Simple Models and Biased Forecasts

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(1) Agents entertain a large and flexible class of time-series models.
  - all state-space models of a given dimension
(2) All models in the class are too simple relative to the truth, i.e., they are misspecified.
  - the models are low-dimensional
(3) Study the long-run limit where agents' estimates have settled.
  - agents settle on pseudo-true models that approximate the true model

The Framework

Individual Problem

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  - for now, assume $\mathbb{P}$ is exogenous (will be endogenous in GE)
  - mean zero, stationary ergodic, and purely non-deterministic
  - finite second moment and entropy rate
  - expectation operator $\mathbb{E}$

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- The agent is attempting to forecast future values of the observables.
  - time-t information set is $\{y_\tau\}_{\tau=-\infty}^{t}$
  - agent uses a model to map past observables to her forecasts:

$$\theta : \{y_\tau\}_{\tau=-\infty}^{t} \mapsto E_t^\theta [\cdot]$$

State-Space Models

Main assumption: the agent can only entertain stationary ergodic distributions P^θ that can be represented by state-space models with at most d states.

$$z_t = A z_{t-1} + w_t, \qquad w_t \sim \text{ i.i.d. } \mathcal{N}(0, Q)$$
$$y_t = B' z_t + v_t, \qquad v_t \sim \text{ i.i.d. } \mathcal{N}(0, R)$$

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- $\theta \equiv (A, B, Q, R)$ is estimated endogenously by the agent.

Cross-Sectional and Times-Series Complexity

A dichotomy: model θ is unconstrained other than the constraint on d.

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Stark assumption, but...

- It allows me to focus on the difficulty of dealing with time-series complexity.
  - Cross-sectional complexity is the focus of rational inattention, sparsity, etc.
- It leads to a useful linear invariance property for expectations.

Models with d running statistics

- Running statistics:

$$s_t \in \mathbb{R}^d$$

- The running statistics are updated linearly over time:

$$s_t = M s_{t-1} + K y_t$$

- Forecasts are linear functions of the running statistics:

$$E_t[y_{t+\tau}] = v_\tau' s_t$$

A Complementary Interpretation: Limited Memory

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An equivalence result: d-state models $\approx$ models with d running statistics

Endogenizing the Agent's Model

Two ways of specifying the agent's model $\theta \equiv (A, B, Q, R)$:

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Focus on the limit where the agent has infinite date:

- The agent (generically) ends up with the same model using both approaches.

- The limiting model is a pseudo-true model that minimizes KLDR:

$$\lim_{t \to \infty} \frac{1}{t} \mathbb{E} \left[\log \left(\frac{\mathbb{f}(y_1, \ldots, y_t)}{f^\theta(y_1, \ldots, y_t)} \right) \right]$$

The Plan

The rest of the talk...

1. (Some) qualitative features of pseudo-true one-state models.
2. (Some) implications for agents' forecasts and actions.
3. Propagation of TFP shocks in the RBC model.
4. Propagation of separation and productivity shocks in the DMP model.

Relation to the Literature

- **Misspecified learning:** Berk (1966), Huber (1967), White(1982, 1994), Shalizi (2009), Esponda–Pouzo (2016, 2021), ...
  - not about learning foundations
  - characterize properties of pseudo-true models

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- **Noisy information/rational inattention/sparsity:** Mankiw–Reis (2002), Sims (2003), Woodford (2003), Gabaix (2014), Angeletos–Lian (2018), ...
  - perfect knowledge of current variables
  - perfect understanding of cross-sectional relationships (a result)
  - can only understand simple time-series relationships

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- **Bounded rationality with pseudo-true models:** Sawa (1978), Bray (1982), Bray–Savin (1986), Rabin–Vayanos (2010), Fuster–Laibson–Mendel (2010), Fuster–Hebert–Laibson (2012), Spiegler (2016), Levy–Razin–Young (2021), Molavi–Tahbaz-Salehi–Vedolin (2023), ...

