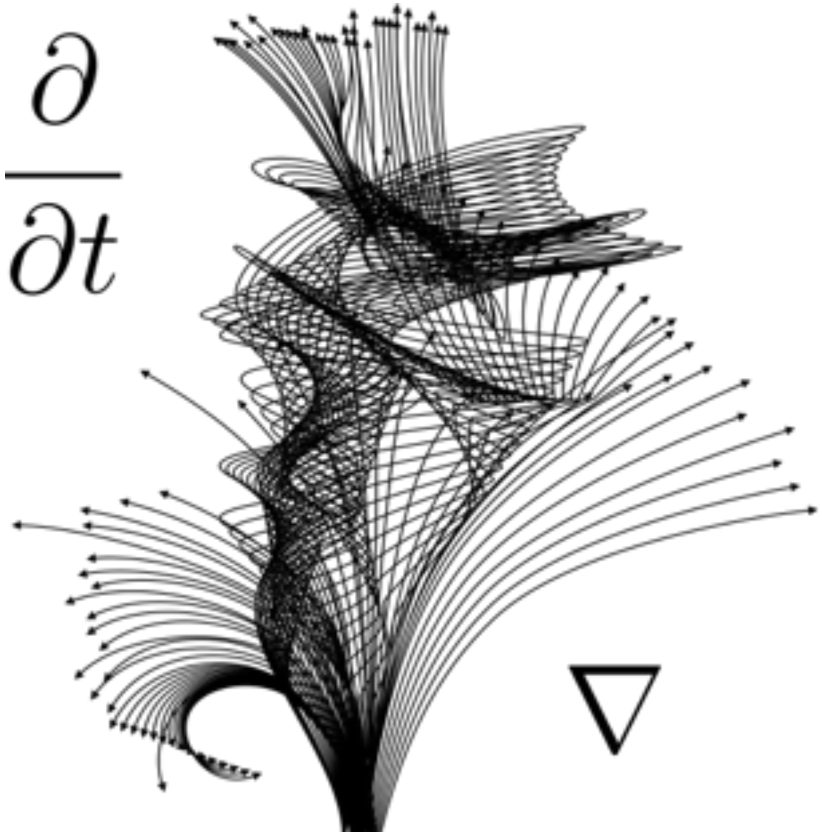
Differential Calculus with Applications to Life Sciences

