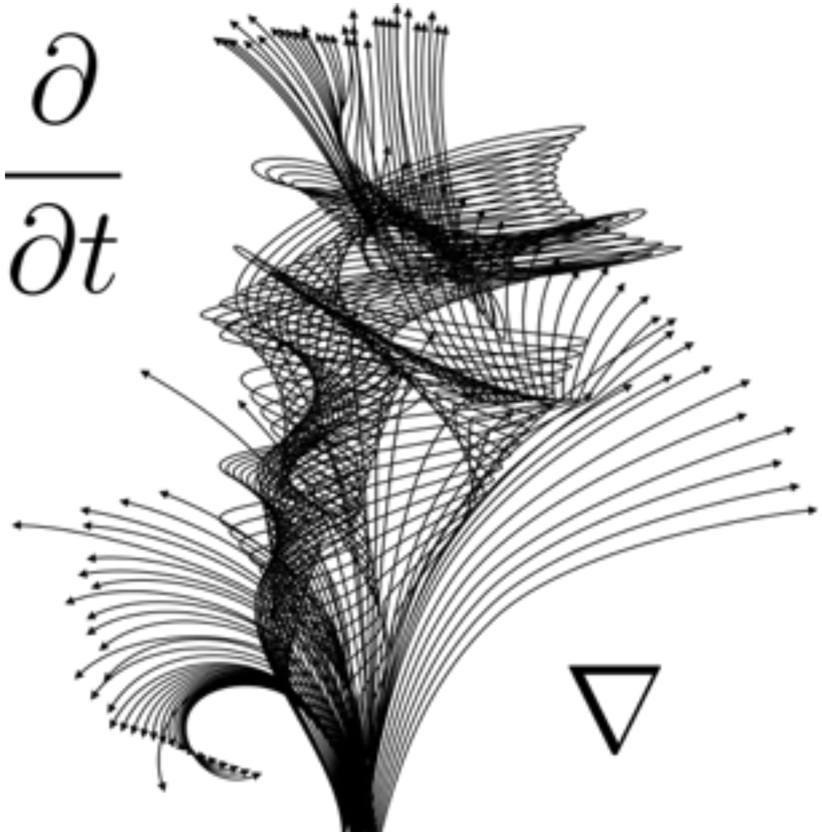
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

Agenda for today:

- Optimization Cont'd
- Finish Kepler's wedding example
- Least squares



Optimization on a closed interval

Given a scenario involving a choice of some number, use calculus to find the best value.

Translate scenario into a mathematical problem, involving a function you need to **optimize** (i.e. find absolute max/min of).

Solve the problem by: finding critical points and boundary points. Check the value of the function at all of them.

Optimization NOT on a closed interval

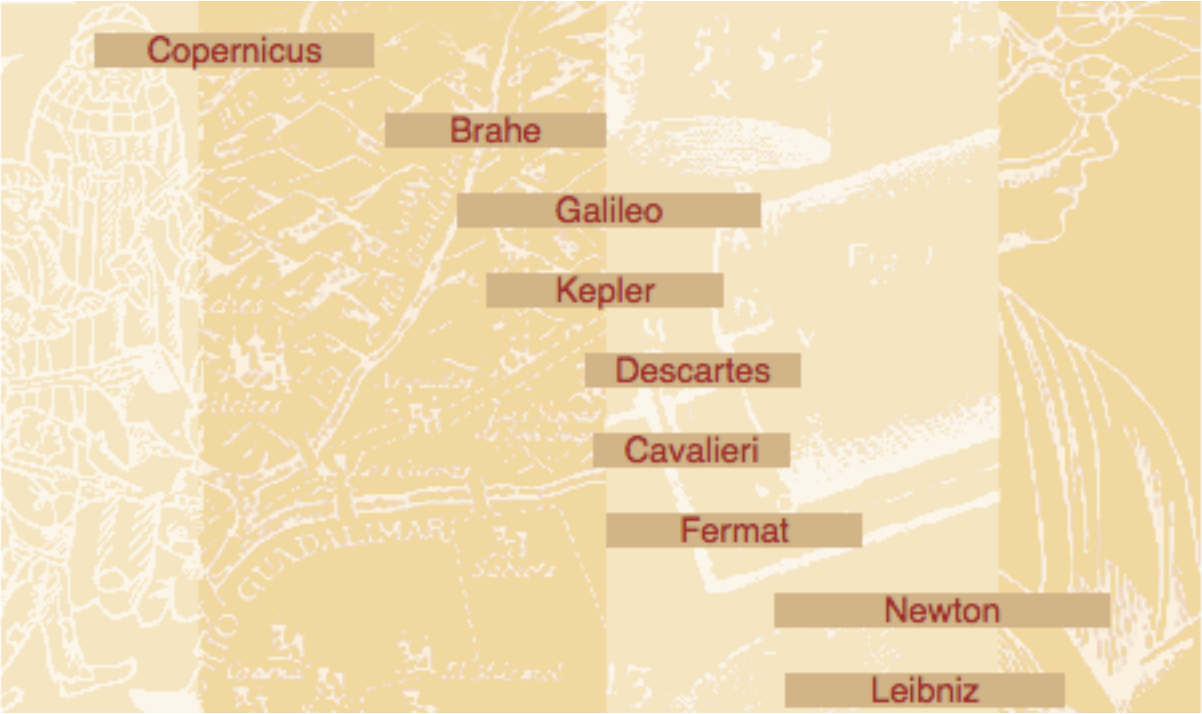
ON A CLOSED INTERVAL:

Solve the problem by finding critical points and boundary points. Check the value of the function at all of them.

