Digital Filtering

87203 – Multimedial Signal Processing 1st Module

Politecnico di Milano –

Polo regionale di Como

Particulars

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Agenda

- Ex1: Digital Linear Time Invariant filters
- Ex2: Poles and Zeros
- Ex3: Digital filter in the frequency domain
- Ex4: FIR vs. IRR filters
- Ex5: Examples of filters

Agenda

- Ex6: Linear phase filter
- Ex7: Minimum phase filter
- Ex8: Allpass filter
- Ex9: Minimum phase/allpass decomposition
- Ex10: Non linear filter
- Ex11: Pass Bass FIR filter

Exercise 1 (1/9)

- Frequency Response: The DTFT of an impulse response is called the Frequency Response/ Transfer Function of a LTI system:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

- Any LTI filter can be implemented by convolving the input signal with the filter impulse response h(n):

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = x(n) * h(n)$$

Exercise 1 (2/9)

- Difference Equation: A LTI system can be described by a linear constant coefficient difference equation:

$$\sum_{k=0}^{N} a_k \, y(n-k) = \sum_{m=0}^{M} b_m \, x(n-m) \quad \forall n$$

if $a_N \neq 0$, then the difference equation is of order N.

Exercise 1 (3/9)

- Difference Equation:

$$y(n) = \underbrace{\underbrace{\sum_{m=0}^{M} b_m \, x(n-m)}_{\text{FIR (MA)}} - \underbrace{\sum_{k=1}^{N} a_k \, y(n-k)}_{\text{IIR (AR)}}}_{\text{IIR (ARMA)}}$$

AutoRegressive Moving Average

Exercise 1 (4/9)

- Goal: computation of the impulse response and the output of a digital filter in accordance with the difference equation

- Matlab provides the function: "y=filter(num,den,x)" that computes the output y of the filter definited from the coefficients "b" and "a" when the input is "x".

  N.B. length(y)=length(x)

Exercise 1 (5/9)

EXAMPLE 2.9 Given the following difference equation

$$y(n) - y(n-1) + 0.9y(n-2) = x(n); \quad \forall n$$

a. Calculate and plot the impulse response $h(n)$ at $n = -20, \ldots, 100$.
b. Calculate and plot the unit step response $s(n)$ at $n = -20, \ldots, 100$.

- HINT: Pay attention to the fact that Matlab indexes start from 1 and not 0 as in Difference Equation

$$y(n) = b_0\, x(n-0) - \sum_{k=1}^{2} a_k\, y(n-k)$$

$$a(1)\, y(n) = -a(2)\, y(n-1) - a(3)\, y(n-2) + b(1)\, x(n)$$

Exercise 1 (6/9)

$$1 * y(n) = 1 * y(n-1) - 0.9 * y(n-2) + 1 * x(n)$$

- a=[1,-1,0.9];
- b=1;
- x=impseq(0,-20,120);   n=[-20:120];

- h=filter(b,a,x);

- N.B.: nh=n

Exercise 1 (7/9)

- Unit step response:

- a=[1,-1,0.9];
- b=1;
- x=stepseq(0,-20,120);   n=[-20:120];

- s=filter(b,a,x);

Exercise 1 (8/9)

- Goal: computation of the impulse response and the output of a digital filter in accordance with the difference equation

$$1 * y(n) = 0.9 * y(n-1) + 1 * x(n) + 1 * x(n-1)$$

```
N = 1000;
b = [ 1 1 ];
a = [ 1 -0.9];
x = randn(N,1);

y = filter(b,a,x);
```

Exercise 1 (9/9)

$$1 * y(n) = 0.9 * y(n-1) + 1 * x(n) + 1 * x(n-1)$$

```
N = 1000;
b = [ 1 1 ];
a = [ 1 -0.9];
delta = [ 1 ; zeros(N-1,1)]';
h = filter(b,a,delta);

y1 = conv(h,x);
y1 = y1(1:length(x));
```

$$y(n) = \sum_{i=0}^{n} x(n)\ h(n-i)$$

Exercise 2 (1/3)

- The transfer function of a linear time-invariant discrete-time filter is defined as:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)}$$

- Where: $$X(z) = \sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$A(z) = 1 + a_1 z^{-1} + \ldots + a_N z^{-N}$$

$$B(z) = b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}$$

Exercise 2 (2/3)

- Direct form

$$H(z)=\frac{B(z)}{A(z)}=\frac{b_0+b_1z^{-1}+\ldots+b_Mz^{-M}}{1+a_1z^{-1}+\ldots+a_Nz^{-N}}$$

- Factored form (cascade form)

$$H(z)=\frac{B(z)}{A(z)}=b_0\frac{(1-q_1z^{-1})(1-q_2z^{-1})\ldots(1-q_Mz^{-1})}{(1-p_1z^{-1})(1-p_2z^{-1})\ldots(1-p_Nz^{-1})}$$

Exercise 2 (3/3)

- Matlab provides the functions:
  - “p=roots(a)” that computes the roots “p” of the polynomial defined from the coefficients “a”.
  - “a=poly(p)” that computes the coefficients “a” of a polynomial whose roots are the elements of “p”.
  - “zplane (z,p)” or equivalently “zplane(b,a)” plot in the complex plane the position of the zeros (°) and poles (x) of the transfer function of the filter.

Exercise 3 (1/2)

- GOAL: compute the transfer function of a IIR filter:

$$H(\omega) = \frac{B(\omega)}{A(\omega)} = \frac{b_0 + b_1 e^{-j\omega} + \ldots + b_M e^{-j\omega M}}{1 + a_1 e^{-j\omega} + \ldots + a_N e^{-j\omega N}}$$

- Matlab provides the function "[H w]=freqz(b,a,N)" that returns the N-point complex frequency response "H" and the N-point frequency vector "w" in radians/sample of the filter given numerator and denominator coefficients in vectors "b" and "a". The frequency response is evaluated at N points equally spaced around the upper half of the unit circle. If N isn't specified, it defaults to 512.

Exercise 3 (2/2)

- Represent also the negative frequencies:
  - w=[-flipud(w(2:end)); w]; % flipping the rows
  - H=[conj(flipud(H(2:end))); H];

- The frequency response equals the transfer function H(z) evaluated on the unit circle in the z plane:
  - B = fft(b,Ns);
  - A = fft(a,Ns);
  - w = 2*pi*[0:Ns-1]/(Ns);
  - H = B./A;

Exercise 4 (1/1)

- **Goal**: have a look on the effects of truncating impulse response of a IIR filter to N samples to obtain a FIR filter.
- Plot the frequency response of the truncated FIR filters, superimposed to the original frequency response.

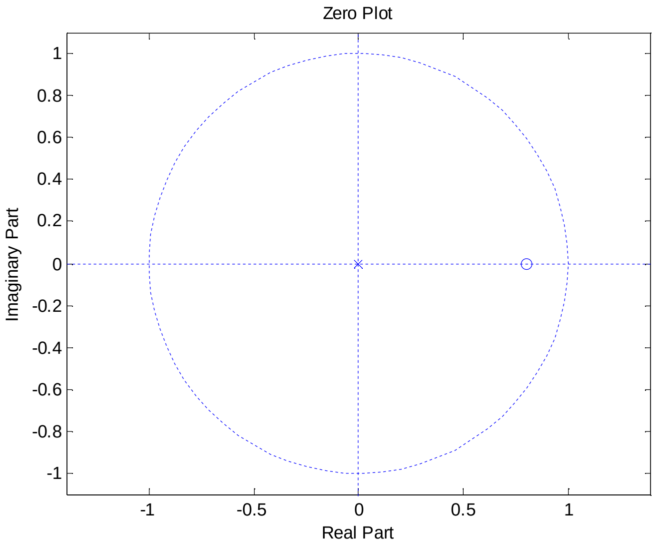
- N.B.:A FIR filter is always stable!