Math 102:105

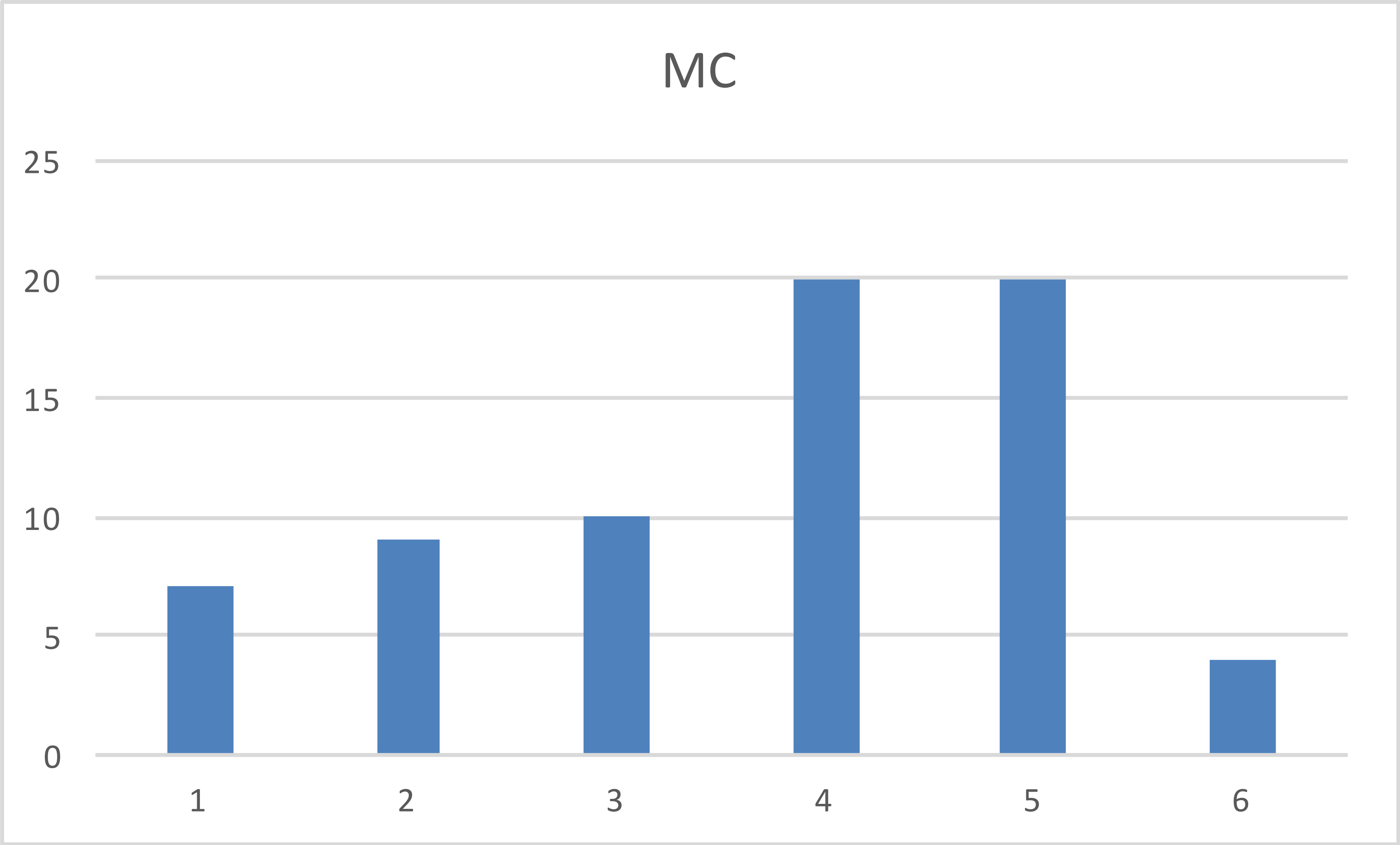
Pooya Ronagh

Agenda for today:

