{1}{x}\right)$$



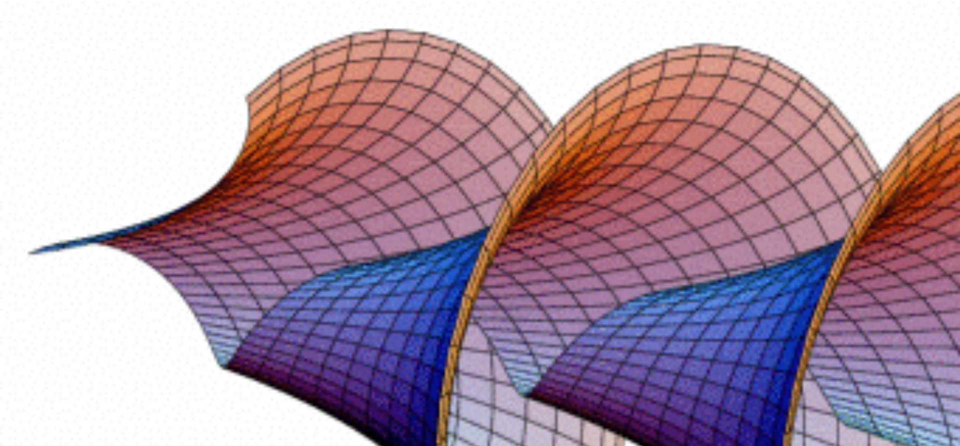
$$f(x) = \frac{1}{x}$$



Continuity

Limit exists if:

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$$



Continuity

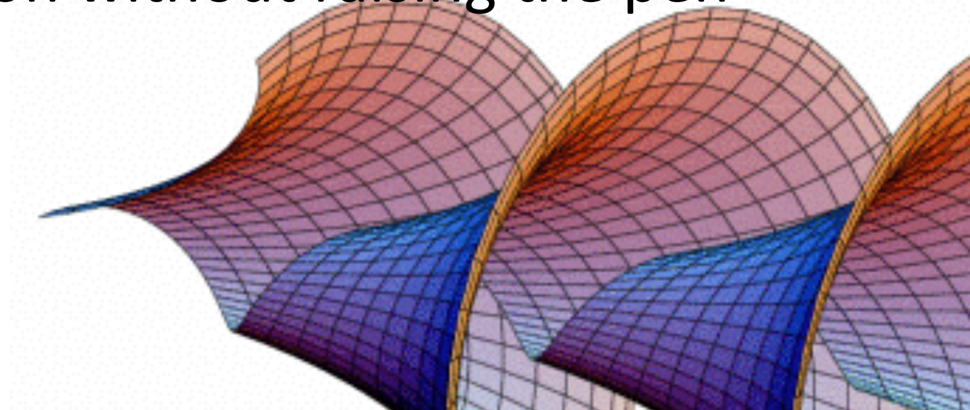
Limit exists if:

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a} f(x)$$

Function is continuous if:

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a} f(x) = f(a)$$

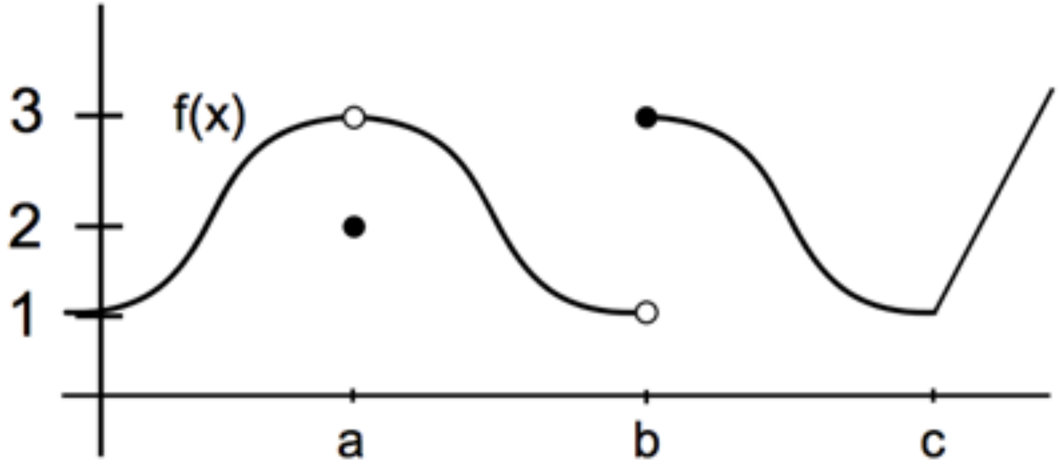
intuitively if we can draw the graph of the function without raising the pen from the paper.