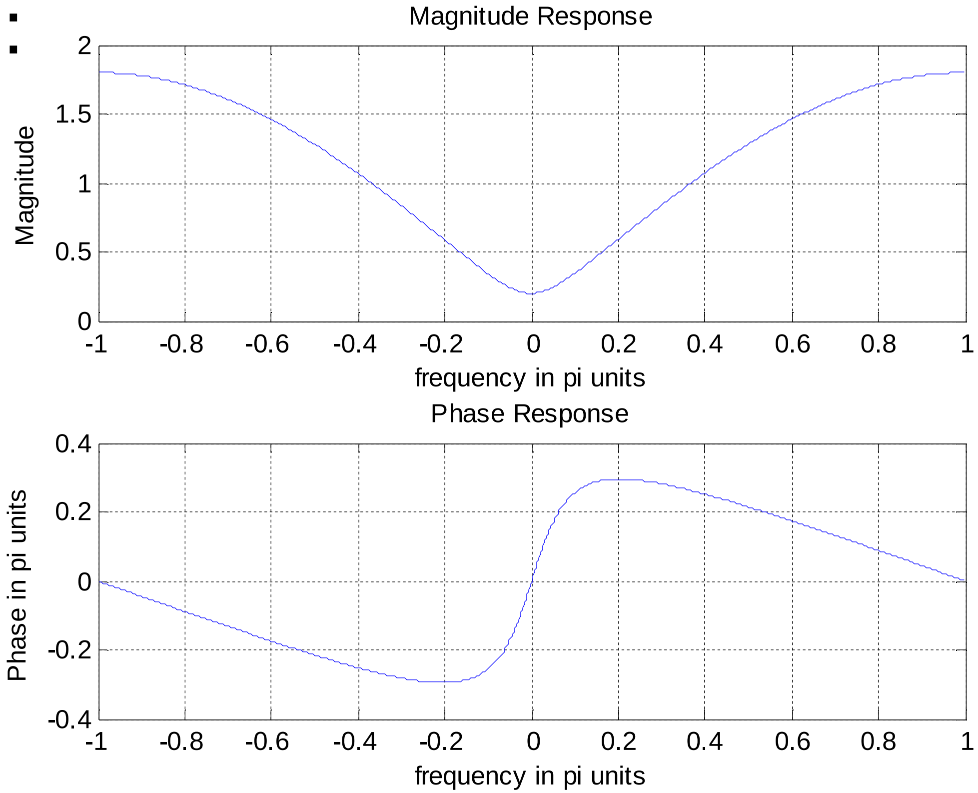
Agenda

- Ex5: FILTERS

Exercise 5 (1/15)

- Filter with one zero:

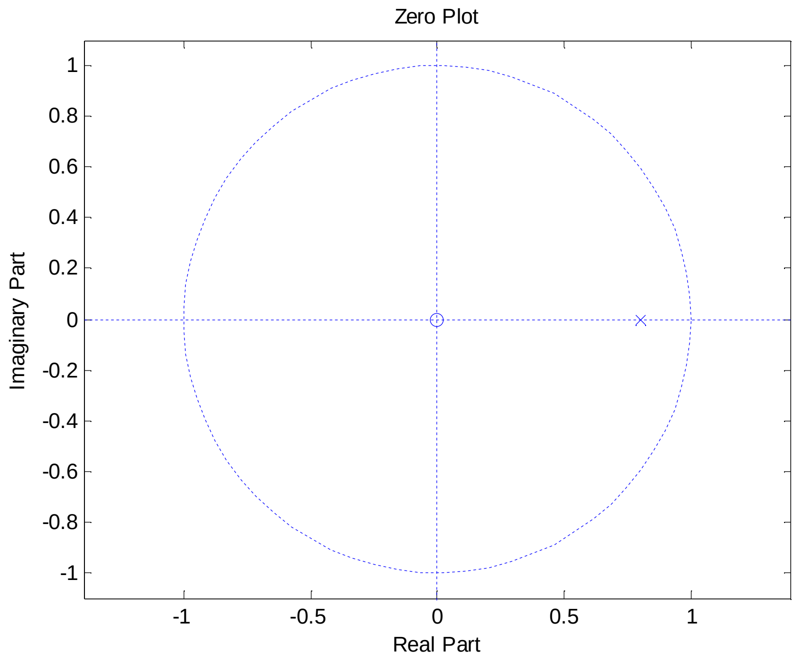


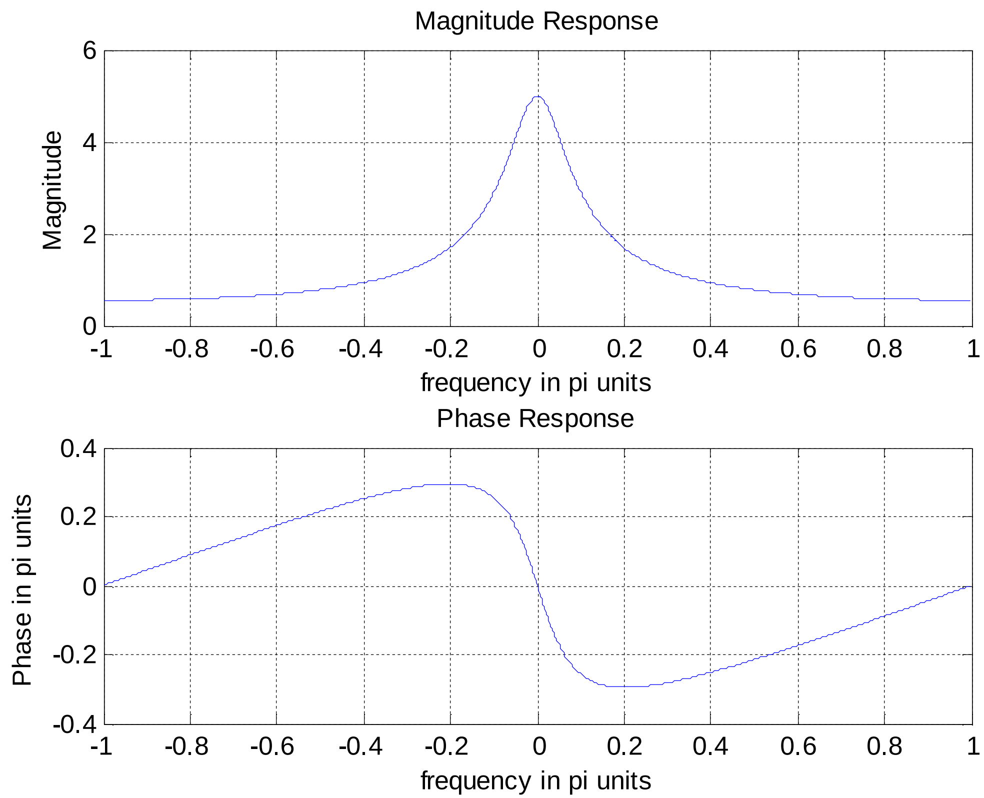


Not selective filter: in the amplitude spectrum the convex part is much more than the concave

Exercise 5 (2/15)

- Filter with one pole:

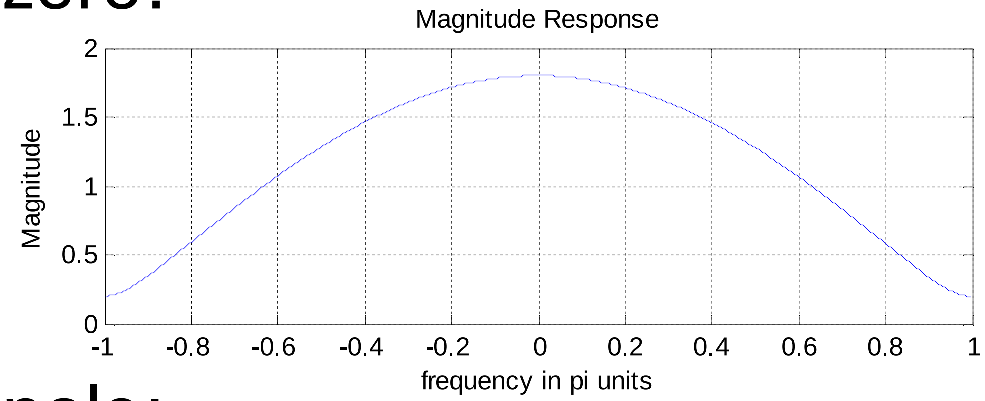




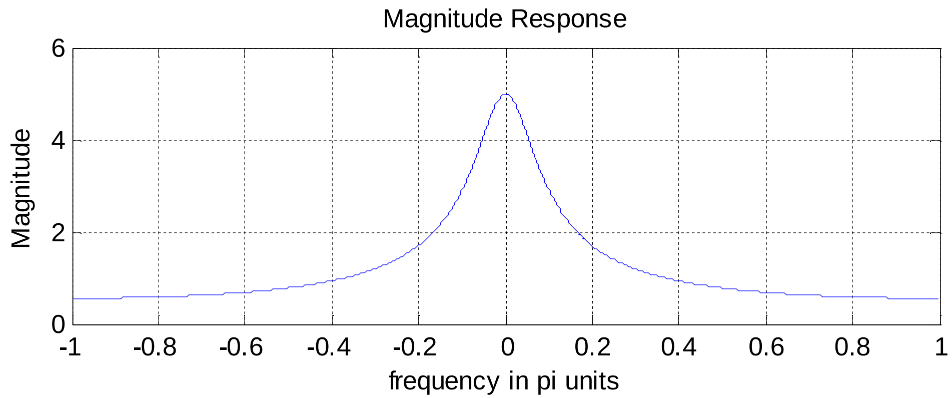
More selective filter: in the amplitude spectrum the convex part is much narrower than the concave

Exercise 5 (3/15)

- Filter with one zero:



