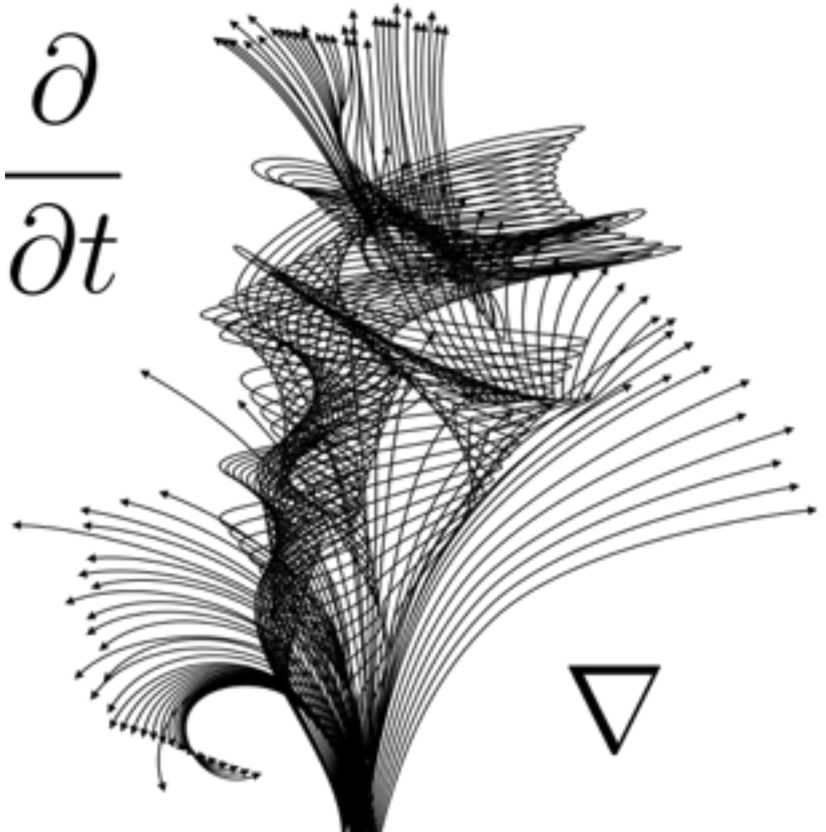
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

Agenda for today:

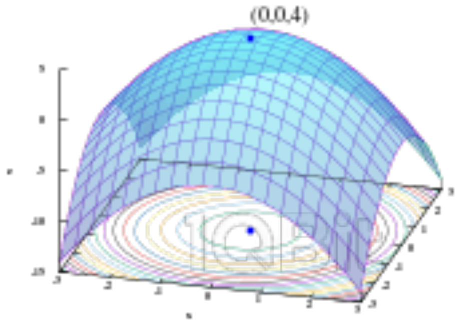
- Related rates
- Implicit differentiation



Chain rule warm-up

If $h(x) = (x^3 - 2x + 1)^6$ then the derivate is…

$$
\begin{aligned}
&(A) \quad 6(x^3 - 2x + 1)^5 \\
&(B) \quad (x^3 - 2x + 1)^6(3x^2 - 2) \\
&(C) \quad 6(x^3 - 2x + 1)^5(3x^2 - 2) \\
&(D) \quad 6(x^3 - 2x + 1)^5(x^3 - 2x + 1)
\end{aligned}
$$



Related rates: Intro

Conclusion: **the fact that we had driven 130km was irrelevant!**

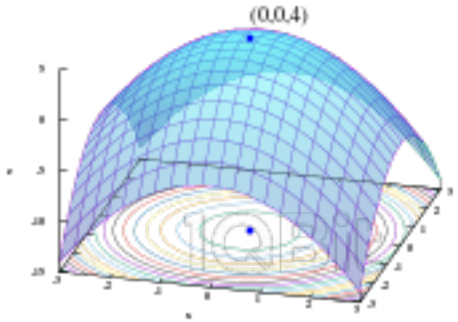
We could have solved the problem by only considering the related rates as follows:

Cost of gas is given by

$$c = 1.25\ell$$

Rate of change of cost is given by

$$\frac{d}{dx}c = 1.25\frac{d}{dx}\ell = 1.25 \times 0.07$$



Another example

The radius of a spherical tumour grows at a constant rate, k. Determine the rate of growth of the volume of the tumour when the radius is one centimetre.