Pseudo-True One-State Models

An Invariance Result

Theorem (linear invariance)

Consider two agents:

- *Agent i observes y_t and uses pseudo-true model θ.*

- *Agent j observes $\tilde{y}_t = T y_t$ and uses pseudo-true model $\tilde{\theta}$.*

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For any full-rank matrix T,

$$E_{jt}^{\tilde{\theta}}[\tilde{y}_{t+s}] = T E_{it}^{\theta}[y_{t+s}]$$

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> *For any full-rank matrix T,*
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- Pseudo-true simple models respect *all* linear intratemporal relationships...

$$E_t^{\theta}[\alpha y_{1,t+s} + \beta y_{2,t+s}] = \alpha E_t^{\theta}[y_{1,t+s}] + \beta E_t^{\theta}[y_{2,t+s}]$$

- *Not* always true in non-RE models.

No Misperception in the Cross Section

> **Theorem**
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- The agent correctly perceived *all* the cross-sectional correlations.
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- Different from rational inattention, noisy information, sparsity, etc.

- **Intuition:** The agent can always match the cross-sectional correlations by an appropriate choice of $\theta = (A, B, Q, R)$. MLE/Bayesian learning leads her to do so.

- Under any pseudo-true one-state model θ,

$$E_t^\theta[y_{t+s}] = a^s(1-\eta)qp' \sum_{\tau=0}^{\infty} a^\tau \eta^\tau y_{t-\tau}$$

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- (a, η, p, q) is the solution to a non-convex optimization problem.

Exponential Ergodicity and Markovian Models

> **Definition (exponential ergodicity)**
>
> The true process is **exponentially ergodic** if
>
> $$\rho\left(\left(\frac{\Gamma_l + \Gamma_l'}{2}\right)\Gamma_0^\dagger\right) \le \rho\left(\left(\frac{\Gamma_1 + \Gamma_1'}{2}\right)\Gamma_0^\dagger\right)^l$$
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> where $\rho(\cdot)$ denotes the spectral radius.

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Theorem

If the true process is exponentially ergodic, then the pseudo-true one-state model:

(a) *is Markovian*

(b) *only depends on Γ_0 and Γ_1*

So, Which Processes are Exponentially Ergodic?

- A spanning linear combination of independent AR(1) processes:

$$f_{it} = \alpha_i f_{i,t-1} + \epsilon_{it}, \qquad i = 1, \ldots, m$$

$$y_t = H' f_t, \qquad\qquad H \;\; \text{rank } m$$

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- Estimated (joint) time-series for output gap, inflation rate, and interest rate:
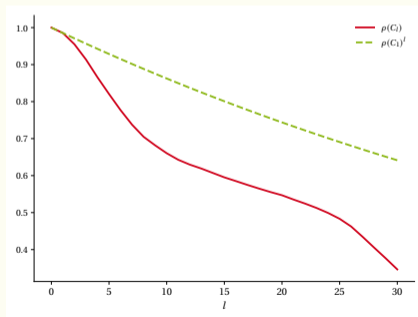
Intuition (1/2)

- Suppose $n = 1$ and the true process is invertible.
- First, suppose $d = \infty$.
- (Pseudo-)true forecasts are given by

$$\mathbb{E}_t [y_{t+1}] = \sum_{\tau=1}^{\infty} \phi_\tau y_{t+1-\tau}$$

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- Coefficients $\{\phi_\tau\}_{\tau=1}^{\infty}$ are all generically non-zero.
- But the optimal $\{\phi_\tau\}_{\tau=1}^{\infty}$ are fine-tuned to the true autocorrelation function (ACF).
  - a one-to-one mapping between the two
  - Yule–Walker equations:

$$\text{the ACF} \longleftrightarrow \{\phi_\tau\}_{\tau=1}^{\infty}$$

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- Only two degrees of freedom. $\implies$ Cannot fine-tune α_τ to fit the true ACF.

- With exp. ergodicity, y_t is much more informative (than $y_{t+1-\tau}$ for $\tau > 1$) about y_{t+1}.

