Differential Calculus with Applications to Life Sciences

Math 102:105

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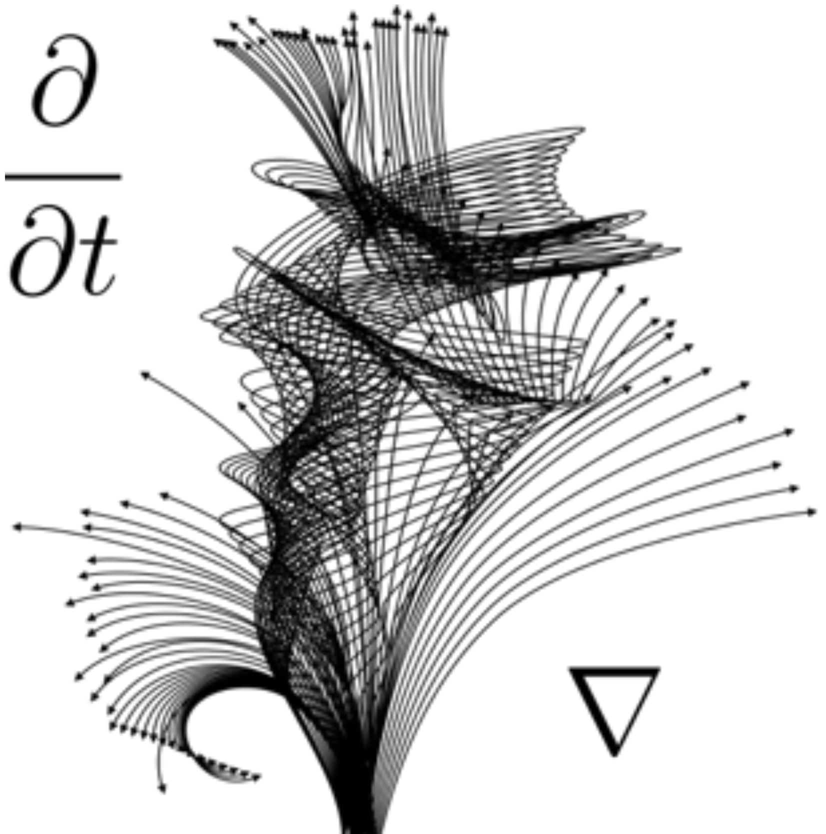
Agenda for today:

- Test of first derivative
- Test of second derivative
- Concavity
- Sketching graphs using the sign table



Increasing/decreasing functions

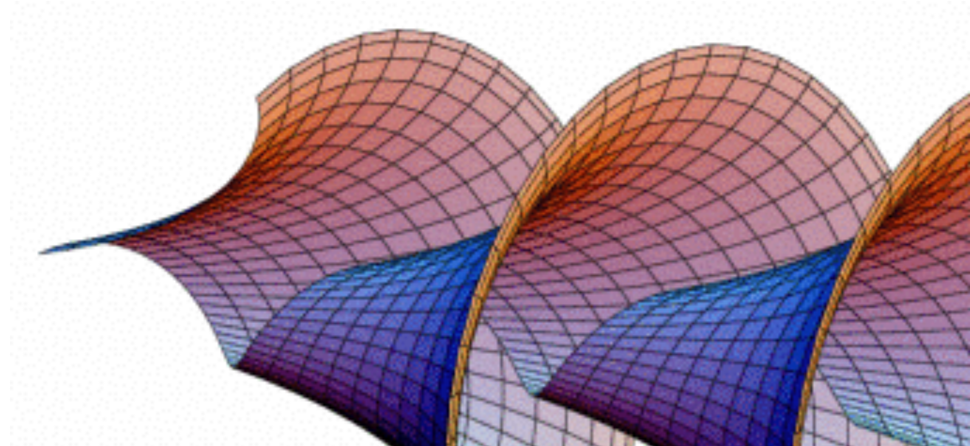
Given a function defined on an interval D :

