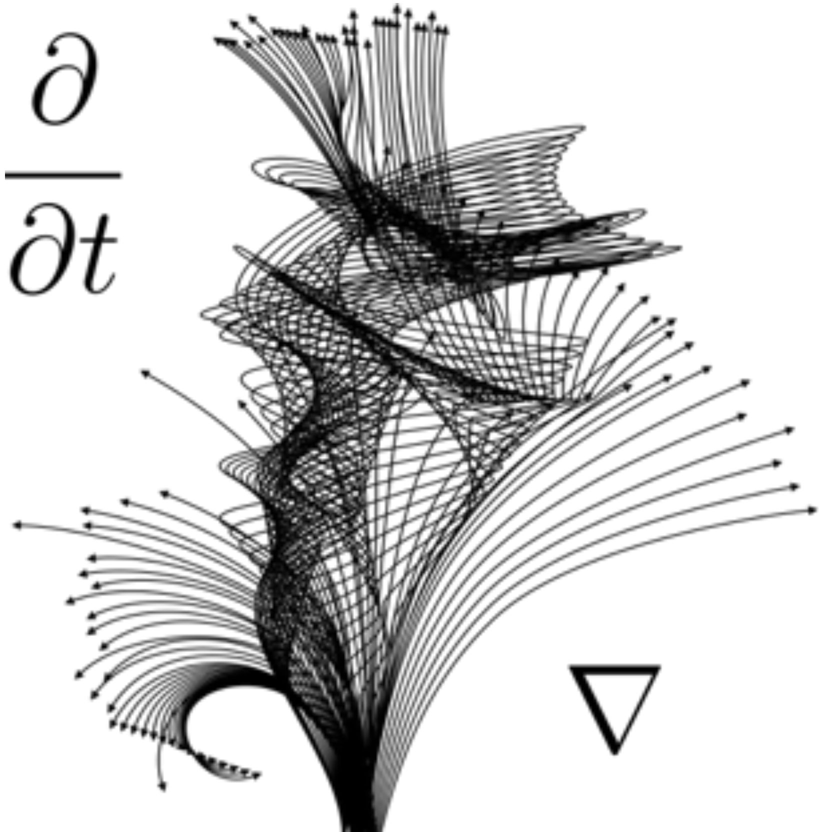
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

Agenda for today:

- Absolute max/min
- Optimization



Terminology

Your book is never talking about "convex" functions or "convexity."

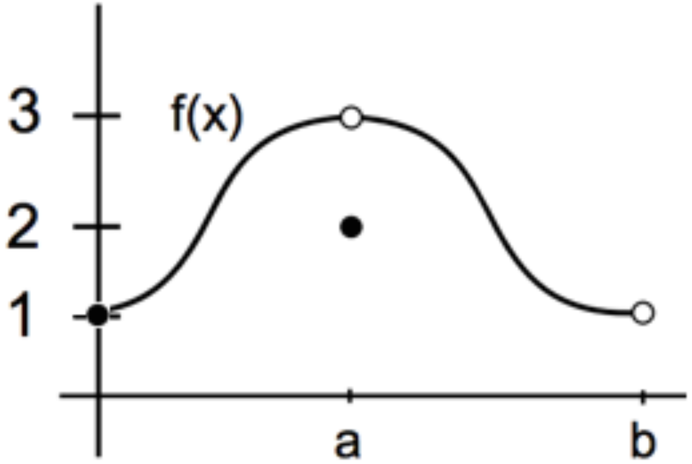
We'll stick to "concavity", and "concave up" and "concave down".

Absolute extrema

An **absolute maximum** is the **largest** value that a function attains.
An **absolute minimum** is the **smallest** value that a function attains.

Question: Does f have an absolute max on [0, a]?

Question: Does f have an absolute min on [a, b)?



Question: Does f have an absolute min on [0, b)?

Absolute extrema

A continuous function on a closed interval [a,b] takes on its absolute extrema.

Absolute extrema

A continuous function on a closed interval [a,b] takes on its absolute extrema.

The absolute extrema are obtained either at a local maximum (minimum) or **at an end point** (x=a or x=b).

Absolute extrema

Where does $f(x)=x^3-x^2$ take on its absolute minimum on the interval [-1,2]?

(A) $x = -1$

(B) $x = 0$

(C) $x = 2/3$

(D) $x = 2$

Optimization on a closed interval

Given a scenario involving a choice of some number, use calculus to find the best value.

Translate scenario into a mathematical problem, involving a function you need to **optimize** (i.e. find absolute max/min of).

Solve the problem by: finding critical points and boundary points. Check the value of the function at all of them.