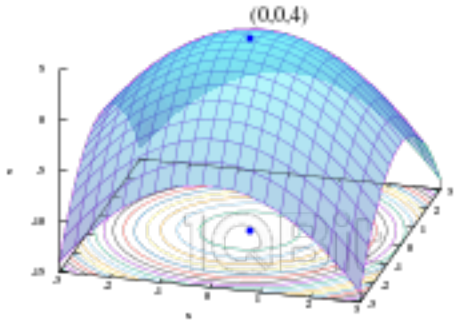
Which one is the useful relation to start from?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2\ k$

(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi$



Another example

The radius of a spherical tumour grows at a constant rate, k. Determine the rate of growth of the volume of the tumour when the radius is one centimetre.

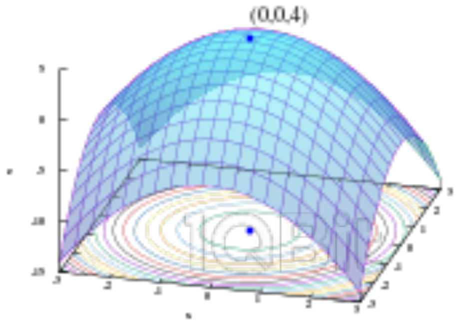
Which one is the correct relation between the related rates?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2\ k$

(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi\ k^3$



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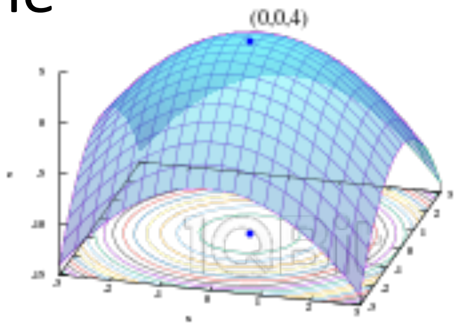
(C) $V' = 4\ \pi\ k^2$

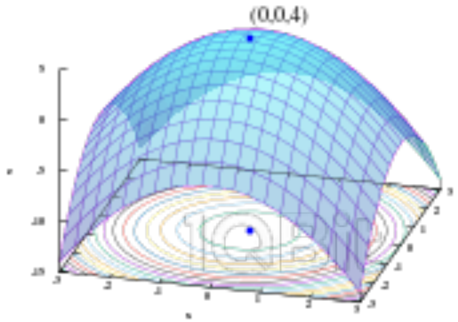
(D) $V = 4/3\ \pi\ k^3$

The fact that the radius of tumour is one was irrelevant in finding the related rates. But we can plug it in!



Last example

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.



Last example

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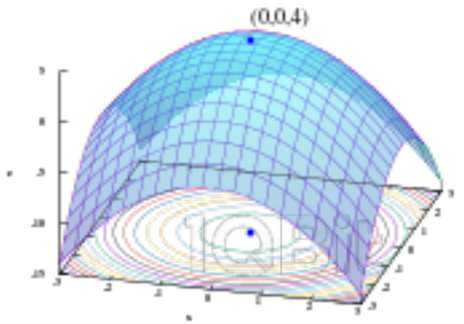
Recall: Volume of a conical shape of radius R and height H is given by

$$V = \frac{1}{3}\pi R^2 H$$



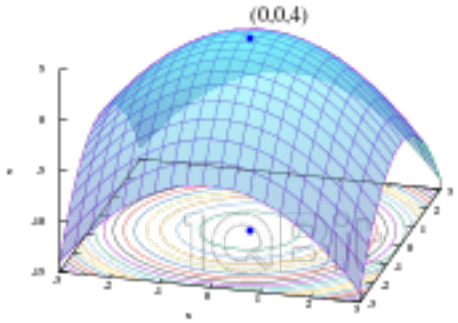
Procedure

Establish expectation(s) based on sketch or otherwise.

