Simple Models and Biased Forecasts

Pooya Molavi

July 23, 2023

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(2) All models in the class are too simple relative to the truth, i.e., they are misspecified.
  - the models are low-dimensional

(3) Study the long-run limit where agents' estimates have settled.
  - agents settle on pseudo-true models that approximate the true model

The Framework

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  - finite second moment and entropy rate
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- The agent is attempting to forecast future values of the observables.
  - time-t information set is $\{y_\tau\}_{\tau=-\infty}^{t}$
  - agent uses a model to map past observables to her forecasts:

$$\theta : \{y_\tau\}_{\tau=-\infty}^{t} \mapsto E_t^\theta[\cdot]$$

State-Space Models

Main assumption: the agent can only entertain stationary ergodic distributions P^{θ} that can be represented by state-space models with at most d states.

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\begin{aligned}
z_t &= A z_{t-1} + w_t, \qquad & w_t &\sim \text{ i.i.d. } \mathcal{N}(0, Q) \\
y_t &= B' z_t + v_t, \qquad & v_t &\sim \text{ i.i.d. } \mathcal{N}(0, R)
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- d is the only free parameter.
- I take d to be (much) smaller than n.
- $\theta \equiv (A, B, Q, R)$ is estimated endogenously by the agent.

Cross-Sectional and Times-Series Complexity

A dichotomy: model θ is unconstrained other than the constraint on d.

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Stark assumption, but...

- It allows me to focus on the difficulty of dealing with time-series complexity.
  - Cross-sectional complexity is the focus of rational inattention, sparsity, etc.
- It leads to a useful linear invariance property for expectations.

A Complementary Interpretation: Limited Memory

Models with d running statistics

- Running statistics:

$$s_t \in \mathbb{R}^d$$

- The running statistics are updated linearly over time:

$$s_t = M s_{t-1} + K y_t$$

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$$E_t[y_{t+\tau}] = v_\tau' s_t$$

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An equivalence result: d-state models $\approx$ models with d running statistics

Endogenizing the Agent's Model

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Focus on the limit where the agent has infinite date:

- The agent (generically) ends up with the same model using both approaches.
- The limiting model is a pseudo-true model that minimizes KLDR:

$$\lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[\log \left(\frac{\mathbb{f}(y_1, \ldots, y_t)}{f^{\theta}(y_1, \ldots, y_t)} \right) \right]$$

The Plan

The rest of the talk...

1. (Some) qualitative features of pseudo-true one-state models.
2. (Some) implications for agents' forecasts and actions.
3. Propagation of TFP shocks in the RBC model.
4. Propagation of separation and productivity shocks in the DMP model.

Relation to the Literature

- **Misspecified learning:** Berk (1966), Huber (1967), White(1982, 1994), Shalizi (2009), Esponda–Pouzo (2016, 2021), ...
    - not about learning foundations
    - characterize properties of pseudo-true models

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  - perfect knowledge of current variables
  - perfect understanding of cross-sectional relationships (a result)
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- **Bounded rationality with pseudo-true models:** Sawa (1978), Bray (1982), Bray–Savin (1986), Rabin–Vayanos (2010), Fuster–Laibson–Mendel (2010), Fuster–Hebert–Laibson (2012), Spiegler (2016), Levy–Razin–Young (2021), Molavi–Tahbaz-Salehi–Vedolin (2023), ...

Pseudo-True One-State Models

An Invariance Result

Theorem (linear invariance)

Consider two agents:

- *Agent i observes y_t and uses pseudo-true model θ.*
- *Agent j observes $\tilde{y}_t = Ty_t$ and uses pseudo-true model $\tilde{\theta}$.*

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- Pseudo-true simple models respect *all* linear intratemporal relationships...

$$E_t^{\theta}[\alpha y_{1,t+s} + \beta y_{2,t+s}] = \alpha E_t^{\theta}[y_{1,t+s}] + \beta E_t^{\theta}[y_{2,t+s}]$$

- *Not* always true in non-RE models.

No Misperception in the Cross Section

Theorem

Under any pseudo-true model θ,

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- The agent correctly perceived *all* the cross-sectional correlations.
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- Different from rational inattention, noisy information, sparsity, etc.

- **Intuition:** The agent can always match the cross-sectional correlations by an appropriate choice of $\theta = (A, B, Q, R)$. MLE/Bayesian learning leads her to do so.

Pseudo-True Forecasts

- Under any pseudo-true one-state model θ,

$$E_t^{\theta}[y_{t+s}] = a^s(1-\eta)qp' \sum_{\tau=0}^{\infty} a^{\tau}\eta^{\tau} y_{t-\tau}$$

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- (a, η, p, q) is the solution to a non-convex optimization problem.

Definition (exponential ergodicity)

The true process is exponentially ergodic if

$$\rho\left(\left(\frac{\Gamma_l + \Gamma_l'}{2}\right)\Gamma_0^\dagger\right) \leq \rho\left(\left(\frac{\Gamma_1 + \Gamma_1'}{2}\right)\Gamma_0^\dagger\right)^l$$

where $\rho(\cdot)$ denotes the spectral radius.