Optimization NOT on a closed interval

ON A CLOSED INTERVAL:

Solve the problem by finding critical points and boundary points. Check the value of the function at all of them.

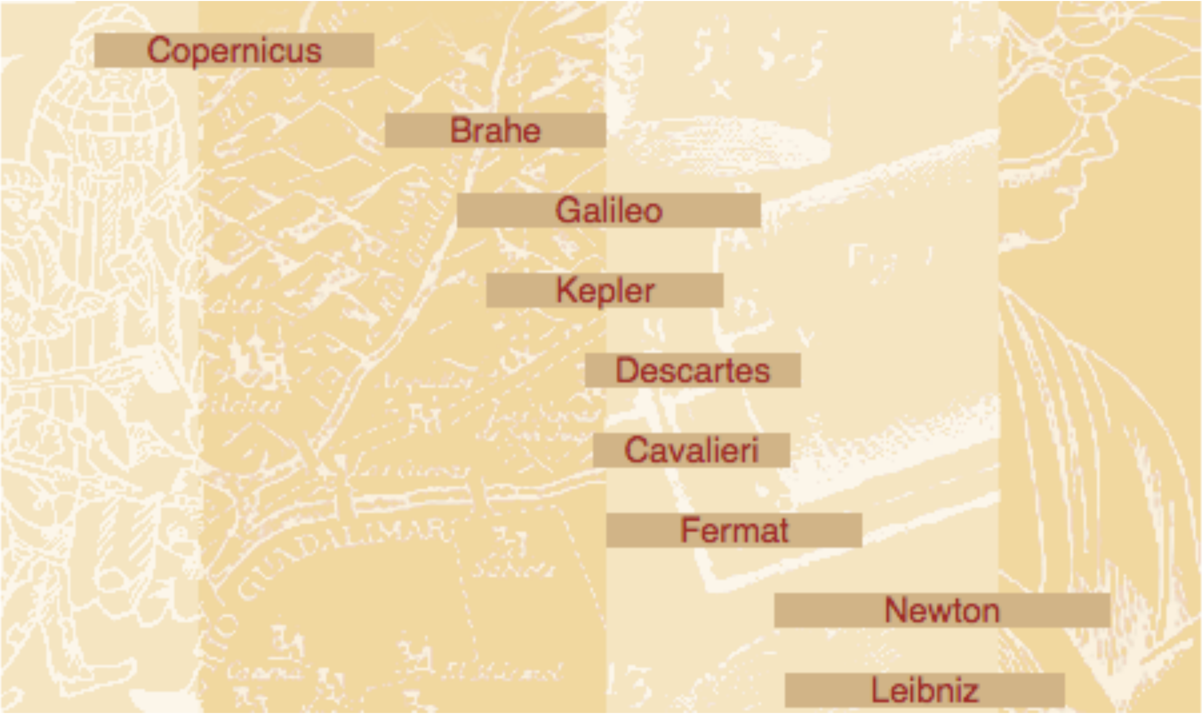
NOT ON A CLOSED INTERVAL:

Find critical points and any boundary points (if exists). Need to run FDT, and SDT for the critical points.