Find equation relating a first quantity and a second quantity.

Take derivatives on both sides (CHAIN RULE).

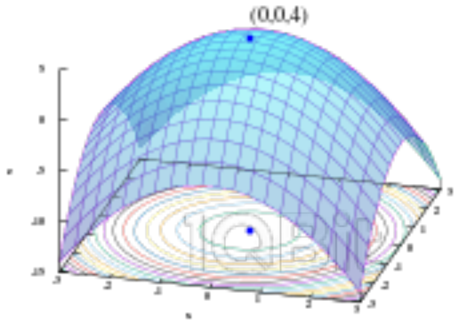
Finally, plug in specific values.



Implicit differentiation

Intro example

Find the equation of the tangent line to $x^2 + y^2 = 25$ at $(3,-4)$.



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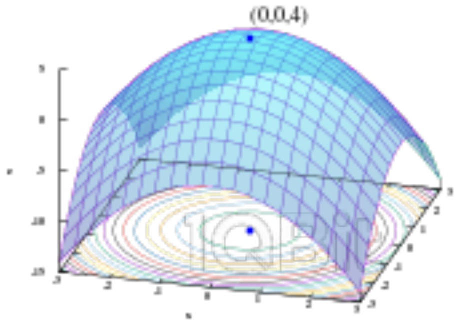


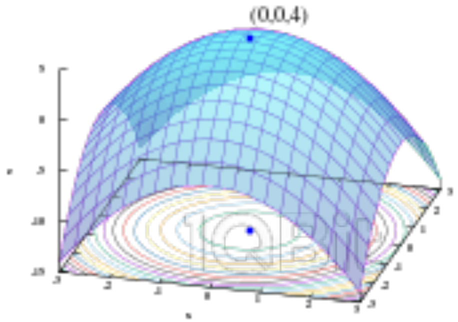
Implicit differentiation

Intro example

Find the equation of the tangent line to $x^2 + y^2 = 25$ at $(3,-4)$.

Problem! What function do we take derivative of?

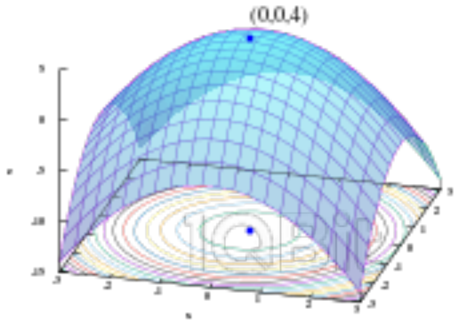
Answer: $x^2 + y^2 = 25$ is a function **almost** everywhere!



Exponential functions

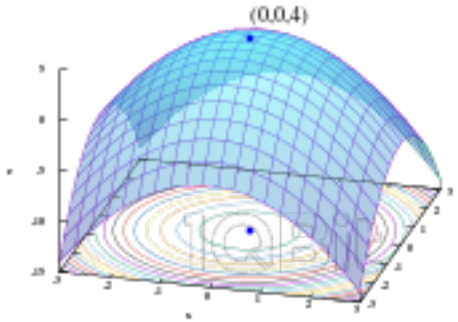
Which of the following is an exponential function?
(Assume: n is a constant, x is a variable)

(A) x^n (B) n^x (C) 2^x (D) e^n (E) $\ln(x)$



Thinking about exponential functions

Millennium Stadium (Cardiff, Wales): Home for Wales national rugby team and venue for the 2017 UEFA Champions League Final.



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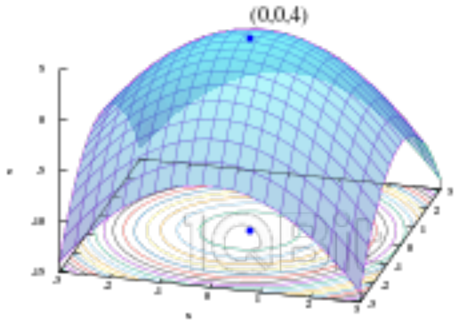
Bowl volume: 1.5 million cubic metres!



