Simple Models and Biased Forecasts

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- But why can we assume that agents know the true model?
- Learning literature: let's "make the agents like econometricians."

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(2) All models in the class are too simple relative to the truth, i.e., they are misspecified.
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(1) Agents entertain a large and flexible class of time-series models.
   - all state-space models of a given dimension
(2) All models in the class are too simple relative to the truth, i.e., they are misspecified.
   - the models are low-dimensional
(3) Study the long-run limit when learning is complete.
   - agents settle on pseudo-true models that approximate the true model

The Framework

- Discrete-time economy with a single agent (for now).

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  - for now, assume $\mathbb{P}$ is exogenous (will be endogenous in GE)
  - mean zero, stationary, and Gaussian
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- The agent is attempting to forecast future values of the observables.
  - time-t information set is $\{y_\tau\}_{\tau=-\infty}^{t}$
  - agent uses a model to map past observables to her forecasts:

$$\theta : \{y_\tau\}_{\tau=-\infty}^{t} \mapsto E_t[\cdot]$$

State-Space Models

Main assumption: the agent can only entertain stationary ergodic distributions P that can be represented by state-space models with at most d states.

$$z_t = A z_{t-1} + w_t, \qquad w_t \sim \text{ i.i.d. } \mathcal{N}(0, Q)$$
$$y_t = B' z_t + v_t, \qquad v_t \sim \text{ i.i.d. } \mathcal{N}(0, R)$$

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- d is the only free parameter.
- $\theta \equiv (A, B, Q, R)$ is estimated endogenously by the agent.

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- The agent can entertain *any* linear cross-sectional relationship between variables.
- But is constrained in the types of time-series relationships she can perceive.

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Stark assumption, but...

- It allows me to focus on the difficulty of dealing with time-series complexity.
  - Cross-sectional complexity is the focus of rational inattention, sparsity, etc.
- It leads to a useful linear invariance property for expectations.

▷ limited memory interpretation

Pseudo-True Simple Models

Goodness-of-fit measure: Kullback–Leibler Divergence Rate

$$\text{KLDR}(\theta) \equiv \lim_{t \to \infty} \frac{1}{t} \mathbb{E}\left[\log\left(\frac{\mathbb{f}(y_1, \ldots, y_t)}{f^\theta(y_1, \ldots, y_t)} \right) \right]$$

- f^θ is the agent's subjective density of $\{y_t\}_t$ under model θ.
- $\mathbb{f}$ is the density and $\mathbb{E}[\cdot]$ is the expectation under the true DGP.

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Definition (Pseudo-True d-State Models)

θ^* is a *pseudo-true d-state model* if

$$\theta^* \in \arg\min_{\theta \in \Theta^d} \text{KLDR}(\theta)$$

where

$$\Theta^d \equiv \{\text{all } d\text{-state models } \theta = (A, B, Q, R)\}$$

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Agent recovers the *true* model if we replace $\theta \in \Theta^d$ with $\theta \in \bigcup_{d=0}^{\infty} \Theta^d$.

Relation to the Literature

- **Factor analysis of business-cycle:** Sargent-Sims (1977), Watson (2004), Angeletos-Collard-Dellas (2020)
  - endogenizes the "main business-cycle shock" of Angeletos et al.
- **Noisy information/rational inattention/sparsity:** Mankiw-Reis (2002), Sims (2003), Woodford (2003), Gabaix (2014), Angeletos-Lian (2018), ...
  - perfect knowledge of current variables
  - perfect understanding of intratemporal relationships
  - can only understand simple intertemporal relationships
- **Learning models in macro:** Marcet-Sargent (1989), Evans-Honkapohja (1995), Adam-Marcet (2011), ...
  - focus on the asymptotics of learning
  - prior rules out the true model
- **Misspecified learning:** Berk(1966), Esponda-Pouzo (2016, 2021), Molavi (2019), ...

The Plan

The rest of the talk...

1. Characterization of pseudo-true 1-state models.
2. (Some) implications for agents' forecasts and actions.
3. Impulse and propagation: the TFP shock in the RBC model.
4. Application to forward guidance in the NK model.

In the paper (but not the talk)...

1. Generalization to the $d > 1$ case.
2. Additional implications.
3. Propagation of productivity and separation shocks in the DMP model.