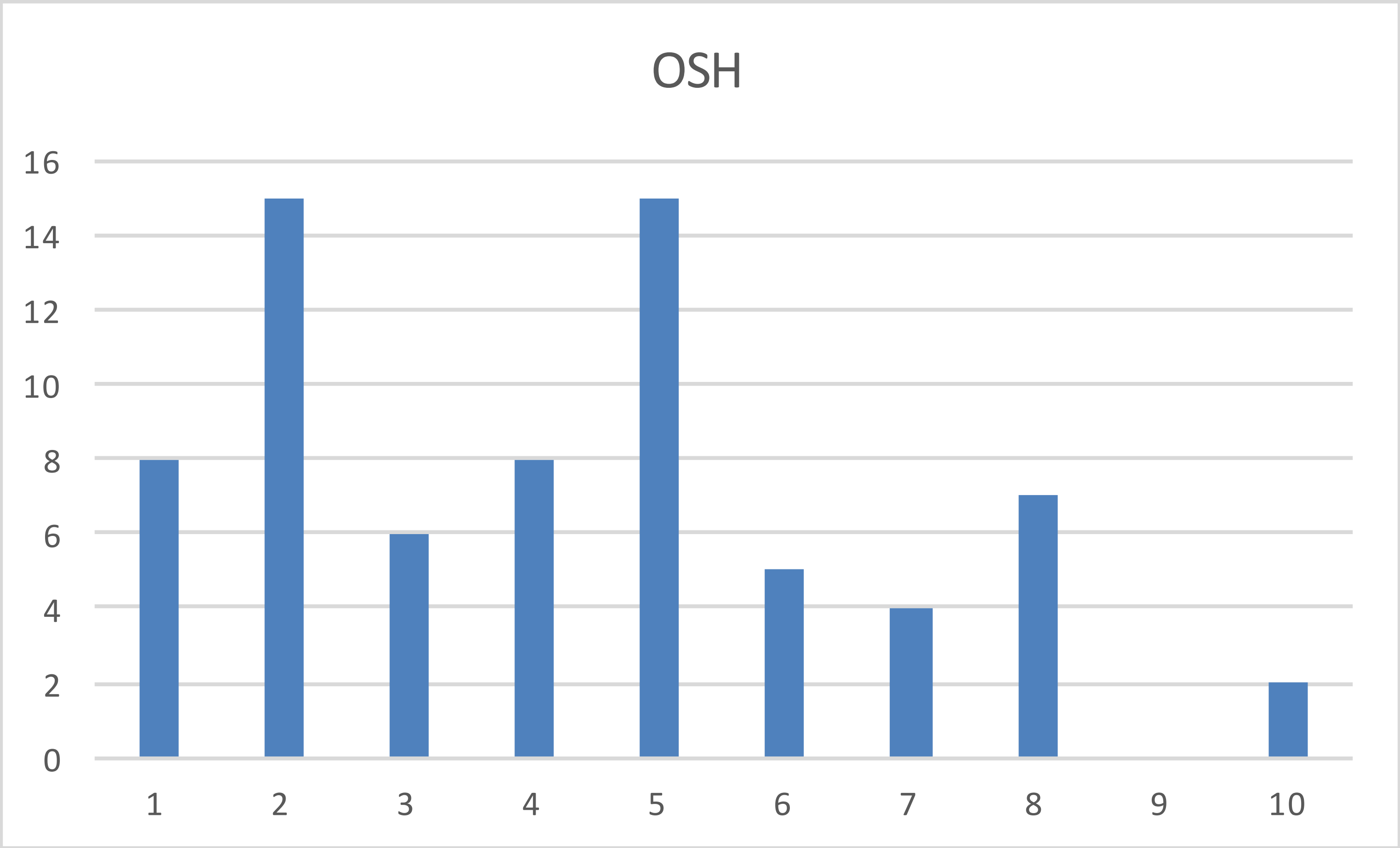
- Linear differential equations
- Trigonometry Review
- Euler's Method



