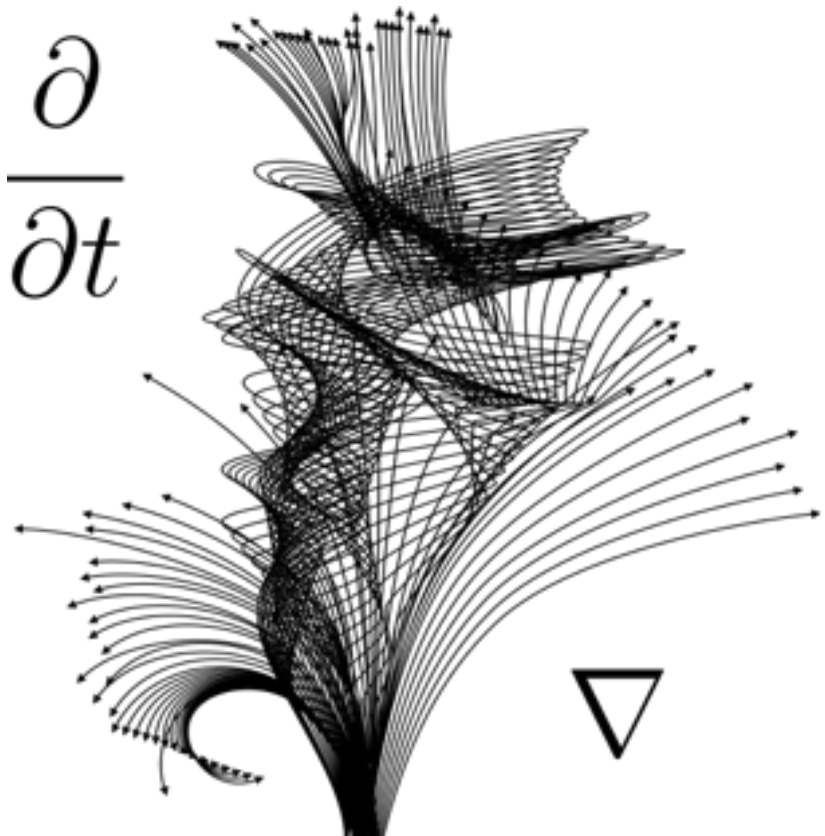
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

Agenda for today:

- Slope fields
- Linear DEs



Stable and unstable

$$x' = x(1 - x)$$

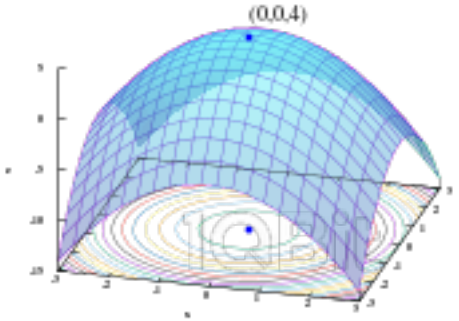
Steady state

x = 0

Steady state

x = 1

Steady states can be **stable** or **unstable**.