Pseudo-True 1-State Models

Theorem (Linear Invariance)

Consider two agents:

- *Agent i observes y_t with distribution $\mathbb{P}$ and uses a pseudo-true model given $\mathbb{P}$.*

- *Agent j observes $\tilde{y}_t = T y_t$ with distribution $\tilde{\mathbb{P}}$ and uses a pseudo-true model given $\tilde{\mathbb{P}}$.*

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For any full-rank matrix T,

$$E_{jt}^*[\tilde{y}_{t+s}] = T E_{it}^*[y_{t+s}]$$

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> *For any full-rank matrix T,*
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Pseudo-true simple models respect *all* linear intratemporal relationships...

$$E_t^*[\alpha y_{1,t+s} + \beta y_{2,t+s}] = \alpha E_t^*[y_{1,t+s}] + \beta E_t^*[y_{2,t+s}]$$

Autocovariances and Autocorrelations

The 1-state pseudo-true model turns out to depend on the true DGP *only* via the true autocorrelations.

- True autocovariance matrices (standard definition):

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- True autocorrelation matrices (not a standard definition):

$$C_l \equiv \Gamma_0^{-1} \left(\frac{\Gamma_l + \Gamma_l'}{2} \right)$$

    - products of two symmetric matrices $\implies$ real eigenvalues
    - reduce to the usual autocorrelations when $n = 1$
    - by linear invariance, can assume without loss that Γ_0 is invertible

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> **Theorem**
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> *Let λ denote the eigenvalue of C_1 largest in magnitude.*
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> *If an ergodicity assumption is satisfied, then given any pseudo-true 1-state model*
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> $$E_t^*[z_t] = p' y_t$$
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> $$E_t^*[z_{t+s}] = \lambda^s E_t^*[z_t]$$
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> $$E_t^*[y_{t+s}] = q E_t^*[z_{t+s}]$$

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A Diagonal Example

- True data-generating process:

$$y_t = \begin{pmatrix} \alpha_1 & 0 & \ldots & 0 \\ 0 & \alpha_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \alpha_n \end{pmatrix} y_{t-1} + \begin{pmatrix} b_1 & 0 & \ldots & 0 \\ 0 & b_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & b_n \end{pmatrix} \epsilon_t, \qquad \epsilon_t \sim \text{i.i.d. } \mathcal{N}(0, I)$$

where

$$|\alpha_1| > |\alpha_2| > \cdots > |\alpha_n|$$

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  where

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- The autocorrelation matrix at lag $l = 1$:

$$C_1 = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_n \end{pmatrix}$$

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$$E_t^*[y_{1,t+s}] = \alpha_1^s y_{1t}$$
$$E_t^*[y_{j,t+s}] = 0, \qquad \forall j \neq 1$$

Pseudo-True Model in the Diagonal Example

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- Forecasts:

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- Persistence bias: forecasts are anchored to the most persistent observable.
  - Forecasts of y_1 coincide with the RE.
  - Forecast y_j for $j \neq 1$ as if i.i.d.

> **Theorem**
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> *The subjective variance under the pseudo-true model coincides with the true variance:*
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> $$E^*[y_t y_t'] = \mathbb{E}[y_t y_t']$$

Second Moments

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- PCA:
    - Project onto the dominant eigenvectors of the variance-covariance matrix, Γ_0.
    - Purely cross-sectional; uses no information about the serial correlations.

- Pseudo-true simple models:
    - Project onto the dominant eigenvectors of the first autocorrelation matrix, C_1.
    - No simplification in the cross section; perfectly matches Γ_0.

Some Implications

- Recall that...

$$E_t^*[y_{t+s}] = \lambda^s q p' y_t$$

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- Update expectations in response to $y_t^{\|}$.
- Does not update expectations *at all* in response to $y_t^{\perp}$.
- Forecasts behave as if the economy is driven by a single "main shock."

Linear best responses:

$$x_{jt} = E_t \left[\sum_{s=1}^{\infty} \beta_j^s c_j' y_{t+s} \right]$$

Comovement

Linear best responses:

$$x_{jt} = E_t \left[\sum_{s=1}^{\infty} \beta_j^s c_j' y_{t+s} \right]$$

Result: for any two forward-looking choices j and k

$$1 = \left| \mathbb{corr} \left(x_{jt}^*, x_{kt}^* \right) \right| \geq \left| \mathbb{corr} \left(x_{jt}^{\mathrm{RE}}, x_{kt}^{\mathrm{RE}} \right) \right|$$

Intuition:

- Expectations are unresponsive to $y_t^{\perp}$.
- It is as if there is a single shock $y_t^{\parallel}$ driving everything.
- This increases the comovement of different choices.