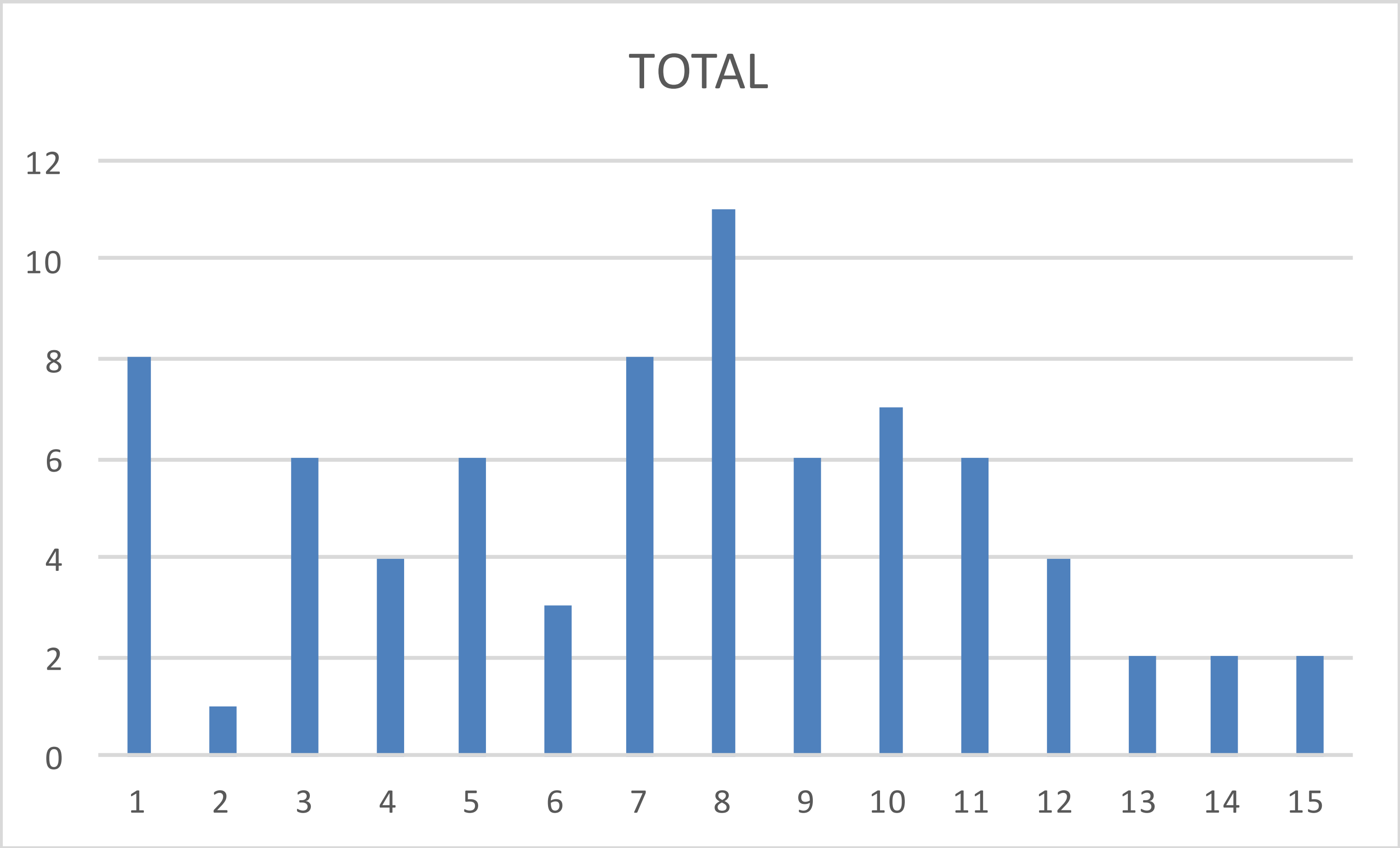
Quiz 3 Stats



Quiz 3 Stats



Quiz 3 Stats



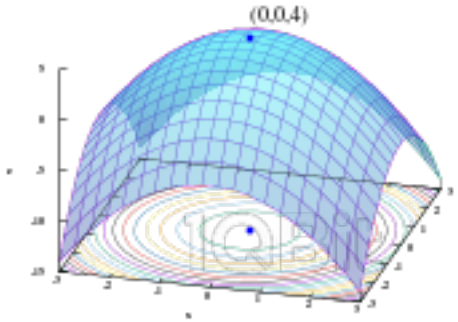
Solutions of y' = a - by

$$y(t) = \frac{a}{b} + \left(y_0 - \frac{a}{b}\right) e^{-bt}$$

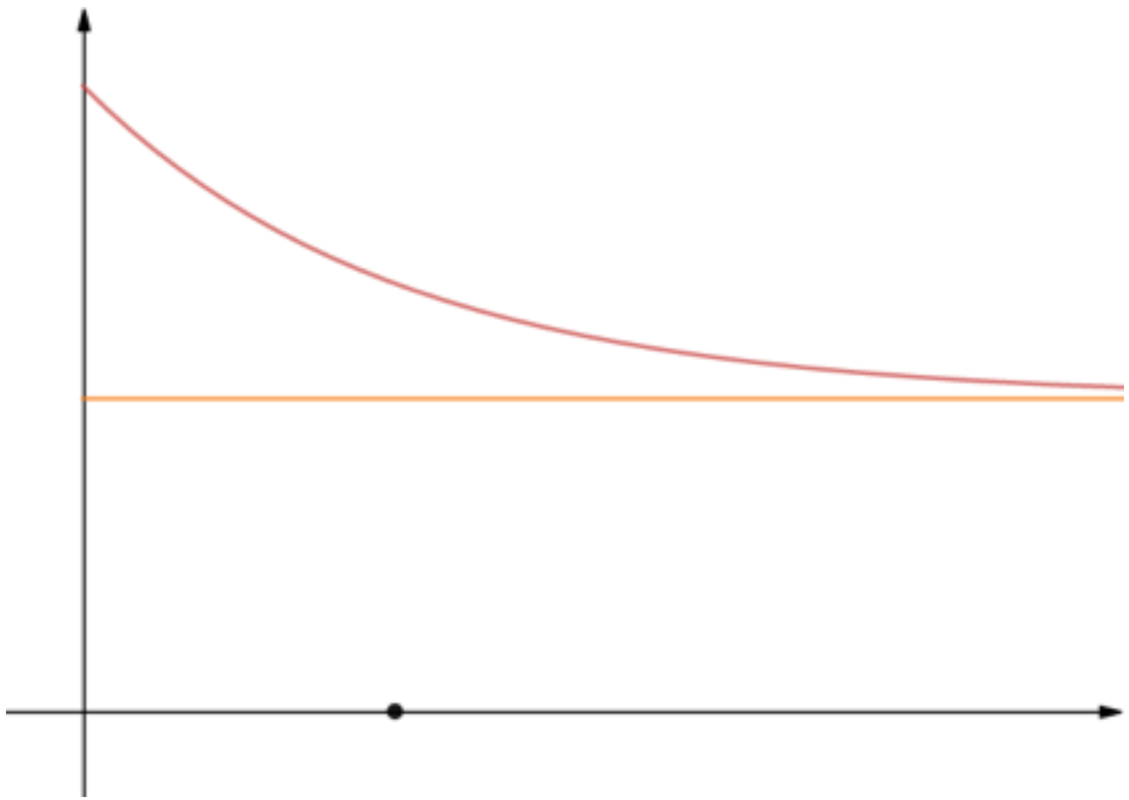
Question: Let’s assume b>0. What is a horizontal asymptote for a solution?

Question: What is a steady state solution?

Question: What is a good definition for characteristic time here?