- Optimal to not distort how one uses y_t. Instead set $\eta = 0$, and so, $\alpha_\tau = 0$ for $\tau > 1$.

Persistence Bias

> **Theorem (persistence bias)**
>
> *If the true process is exponentially ergodic, under any pseudo-true model θ,*
>
> $$E_t^{\theta}[y_{t+s}] = a^s q p' y_t$$
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> *a is the top eigenvalue and p and q are the corresponding left and right eigenvectors of*
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- The agent's forecasts only depend on the most persistent component of y_t.

A Diagonal Example

- True data-generating process:

$$y_t = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_n \end{pmatrix} y_{t-1} + \begin{pmatrix} b_1 & 0 & \dots & 0 \\ 0 & b_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & b_n \end{pmatrix} \epsilon_t, \qquad \epsilon_t \sim \text{i.i.d. } \mathcal{N}(0, I)$$

where

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where

$$|\alpha_1| > |\alpha_2| > \dots > |\alpha_n|$$

- The autocorrelation matrix at lag $l = 1$:

$$\left(\frac{\Gamma_1 + \Gamma_1'}{2} \right) \Gamma_0^\dagger = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_n \end{pmatrix}$$

- Eigenvalue and eigenvector...

$$a = \alpha_1, \qquad p = q = (1, 0, \ldots, 0)'$$

Pseudo-True One-State Model in the Diagonal Example

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- Persistence bias:
  - Forecasts of y_1 coincide with the RE.
  - Forecast y_j for $j \neq 1$ as if i.i.d.

Intuition

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Why don't volatilities matter?

- By linear invariance, can scale different components to have the same volatilities!
- Furthermore, the agent can always match the volatilities.

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Why focus on the most persistent component?

- Any component that is ignored is approximated as i.i.d.
- Less persistent components are closer to being i.i.d.

Forward-Looking Decisions

- Consider J agents, with agent j using a pseudo-true d_j-state model θ_j.
- Suppose each agent takes an action of the form

$$x_{jt} = E_{jt}^{\theta_j} \left[\sum_{s=1}^{\infty} c_{js}' y_{t+s} \right]$$

Increased Comovement

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A main shock:

- The economy looks *as if* it is driven by a single "main shock."
- Angeletos, Collard, and Dellas (2020) find a "main business cycle shock."

TFP Shocks in the RBC Model

- TFP:

$$a_t = \rho a_{t-1} + \epsilon_t$$

- Capital:

$$k_t = (1 - \delta)k_{t-1} + \delta i_{t-1}$$

- Consumption (permanent income hypothesis):

$$c_t = \frac{\chi}{\beta}k_t + \chi r_t + \chi\zeta w_t + (\chi - \beta\sigma)\sum_{s=1}^{\infty}\beta^s E_t[r_{t+s}] + \chi\zeta\sum_{s=1}^{\infty}\beta^s E_t[w_{t+s}]$$

  - true for arbitrary expectations that satisfy the LIE.
  - the aggregate Euler equation may *not* be valid away from RE (Preston, 2005)

- r_t, w_t, and i_t are linear functions of k_t, a_t, and c_t.

Primitives:

- Each period is a quarter.
- Textbook calibration of primitives:

$$\beta = 0.99, \qquad \sigma = \phi = 1, \qquad \delta = 0.012, \qquad \alpha = 0.3, \qquad \rho = 0.95$$

Expectations:

- Agents have full information: perfectly observe everything.

Calibration

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Expectations:

- Agents have full information: perfectly observe everything.

- d is the *only* additional free parameter.
- $d \geq 2 \implies$ REE
- So, we only need to consider $d = 1$.

Pseudo-True One-State Model

The (endogenously-determined) main shock:

$$0.947k_t + 0.053a_t$$

- Almost perfectly correlated with the capital stock.
- In equilibrium the capital stock is more persistent than TFP (persistence bias).
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Sluggishness:

- Expectations move almost one-for-one with capital.
- Capital is sluggish, so expectatations are sluggish.
- Consumption is forward-looking, so it is also sluggish.