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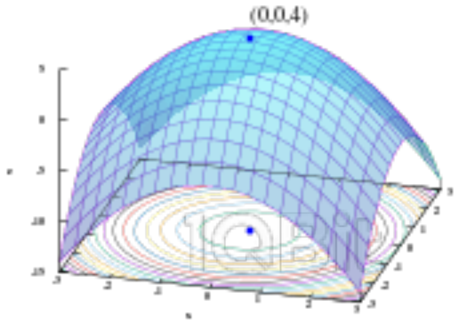
Volume of a drop of water: 0.05 mL



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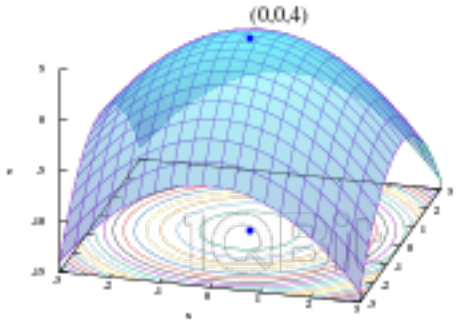
1 cube meter = 1000 litres



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Bowl volume: 1.5 million cubic metres = 1500000000000 mL

Volume of a drop of water: 0.05 mL

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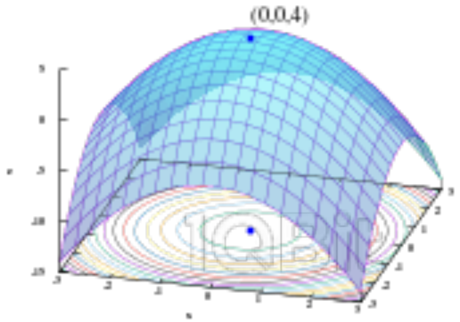

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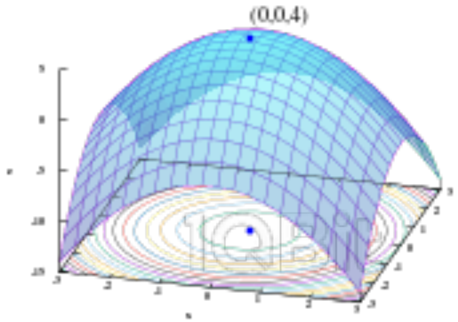
Magic drop = replicating itself in every second



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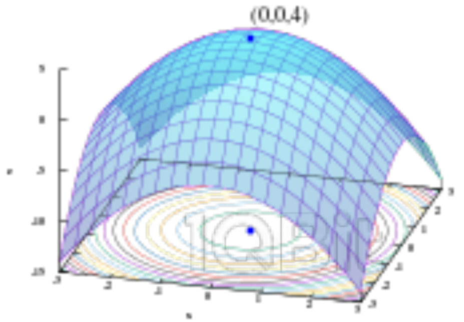
After 2 seconds: 0.1mL



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Volume of a drop of water: 0.05 mL

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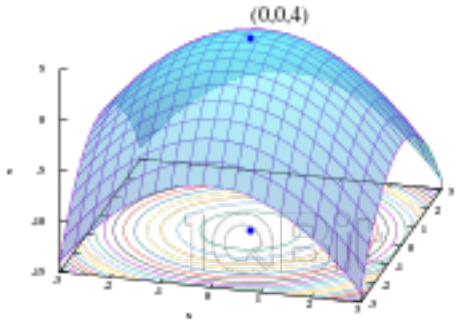
After 10 seconds: 51mL



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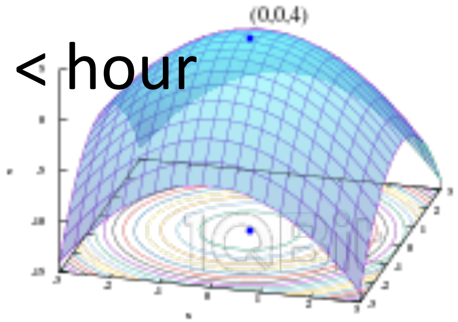
Bowl volume: 1.5 million cubic metres = 1500000000000 mL

Volume of a drop of water: 0.05 mL

Magic drop = replicating itself in every second

Question: How much time do you have to scape the stadium before the magic drop drowns you?

