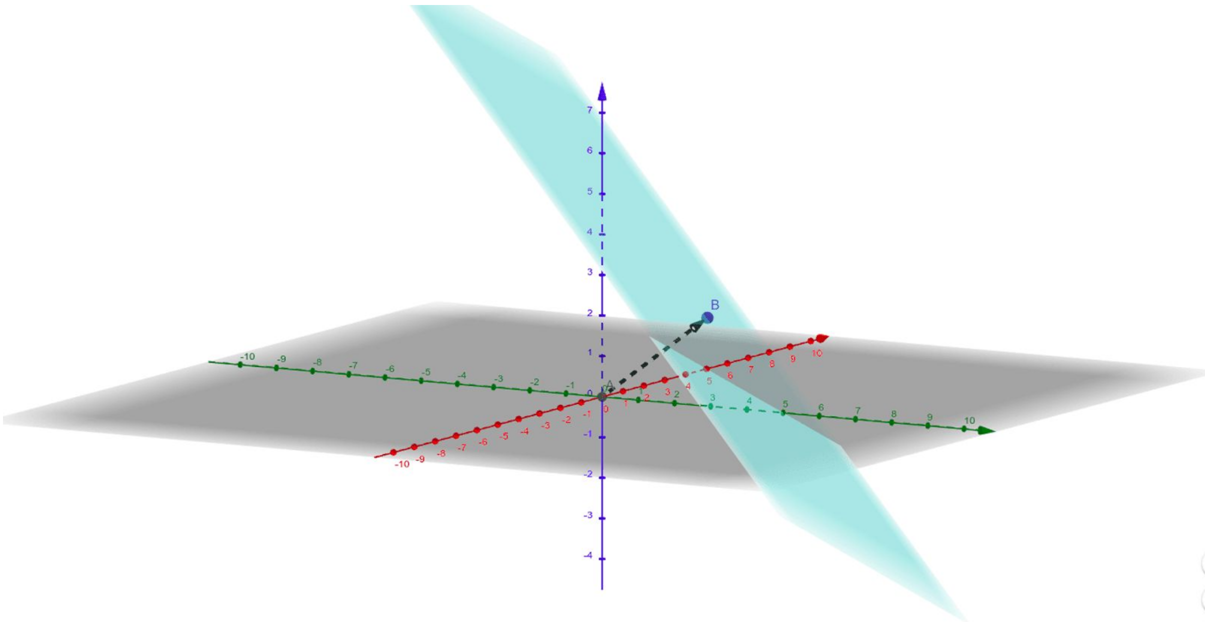
Dear Mr. Khan,
I hope that all is well with you.

I was watching your video about defining a plane in R3 with a point and a normal vector and I start to realizing that we are making a plane with 2 points, which one of them is a vector perpendicular to the plane and the other is just a point from the plane. We can do it by this equation:

$[n_1,n_2,n_3]^T \cdot [x-x_1,y-y_1,z-z_1]^T = 0$

It gives us an equation for a plane where the first vector is the perpendicular vector to the plane and the second one just the x, y and z coordinate minus a point from the plane.

I was always told that we need 3 points for making a plane where each point is a point picked from the plane, but here we are actually making a plane just with 2 point, but the way that we see the points are different. I was so excited because we decrease the size of the necessary information for making a plane, and also we can make all unique planes in R3, but I start to thinking is that the minimum information that we need in order to make all unique planes in R3? I found out that it is not! The vector that we are using can start at any point(the things that are important to us are the length and the orientation of the vector), but if we assume that the vector is started at the origin we can make all unique plane that do not cross the origin by assuming that the vector is perpendicular to the plane and it crosses at the end of the perpendicular vector.

