(1)

$A \rightarrow -0.1x_1 - 0.1x_2 > -0.15$

$B \rightarrow -0.1x_1 - 0.1x_2 > 0.05$

ب)

A) جدول

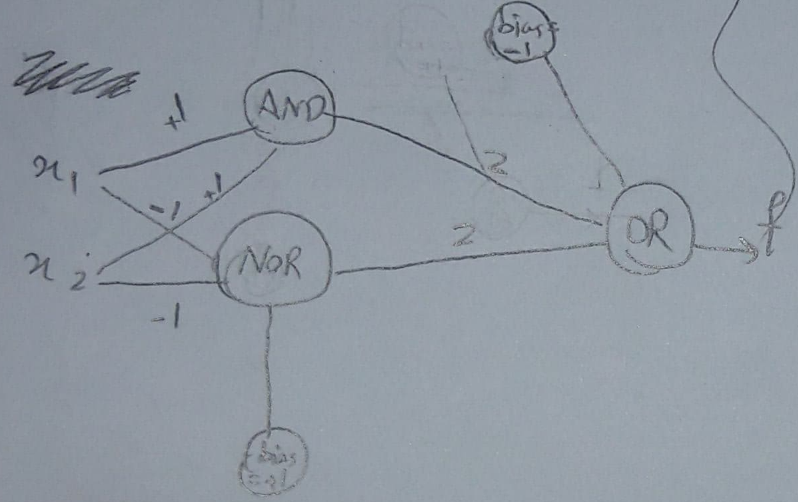
x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

B

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

A نشان‌دهنده "NAND" است

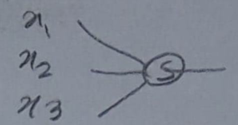
B نشان‌دهنده "NOR" است



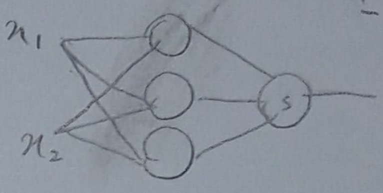
الف) در بخش a : جداسازی با یه خط انجام شده است.

داریم:

$$S = \begin{cases} 1 & net > t \\ 0 & -t < net < t \\ -1 & net < -t \end{cases}$$



در بخش b اینجا multilayer است و چون در اینجا چند خط (خط) را پیش‌بینی کار را انجام می‌دهیم.



ج)

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

x_1 xnor x_2

با ترکیب (OR، AND، NOR) ← داریم

(2)

$S = \frac{1}{1+e^{-Z}}$

Sigmoid

$Z_1 = w_{11}x_1 + w_{21}x_2 + w_{01}$

$Z_2 = w_{12}x_1 + w_{22}x_2 + w_{02}$

$$\rightarrow \begin{cases} \frac{1}{1+\exp[-Z_1]} \\ \frac{1}{1+\exp[-Z_2]} \end{cases}$$

$Y = S_1V_1 + S_2V_2 + V_0$

داریم $\rightarrow \begin{cases} Z_1 = (0.7)(1) + 0 + 0.4 = \boxed{1.1} \\ Z_2 = (-0.1)(1) + 0 + 0.6 = \boxed{0.2} \end{cases} \rightarrow \boxed{\begin{cases} S_1 \simeq 0.752 \\ S_2 \simeq 0.552 \end{cases}}$

$Y = (0.752)(0.5) + (0.552)(0.1) - 0.3 = \boxed{0.1311}$

$w_i' = w_i + \alpha(t - net)x_i \rightarrow \begin{cases} w_{11}' = 0.7 + 0.2(0.1311) \simeq \boxed{0.726} \\ w_{12}' = -0.2 + 0.2(0) = \boxed{-0.2} \\ w_{21}' = -0.4 + 0.2(0.1311) = \boxed{-0.378} \\ w_{22}' = 0.3 + 0 = \boxed{0.3} \end{cases}$

③

حجم: $H(y|p) = \frac{3}{5}\left(\lg\frac{4}{6} + \frac{2}{6}\lg\frac{2}{6}\right) + \frac{2}{5}\left(\frac{1}{2}\lg\frac{1}{2} + \frac{1}{2}\lg\frac{1}{2}\right) = \boxed{0.951}$

جنس: $H(y|m) = \frac{3}{5}\left(-\frac{5}{6}\lg\frac{5}{6} - \frac{1}{6}\lg\frac{1}{6}\right) + \frac{2}{5}\left(-\frac{3}{4}\lg\frac{3}{4} - \frac{1}{4}\lg\frac{1}{4}\right) = \boxed{0.7138}$

رنگ: $H(y|c) = \frac{2}{5}\left(-\frac{3}{4}\lg\frac{3}{4} - \frac{1}{4}\lg\frac{1}{4}\right) + \frac{1}{5}(0) + \frac{2}{5}\left(-\frac{3}{4}\lg\frac{3}{4} - \frac{1}{4}\lg\frac{1}{4}\right) = \boxed{0.6488}$