(A) 1 year (B) 1 week (C) about 1 day (D) about 10 hours (E) < hour



(A) 1 year (B) 1 week (C) about 1 day (D) about 10 hours (E) < hour

Pre-calc recall

If $a > 1$ which one is correct about the functions $f(x) = a^x$?

(A) All go through the point (1, 1).
(B) All go through the point (0, 0).
(C) All go through the point (1, 0).
(D) If $a < b$ then $a^x < b^x$ for all $x > 0$ and $a^x > b^x$ for all $x < 0$.
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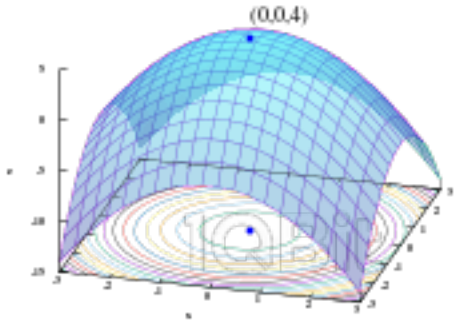

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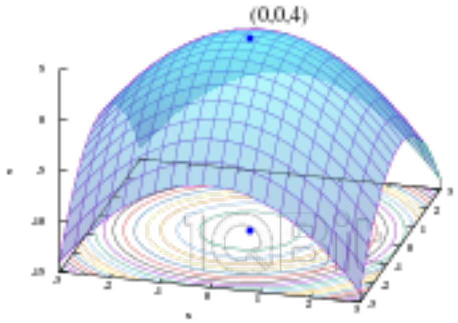
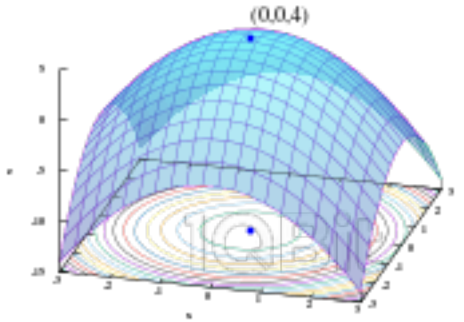
What can you say when $a < 1$?



Derivative of exponential functions

The derivative of $f(x) = a^x$ is of the form

(A) $f'(x) = x\, a^{x-1}$
(B) $f'(x) = a\, x^{a-1}$
(C) $f'(x) = a^x$
(D) $f'(x) = C\, a^x$.



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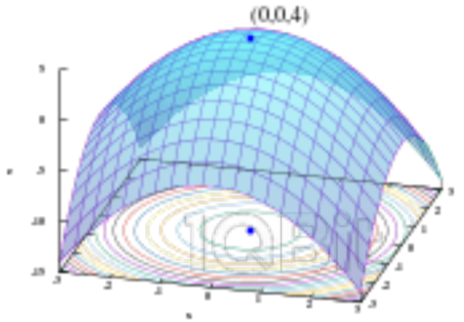
Consider the definition of the derivative, we get

$$C = \lim_{h \to 0} \frac{a^h - 1}{h}$$



The Number e

Definition: The number a for which this limit becomes 1.