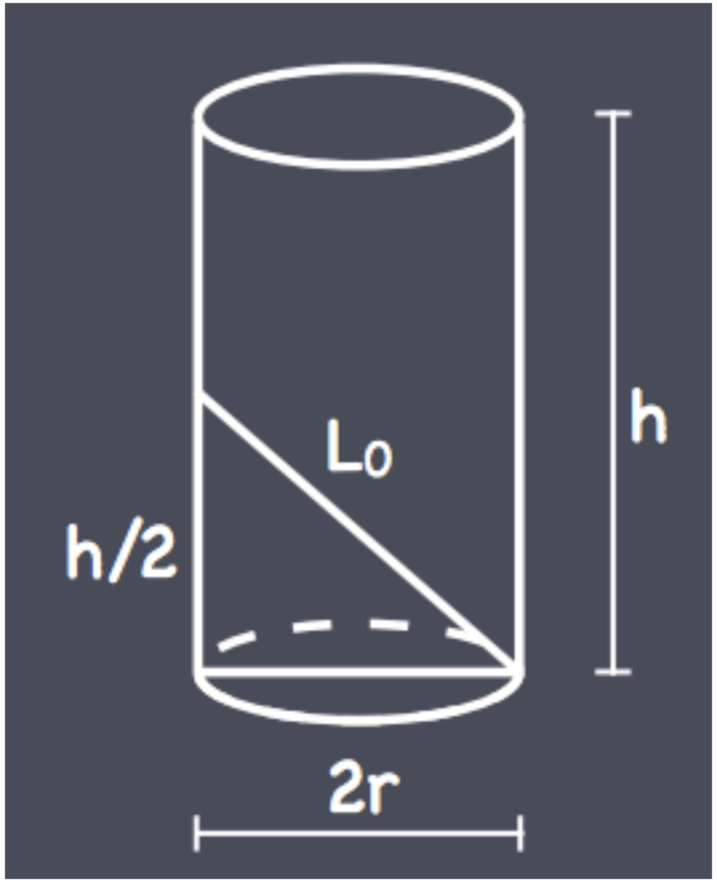
NOT ON A CLOSED INTERVAL:

Find critical points and any boundary points (if exists). Need to run FDT, and SDT for the critical points.

Kepler’s Wine Barrel



Kepler's Wine Barrel



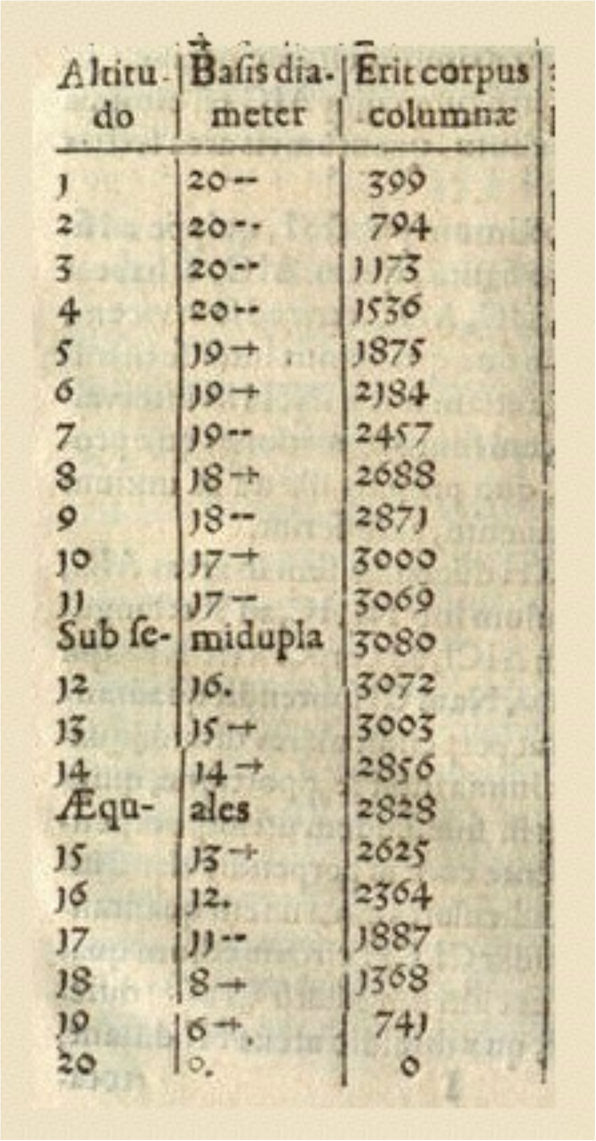
For a fixed budget the Kepler had, he could only buy a barrel with SD= L0. What was the best barrel?

What is the barrel with maximize volume for a given fixed length of the rod?

Kepler’s Wine Barrel

Thus, while the Austrian method of price determination, if applied to Rhenish barrels, would be a clear fraud, it was quite legitimate for Austrian barrels. The Austrian shape had the advantage of permitting such a quick and simple method. So Kepler relaxed in this instance.