Exponential Ergodicity and Markovian Models

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Theorem

If the true process is exponentially ergodic, then the pseudo-true one-state model:

(a) *is Markovian*

(b) *only depends on* Γ_0 *and* Γ_1

So, Which Processes are Exponentially Ergodic?

- A spanning linear combination of independent AR(1) processes:

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\begin{aligned}
f_{it} &= \alpha_i f_{i,t-1} + \epsilon_{it}, && i = 1, \ldots, m \\
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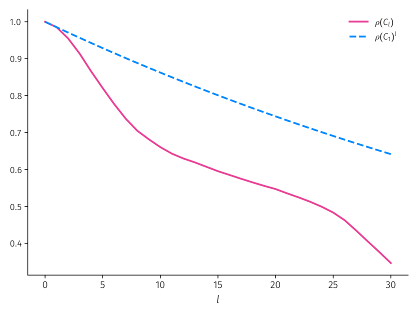
- Estimated (joint) time-series for output gap, inflation rate, and interest rate:



Intuition (1/2)

- Suppose $n = 1$ and the true process is invertible.
- First, suppose $d = \infty$.
- (Pseudo-)true forecasts are given by

$$\mathbb{E}_t[y_{t+1}] = \sum_{\tau=1}^{\infty} \phi_\tau y_{t+1-\tau}$$

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- Coefficients $\{\phi_\tau\}_{\tau=1}^{\infty}$ are all generically non-zero.
- But the optimal $\{\phi_\tau\}_{\tau=1}^{\infty}$ are fine-tuned to the true autocorrelation function (ACF).
  - a one-to-one mapping between the two
  - Yule–Walker equations:

$$\text{the ACF} \longleftrightarrow \{\phi_\tau\}_{\tau=1}^{\infty}$$

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- Only two degrees of freedom. $\implies$ Cannot fine-tune α_τ to fit the true ACF.
- With exp. ergodicity, y_t is much more informative about y_{t+1} than y_{t-l} for $l \geq 1$.
- Optimal to not distort how one uses y_t. Instead set $\eta = 0$, and so, $\alpha_l = 0$ for $l \geq 1$.

Theorem (persistence bias)

If the true process is exponentially ergodic, under any pseudo-true model θ,

$$E_t^{\theta}[y_{t+s}] = a^s q p' y_t$$

a is the top eigenvalue and p and q are the corresponding left and right eigenvectors of

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- The agent's forecasts only depend on the most persistent component of y_t.

A Diagonal Example

- True data-generating process:

$$y_t = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_n \end{pmatrix} y_{t-1} + \begin{pmatrix} b_1 & 0 & \dots & 0 \\ 0 & b_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b_n \end{pmatrix} \epsilon_t, \qquad \epsilon_t \sim \text{i.i.d. } \mathcal{N}(0, I)$$

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- The autocorrelation matrix at lag $l = 1$:

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Pseudo-True One-State Model in the Diagonal Example

- Eigenvalue and eigenvector...

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- Persistence bias:
  - Forecasts of y_1 coincide with the RE.
  - Forecast y_j for $j \neq 1$ as if i.i.d.

Intuition

Agents behave *as if* only the most persistent component of y_t is relevant.

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Why don't volatilities matter?

- By linear invariance, can scale different components to have the same volatilities!
- Furthermore, the agent can always match the volatilities.

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Why focus on persistent component?

- Any component that is ignored is approximated as i.i.d.
- Less persistent components are closer to being i.i.d.

Forward-Looking Decisions

- Consider J agents, with agent j using a pseudo-true d_j-state model θ_j.
- Suppose each agent takes an action of the form

$$x_{jt} = E_{jt}^{\theta_j}\left[\sum_{s=1}^{\infty} c'_{js} y_{t+s}\right]$$

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Theorem

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A main shock:

- The economy looks *as if* it is driven by a single "main shock."
- Angeletos, Collard, and Dellas (2020) find a "main business cycle shock."

TFP Shocks in the RBC Model

The Loglinearized RBC Model

- TFP:

$$a_t = \rho a_{t-1} + \epsilon_t$$

- Capital:

$$k_t = (1-\delta)k_{t-1} + \delta i_{t-1}$$

- Consumption (permanent income hypothesis):

$$c_t = \frac{\chi}{\beta}k_t + \chi r_t + \chi\zeta w_t + (\chi - \beta\sigma)\sum_{s=1}^{\infty}\beta^s E_t[r_{t+s}] + \chi\zeta\sum_{s=1}^{\infty}\beta^s E_t[w_{t+s}]$$

  - true for arbitrary expectations that satisfy the LIE.
  - the aggregate Euler equation may *not* be valid away from RE (Preston, 2005)

- r_t, w_t, and i_t are linear functions of k_t, a_t, and c_t.

