Deep Learning of High Dimensional PDEs

Master's Thesis Abstract

The numerical solution of differential equations in high dimensions has always been a challenge and has been associated with various computational difficulties. These equations appear naturally in a variety of problems such as financial mathematics, control, and physics, and their optimal solution with high accuracy and speed can open new windows on new applications. Conventional methods such as finite element and finite difference method in high dimensions lose their efficiency, which is a barrier to fast and accurate calculation of these equations. In this dissertation, first, we review some theoretical and practical aspects of deep neural networks and then we try to examine the recent achievements of machine learning methods in solving a set of equations like the ten-dimensional American option. The deep neural network is trained to satisfy the differential operator, initial condition, and boundary conditions using stochastic gradient descent at randomly sampled spatial points. By randomly sampling spatial points, we avoid the need to form a mesh (which is infeasible in higher dimensions) and instead convert the PDE problem into a machine learning problem. We implement the discussed methods on a set of equations and examine their strengths and weaknesses by comparing them with conventional methods.

Key words: PDE, Deep Neural Network, Numerical Solution, Free boundary problem