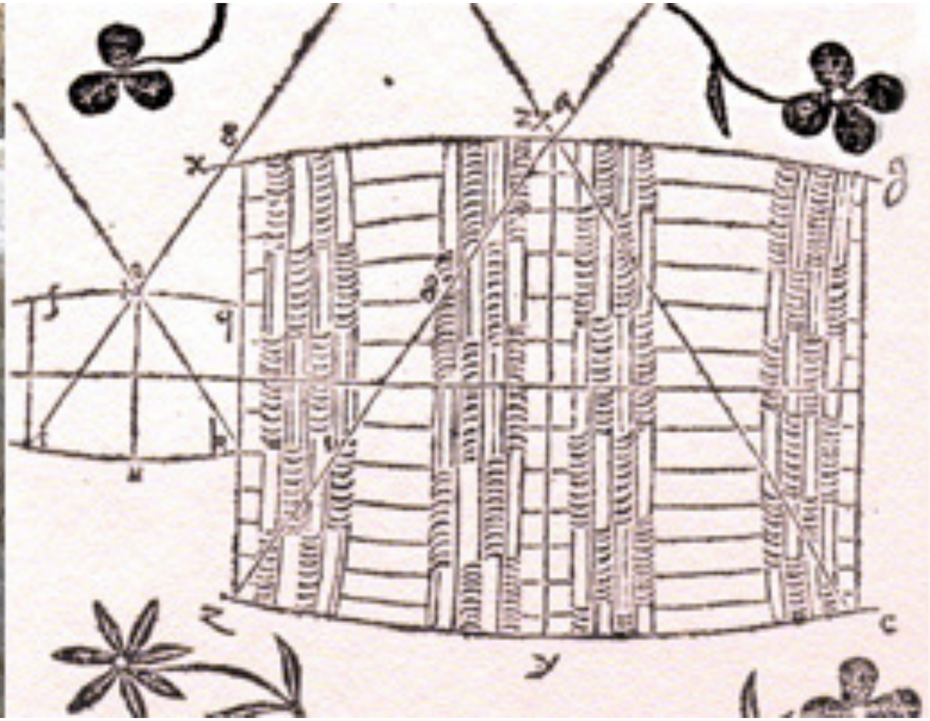
Kepler’s Wine Barrel

Kepler had several children before his first wife died. In 1613, he married for the second time in a celebration in Linz, Austria. Kepler bought a barrel of wine for the wedding but questioned the method the wine merchant used to measure the volume of the barrel and thus determine the price.

Kepler’s Wine Barrel

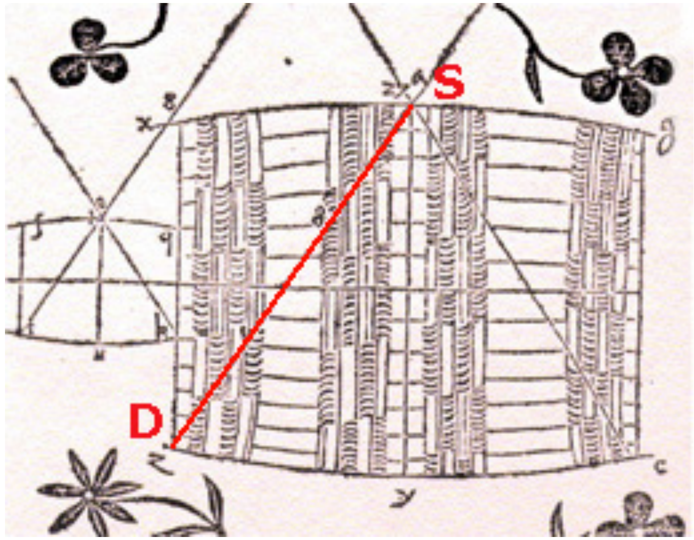


Kepler’s Wine Barrel



Kepler’s Wine Barrel

Then he read off the length SD and set the price accordingly. This outraged Kepler who saw that a narrow, high barrel might have the same SD as a wide one and would indicate the same wine price, though its volume would be ever so much smaller.



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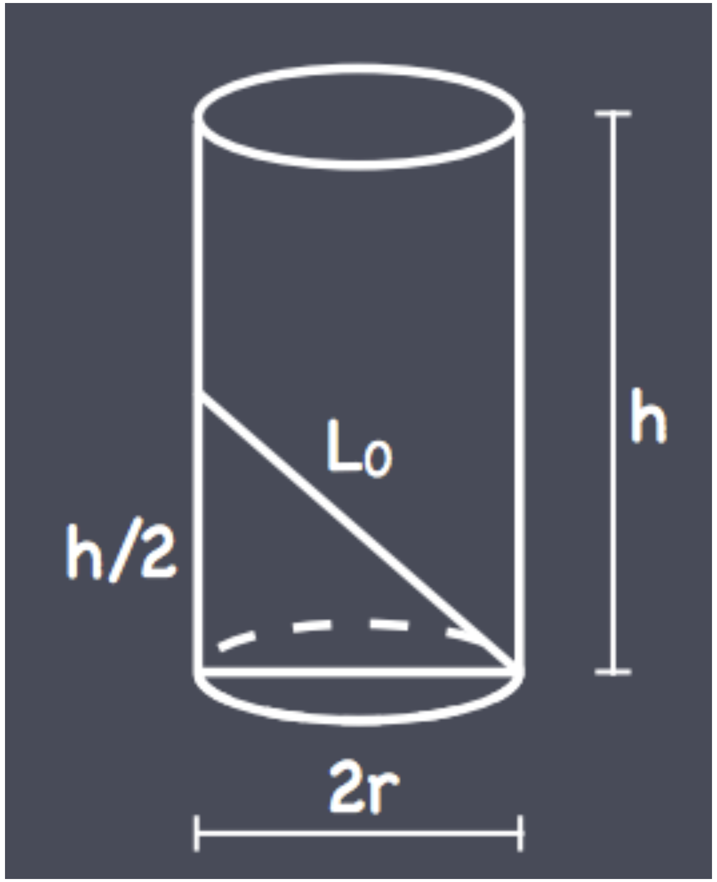
http://www.maa.org/press/periodicals/convergence/kepler-the-volume-of-a-wine-barrel-introduction