Impulse Response Functions
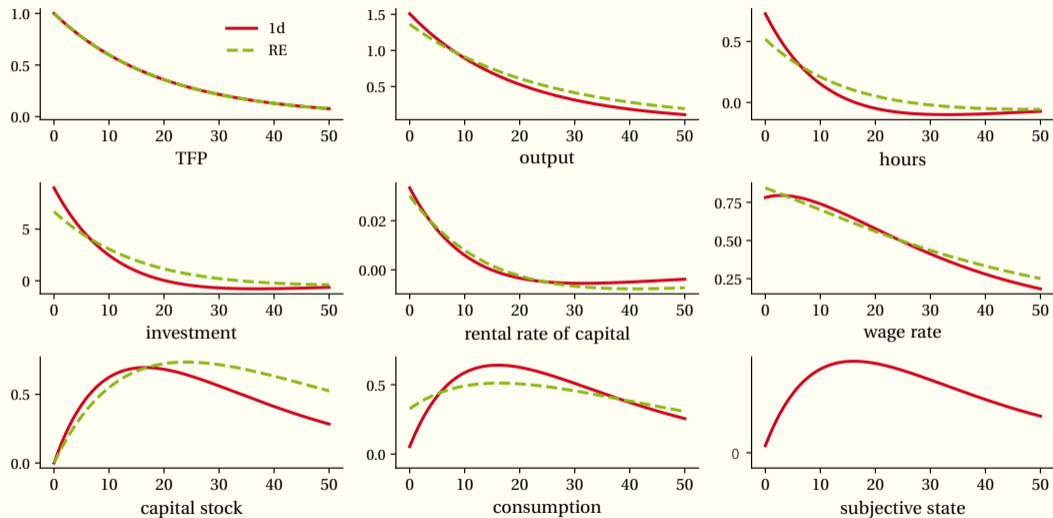
Productivity and Separation Shocks in the DMP Model

Primitives:

- Standard undirected labor search model: discrete-time version of Shimer (2005).
- The only interesting decision: firms' vacancy-posting decision.

Primitives and Calibration

Primitives:

- Standard undirected labor search model: discrete-time version of Shimer (2005).
- The only interesting decision: firms' vacancy-posting decision.

Calibration:

- Almost everything is calibrated as in Shimer (2005).
- Parameters that don't matter under RE:
  - correlation of the two shocks: −0.4
  - SDs of the shocks: ten times larger for productivity

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Expectations:

- Agents have full information.
- Set $d = 1$ (only other free parameter).

Pseudo-True One-State Model

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Intuition: "Keynesian" complementarity between the firms' decisions and models...

- Suppose $u \uparrow + a \downarrow + s \uparrow$ make firms "pessimistic."

- Pessimistic firms post few vacancies. $\implies$ Persistent recession after $u \uparrow + a \downarrow + s \uparrow$.

- In equilibrium, the most persistent combination has $u \uparrow + a \downarrow + s \uparrow$.

Impulse Response Functions to Separation Shock
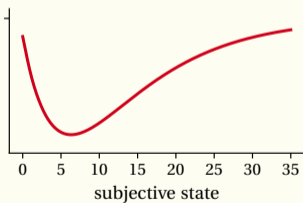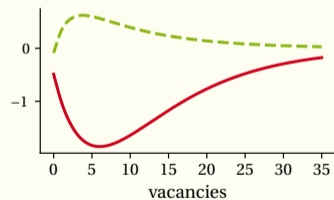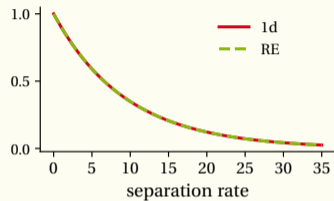
Concluding Remarks

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- Illustration in the context of the RBC and DMP models.
- The framework can be embedded in workhorse macro models.

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- The Julia code is available on my website.
- Paper with Alireza Tahbaz-Salehi and Andrea Vedolin: asset-pricing implications.