Characteristic time



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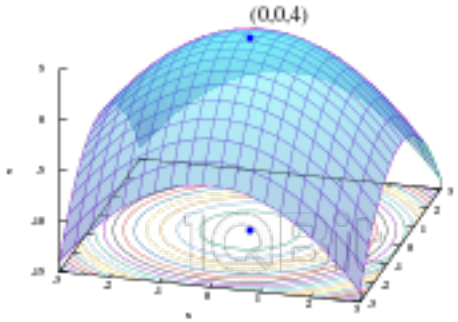
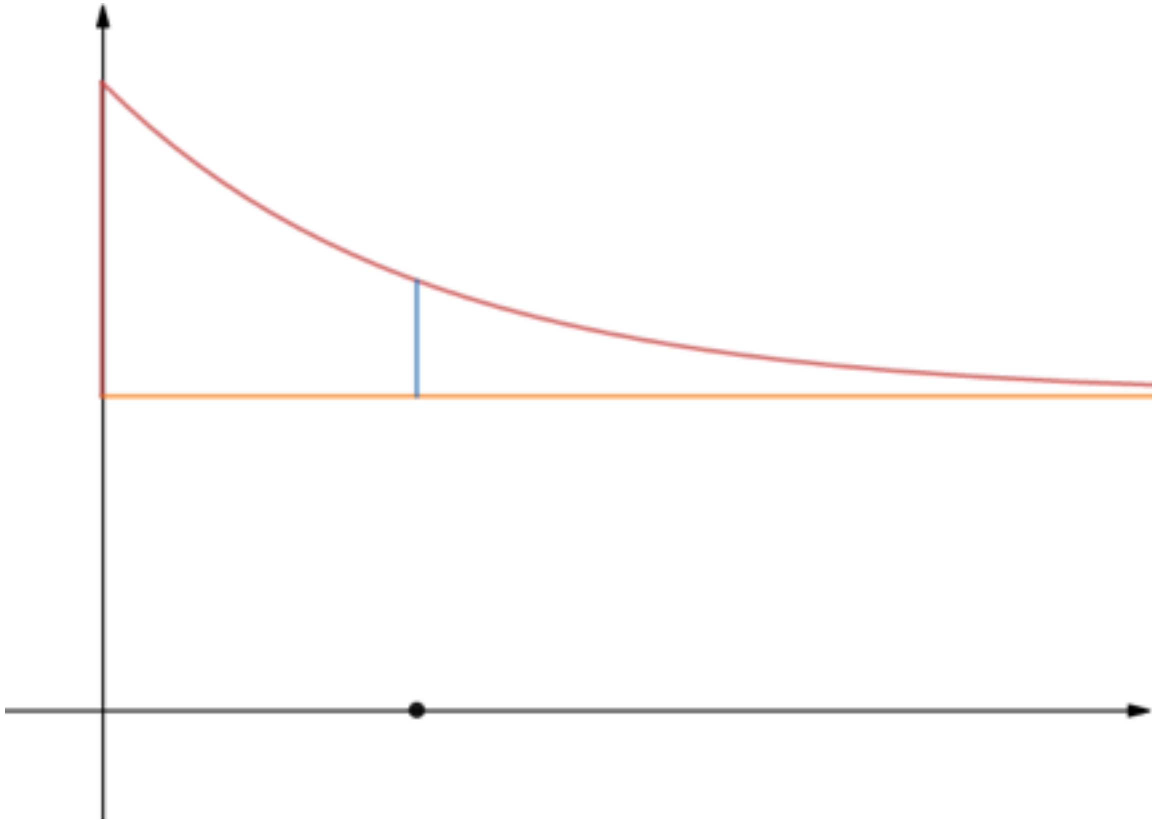