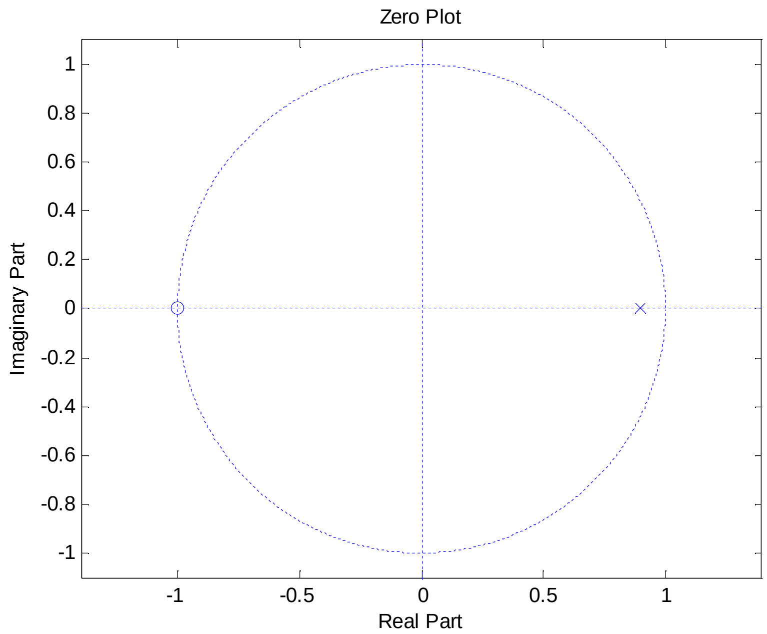
- Filter with one pole:



Exercise 5 (4/15)

- Pass Bass filter:
  - p = 0.9
  - z = -1

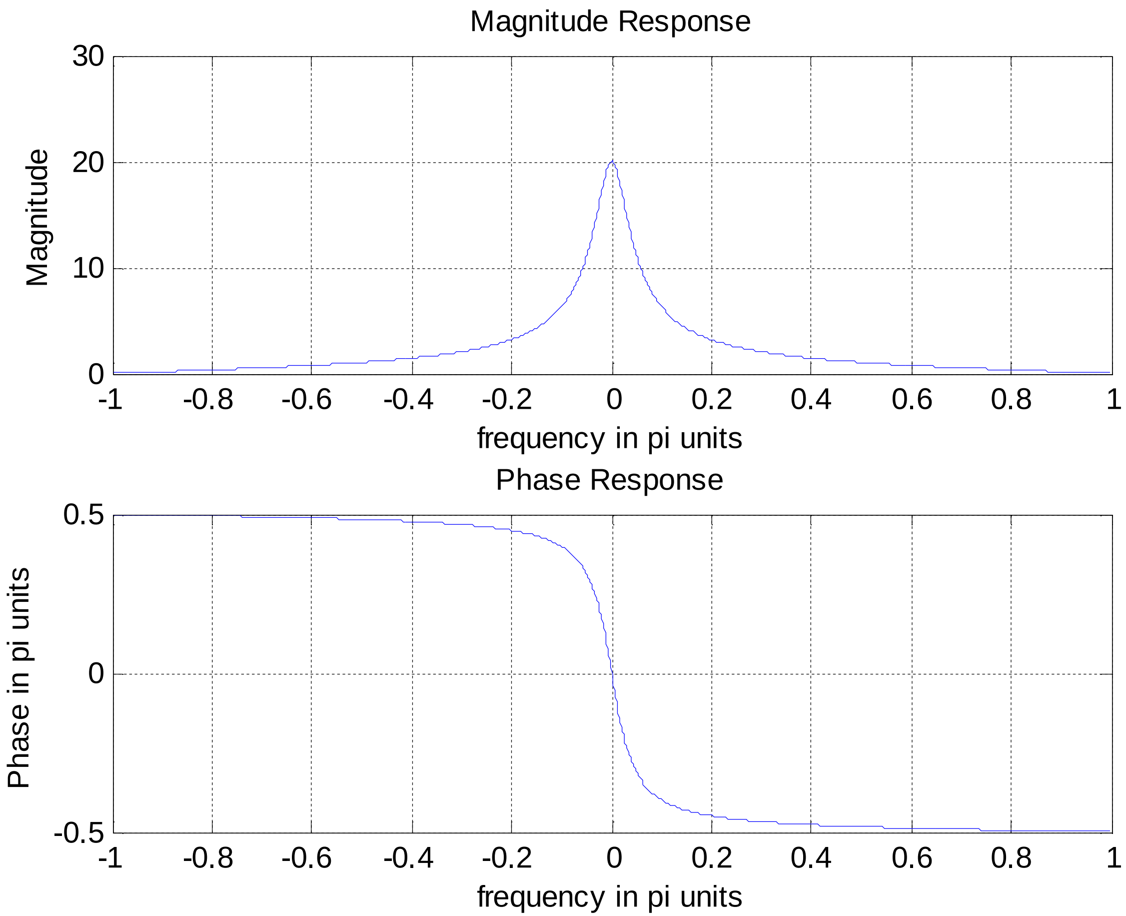
$$H(z) = \frac{B(z)}{A(z)} = \frac{1+z^{-1}}{1-0.9z^{-1}}$$



Exercise 5 (5/15)

- Pass Bass filter:
  - p = 0.9
  - z = -1

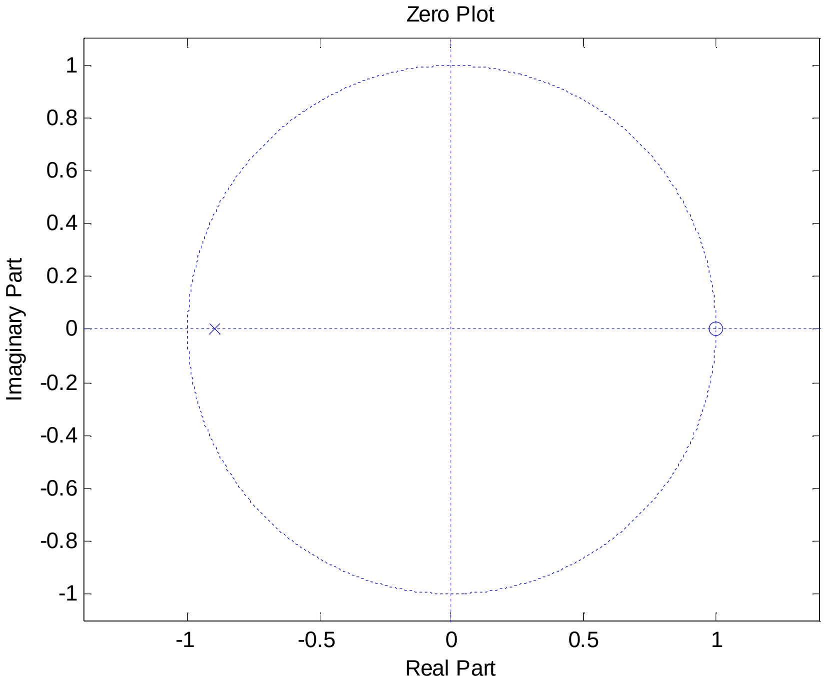
$$H(z) = \frac{B(z)}{A(z)} = \frac{1+z^{-1}}{1-0.9z^{-1}}$$



Exercise 5 (6/15)

- High Pass filter:
  - p = - 0.9
  - z = +1

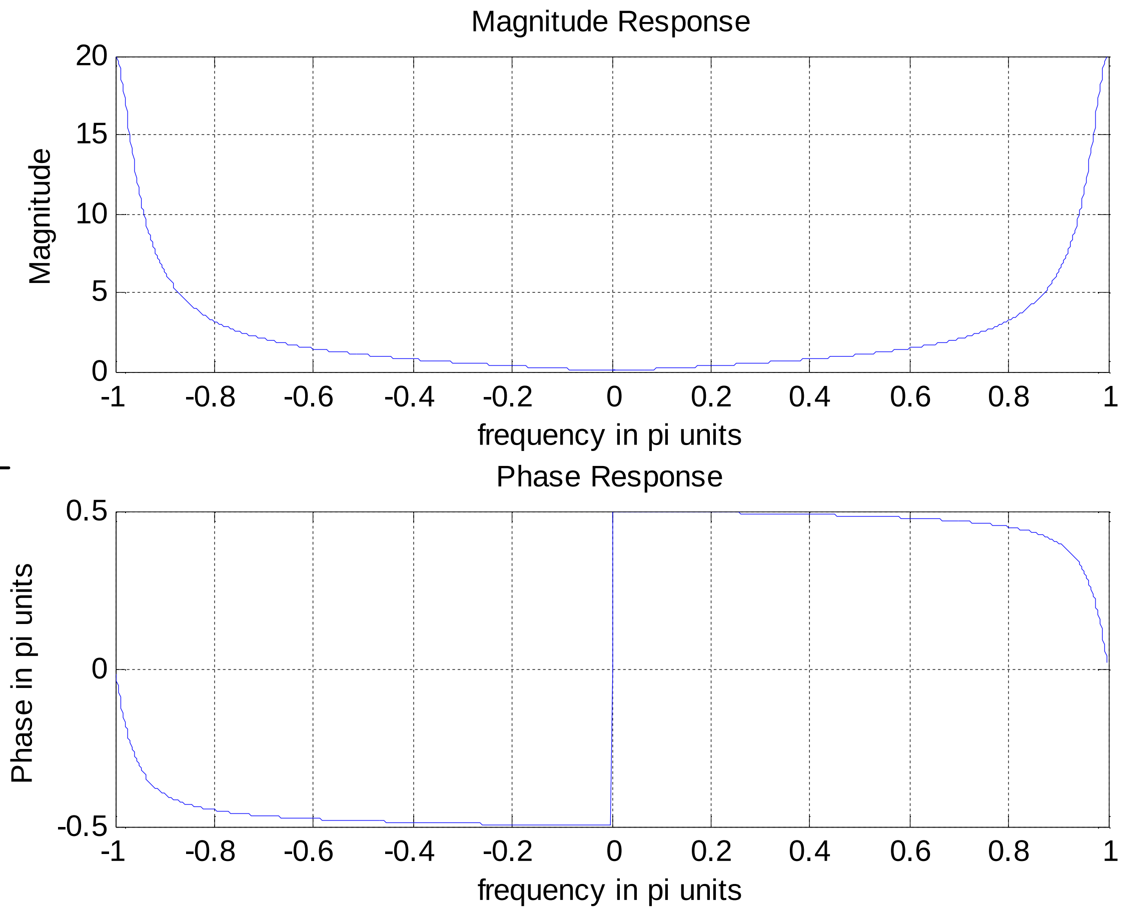
$$H(z) = \frac{B(z)}{A(z)} = \frac{1 - z^{-1}}{1 + 0.9z^{-1}}$$