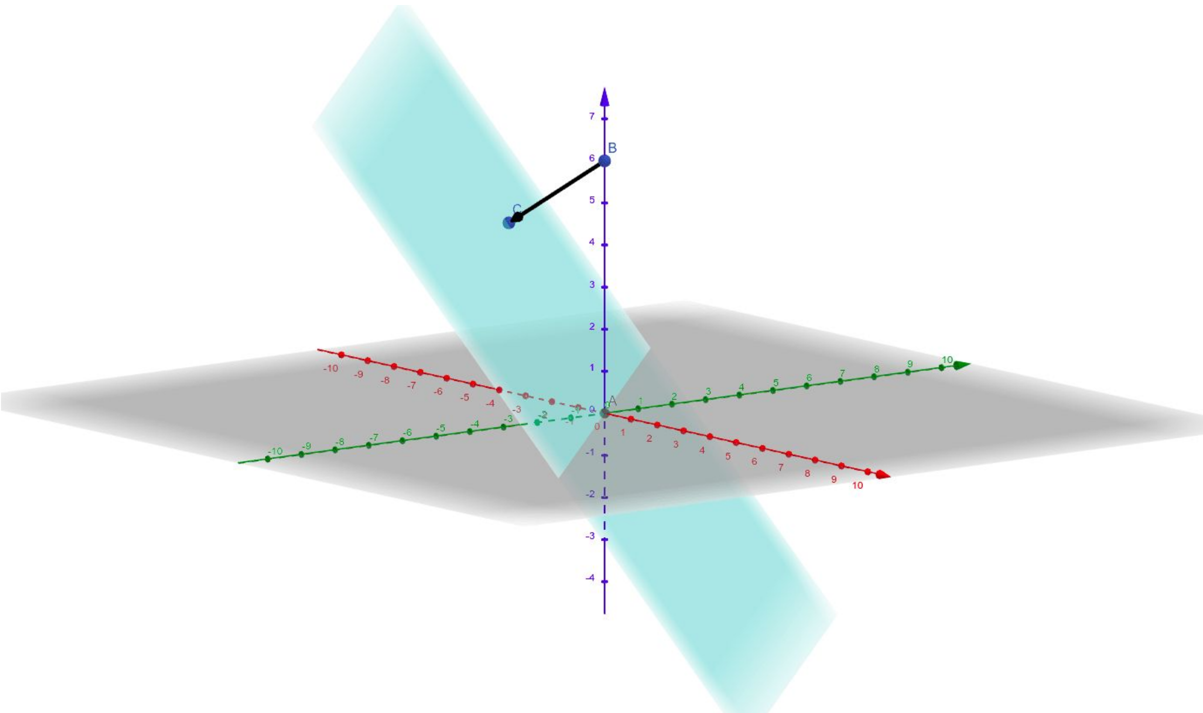


So we can just make a plane by this equation:

$[n_1,n_2,n_3]^T \cdot [x-n_1,y-n_2,z-n_3]^T = 0$

It gives us an equation for a plane where the first vector is the perpendicular vector to the plane and the second one is just the x, y and z coordinates minus a point from the plane, which is located at the end of the vector that is perpendicular to the plane (the length of the perpendicular vector matters). So we basically made a plane by using only 1 point. Actually more than just using the perpendicularity of the first vector (orientation) I also use the length (by changing the length the plane will change).

The problem is though, we can not make planes that go through the origin(we can not specify the plane. we can not make a unique plane), so we need a binary number that tells us if the plane is going through the origin or not and in the case that it is going through the origin we just use a perpendicular vector (it does not depend on length of the perpendicular vector) and since we know that the plane is going from the origin we can make the plane by a perpendicular vector and a point(which is origin).



Then we can make a plane by this equation:
$[n_1,n_2,n_3]^T \cdot [x-0,y-0,z-0]^T = 0$
It gives us an equation of a plane for where the first vector is the perpendicular vector to the plane and the second one just the x,y and z coordinate minus the zero vector.

It shows that we can make a plane in R3 just by a point $[n_1,n_2,n_3]^T$, which represent the perpendicular vector and a binary number b, that tells us if the plane is going through the origin or not. If b = 0 (which tells that the plane is not going through the origin, by condition) then the length of the perpendicular vector matters, and if b = 1 (which tells that the plane is going through the origin) the length of the perpendicular vector does not matter.

For example given vector = $[1,3,2]^T$ and b = 0 (which tells that the plane is not going through the origin) we can make a unique plane by:
$[1,3,2]^T \cdot [x-1,y-3,z-2]^T = 0$
(by considering that the plane is going through $[1,3,2]^T$)

Another example would be vector = $[2,5,1]^T$ and b = 1 (which tells that the plane is going through the origin) we can make a unique plane by:
$[2,5,1]^T \cdot [x-0,y-0,z-0]^T = 0$
(by considering that the plane is going through origin)

We can extend this to R^n and we would need a vector in R^n ($[n_1,n_2,...,n_n]$) which represent the perpendicular vector and a binary number that tells us if the plane goes through the origin or not.

Thank you for your time. I look forward to hearing from you.

Sincerely.
Pooya Kooshanfar