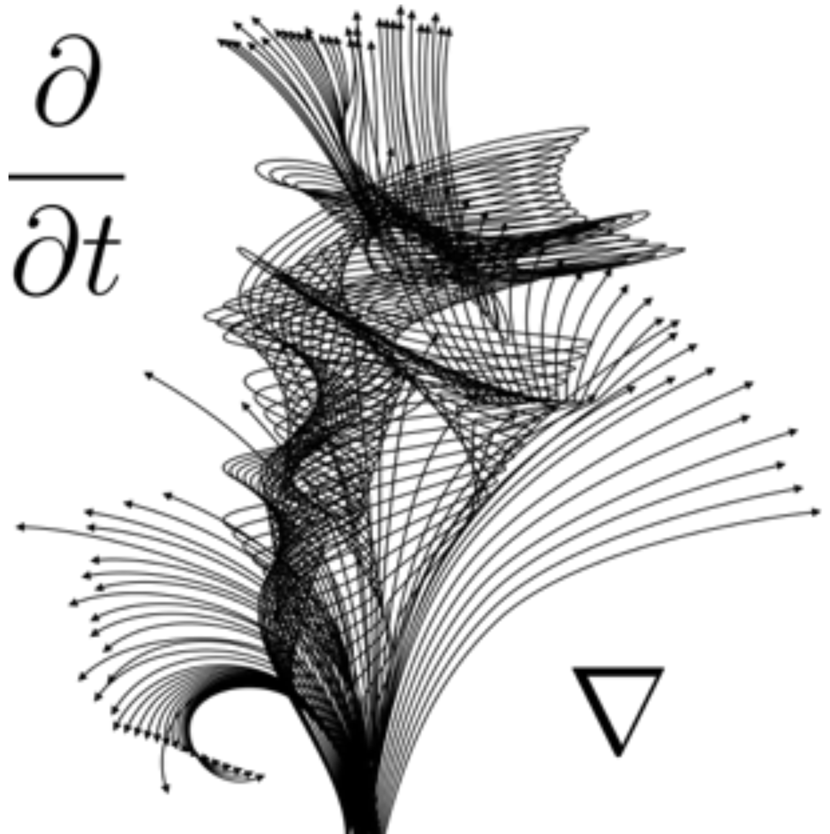
Differential Calculus with Applications to Life Sciences

Math 102:105

Pooya Ronagh

Agenda for today:

- Euler's Method
- Disease dynamics
- Trig derivatives



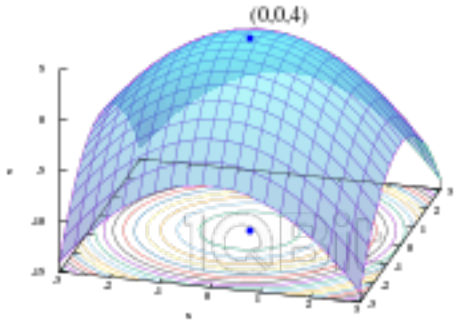
Last time: Euler's Method

$$\frac{dy}{dx} = f(y)$$

$$y_{i+1} = y_i + \Delta \times f(y_i)$$

Example: Approximate the solution of the IVP below at x= 2.