The function is increasing on D, if for every choice of a and b with a < b,

$$f(a) \leq f(b)$$

The function is decreasing on D, if for every choice of a and b with a < b,

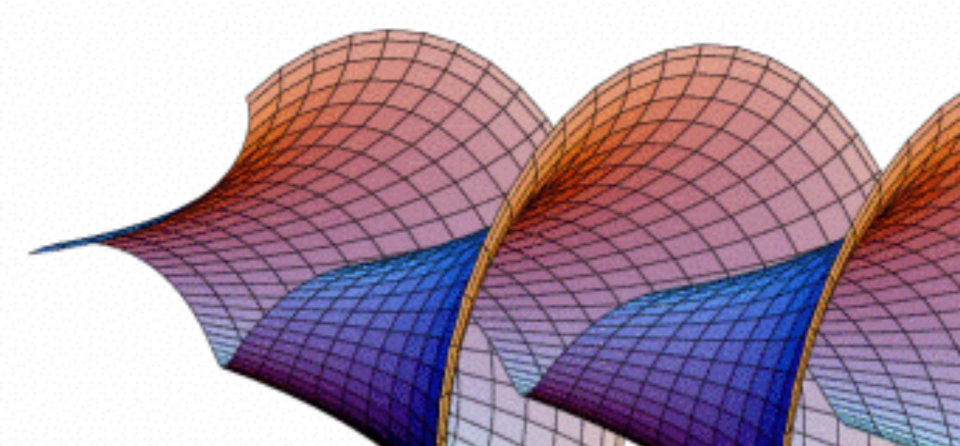
$$f(a) \geq f(b)$$



Increasing/decreasing functions

If $f'(x) \geq 0$ on D then f is **increasing** on D.

If $f'(x) \leq 0$ on D then f is **decreasing** on D.



Local min/max

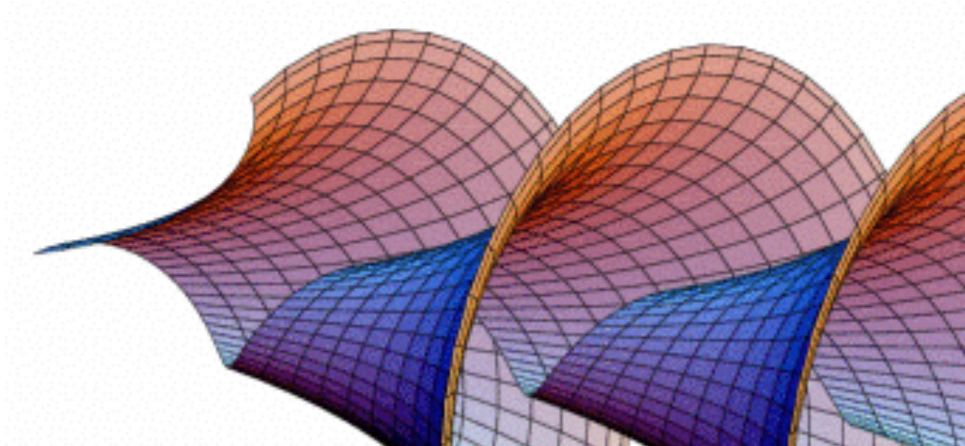
Given a function defined on an interval D and a is a point in D :

The function has a **local minimum** at a if for all x in D:

$$f(a) \leq f(x)$$

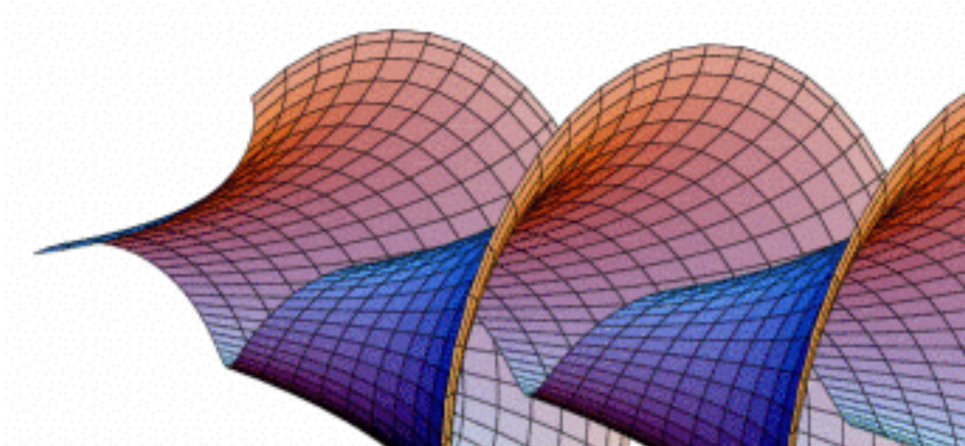
The function has a **local maximum** at a if for all x in D:

$$f(a) \geq f(x)$$



Local min/max

If f' changes sign at x= a then a is a local min/max (aka **local extremum**).