Exercise 5 (7/15)

- High Pass filter:
  - p = - 0.9
  - z = +1

$$H(z)=\frac{B(z)}{A(z)}=\frac{1-z^{-1}}{1+0.9z^{-1}}$$

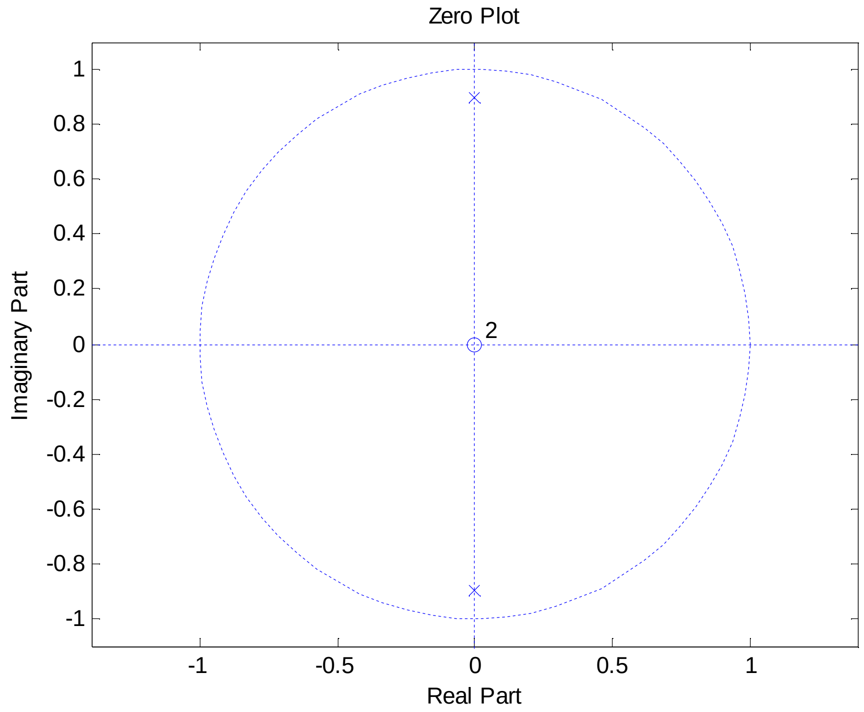


Exercise 5 (8/15)

- Band Pass filter

$$p_1 = \rho e^{j\phi} \qquad p_2 = p_1^*$$

$$H(z) = \frac{B(z)}{A(z)} = \frac{1}{\left(1 - p_1 z^{-1}\right)\left(1 - p_1^* z^{-1}\right)}$$

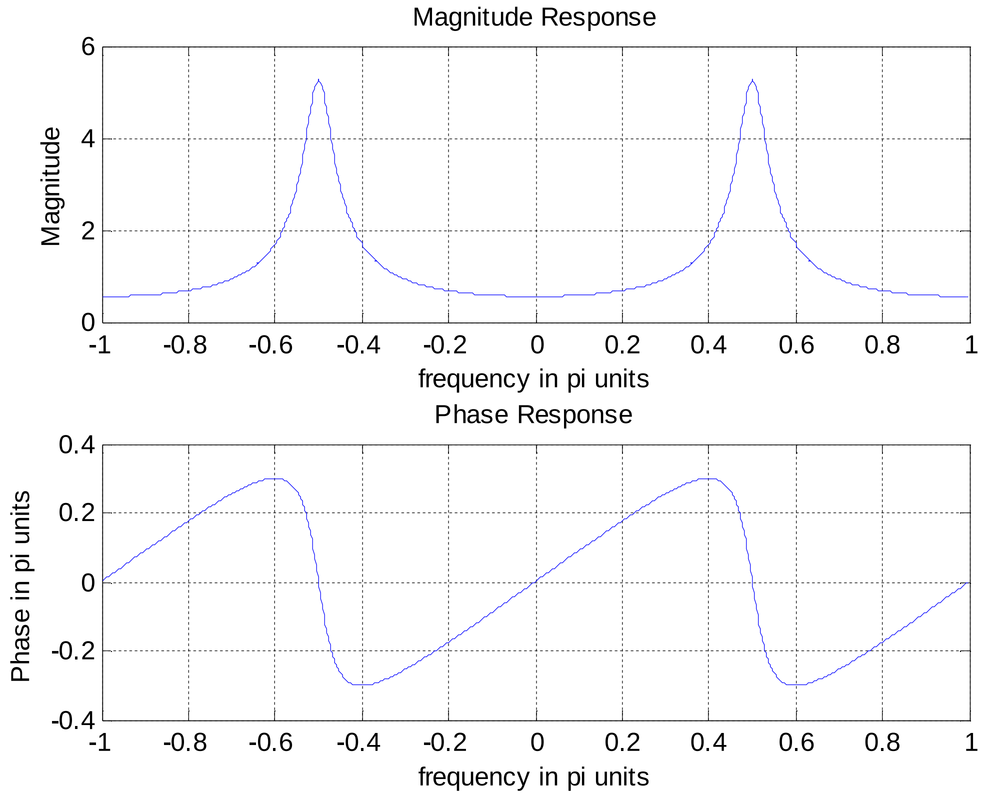


$$p_1 = 0.9 e^{j\pi/2}$$

Exercise 5 (9/15)

- Band Pass filter $H(z)=\dfrac{B(z)}{A(z)}=\dfrac{1}{\left(1-\rho e^{j\phi}z^{-1}\right)\left(1-\rho e^{-j\phi}z^{-1}\right)}$

$$=\frac{1}{1-2\rho\cos\left(\phi\right)z^{-1}+\rho^{2}z^{-2}}$$



Exercise 5 (10/15)

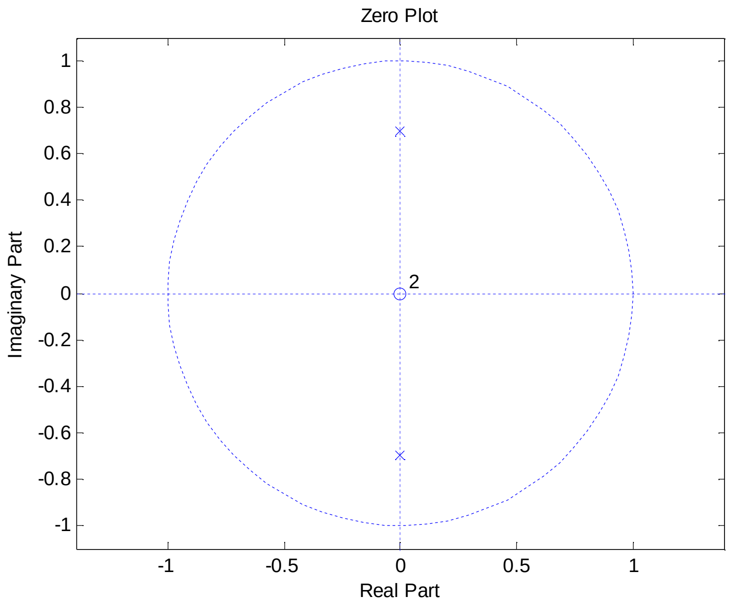
- How does it change, changing ρ and φ?