Kepler's Wine Barrel



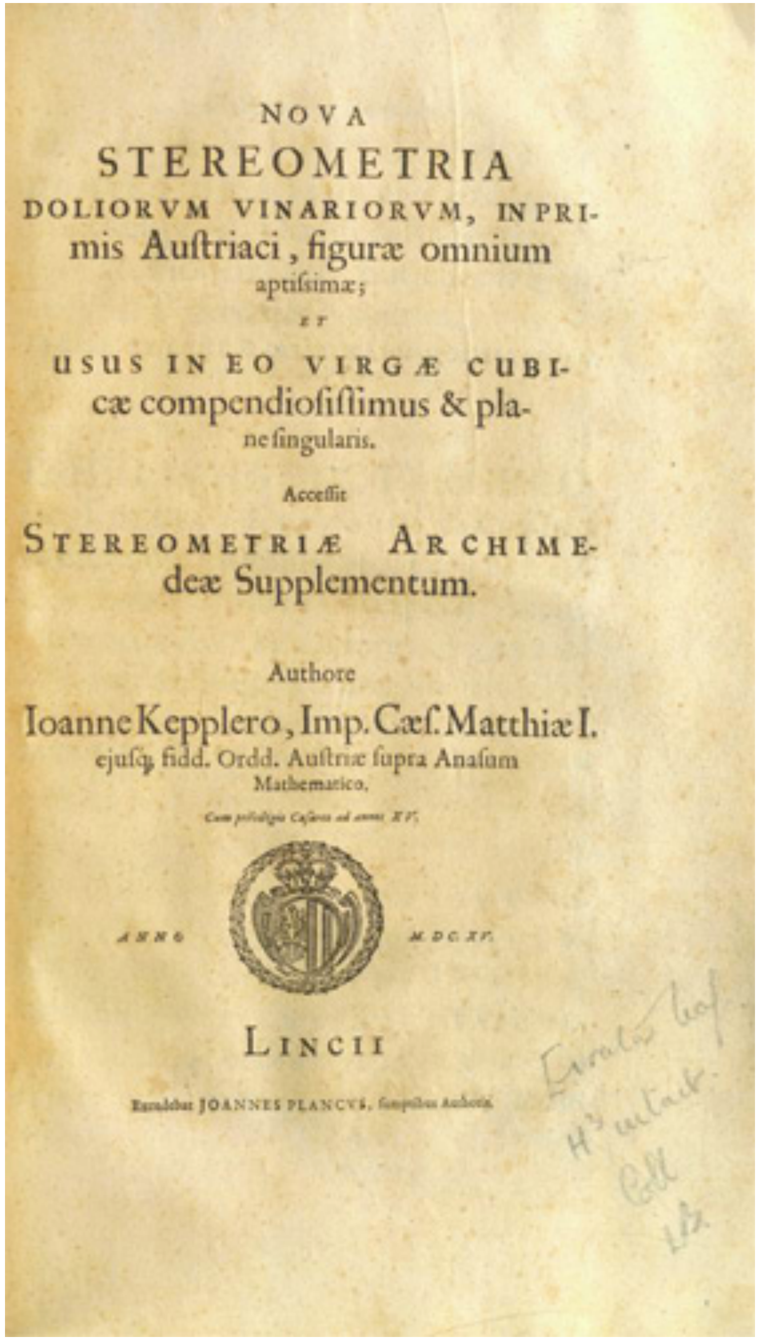
For a fixed budget the Kepler had, he could only buy a barrel with SD= L0. What was the best barrel?

What is the barrel with maximize volume for a given fixed length of the rod?

Kepler’s Wine Barrel

Thus, while the Austrian method of price determination, if applied to Rhenish barrels, would be a clear fraud, it was quite legitimate for Austrian barrels. The Austrian shape had the advantage of permitting such a quick and simple method. So Kepler relaxed in this instance.

Otto Toeplitz, The Calculus: A Genetic Approach, University Of Chicago Press, 1963



NOVA
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Accessit
STEREOMETRIÆ ARCHIME-
deæ Supplementum.
Authore
Ioanne Kepplero, Imp. Cæs. Matthiæ I.
ejusq; fidd. Ordd. Austriæ supra Anasum
Mathematico.
ANNO M. DC. XV.
LINCII
JOANNES PLANCVS,

For these types of problems…

Draw some sketches!

Determine the objective function. Determine the constraint.

Use constraint to change the objective function into a function of one variable.

Find end points and all critical points. Evaluate all critical points and compare.

See you next week!

Oct 14	OSH 3
Oct 17	PL7.1
Oct 18	MIDTERM