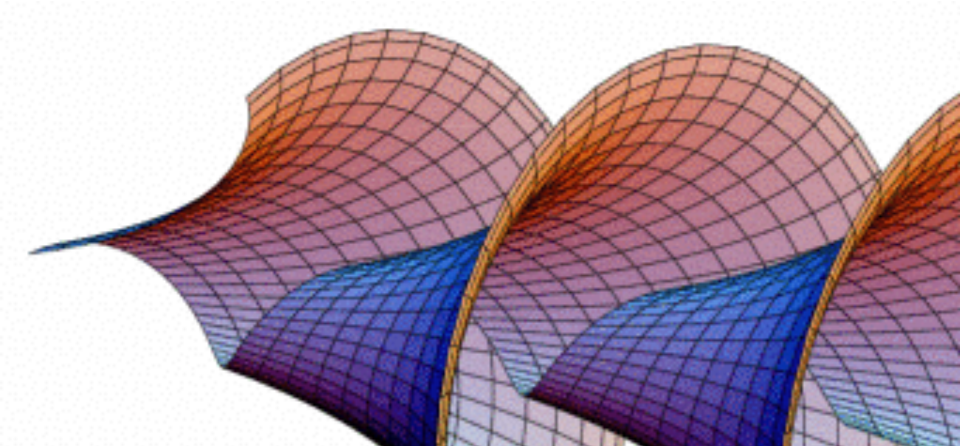


Critical points

Critical points are point for which either

(1) f’(a) = 0 or
(2) f’ is undefined at a even though f(a) is defined

If f’(x) **changes sign** at a critical point, then the critical point is a local extremum of f(x).

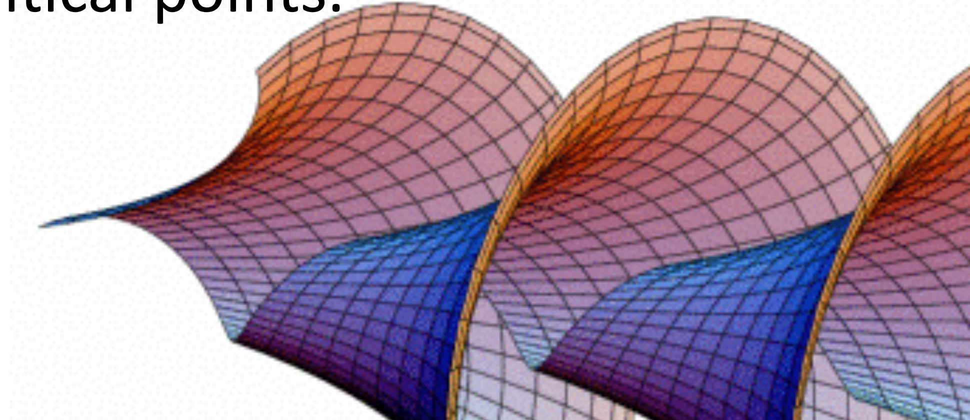


Critical points

Critical points are point for which either

(1) f’(a) = 0 or
(2) f’ is undefined at a even though f(a) is defined

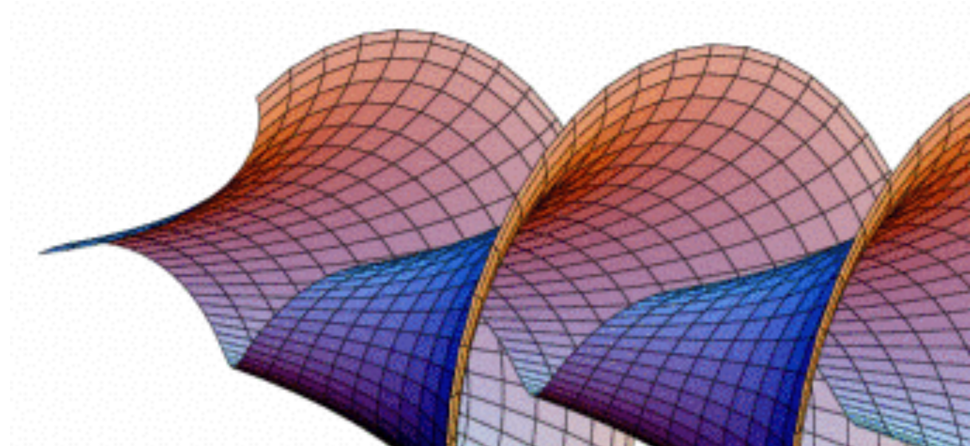
So: to find local max/min, first step is finding the critical points, and second step is determining the sign of f away from the critical points.