Stable and unstable

$$x' = x(1 - x)$$

Steady state

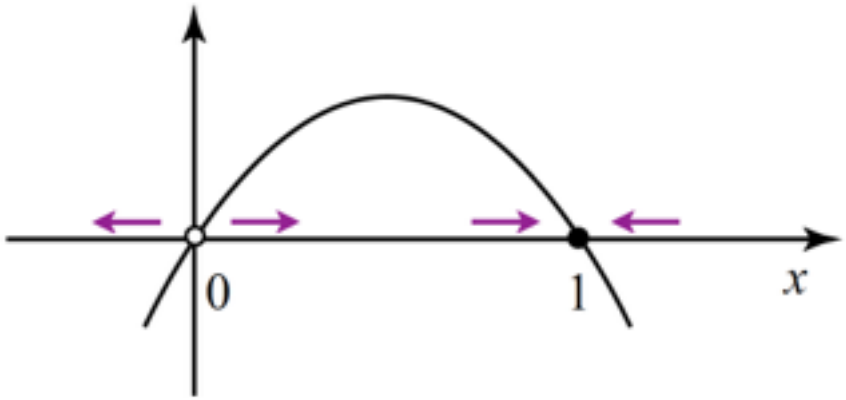
x = 0

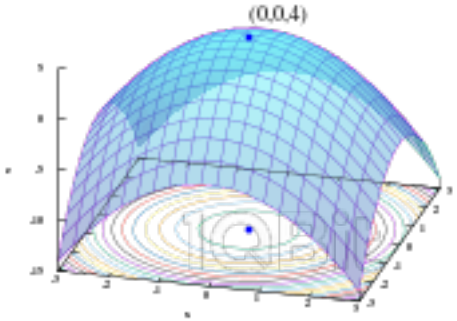
Unstable

Steady state

x = 1

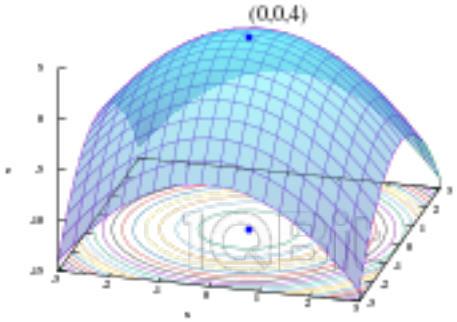
Stable