TFP Shocks in the RBC Model

- TFP:

$$a_t = \rho a_{t-1} + \epsilon_t$$

- Capital:

$$k_t = (1 - \delta)k_{t-1} + \delta i_{t-1}$$

- Consumption (permanent income hypothesis):

$$c_t = \frac{\chi}{\beta} k_t + \chi r_t + \chi \zeta w_t + (\chi - \beta \sigma) \sum_{s=1}^{\infty} \beta^s E_t[r_{t+s}] + \chi \zeta \sum_{s=1}^{\infty} \beta^s E_t[w_{t+s}]$$

- True for arbitrary expectations that satisfy the LIE.
  - the aggregate Euler equation may *not* be valid away from RE (Preston, 2005)

- r_t, w_t, and i_t are linear functions of k_t, a_t, and c_t.

Calibration and Agents' Pseudo-True Model

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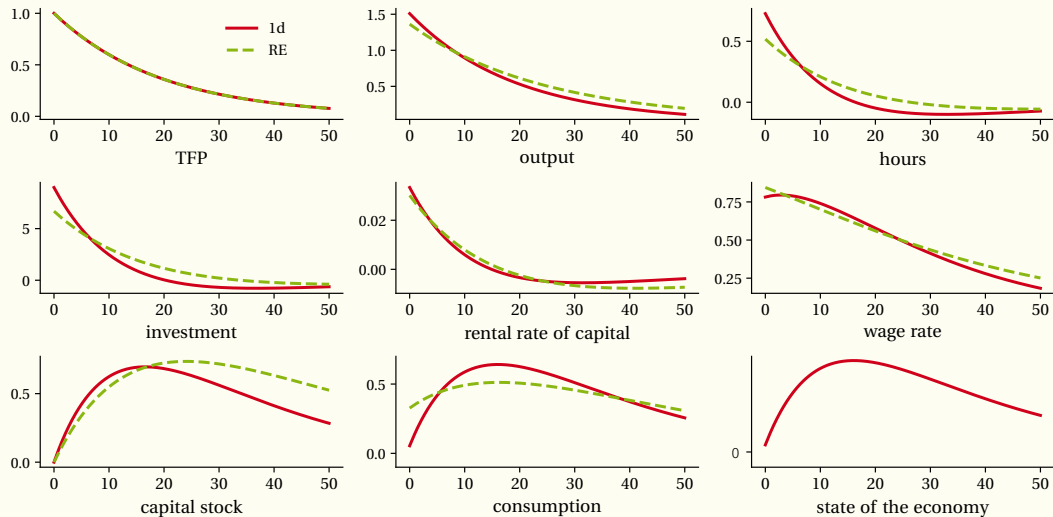- Agents have full information.

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- Agents' nowcast of the subjective state

$$E_t^*[z_t] = p'y_t = 0.947k_t + 0.053a_t$$

- Expectations move (almost) one-for-one with changes in capital stock.
  - almost no (direct) response to changes in TFP
  - persistence bias

Impulse Response Functions

Forward Guidance

The Two-Equation New-Keynesian Model

- Dynamic IS curve:

$$\hat{x}_t = -\sigma \left(\hat{i}_t - r_t^n\right) + E_t^h \left[\sum_{s=1}^{\infty} \beta^s \left(\frac{1-\beta}{\beta}\hat{x}_{t+s} - \sigma \left(\hat{i}_{t+s} - r_{t+s}^n\right) - \frac{\sigma}{\beta}\hat{\pi}_{t+s}\right)\right]$$

- NK Phillips curve:

$$\hat{\pi}_t = \kappa \hat{x}_t + \mu_t + E_t^f \left[\sum_{s=1}^{\infty} (\beta\delta)^s \left(\kappa \hat{x}_{t+s} + \frac{1-\delta}{\delta}\hat{\pi}_{t+s} + \mu_{t+s}\right)\right]$$

- No need for the Taylor principle.
    - assume observables *cannot* include sunspots
    - equilibrium is determinate as long as actions are measurable with respect to observables
- Natural rate, cost push-up, and interest-rate shocks.

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- d is the *only* additional free parameter.
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- Choose the DGP for $(r_t^n, \mu_t, \hat{i}_t)$ to target the autocovariance of $(\hat{x}_t, \hat{\pi}_t, \hat{i}_t)$ at lags 0, 1.
    - only identifies the autocovariance of $(r_t^n, \mu_t, \hat{i}_t)$ at lags 0, 1
    - sufficient for my analysis
    - can hit the target perfectly

Pseudo-True 1-State Model

Agents' nowcast of the subjective state:

$$E_t^*[z_t] = p'y_t = 0.022\hat{x}_t - 0.42\hat{\pi}_t - 0.014\hat{\imath}$$

- High inflation and high output gap have opposite effects on the agents' nowcast.
- Expectations respond a lot inflation, not so much to the nominal rate.