First derivative test

A critical point x=a is a local extremum when f'(x) changes sign at x=a.

- If f'(x) goes from - to 0 to + then x=a is a min of f(x).
- If f'(x) goes from + to 0 to - then x=a is a max of f(x).



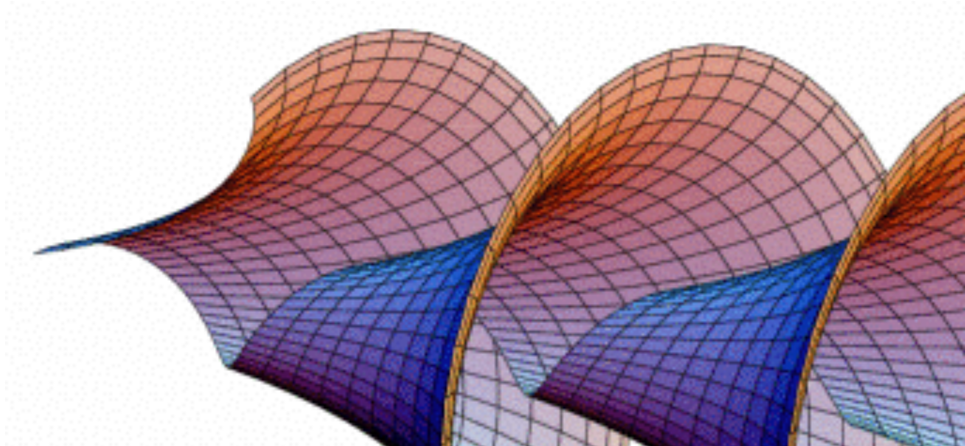
Concavity

We say a function is concave up on some interval if f’(x) is increasing on that interval.

When f”(x) exists, same as f”(x)>0.

We say a function is concave down on some interval if f’(x) is decreasing on that interval.

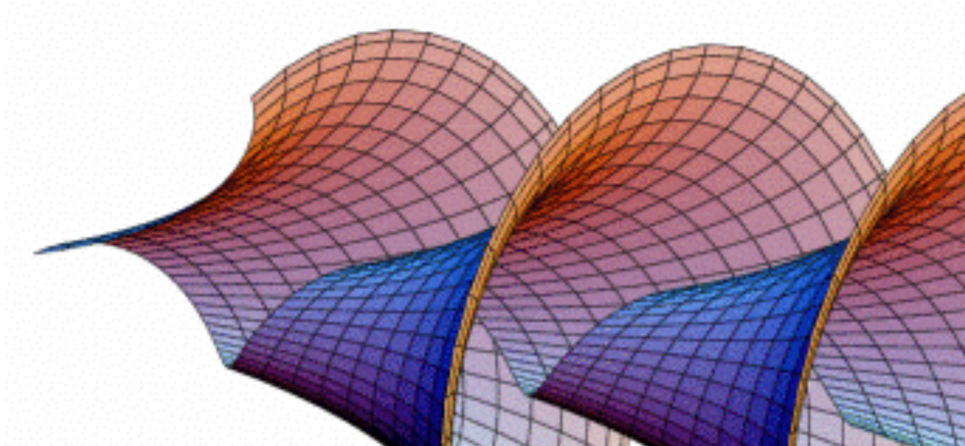
When f”(x) exists, same as f”(x)<0.



Second derivative test

If $f'(x)$ is differentiable at $x=a$, then $x=a$ is a local extremum when $f''(a) \neq 0$.

- If $f''(a) > 0$, then $f'(x)$ goes from - to 0 to + so $x=a$ is a min of $f(x)$.
- If $f''(a) < 0$, then $f'(x)$ goes from + to 0 to - so $x=a$ is a max of $f(x)$.



Happy thanksgiving!

Oct 6	WW 4
Oct 12	PL6.2