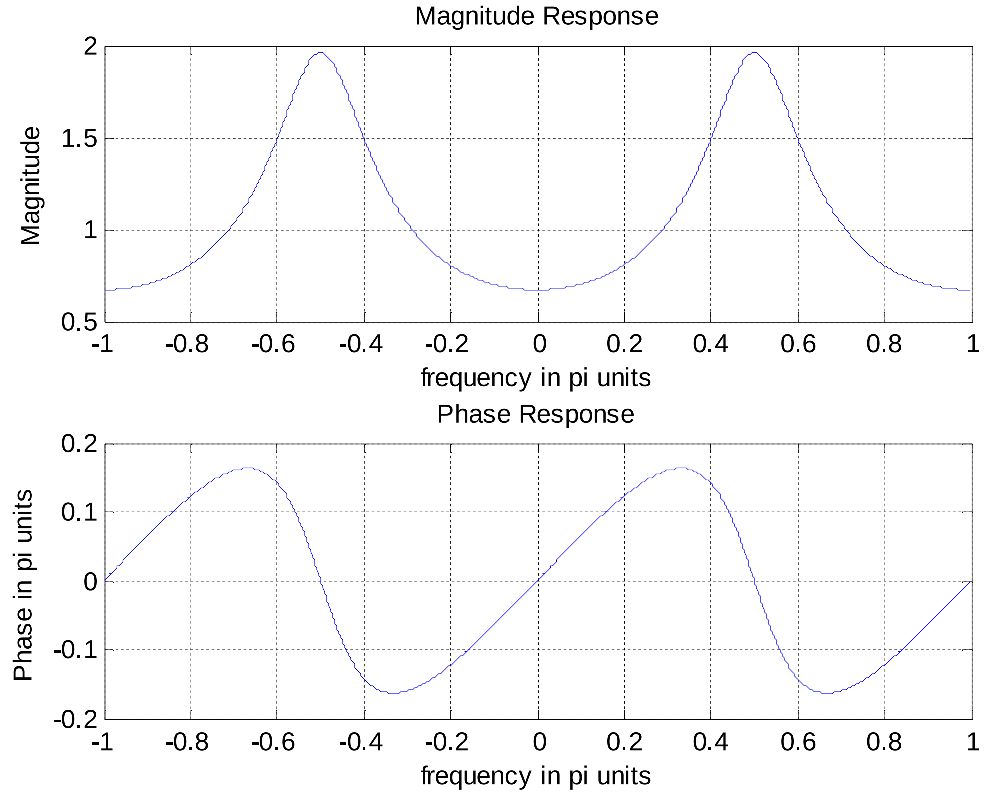
$$\left|H(z)\right|_{\substack{\max \\ z=e^{\pm j\phi}}} = \left|\frac{1}{(1-\rho)\left(1-\rho e^{-2j\phi}\right)}\right| \cong \frac{1}{|1-\rho|\left|2jsen(\phi)\right|} = \frac{1}{\left|2\varepsilon sen(\phi)\right|}$$

$$\Delta\phi_{3dB} = 2\varepsilon$$

Exercise 5 (11/15)

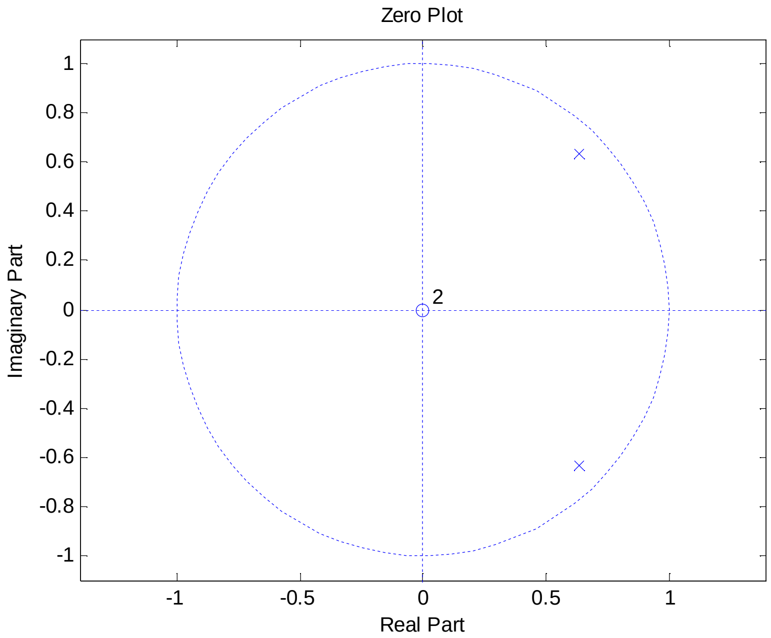
- Band Pass filter: $p_1 = 0.7e^{j\pi/2}$

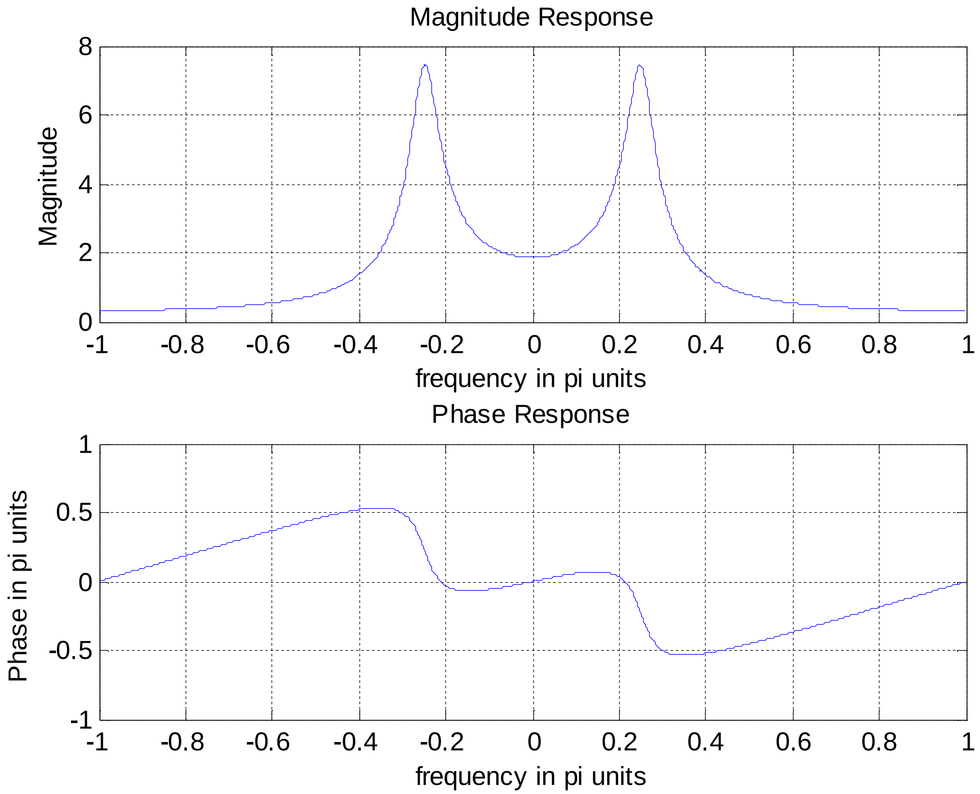




Exercise 5 (12/15)

- Band Pass filter: $p_1 = 0.9e^{j\pi/4}$





Exercise 5 (13/15)

- Band Pass filter:

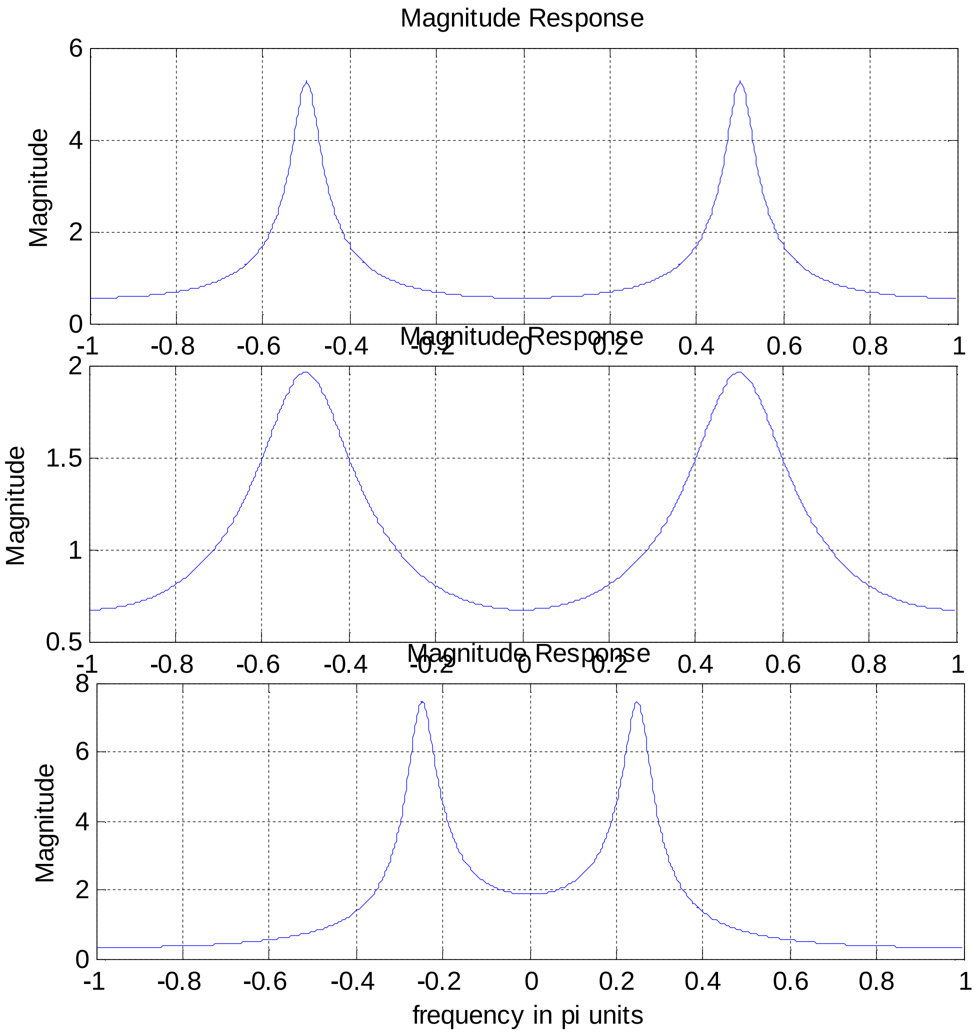
$$p_1 = 0.9e^{j\pi/2}$$

$$\left|H(z)\right|_{\max} = \frac{1}{\left|2\varepsilon sen(\phi)\right|}$$

$$p_1 = 0.7e^{j\pi/2}$$

$$\Delta\phi_{3dB} = 2\varepsilon$$

$$p_1 = 0.9e^{j\pi/4}$$



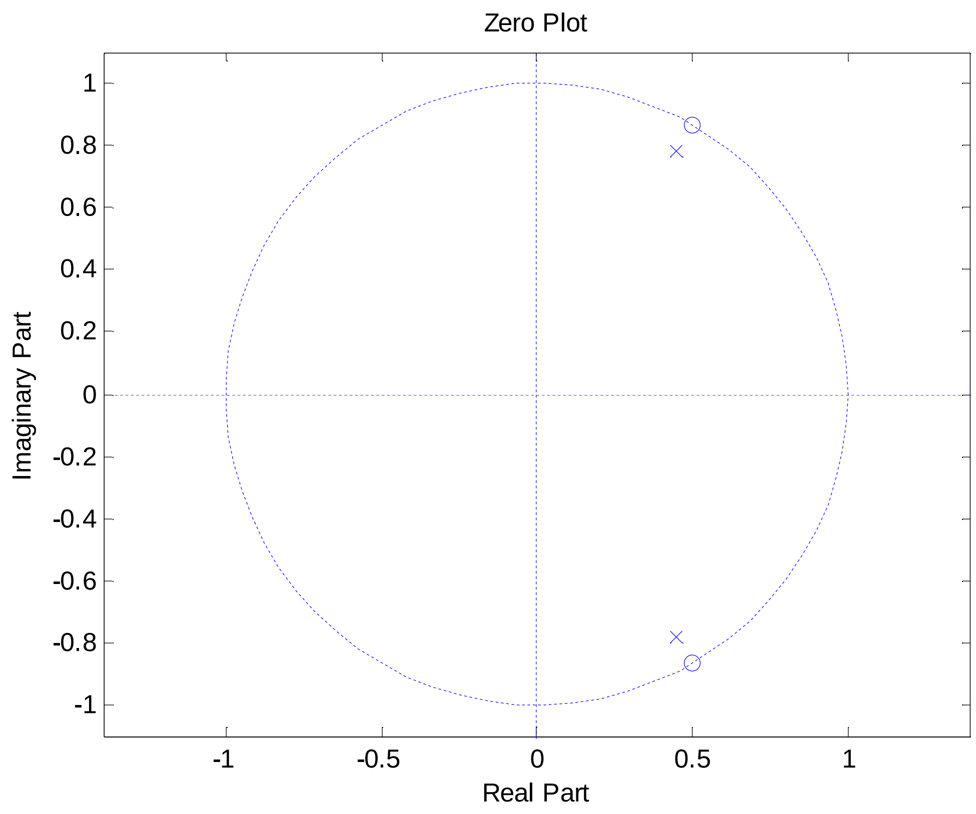
Exercise 5 (14/15)

- Stop Band filter = Notch filter:

$$p_1 = \rho e^{j\phi} \qquad p_2 = p_1^*$$

$$z_1 = e^{j\phi} \qquad z_2 = z_1^*$$

$$p_1 = 0.9e^{j\pi/3} \qquad z_1 = e^{j\pi/3}$$



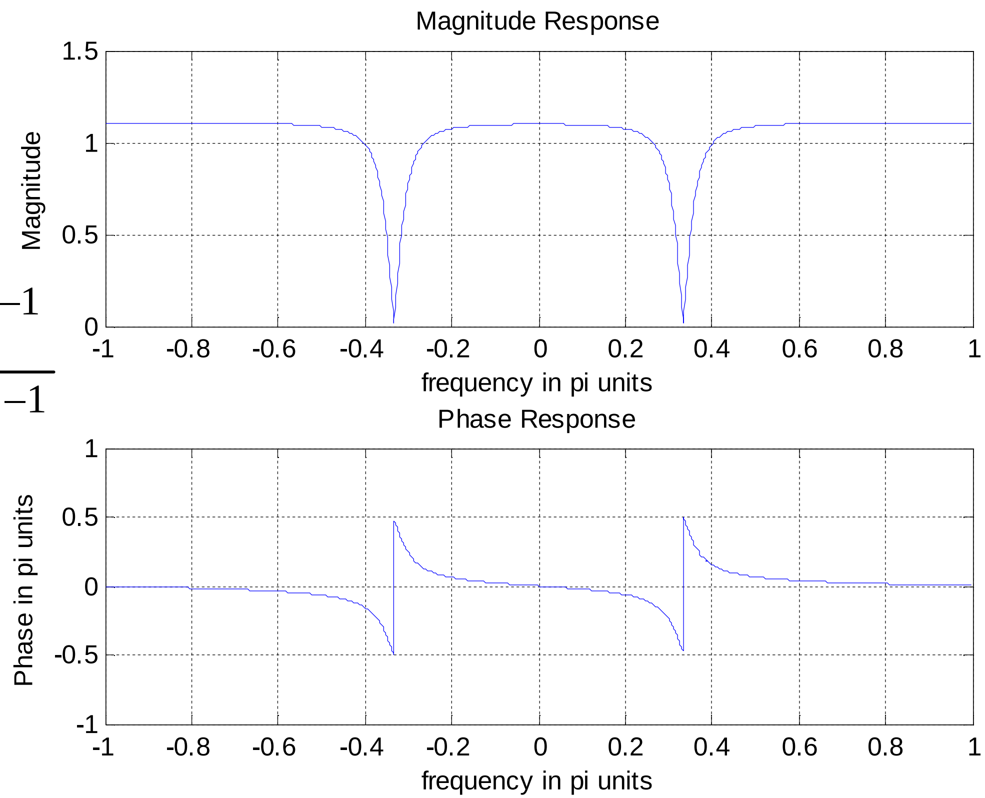
Exercise 5 (15/15)

- Stop Band filter = Notch filter:

$$p_1 = 0.9e^{j\pi/3} \qquad z_1 = e^{j\pi/3}$$

$$H(z) = \frac{B(z)}{A(z)} = \frac{1 - z_1 z^{-1}}{1 - p_1 z^{-1}} \cdot \frac{1 - z_1^{*} z^{-1}}{1 - p_1^{*} z^{-1}}$$

$$\Delta\phi_{3dB} = 2\varepsilon$$



Agenda

- Ex6: Linear phase filter

Exercise 6 (1/21)

- A FIR filter is defined by :

$$H(z) = \frac{B(z)}{A(z)} = 1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

- GOAL: choose b1, b2, b3 that implement a linear phase filter

Exercise 6 (2/21)

- A linear phase filter has a phase response that is linear function of the frequency:

$$\angle H(e^{j\omega}) = \beta - \alpha\omega, \quad -\pi < \omega \le \pi$$

- For causal linear phase filters [0,N-1], the linear phase condition impose a symmetric impulse response:

$$h(n) = h(N-1-n) \quad if\ \beta = 0,\ 0 \le n \le N-1$$

$$h(n) = -h(N-1-n) \quad if\ \beta = \pm \pi/2,\ 0 \le n \le N-1$$

Exercise 6 (3/21)

- Solution:

$$y(n) = 1x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3)$$

- Symmetric impulse response means that:

$$h(n) = h(N-1-n) = h(3-n)$$

- i.e.:

$$h(0) = 1 = h(3) = b_3$$

$$h(1) = b_1 = h(2) = b_2$$

Exercise 6 (4/21)

- The zeros of each linear phase FIR filter possess certain symmetries (that are due to the symmetry constrains on h(n).
- If H(z) has a zero at: $z = z_1 = re^{j\theta}$
  - For linear phase there must be a zero at: $z_2 = \dfrac{1}{z_1} = \dfrac{1}{r}e^{-j\theta}$
  - For a real valued filter there must be the conjugate zeros:

$$z_3 = z_1^* = re^{-j\theta} \qquad z_4 = z_2^* = \frac{1}{r}e^{j\theta}$$

Exercise 6 (5/21)

- If $r=1 \rightarrow r=\frac{1}{r}=1$, hence the zeros on the unit circle occur in pairs: $e^{j\theta}$ $e^{-j\theta}$
- If $\theta=0 or \pi$ (the zeros are on the real line), then occur in pairs: r $\frac{1}{r}$
- If $\theta=0 or \pi$ and r=1, the zeros are either at z=1 or z=-1.