$$\begin{cases} \frac{dy}{dx} = y + 1 \\ y(0) = 2 \end{cases}$$



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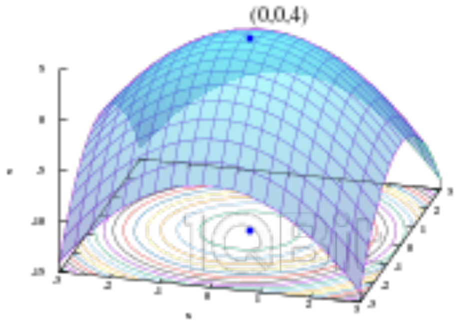
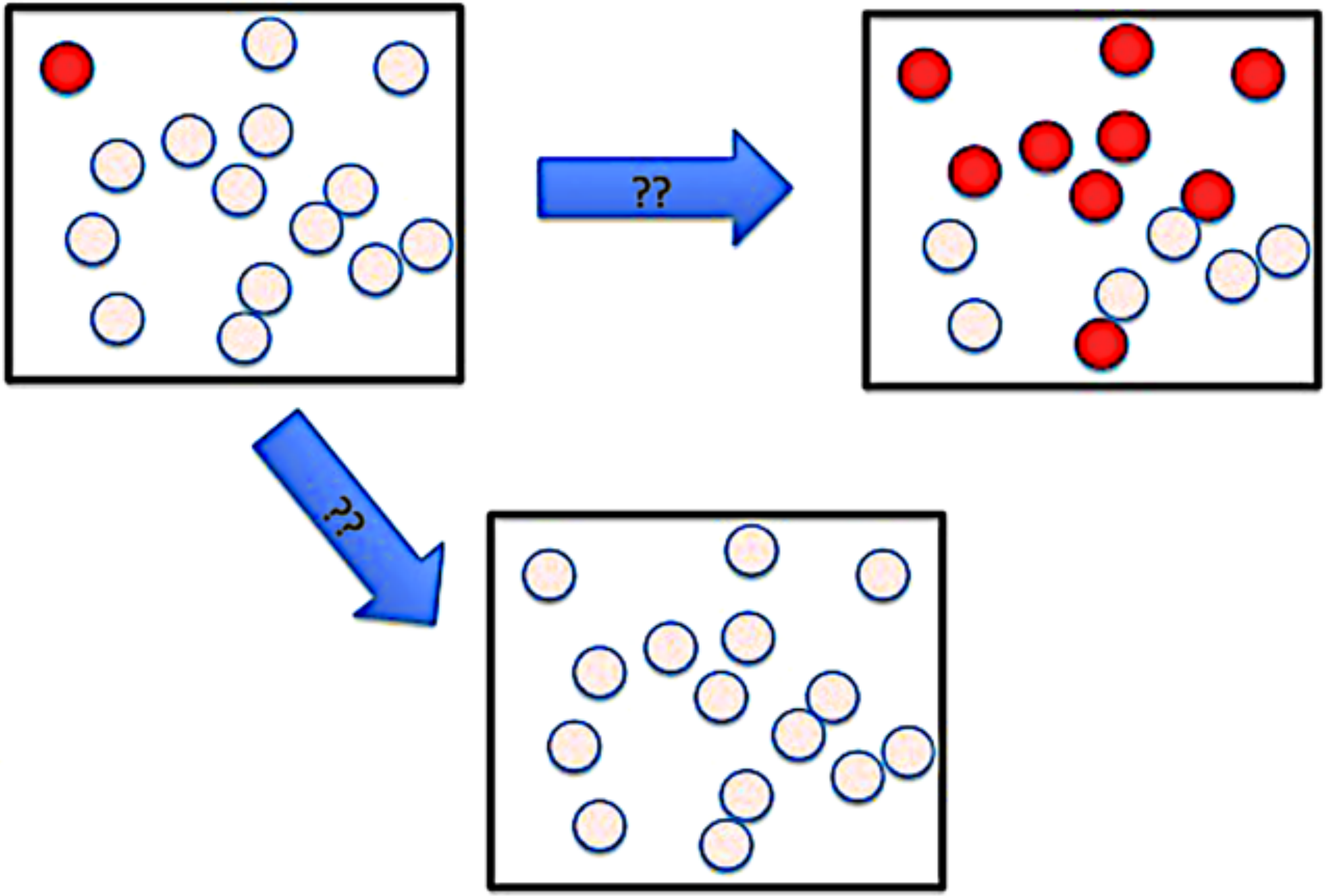
Watch video link [45] on using spreadsheet

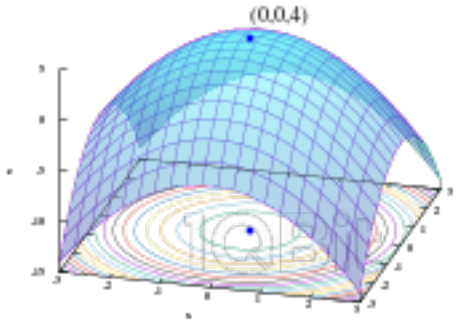


Disease dynamics



Disease dynamics

N: number of individuals in the population (no birth, death or migration)



Disease dynamics

N: individuals in the population (no birth, death or migration)