Characteristic time



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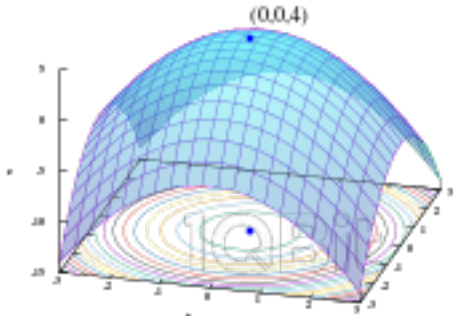
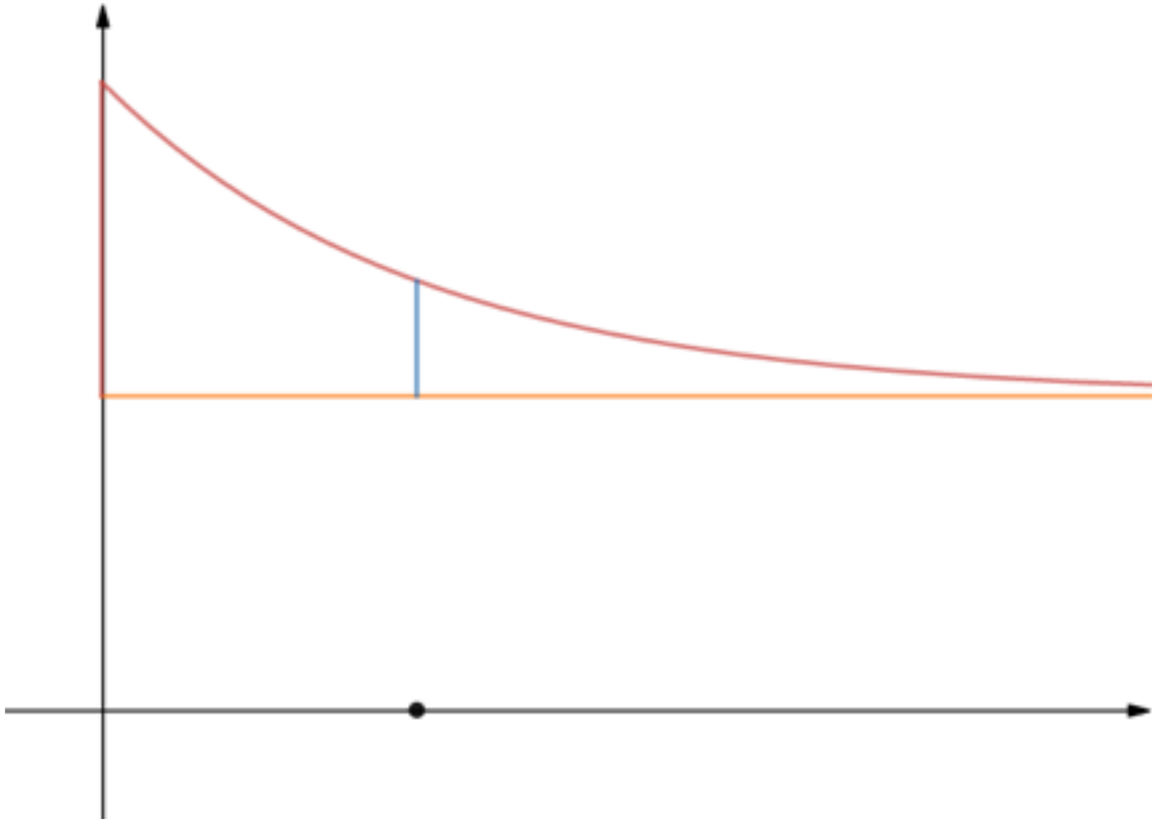


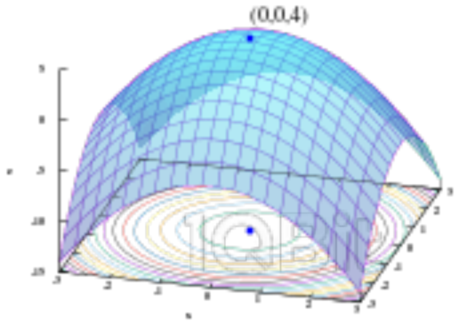
Characteristic time



$$y(t) = \frac{a}{b} + \left(y_0 - \frac{a}{b}\right) e^{-bt}$$

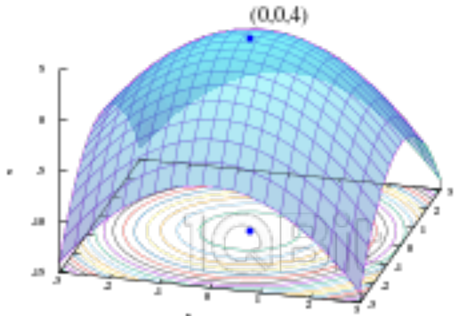
Characteristic time=

time the solution is 1/e of its way to the steady state is it approaching!



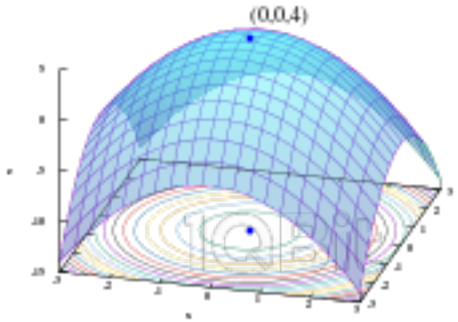
Example

A drug delivered by IV accumulates at a constant rate k_{IV}. The body metabolizes the drug proportional to the amount of the drug. Find the IVP that models this scenario and the solution for it.