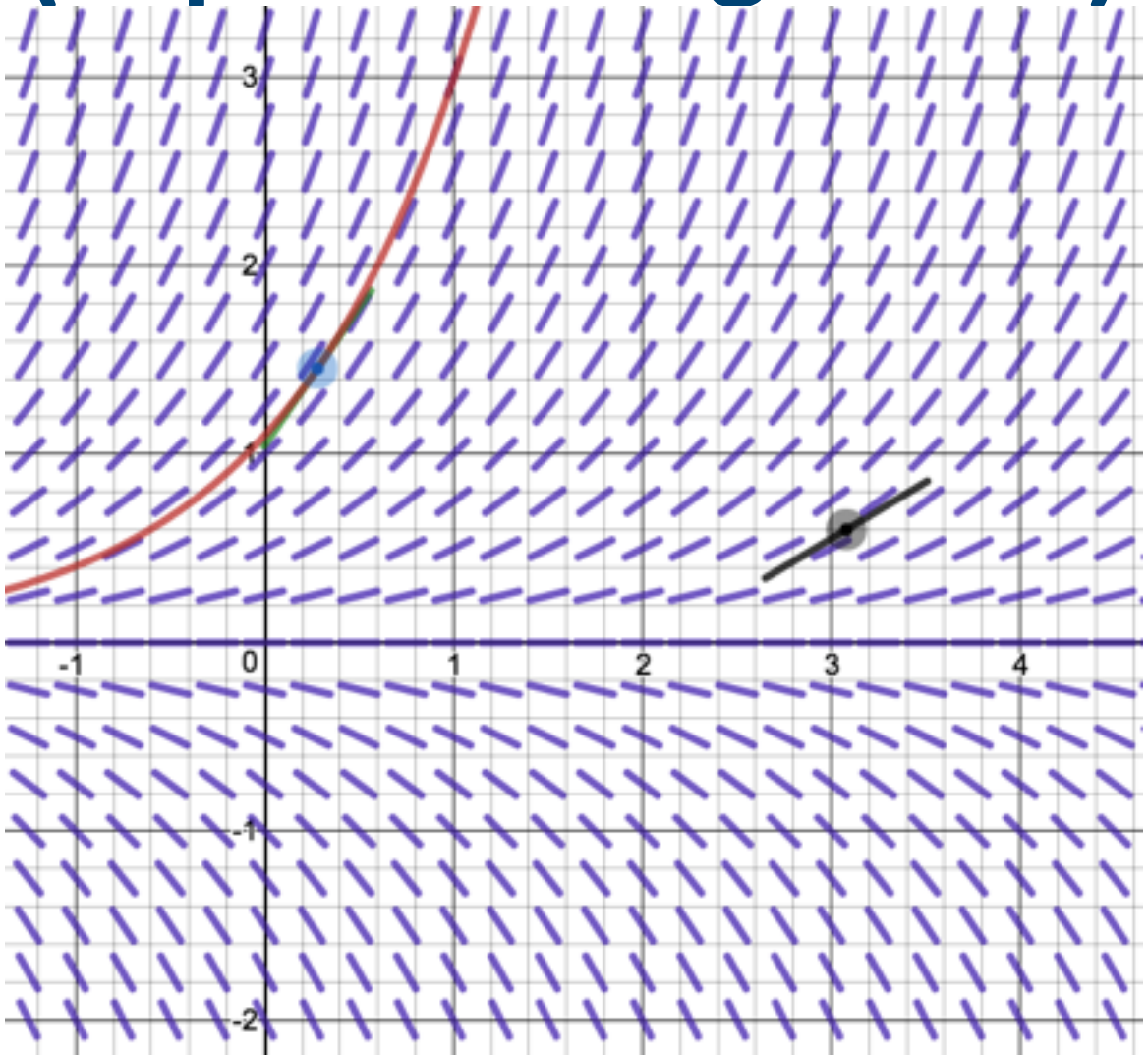


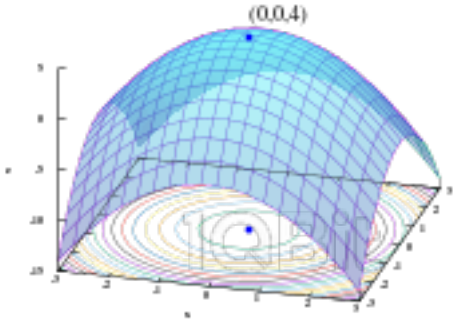
Slope field

Let’s look at our simple DE again: $f' = f$

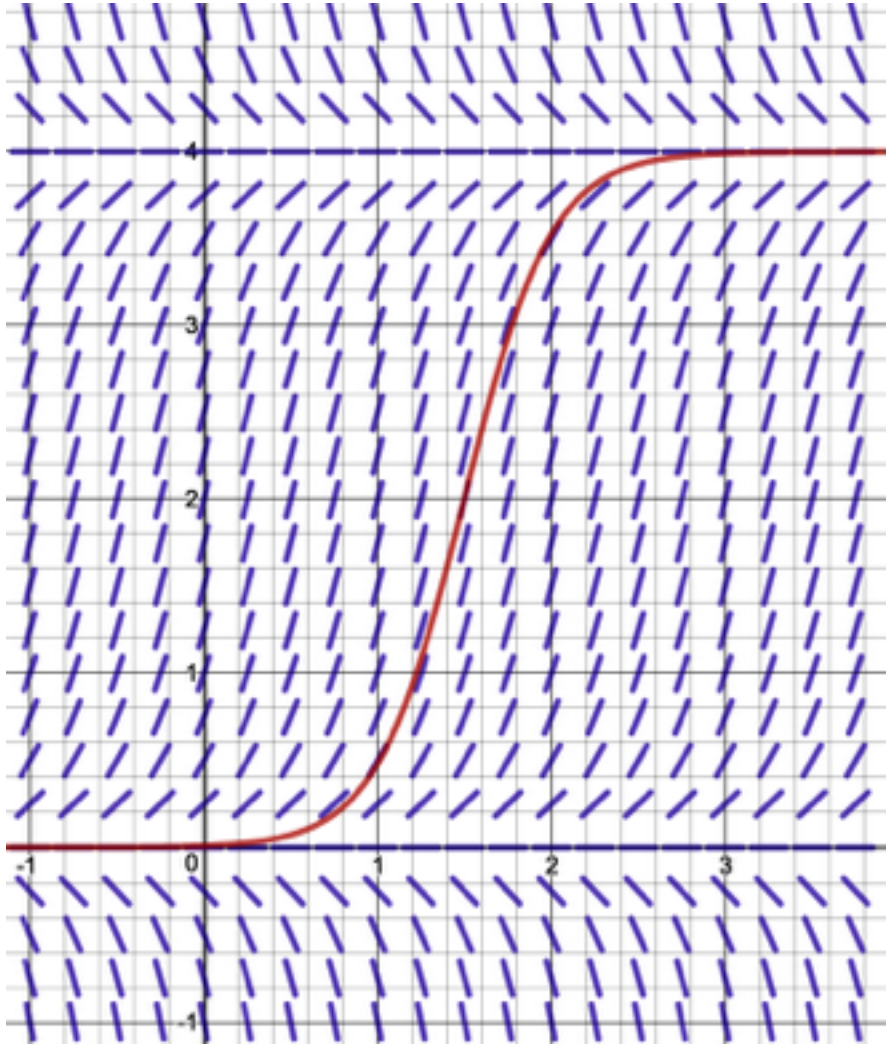


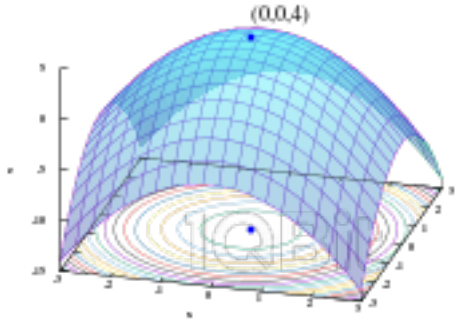
Slope field (exponential growth)