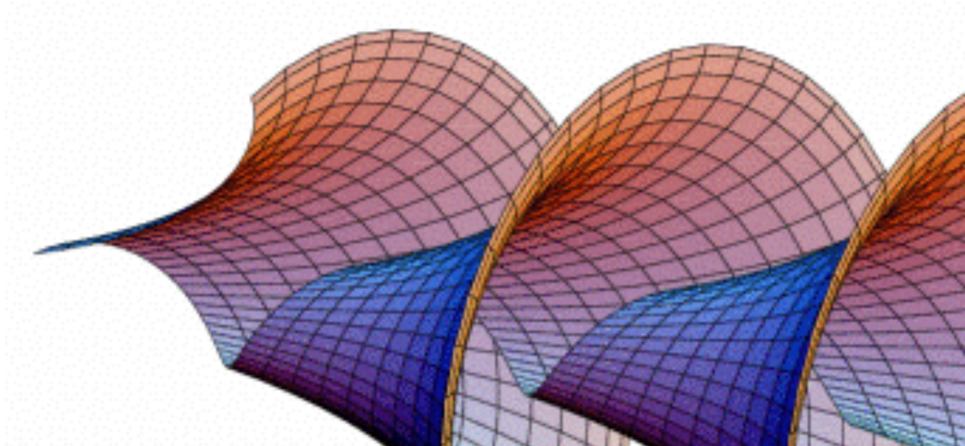


Continuity

At which point

(A

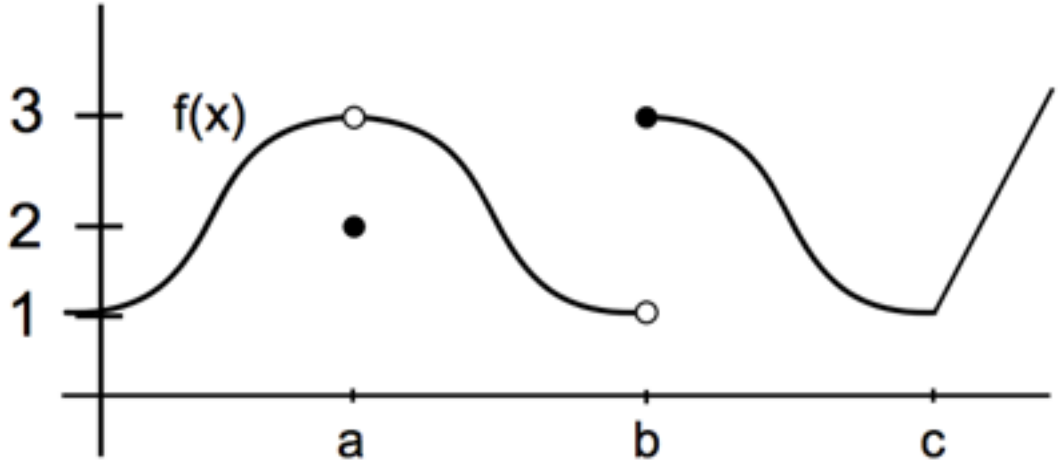


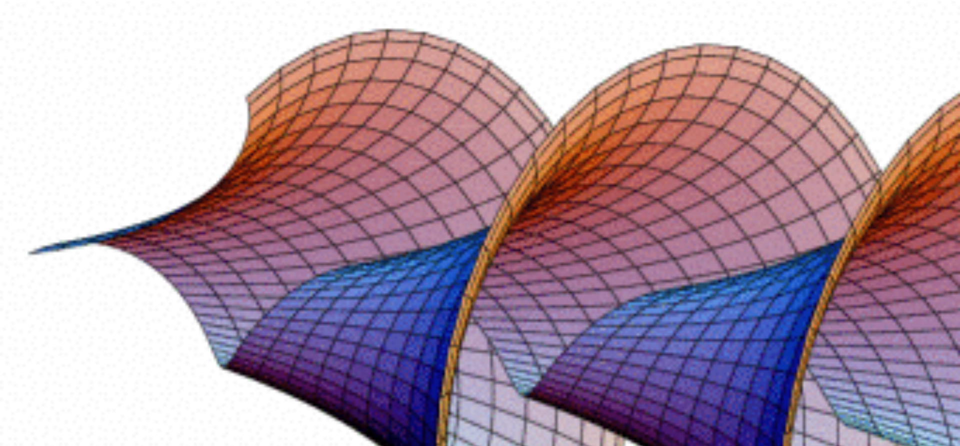


Continuity

At which point [...] ists?

(A





Have a good weekend!

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Sunday Sept 18

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