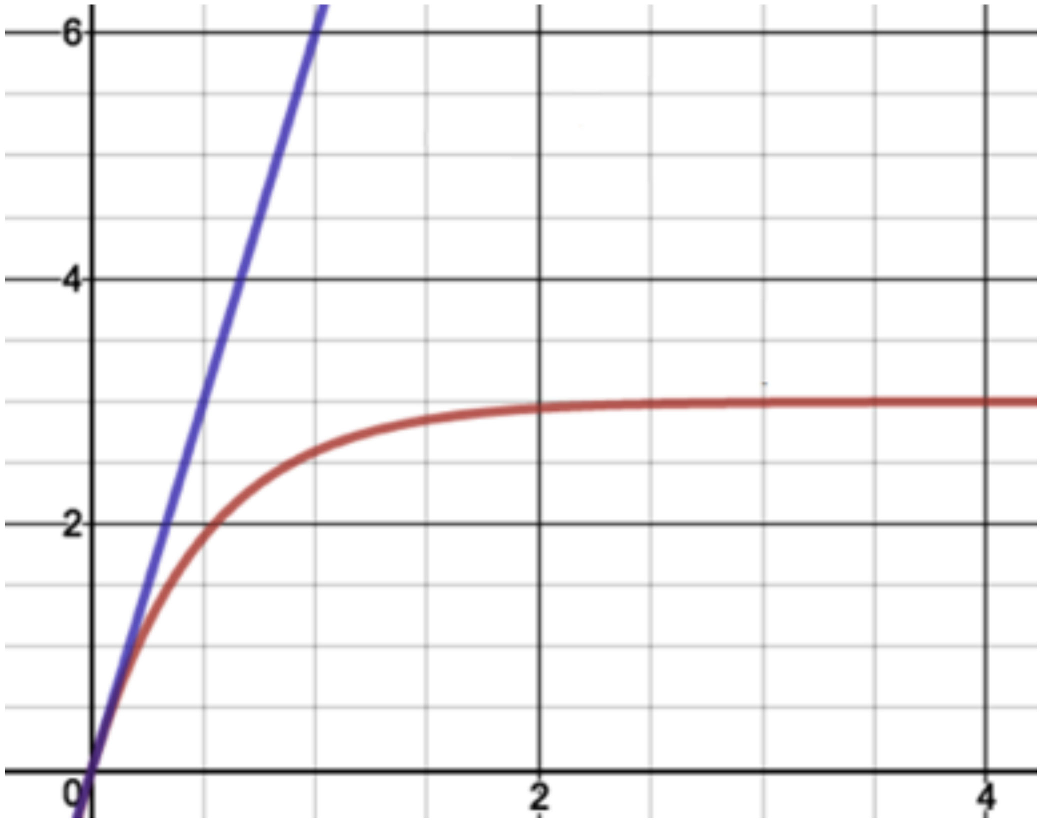
Example

A drug delivered by IV accumulates at a constant rate k_{IV}. The body metabolizes the drug proportional to the amount of the drug. Find the IVP that models this scenario and the solution for it. (Hint: **Consider this as an IVP**!)



Parameter study of a phenomena

You measure the mass of drug in the patient's body as a function of time, d(t), and plot it. You arrive at



What is the constant k_{IV} ?

(A) 1 (B) 2 (C) 3 (D) 6

1QBit

Parameter study of a phenomena

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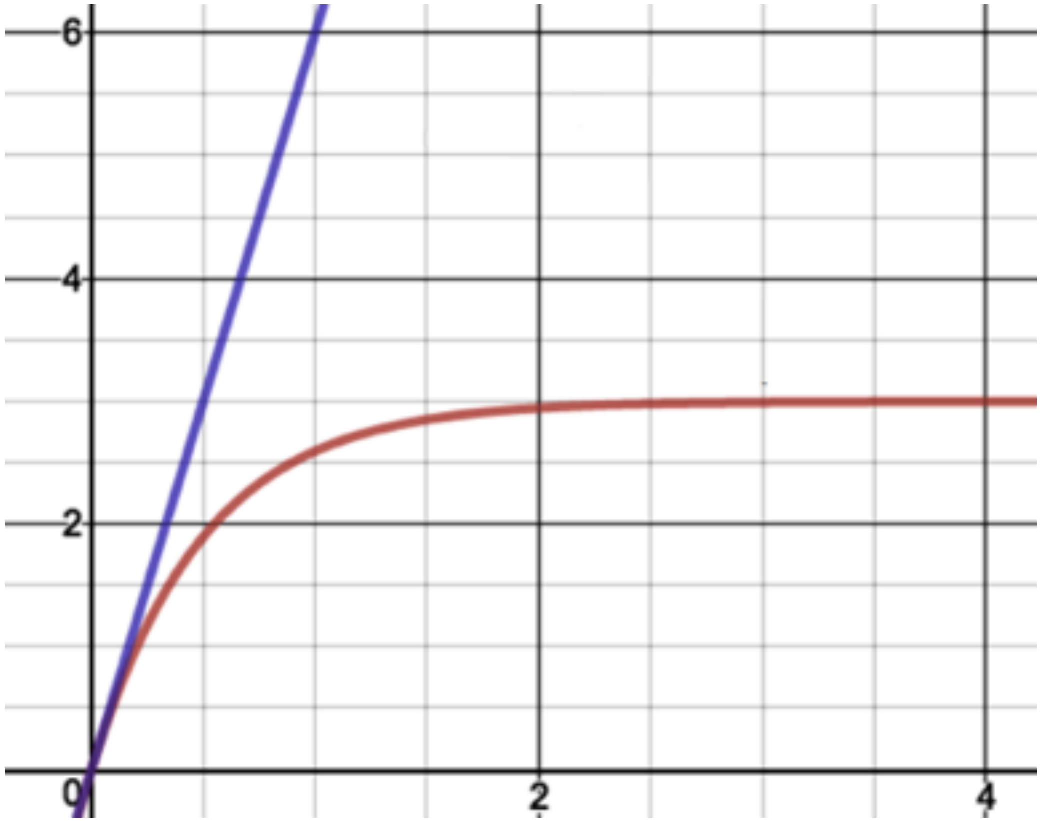