$$1 = \lim_{h \to 0} \frac{e^h - 1}{h}$$

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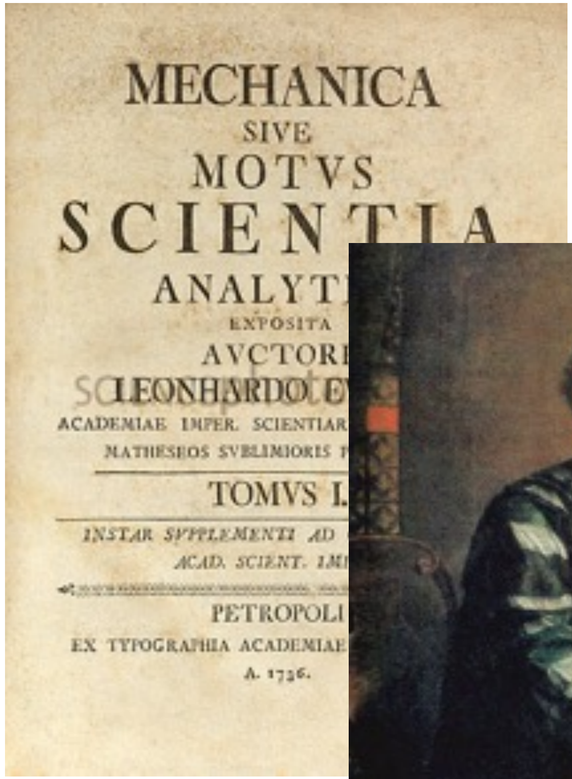

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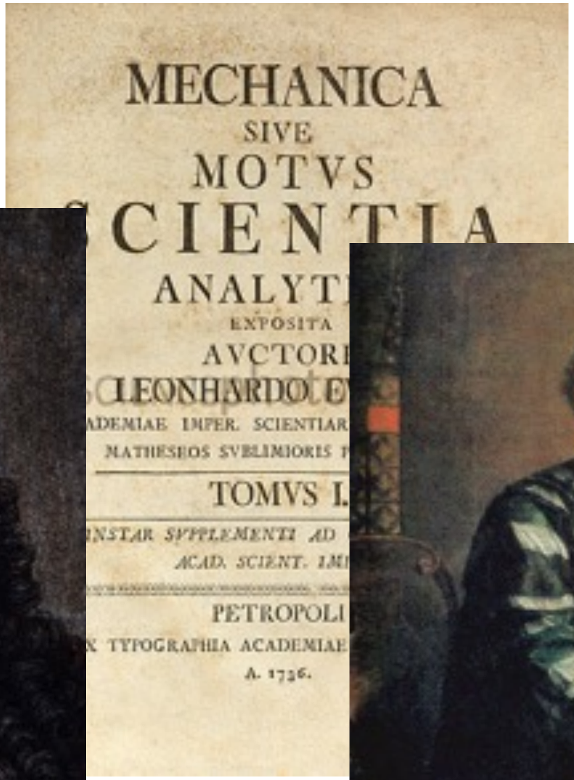

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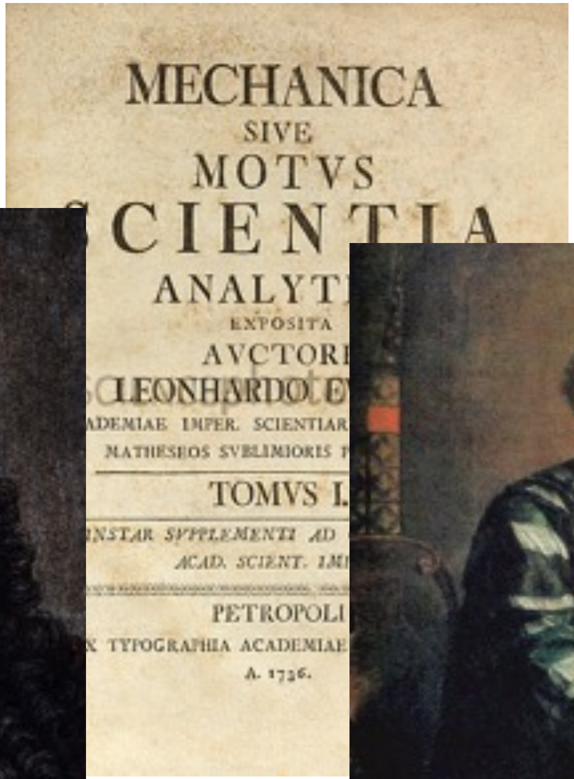

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$$1 = \lim_{h \to 0} \frac{e^h - 1}{h}$$

The Number e

t bec

-1

You have 45 seconds to scape now!

Oct 27	WW 7
Oct 28	OSH 4
Oct 31	PL9.1

$$\frac{\partial}{\partial t}$$