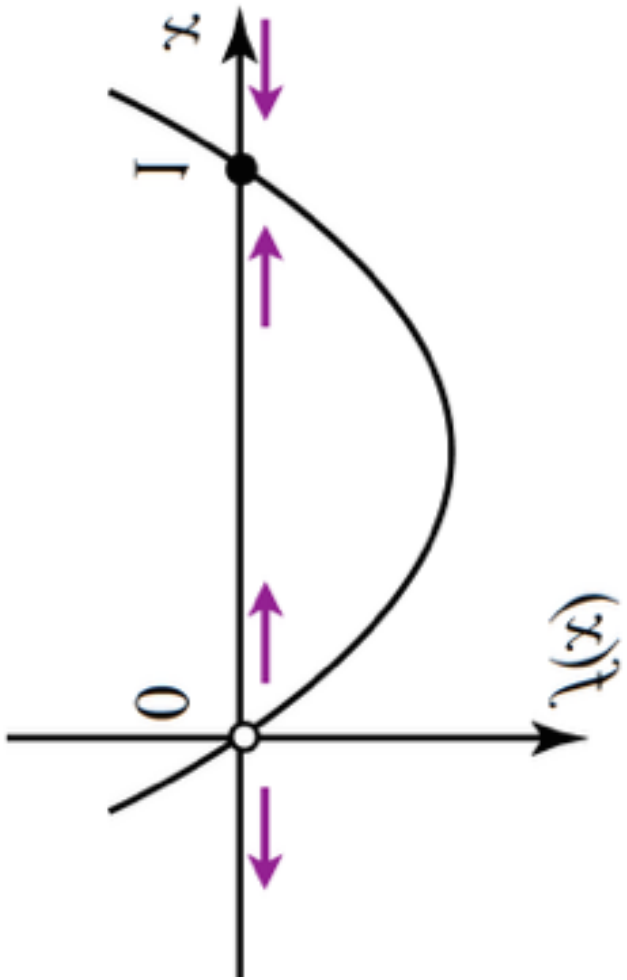


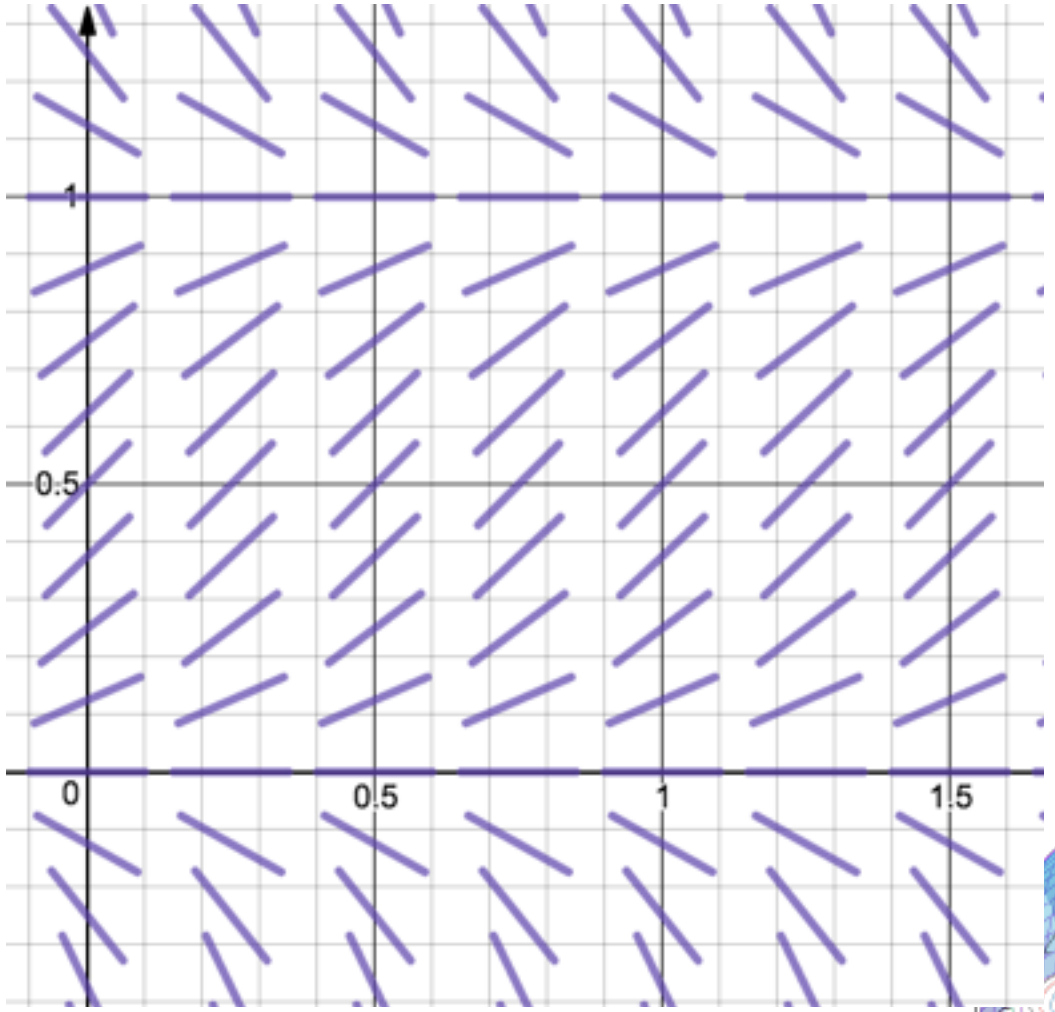
Slope field (logistic equation)