Otto Toeplitz, The Calculus: A Genetic Approach, University Of Chicago Press, 1963



Altitudo	Basis diameter	Erit corpus columnæ
1	20 --	399
2	20 --	794
3	20 --	1173
4	20 --	1536
5	19 +	1875
6	19 +	2184
7	19 --	2457
8	18 +	2688
9	18 --	2871
10	17 +	3000
11	17 --	3069
Sub se-	midupla	3080
12	16.	3072
13	15 +	3003
14	14 +	2856
Æqu-	ales	2828
15	13 +	2625
16	12.	2364
17	11 --	1887
18	8 +	1368
19	6 +.	741
20	0.	0

For these types of problems…

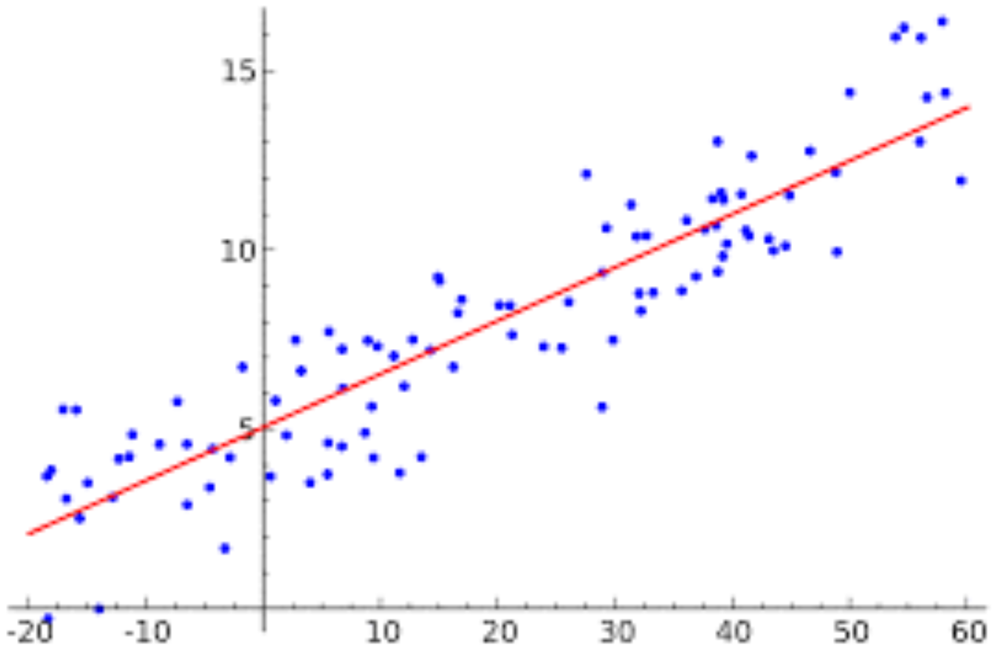
Draw some sketches!

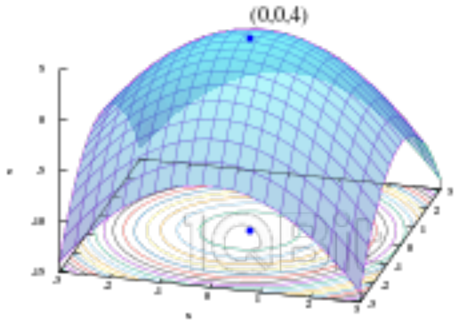
Determine the objective function. Determine the constraint.

Use constraint to change the objective function into a function of one variable.

Find end points and all critical points. Evaluate all critical points and compare.

Least square model fitting





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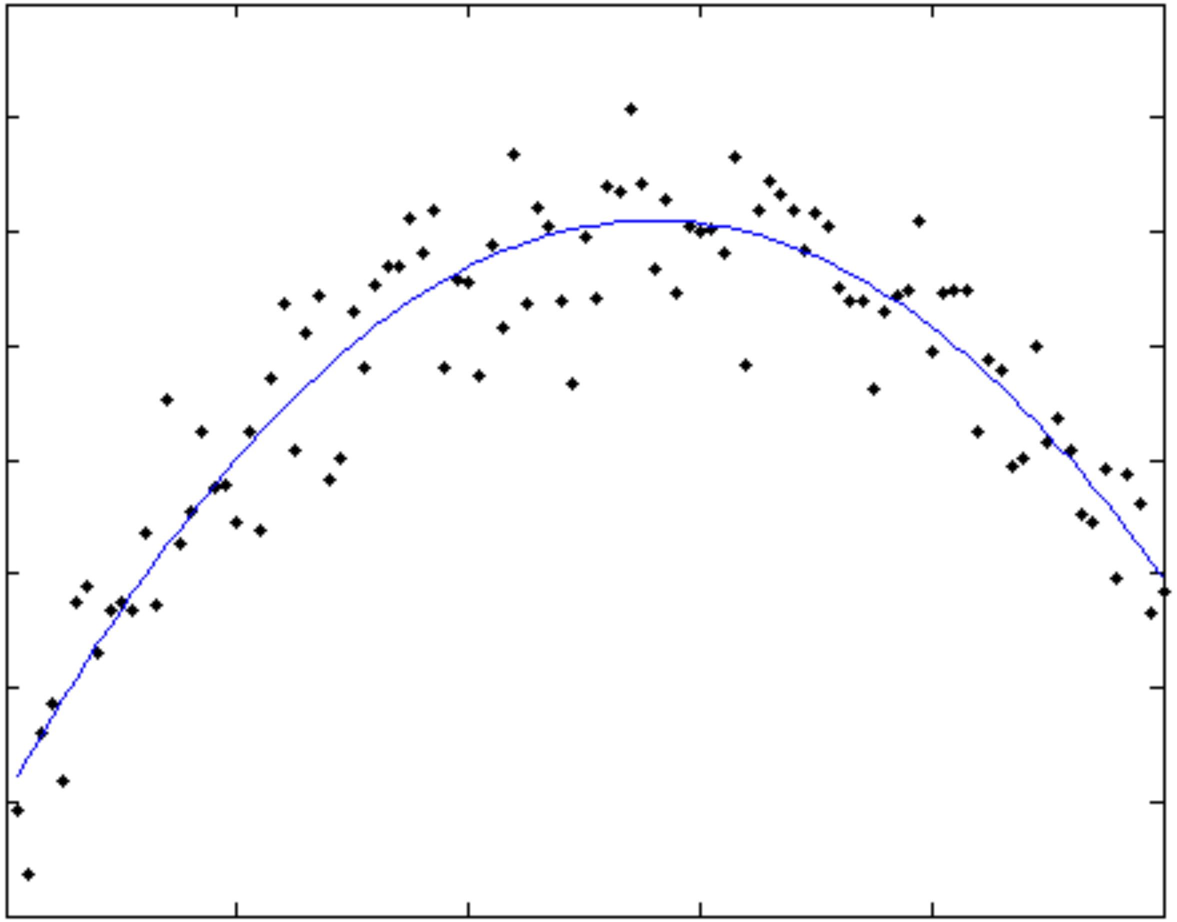