Exercise 6 (6/21)

- We can have 4 different kind of causal linear phase FIR filter:

$$\alpha = \frac{M-1}{2} = index\ of\ \ symmetry$$

$$\beta = 0\ \ or \pm\frac{\pi}{2}$$

  - β=0 and M odd
  - β=0 and M even
  - β=±π/2 and M odd
  - β=±π/2 and M even

Exercise 6 (7/21)

- If β=0, then $\angle H(e^{j\omega}) = -\alpha\omega$ where α =constant phase delay $\dfrac{\angle H(e^{j\omega})}{\omega} = -\alpha = const$
  - β=0 and M odd $h(n) = h(N-1-n) \quad \alpha \in N$
  - β=0 and M even $h(n) = h(N-1-n) \quad \alpha \notin N$

Exercise 6 (8/21)

- If β =±π/2, then $\angle H(e^{j\omega}) = \beta - \alpha\omega$. In this case α is the constant group delay:

$$\frac{d\angle H(e^{j\omega})}{d\omega} = -\alpha = const$$

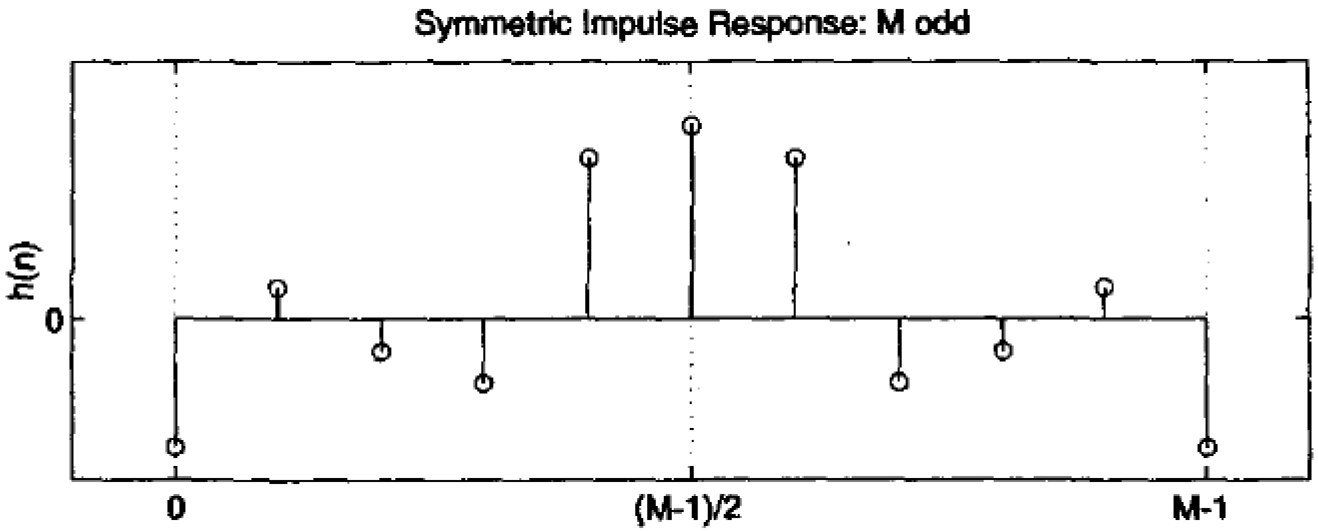
  - β=±π/2 and M odd $h(n) = -h(N-1-n) \quad \alpha \in N$
  - β=±π/2 and M even $h(n) = -h(N-1-n) \quad \alpha \notin N$

Exercise 6 (9/21)

$$\angle H(e^{j\omega}) = -\alpha\omega$$

- β=0 and M odd

$$h(n) = h(N-1-n) \quad \alpha \in N$$



Exercise 6 (10/21)

$$\angle H(e^{j\omega}) = -\alpha\omega$$

- β=0 and M odd

$$H(e^{j\omega}) = \left[\sum_{n=0}^{(M-1)/2} a(n)\cos(\omega n)\right] e^{-j\omega(M-1)/2}$$

$$a(0) = h\left(\frac{M-1}{2}\right)$$

$$a(n) = 2h\left(\frac{M-1}{2} - n\right)$$

$$\angle H(e^{j\omega}) = -\alpha\omega$$

Exercise 6 (11/21)

- β=0 and M odd

EXAMPLE 7.4 Let $h(n) = \{\underset{\uparrow}{-4}, 1, -1, -2, 5, 6, 5, -2, -1, 1, -4\}$. Determine the amplitude response $H_r(\omega)$ and the locations of the zeros of $H(z)$.

ion

Since $M = 11$, which is odd, and since $h(n)$ is symmetric about $\alpha = (11-1)/2 = 5$, this is a Type-1 linear-phase FIR filter. From (7.7) we have

$$a(0) = h(\alpha) = h(5) = 6, \quad a(1) = 2h(5-1) = 10, \quad a(2) = 2h(5-2) = -4$$

$$a(3) = 2h(5-3) = -2, \quad a(4) = 2h(5-4) = 2, \quad a(5) = 2h(5-5) = -8$$

From (7.8), we obtain

$$\begin{aligned} H_r(\omega) &= a(0) + a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega + a(4)\cos 4\omega + a(5)\cos 5\omega \\ &= 6 + 10\cos\omega - 4\cos 2\omega - 2\cos 3\omega + 2\cos 4\omega - 8\cos 5\omega \end{aligned}$$

Exercise 6 (12/21)

$$\angle H(e^{j\omega}) = -\alpha\omega$$

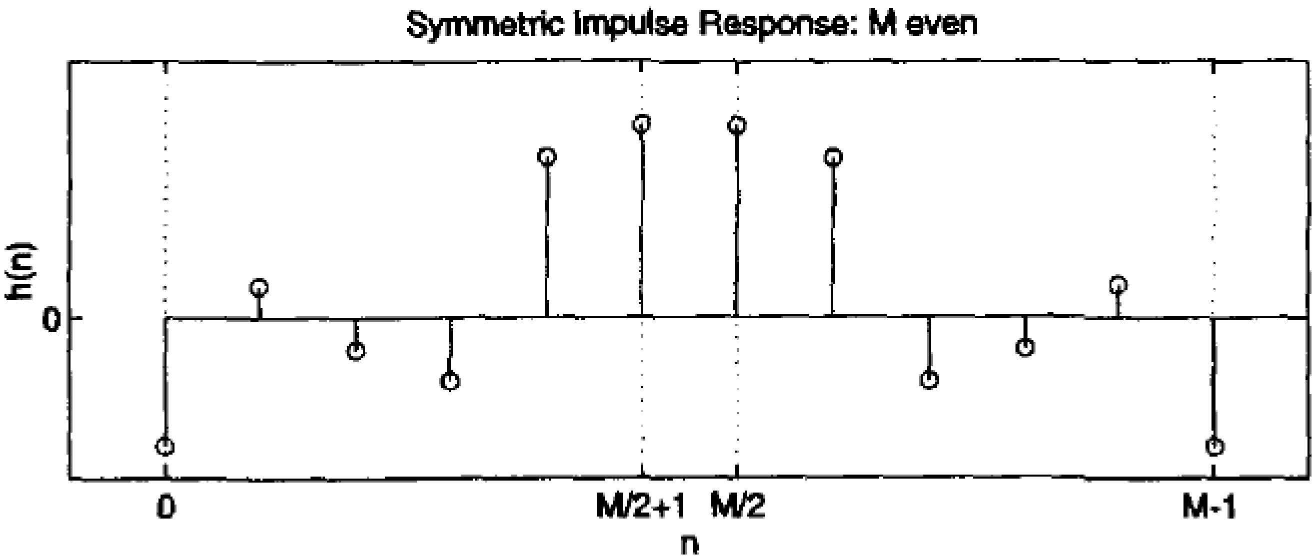
- Matlab code for computing Amplitude response Hr(w) of a Type-1 LP FIR filter

  - **function [Hr,w,a,L] = Hr_Type1(h);**
  - M = length(h);
  - L = (M-1)/2;
  - a = [h(L+1) 2*h(L:-1:1)]; % 1x(L+1) row vector
  - n = [0:1:L]; % (L+1)x1 column vector
  - w = [0:1:500]'*pi/500;
  - Hr = cos(w*n)*a';