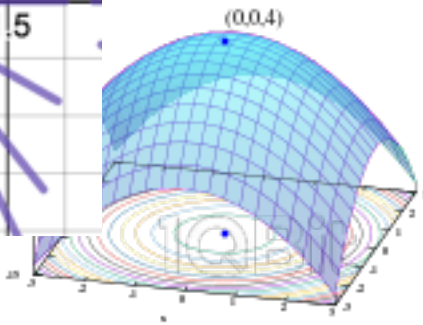




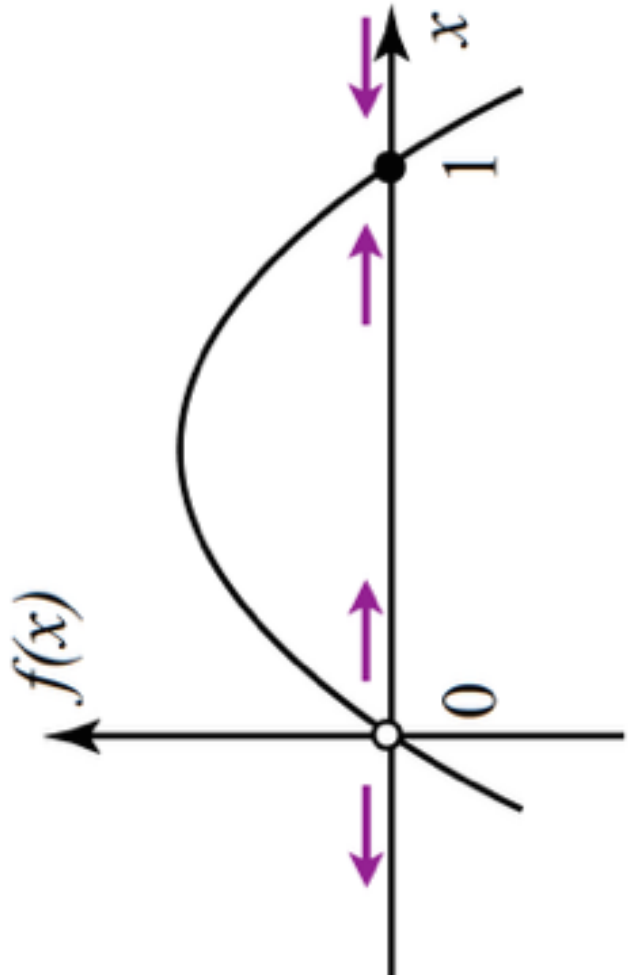
Slope field (logistic equation)