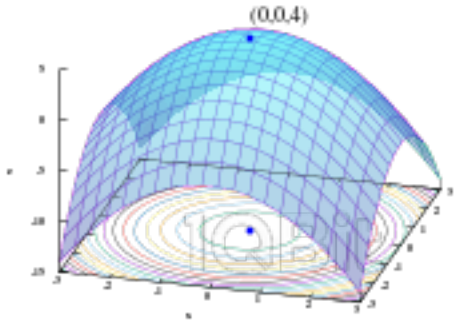

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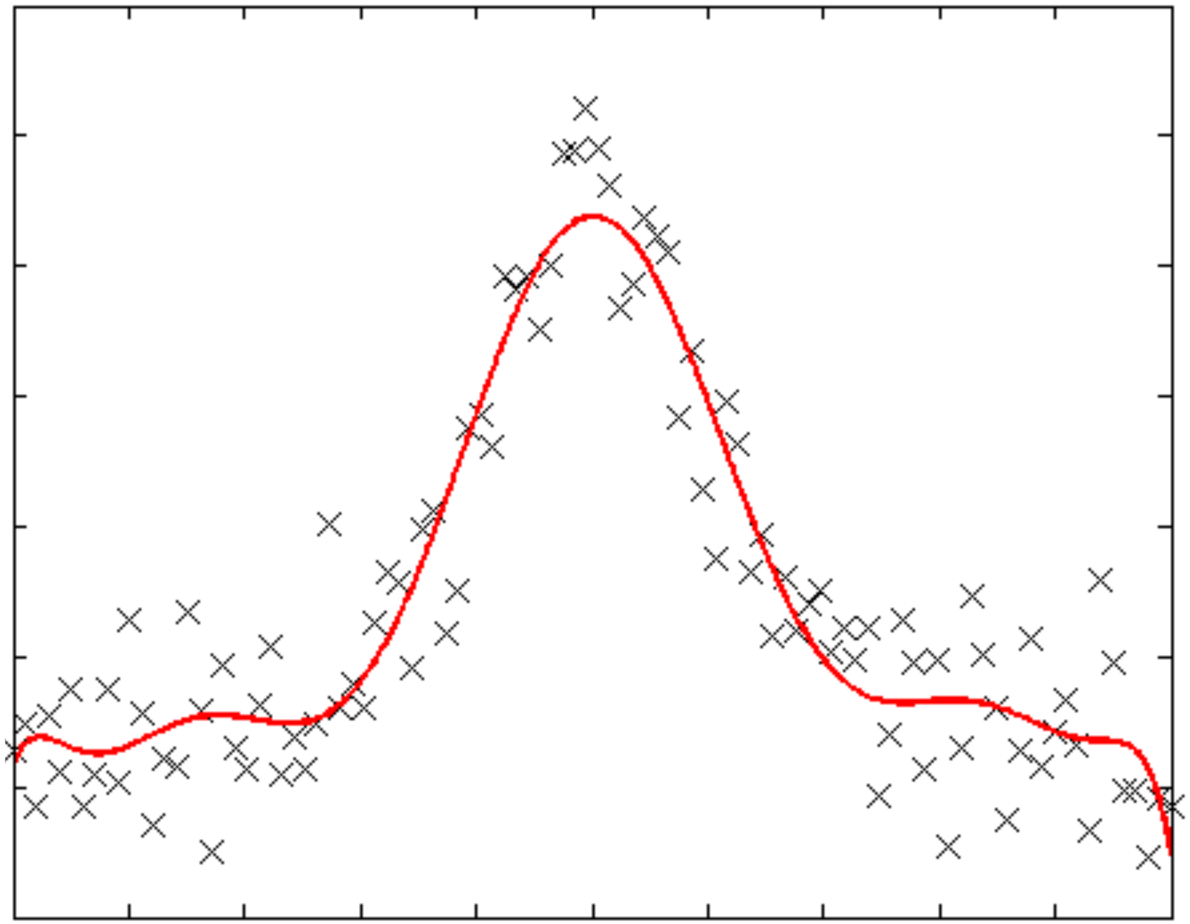
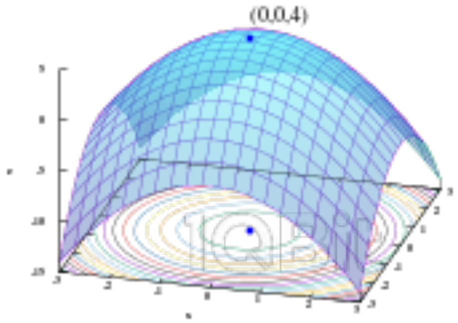