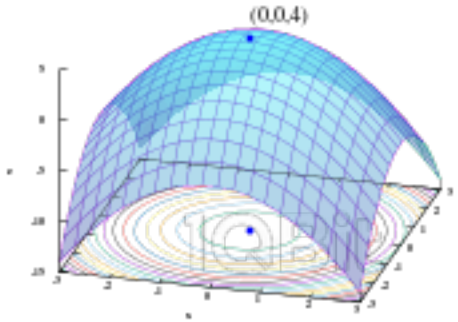
I: infected individuals

S: susceptible individuals

The disease spreads with a rate proportional to the product of two types in the population, which DE described the spread of the disease?

$$(A) \quad \frac{dI}{dt} = -bI(N-I) \qquad (B) \quad \frac{dI}{dt} = bI(N-I)$$

$$(C) \quad \frac{dS}{dt} = -bSI \qquad (D) \quad \frac{dI}{dt} = bSI$$



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Rate of transmission

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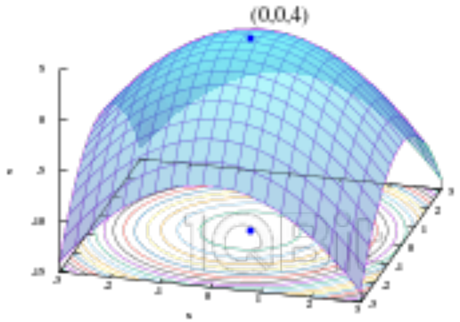

Disease dynamics

N: individuals in the population (no birth, death or migration)

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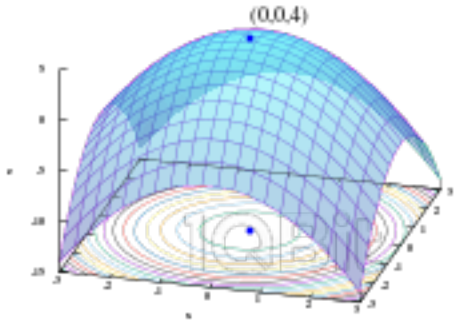
The disease spreads with a rate proportional to the product of two types in the population, now also consider that each individual recovers from the disease.

Rate of recovery

$$\frac{dI}{dt} = bI(N - I) - \mu I$$



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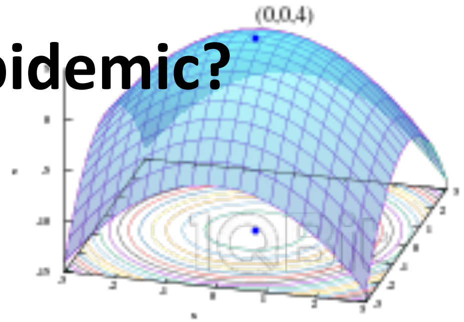
Question: For what values of b and μ does the disease become epidemic?

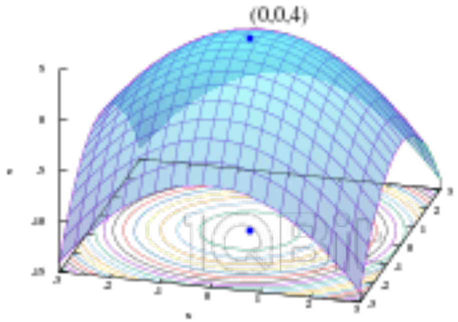


Euler's Method Revisited

$$\frac{dy}{dx} = f(y) \qquad\qquad y_{i+1} = y_i + \Delta \times f(y_i)$$

Example: Approximate the solution of the logistic DE as an IVP.

$$\begin{cases} y' = ry(1 - \frac{1}{K}y) \\ y(0) = 0.01 \end{cases}$$



Last time…

tan θ = sin θ / cos θ

cot θ = 1 / tan θ

sec θ = 1 / cos θ

csc θ = 1 / sin θ

Which of the following is not a trig identity?

(A) $1 + \cot^2\theta = \csc^2\theta$

(B) $\tan^2\theta + 1 = \sec^2\theta$

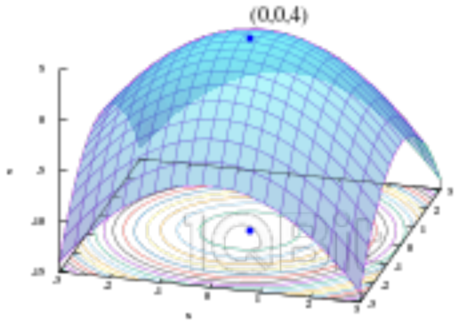
(C) $\sin(2\theta) = 2\sin\theta\cos\theta$