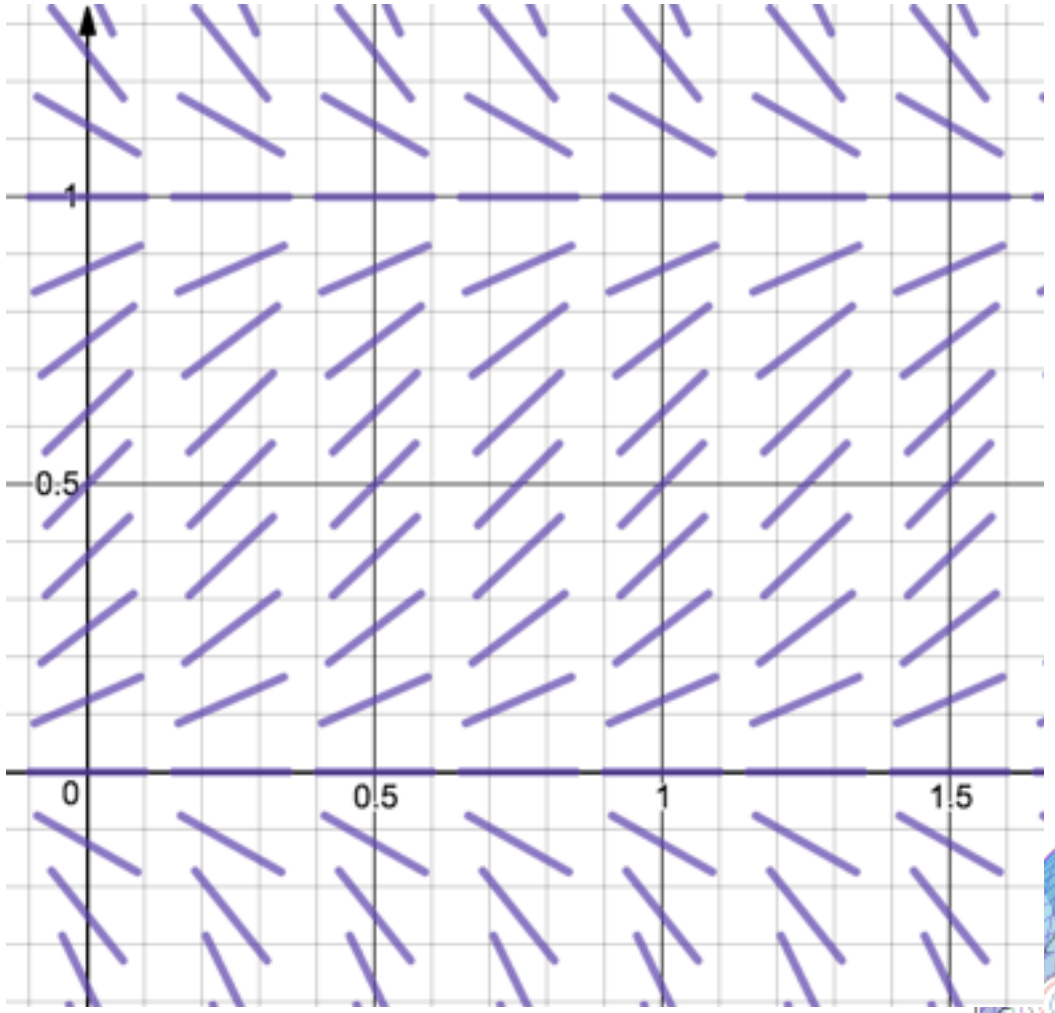


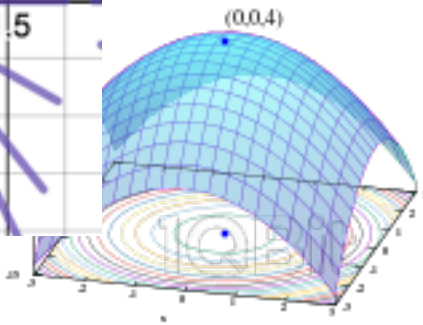




Slope field (logistic equation)