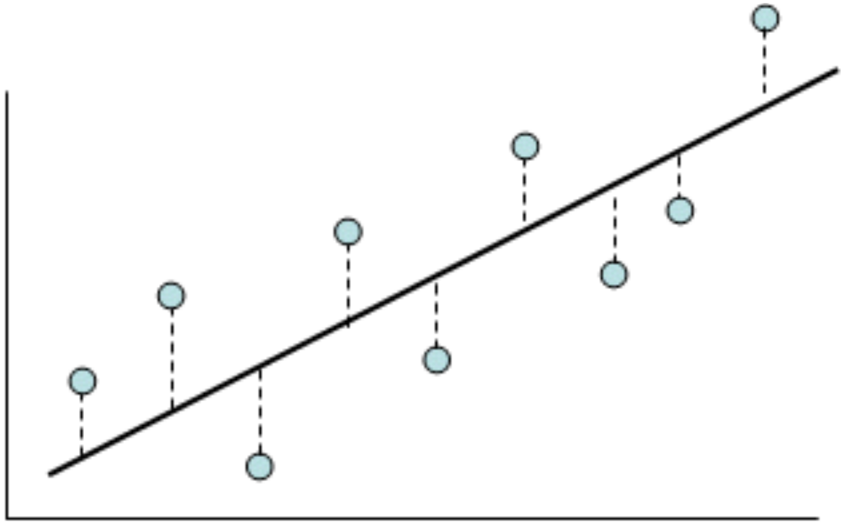


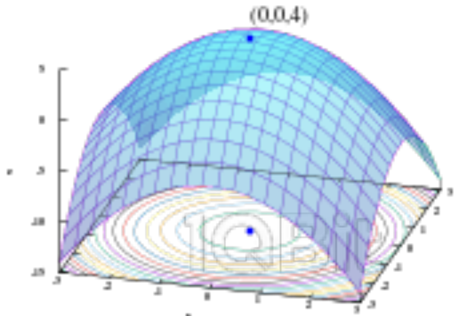
Least square model fitting

Each dotted bar is called a residual.

We want all the residuals to be as small as possible.

What is the value of a residual?



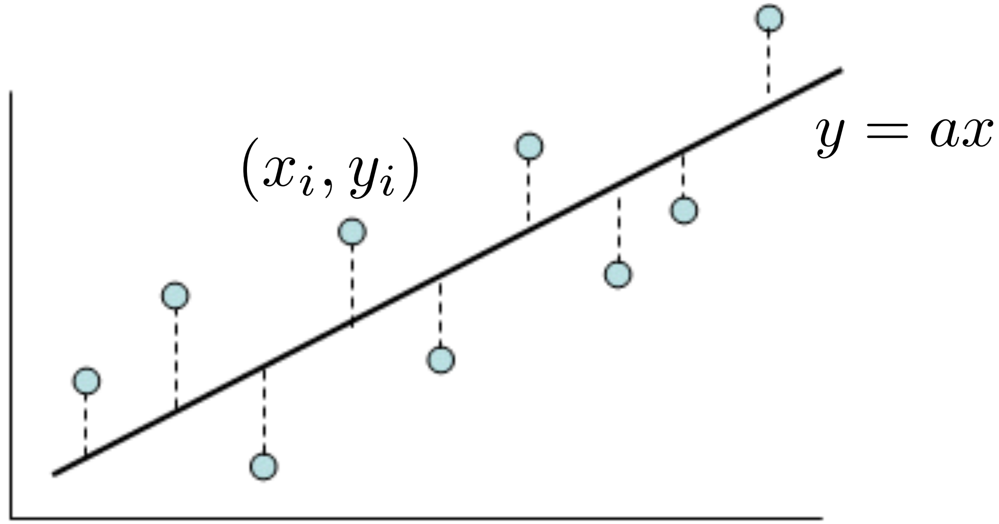


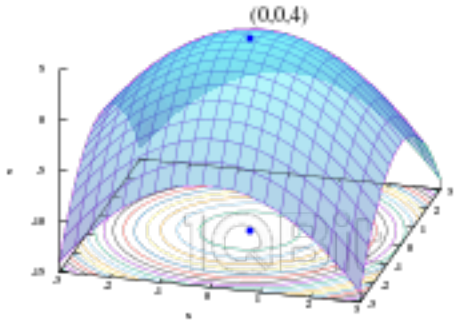
Least square model fitting

Each red bar is called a residual.

We want all the residuals to be as small as possible.

What is the value of a residual?





Least square model fitting

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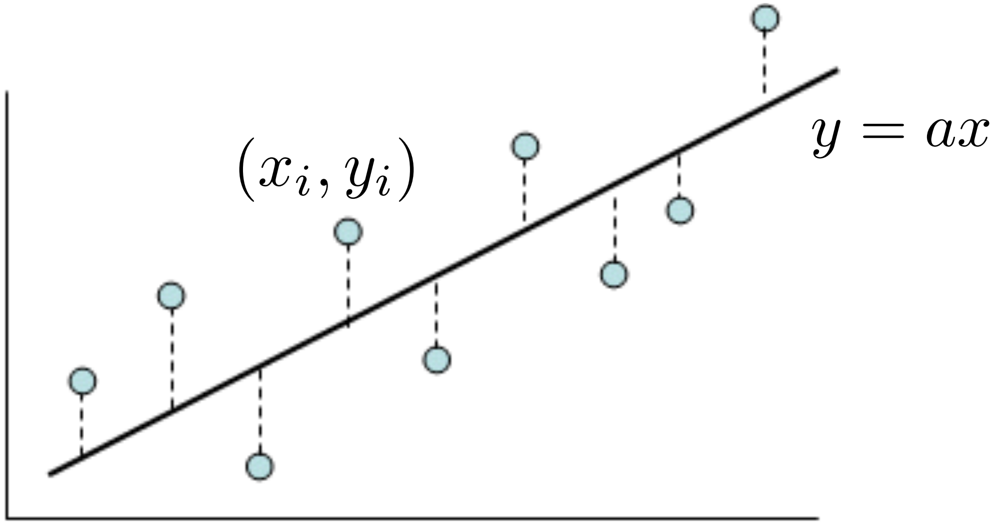
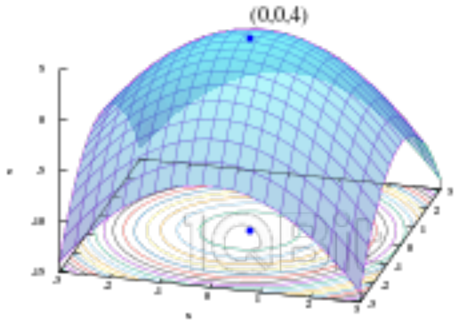
What is the value of a residual?