(D) $\cos(\theta) = \sin(\theta-\pi/2)$

(E) $\sin(\theta) = \cos(\theta-\pi/2)$

Pythagorus Theorem:

$$\sin(\theta)^2 + \cos(\theta)^2 = 1$$



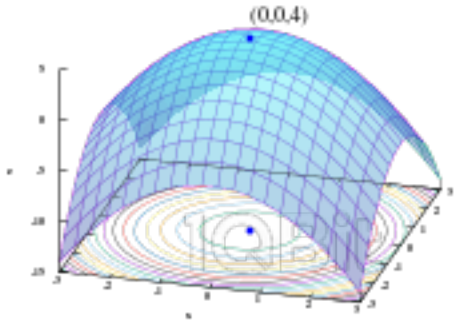
Famous Angles

What is cos(2π/3)?

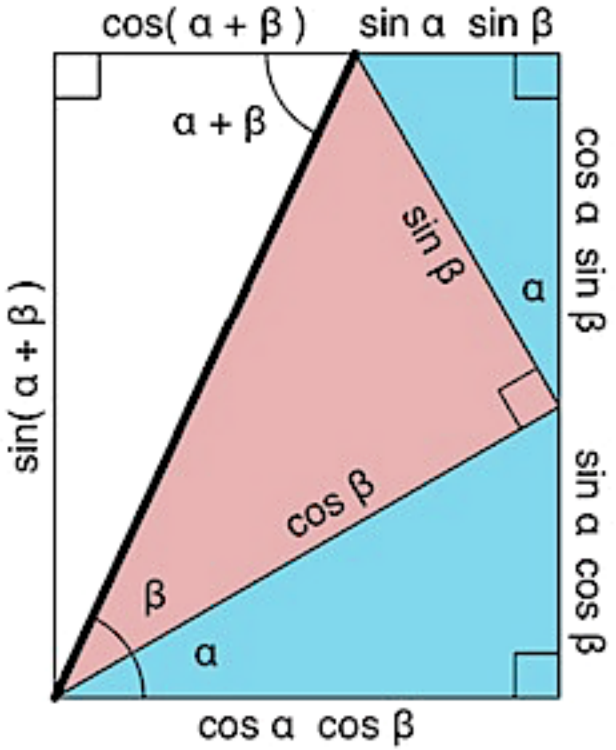
$$(A)\quad \frac{\sqrt{3}}{2} \qquad (B)\quad -\frac{\sqrt{3}}{2} \qquad (C)\quad \frac{1}{2} \qquad (D)\quad -\frac{1}{2}$$

What is tan(π/4)?

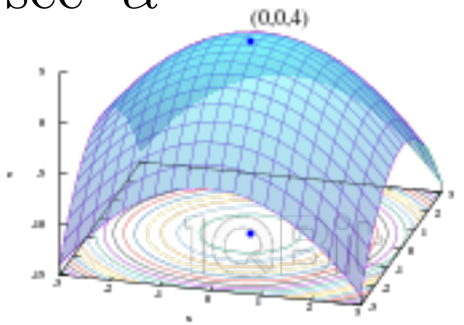
$$(A)\quad \frac{1}{\sqrt{2}} \qquad (B)\quad 1 \qquad (C)\quad 45 \qquad (D)\quad \frac{1}{2}$$



Sums of angles

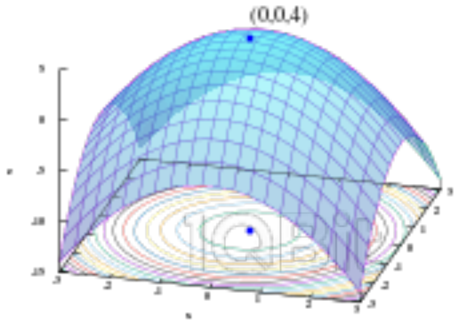


$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$$
$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$
$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$
$$1+\cot^2\alpha = \csc^2\alpha$$
$$1+\tan^2\alpha = \sec^2\alpha$$