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Persistence of the subjective state:

$$E_t^*[z_{t+1}] = 0.985 E_t^*[z_t]$$

- Larger than the estimated persistence of any of the shocks.
- But *not* unit root.

Forward Guidance

- A credible commitment by monetary authority at time t to

$$\{\hat{i}_{t+1}, \hat{i}_{t+2}, \ldots, \hat{i}_{t+T}\}$$

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- Agents' information set at time t under forward guidance:

$$\omega_T \equiv \{\ldots, y_{t-1}, y_t, \hat{i}_{t+2}, \ldots, \hat{i}_{t+T}\}$$

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$$E_t^{*-\text{FG}}[\cdot] = E^*[\cdot|\omega_T]$$

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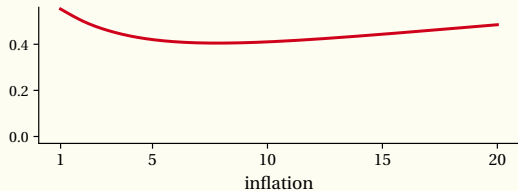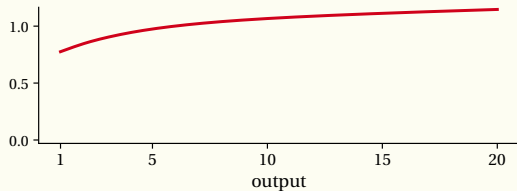$$E_t^{*-\text{FG}}[\cdot] = E^*[\cdot|\omega_T]$$

- Agents' subjective model is Gaussian. $\implies$ Conditioning is easy!

$$E^*[\zeta_{t+s}|\omega_T] = \Sigma_{\zeta_s \omega_T} \Sigma_{\omega_T \omega_T}^{-1} \omega_T$$

$$\hat{x}_t = v_{xi}^{(T)} \hat{i}_t + v_{xn}^{(T)} r_t^n + v_{x\mu}^{(T)} \mu_t + \sum_{s=1}^{T} v_{xi_s}^{(T)} \hat{i}_{t+s}$$

$$\hat{\pi}_t = v_{\pi i}^{(T)} \hat{i}_t + v_{\pi n}^{(T)} r_t^n + v_{\pi\mu}^{(T)} \mu_t + \sum_{s=1}^{T} v_{\pi i_s}^{(T)} \hat{i}_{t+s}$$

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$$\hat{\pi}_t = v_{\pi i}^{(T)} \hat{i}_t + v_{\pi n}^{(T)} r_t^n + v_{\pi\mu}^{(T)} \mu_t + \sum_{s=1}^{T} v_{\pi i_s}^{(T)} \hat{i}_{t+s}$$

output

inflation

Concluding Remarks

- I assume agents forecast using simple models that are fit to the DGP.
- Illustration in the context of the RBC and NK models.
- The framework can be embedded in workhorse macro models.

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- Illustration in the context of the RBC and NK models.
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- The Julia code is available on my website.
- Paper with Alireza Tahbaz-Salehi and Andrea Vedolin: asset-pricing implications.

Models with d running statistics

- The summary statistics

$$s_t \in \mathbb{R}^d$$

- The running statistics are updated linearly over time:

$$s_t = M s_{t-1} + K y_t$$

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An equivalence result: d-state models $\approx$ models with d running statistics

Learning Foundations

> **Theorem (Huber, White, Douc-Moulines, ...)**
> *Assume the agent estimates the parameters of the model $\theta \equiv (A, B, Q, R)$ using (quasi-)MLE. Then asymptotically her point estimate converges to a pseudo-true model.*

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Assume the agent estimates the parameters of the model $\theta \equiv (A, B, Q, R)$ using (quasi-)MLE. Then asymptotically her point estimate converges to a pseudo-true model.

Theorem (Berk, Bunke-Milhaud, Shalizi, ...)
Assume the agent starts with a full-support prior over the set of d-state models and updates her prior over time using Bayes' rule. Asymptotically the agent's belief concentrates over the set of pseudo-true models.

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Ergodicity Assumption

Assumption

For all $l \geq 1$

$$\rho(C_l) \leq \rho(C_1)^l$$

where

$$\rho(C_l) = \max\left\{|\lambda| : \lambda \text{ is an eigenvalue of } C_l\right\}$$

- Requires autocorrelations to decay sufficiently fast.
- Satisfied for many commonly used specifications.
- Satisfied in all the applications studied in this talk.

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