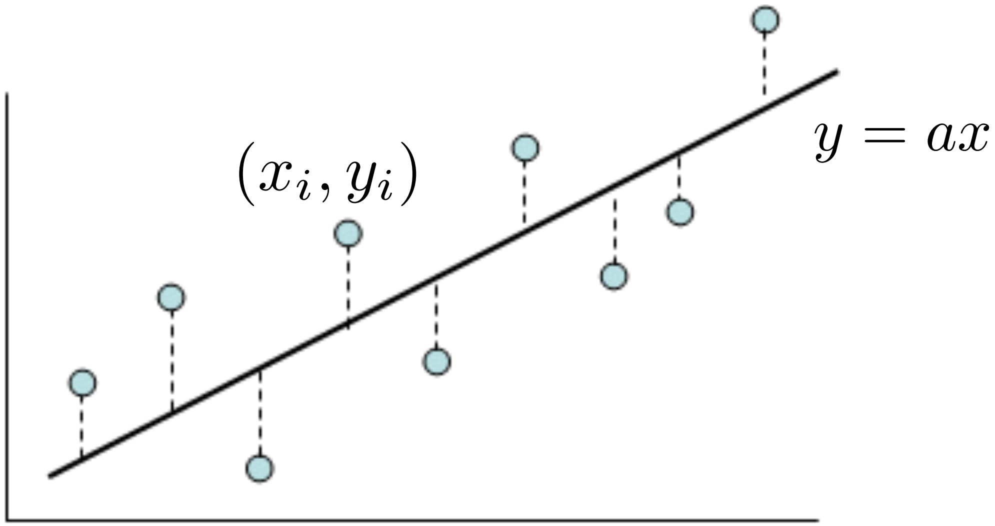
$$r_i = y_i - ax_i$$



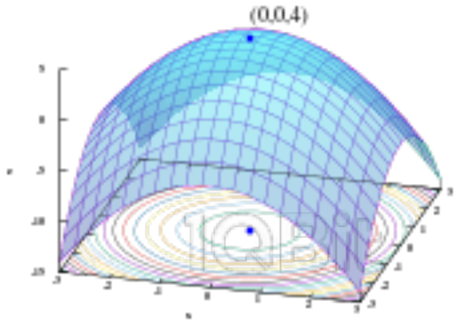


Least square model fitting

Which objective function should we optimize?



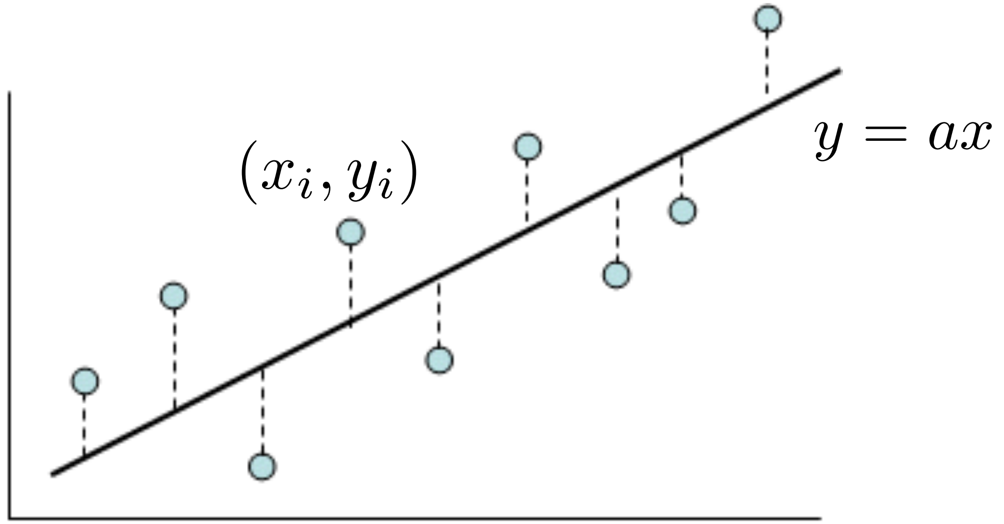
$$
\begin{aligned}
&(A) \quad f(a) = |y_1 - ax_1| + |y_2 - ax_2| + \cdots + |y_n - ax_n| \\
&(B) \quad f(a) = (y_1 - ax_1)^2 + (y_2 - ax_2)^2 + \cdots + (y_n - ax_n)^2 \\
&(C) \quad f(a) = (y_1 - ax_1)(y_2 - ax_2) \cdots (y_n - ax_n) \\
&(D) \quad f(a) = (ay_1 - x_1)^2 + (ay2 - x2)^2 + \cdots + (ay_n - x_n)^2
\end{aligned}
$$

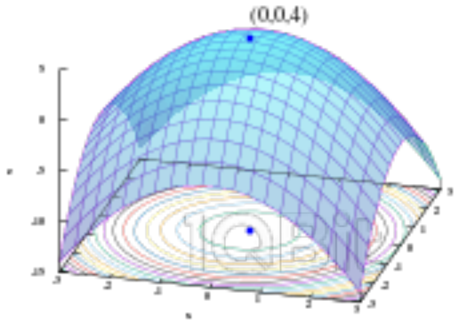


Least square model fitting

Find the slope a such that y = a x fits the points (4, 5) and (6, 7) in the least square sense.