Derivative of trig functions

Use the definition of derivative at a point x on the x-axis…



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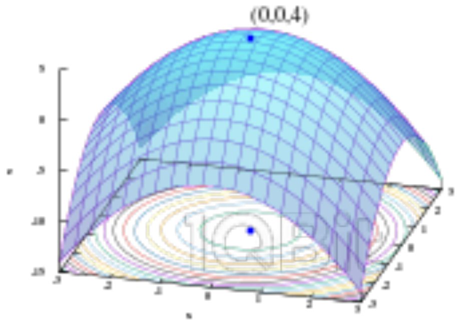
Need to know the following limits:

$$\lim_{h \to 0} \frac{\cos(h) - 1}{h} \qquad \lim_{h \to 0} \frac{\sin(h)}{h}$$

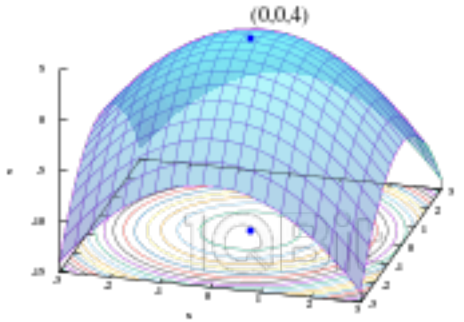


Derivative of trig functions

Use the definition of derivative at a point x on the x-axis…

Need to know the following limits:

$$\lim_{h\to 0} \frac{\cos(h) - 1}{h} = 0 \qquad \lim_{h\to 0} \frac{\sin(h)}{h} = 1$$



Derivative of trig functions

What is the derivative of cot(x)?

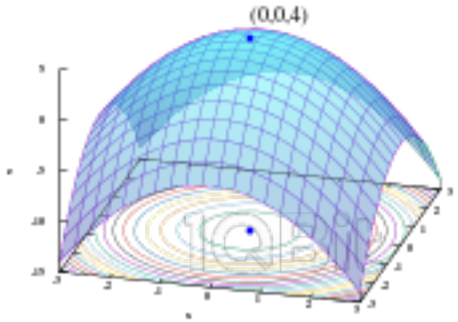
(A) csc(x)cot(x)

(B) -csc(x)cot(x)

(C) $\csc^2(x)$

(D) $-\csc^2(x)$

(E) $\sec^2(x)$



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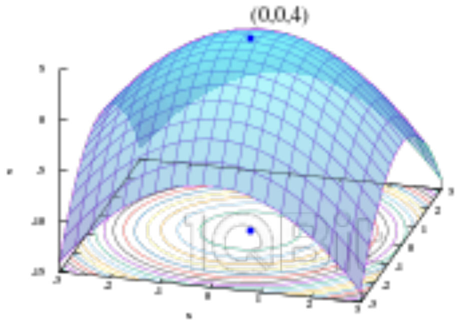
$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$
$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$



Periods of trig functions

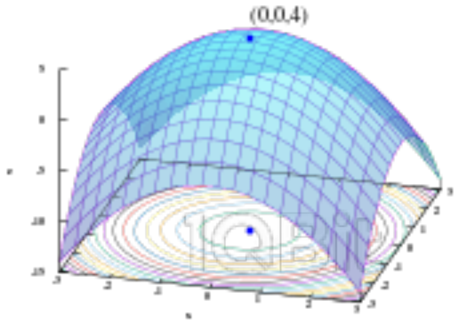
The **period** of the function y = f(x) is the smallest number T for which

$$f(t + T) = f(t)$$

for all values of t.

Example: What is the period of $f(x) = 1 + 2 \sin (3x - 1)$?

Question: What is the **phase-shift** of this function?



Amplitude of trig functions

The **amplitude** of a trig function y = f(x) is defined as

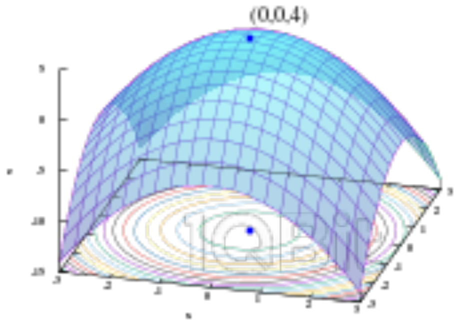
(max f - min f)/2.

Example: What is the amplitude of f(x) = 1 + 2 sin (3x - 1)?

The **midline** (or average) of a trig function y = f(x) is defined as

(max f + min f)/2.

Example: What is the midline of f(x) = 1 + 2 sin (3x - 1)?



Next time: final period office hours

Nov 23	PL12.2
Nov 24	WW 11
Nov 25	OSH 6