Properties of the solutions

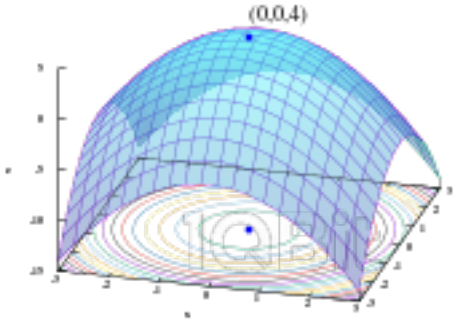
$$x' = x(1-x)$$

Question: Can a solution to this DE have a local maximum?

Question: If x(t) is a solution is a translation x(t-c) a solution?

Question: If x(t) is a solution is a translation x(t)+C a solution?

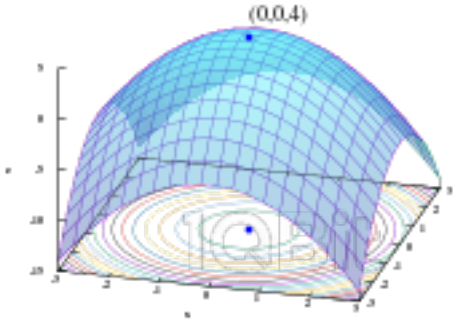
Question: If $x_1(t)$ and $x_2(t)$ are two solutions, can they cross?



Exercise

$$y' = -y(y-1)(y+1)$$

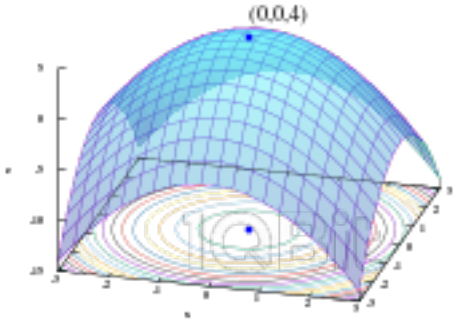
- What are the steady states of this equation?
- Draw the slope fields for this equation.
- For each steady state, determine its stability.
- What value does a solution starting at y(0)=0.2 approach for large t?
- Does that solution have any inflection points?
- Sketch it.



Back to linear DEs

The rate of change of an object's temperature is proportional to the difference between the objects temperature and the surrounding environment. Find a differential equation that expresses this phenomena.

Write down your solution!

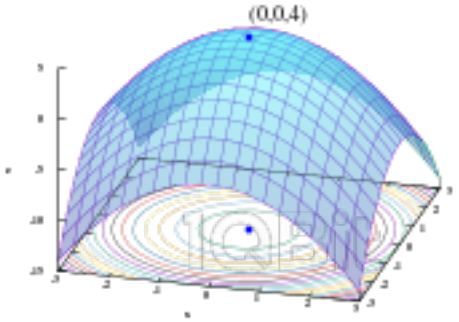


Newton's law of cooling

The rate of change of an object's temperature is proportional to the difference between the objects temperature and the surrounding environment. Find a differential equation that expresses this phenomena.

My differential equation looks like

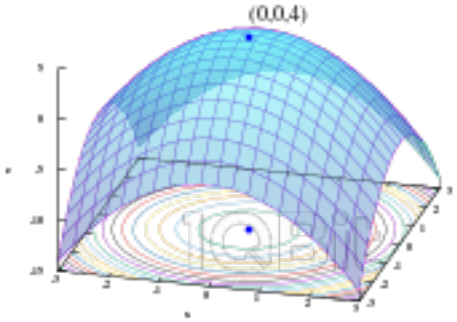
$$(A) \quad T'(t) = kT(t)$$
$$(B) \quad T'(t) = E - kT(t)$$
$$(C) \quad T'(t) = kT(t) - E$$
$$(D) \quad T'(t) = k(T(t) - E)$$
$$(E) \quad T'(t) = k(E - T(t))$$



Free fall in a liquid

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

Write down your differential equation!