(A) a = 7/6
(B) a = 5/4
(C) a = (7/6 + 5/4) / 2
(D) a = 31/26





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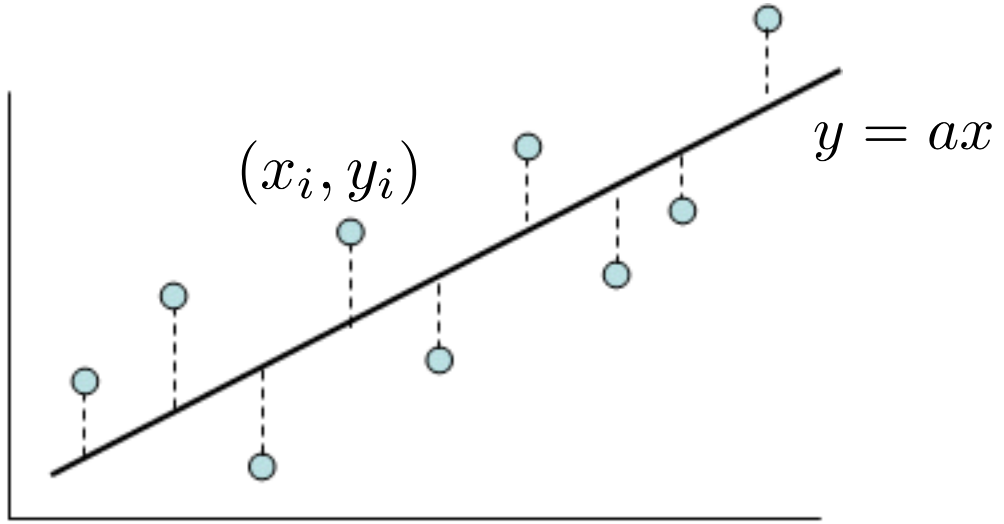

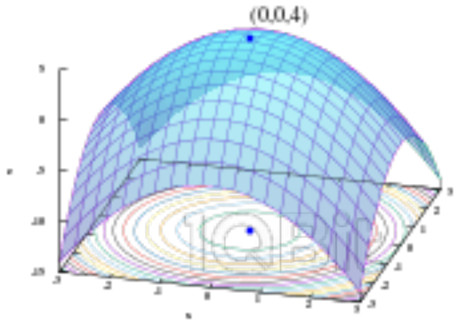

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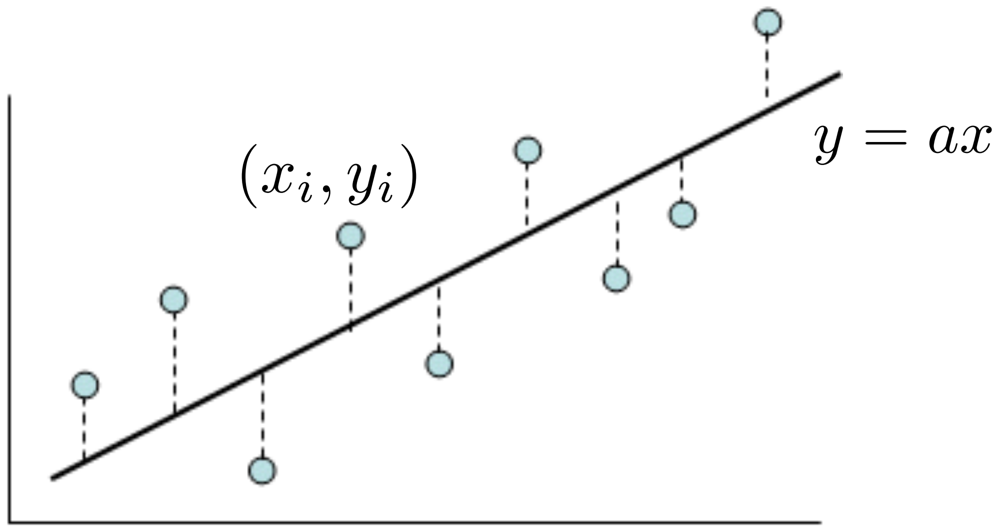
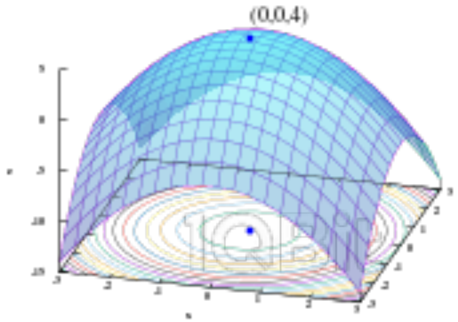
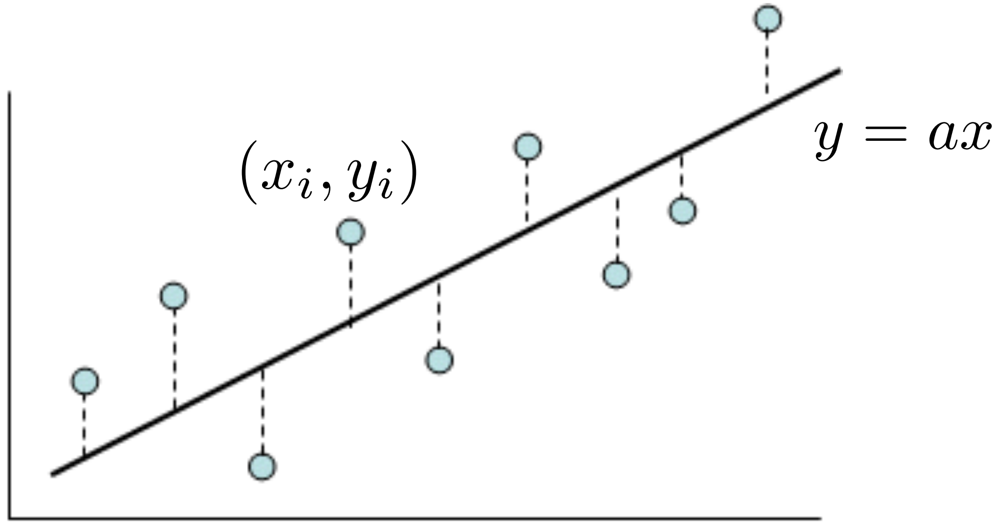
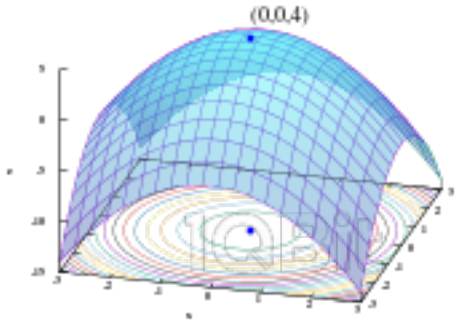