Calibration

Primitives:

- Each period is a quarter.
- Textbook calibration of primitives:

$$\beta = 0.99, \qquad \sigma = \phi = 1, \qquad \delta = 0.012, \qquad \alpha = 0.3, \qquad \rho = 0.95$$

Expectations:

- Agents have full information: perfectly observe everything.

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Expectations:

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- d is the *only* additional free parameter.
- $d \geq 2 \implies$ REE
- So, we only need to consider $d = 1$.

Pseudo-True One-State Model

The (endogenously-determined) main shock:

$$0.947k_t + 0.053a_t$$

- Almost perfectly correlated with the capital stock.
- In equilibrium the capital stock is more persistent than TFP (persistence bias).
- Agents almost ignore innovations to the TFP.

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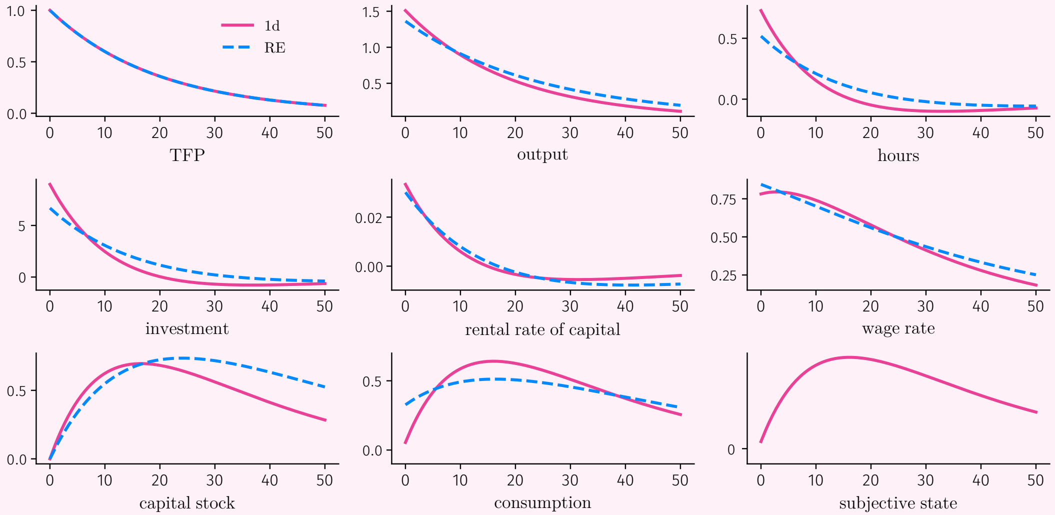
Sluggishness:

- Expectations move almost one-for-one with capital.
- Capital is sluggish, so expectatations are sluggish.
- Consumption is forward-looking, so it is also sluggish.

Impulse Response Functions



Productivity and Separation Shocks in the DMP Model

Primitives and Calibration

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Expectations:

- Agents have full information.
- Set $d = 1$ (only other free parameter).

Pseudo-True One-State Model

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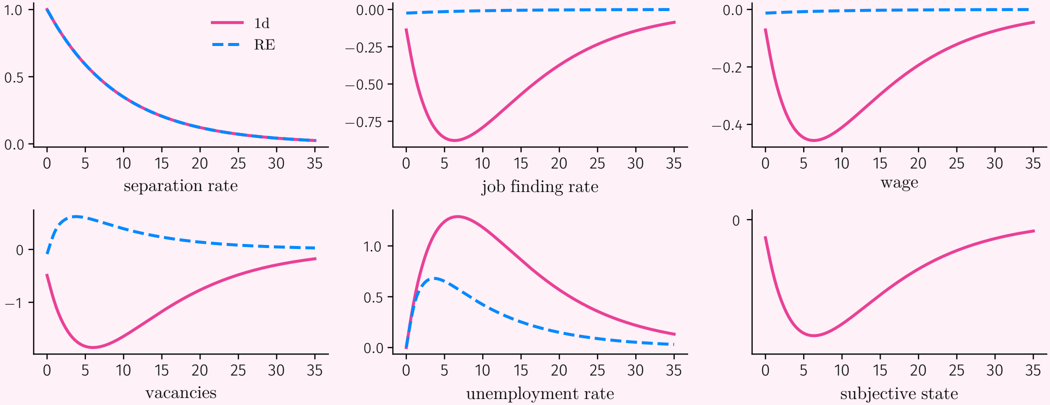
Intuition: "Keynesian" complementarity between the firms' decisions and models...

- Suppose $u\uparrow + a\downarrow + s\uparrow$ make firms "pessimistic."
- Pessimistic firms post few vacancies. $\implies$ Persistent recession after $u\uparrow + a\downarrow + s\uparrow$.
- In equilibrium, the most persistent combination has $u\uparrow + a\downarrow + s\uparrow$.

Impulse Response Functions to Separation Shock



Concluding Remarks

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- The framework can be embedded in workhorse macro models.

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- The Julia code is available on my website.
- Paper with Alireza Tahbaz-Salehi and Andrea Vedolin: asset-pricing implications.