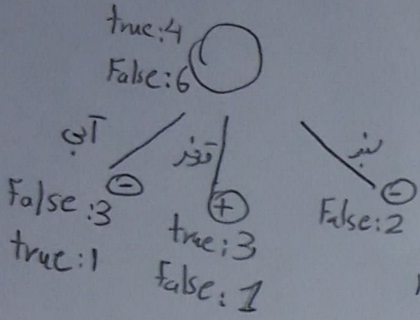
جنس ← m

رنگ ← c

حجم ← p

قابل خرید ← y

← آنتروپی پایین‌تر ← ویژگی «رنگ» انتخاب می‌شود



predicted\actual	+	−
+	tp = 3	FN = 1
−	FP = 1	TN = 5

$recall = \frac{TP}{P} = \boxed{\frac{3}{4}}$

$prec = \frac{TP}{tp+FP} = \boxed{\frac{3}{4}}$

$acc = \frac{TP+TN}{n} = \boxed{\frac{4}{5}}$

$spec = \frac{TN}{N} = \boxed{\frac{5}{6}}$

---

④

$$\varphi(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$\bar{P}_n(x) = \begin{cases} \frac{1}{a}\left(1 - e^{-\left(\frac{x}{h_n}\right)}\right) & x \leq 0 \\ \frac{1}{a}\left(e^{\frac{a}{h_n}} - 1\right) e^{-\frac{x}{h_n}} & a \leq x \end{cases}$$

الف

ب) h_n

$$p(x) = \begin{cases} 1/a, & 0 \leq x \leq a \\ 0, & o.w \end{cases}$$

الف) $\bar{p}_n(x) = E\left(\frac{1}{nh_n}\sum_{i=1}^{n}\varphi\left(\frac{x-x_i}{h_n}\right)\right) = \frac{1}{h_n}\int\varphi\left(\frac{x-v}{h_n}\right)p(v)\,dv = \frac{1}{h_n}\exp\left(\frac{-(x-v)}{h_n}\right)p(v)\,dv =$

$$= \begin{cases} 0, & x < 0 \\ \frac{e^{-x/h_n}}{ah_n}\int_0^x e^{v/h_n}\,dv, & 0 \leq x \leq a \\ \frac{e^{-x/h_n}}{ah_n}\int_0^a e^{v/h_n}\,dv, & x \geq a \end{cases}$$

$$p(x) - \bar{p}_n(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{a} - \frac{1}{a}\left(1 - e^{-\frac{x}{h_n}}\right) & 0 \leq x \leq a \\ 0 - \frac{1}{a}\left(e^{a/h_n} - 1\right)e^{-\frac{x}{h_n}} & x \geq a \end{cases} = \begin{cases} 0, & x < 0 \\ \frac{1}{a}e^{-x/h_n}, & 0 \leq x \leq a \\ -\frac{1}{a}\left(e^{a/h_n} - 1\right)e^{-x/h_n} & x \geq a \end{cases}$$

ب)

$$\frac{p(x) - \bar{p}(x)}{p(x)} \leq 0.01 \qquad \frac{\frac{1}{a}e^{-x/h_n}}{\frac{1}{a}} \leq 0.01 \qquad h_n \leq \frac{0.01a}{\ln(100)}$$

for $a=1$ $\longrightarrow h_1 = \frac{0.01}{\ln(100)} = \boxed{0.0022}$

5

الف) $P_n(e) = \frac{1}{2^n}\sum_{0}^{\frac{k-1}{2}} \binom{n}{j}$

الا) $p(w_1|x) = \begin{cases} 1, & \|x\| < -1 \\ 0, & ow \end{cases}$ $\quad p(w_2|x) = \begin{cases} 1, & \|x-a\| < 1 \\ 0, & ow \end{cases}$

ب) $P_n(e) = P_r[w_1(true) | w_2(labled)] + P_r[w_2(true) | w_1(labeled)] =$

ج) $2\,p(w_1)\,p$ [label of "w_1" for fewer than $\frac{k-1}{2}$ points and the remaining points labled as "w_2"] $= 2(0.5)\sum_{0}^{k-1/2} P_r(\binom{n}{j}$ points w_1, remaining $w_2) =$

$$\sum_{j=0}^{k-1/2} \binom{n}{j} \frac{1}{2^j} \cdot \frac{1}{2^{(n-j)}} = \boxed{\frac{1}{2^n}\sum_{0}^{k-1/2} \binom{n}{j}}$$

د) $P_n(e) = P_n(e;k)$ $\quad \boxed{P_n(e;1) = \frac{1}{2^n} < P_n(e;k) = \frac{1}{2^n}\sum_{0}^{k-1/2}\binom{n}{j} \quad k>0,\ k>1}$

ه) $P_n(e) = \frac{1}{2^n}\sum_{0}^{k-1/2}\binom{n}{j} = P_r(Y_1 + \dots Y_n < \frac{k-1}{2}) \Longrightarrow$ $\quad P_n(e) \leq P(Y_1 + \dots + Y_n \leq \frac{\frac{a}{\sqrt{n}} - 1}{2})$

$P(Y_i = 0) = P(Y_i = 1) = 0.5$

منفی می‌شود پس حد $P_n(e)$

$\Longrightarrow P_n(e) \leq P(Y_1 + \dots + Y_n \leq 0) \Longrightarrow \boxed{P_n(e) = 0}$

احتمال صفر $\to 0$ $\quad n \to \infty$