Least square model fitting

What is the general formula?





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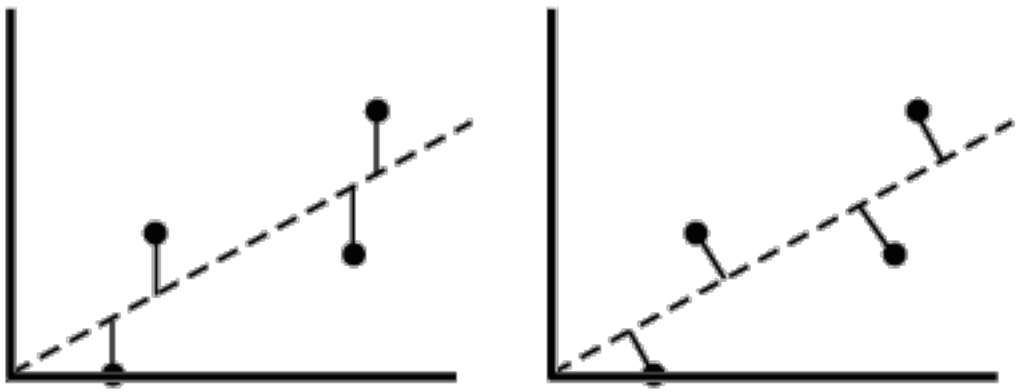

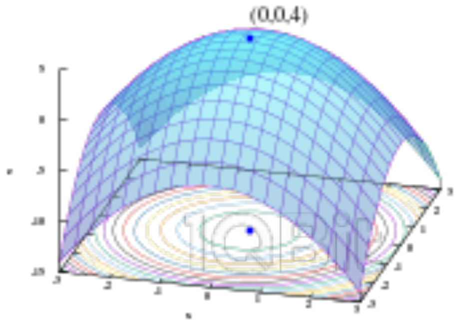

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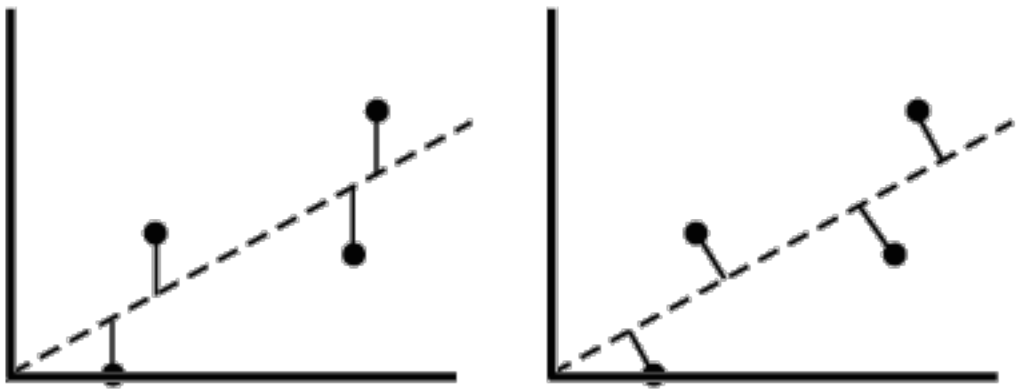
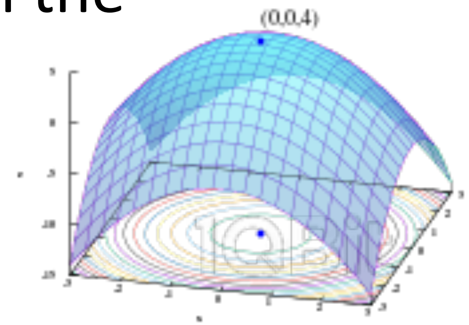


Which one gives a better fitting?

Bonus: what is the objective function to be optimized for SSR when the residual is the perpendicular offset from y = ax?



See you next week!

Oct 24	PL8.1
Oct 25	WW 6