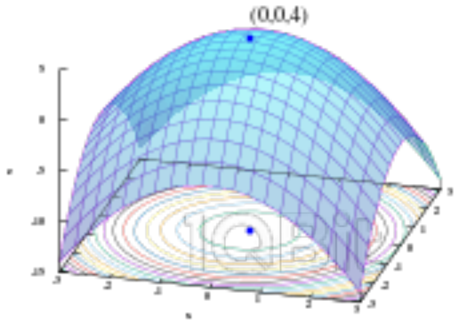
What is the constant k_m ?

(A) 1 (B) 2 (C) 3 (D) 6

1QBit

What you need to know…

- Write down a linear differential equation describing a word problem.
- Use the shift substitution to get z'= r z and write down the solution for the new differential equation.
- Substitute back to find solution to the original differential equation.
- Determine the initial condition of the original solution using the initial condition of the shifted solution.
- Answer questions about the resulting solution.

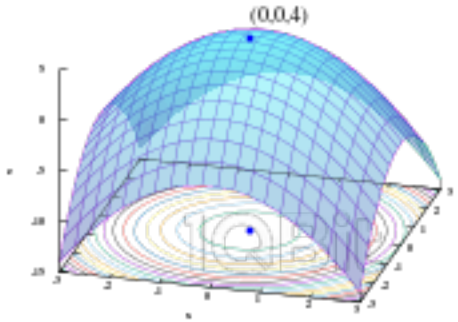


Euler's Method for Solving DEs

$$\frac{dy}{dx} = f(y) \qquad\qquad y_{i+1} = y_i + \Delta \times f(y_i)$$

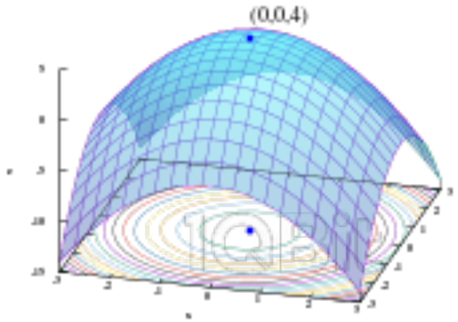
Example: Approximate the solution of the IVP below at x= 2.

$$\begin{cases} \frac{dy}{dx} = y + 1 \\ y(0) = 2 \end{cases}$$



Trigonometry Review

- Sin(θ) and Cos(θ) defined using the unit circle
- Using the unit circle, find values of:
  - Cos(π - θ) = ?
  - Sin(π - θ) = ?
  - Cos(π + θ)= ?
  - Sin(π + θ)= ?
  - Cos(2π - θ) = ?
  - Sin(2π - θ) = ?



Other trigonometric functions

$\tan\theta = \sin\theta / \cos\theta$

$\cot\theta = 1 / \tan\theta$

$\sec\theta = 1 / \cos\theta$

$\csc\theta = 1 / \sin\theta$

Which of the following is not a trig identity?

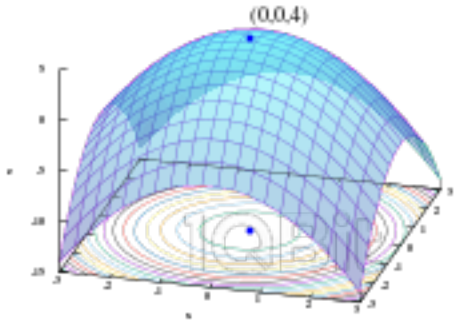
(A) $1 + \cot^2\theta = \csc^2\theta$

(B) $\tan^2\theta + 1 = \sec^2\theta$

(C) $\sin(2\theta) = 2\sin\theta\cos\theta$

(D) $\cos(\theta) = \sin(\theta - \pi/2)$

(E) $\sin(\theta) = \cos(\theta - \pi/2)$



Next time: final period office hours

Nov 23	PL12.2
Nov 24	WW 11
Nov 25	OSH 6