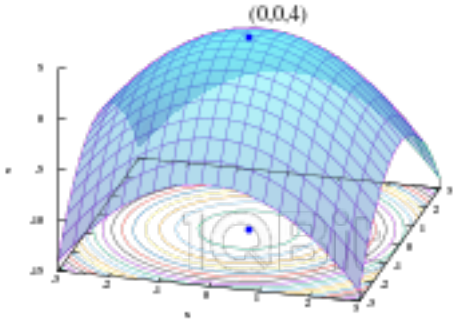


Free fall in a liquid

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

My differential equation looks like:

$$(A) \quad v'(t) = \delta(v(t) - g)$$
$$(B) \quad v'(t) = \delta(g - v(t))$$
$$(C) \quad v'(t) = \delta v(t) - g$$
$$(D) \quad v'(t) = g - \delta v(t)$$
$$(E) \quad \text{something better}$$



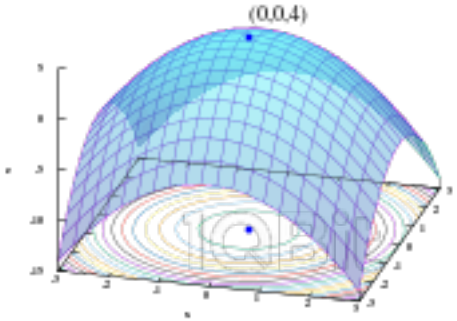
Free fall in a liquid

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

My differential equation looks like:

$$(A) \quad v'(t) = \delta(v(t) - g)$$
$$(B) \quad v'(t) = \delta(g - v(t))$$
$$(C) \quad v'(t) = \delta v(t) - g$$
$$(D) \quad v'(t) = g - \delta v(t)$$
$$(E) \quad \text{something better}$$

Every term has to have the same unit!
What should be the unit of g?



Free fall in a liquid

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

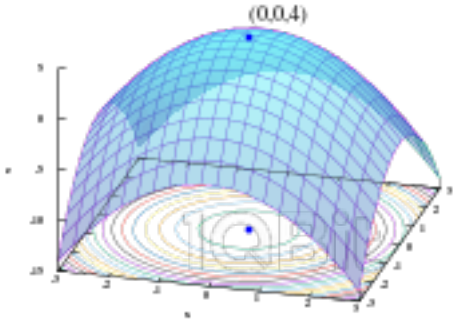
My differential equation looks like:

$$(A) \quad v'(t) = \delta(v(t) - g)$$
$$(B) \quad v'(t) = \delta(g - v(t))$$
$$(C) \quad v'(t) = \delta v(t) - g$$
$$(D) \quad v'(t) = g - \delta v(t)$$
$$(E) \quad \text{something better}$$

Every term has to have the same unit!

What should be the unit of g?

What should be the unit of δ?



Free fall in a liquid

An object dropped in water will accelerate under the influence of the constant downward force of gravity and an upward drag force proportional to the velocity.

My differential equation looks like:

$$(A) \quad v'(t) = \delta(v(t) - g)$$
$$(B) \quad v'(t) = \delta(g - v(t))$$
$$(C) \quad v'(t) = \delta v(t) - g$$
$$(D) \quad v'(t) = g - \delta v(t)$$
$$(E) \quad \text{something better}$$

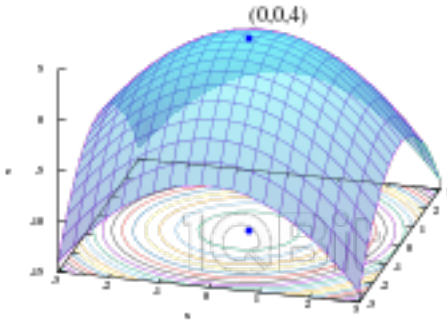
Every term has to have the same unit!

What should be the unit of g?

What should be the unit of δ?

Using Newton's 2nd law of motion:

$$\begin{aligned} ma &= F_g + F_{\text{drag}} \\ &= mg - \gamma v(t) \\ a &= g - \frac{\gamma}{m} v(t) \end{aligned}$$



Have a great long weekend =)

Nov 11	OSH 5
Nov 14	PL11.1
Nov 16	PL11.2