Exercise 6 (13/21)

$$\angle H(e^{j\omega}) = -\alpha\omega$$

- β=0 and M even $\quad h(n) = h(N-1-n) \quad \alpha \notin N$



Exercise 6 (14/21)

$$\angle H(e^{j\omega}) = -\alpha\omega$$

- β=0 and M even

$$H(e^{j\omega}) = \left[\sum_{n=1}^{M/2} b(n)\cos\left(\omega\left(n-\frac{1}{2}\right)\right)\right]e^{-j\omega(M-1)/2}$$

$$b(n) = 2h\left(\frac{M}{2}-n\right)$$

Exercise 6 (15/21)

$$\angle H(e^{j\omega}) = -\alpha\omega$$

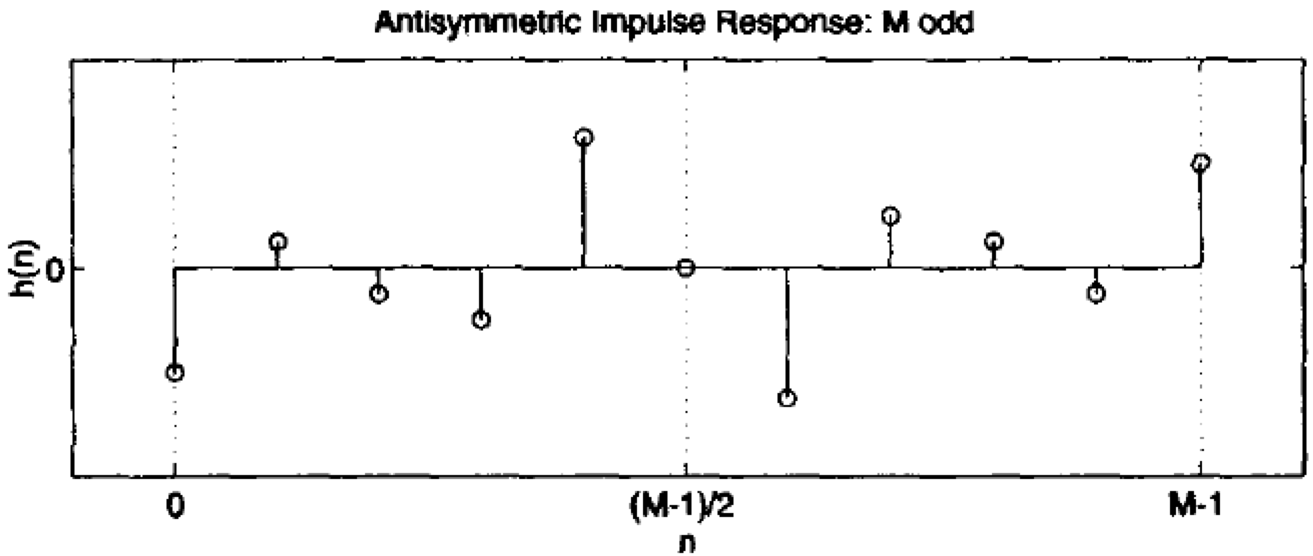
- β=0 and M even

EXAMPLE 7.5 Let $h(n) = \{-4, 1, -1, -2, 5, \underset{\uparrow}{6}, 6, 5, -2, -1, 1, -4\}$. Determine the amplitude response $H_r(\omega)$ and the locations of the zeros of $H(z)$.

- **function [Hr,w,b,L] = Hr_Type2(h);**
- M = length(h);   L = M/2;
- b = 2*[h(L:-1:1)];
- n = [1:1:L]; n = n-0.5; w = [0:1:500]'*pi/500;
- Hr = cos(w*n)*b‘;

Exercise 6 (16/21) $\angle H(e^{j\omega}) = \beta - \alpha\omega$

- β=±π/2 and M odd $\quad h(n) = -h(N-1-n) \quad \alpha \in N$



Note that the sample $h(\alpha)$ at $\alpha = (M-1)/2$ must necessarily be equal to zero, i.e., $h((M-1)/2) = 0$.

Exercise 6 (17/21)

$$\angle H(e^{j\omega}) = \beta - \alpha\omega$$

- β=±π/2 and M odd

$$H(e^{j\omega}) = \left[\sum_{n=0}^{(M-1)/2} c(n)\sin(\omega n)\right] e^{j\left[\frac{\pi}{2} - \frac{M-1}{2}\omega\right]}$$

$$c(0) = h\left(\frac{M-1}{2}\right)$$

$$c(n) = 2h\left(\frac{M-1}{2} - n\right)$$

Exercise 6 (18/21) $\angle H(e^{j\omega}) = \beta - \alpha\omega$

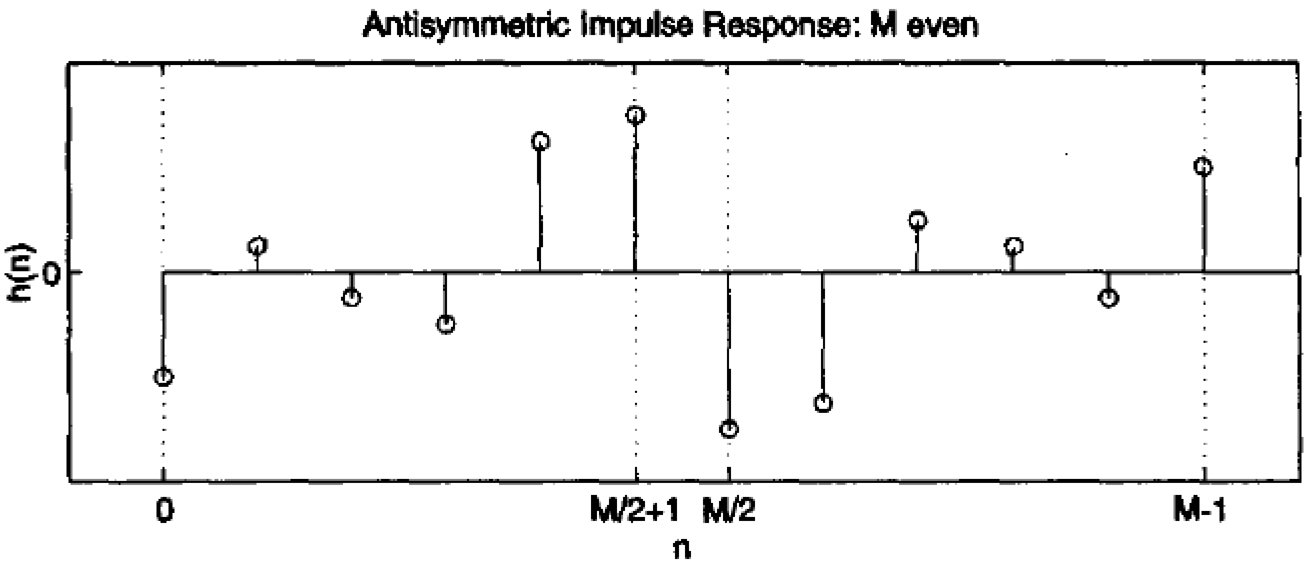
- β=±π/2 and M odd

EXAMPLE 7.6 Let $h(n) = \{-4, 1, -1, -2, 5, 0, -5, 2, 1, -1, 4\}$. Determine the amplitude response $H_r(\omega)$ and the locations of the zeros of $H(z)$.

- **function [Hr,w,b,L] = Hr_Type3(h);**
- M = length(h); L = (M-1)/2;
- c = [h(L+1) 2*h(L:-1:1)];
- n = [0:1:L];   w = [0:1:500]'*pi/500;
- Hr = sin(w*n)*c‘;

Exercise 6 (19/21) $\angle H(e^{j\omega}) = \beta - \alpha\omega$

- β=±π/2 and M even $\quad h(n) = -h(N-1-n) \quad \alpha \notin N$



Exercise 6 (20/21) $\angle H(e^{j\omega}) = \beta - \alpha\omega$

- β=±π/2 and M even

$$H(e^{j\omega}) = \left[\sum_{n=1}^{M/2} d(n)\sin\left(\omega\left(n-\frac{1}{2}\right)\right)\right] e^{j\left[\frac{\pi}{2}-\frac{M-1}{2}\omega\right]}$$

$$d(n) = 2h\left(\frac{M}{2}-n\right)$$

Exercise 6 (21/21) $\angle H(e^{j\omega}) = \beta - \alpha\omega$

- β=±π/2 and M even

EXAMPLE 7.7 Let $h(n) = \{\underset{\uparrow}{-4}, 1, -1, -2, 5, 6, -6, -5, 2, 1, -1, 4\}$. Determine the amplitude response $H_r(\omega)$ and the locations of the zeros of $H(z)$.

- **function [Hr,w,b,L] = Hr_Type4(h);**
- M = length(h); L = M/2;
- d = 2*h(L:-1:1);
- n = [1:1:L];   n=n-0.5;   w = [0:1:500]'*pi/500;
- Hr = sin(w*n)*d‘;

Agenda

- Ex7: Minimum phase filter

Exercise 7 (1/2)

- A LTI filter is defined by :

$$H(z) = \frac{B(z)}{A(z)} = \frac{1 + cz^{-1}}{1 - az^{-1} - bz^{-2}}$$

- GOAL: choose a, b, c that implement a minimum phase filter

- Hint: An LTI filter H(z) = B(z)/A(z) is said to be minimum phase if all its poles and zeros are inside the unit circle |z| = 1 (excluding the unit circle itself).

Exercise 7 (2/2)

- Solution:

$$H(z)=\frac{B(z)}{A(z)}=\frac{1+cz^{-1}}{1-az^{-1}-bz^{-2}}$$

  - Zero is – c
  - Poles are p1 and p2:

$$1-az^{-1}-bz^{-2}\frac{z^2}{z^2}=\frac{z^2-az-b}{z^2}=\frac{(p_1-z)}{z}\frac{(p_2-z)}{z}$$

$$1-az^{-1}-bz^{-2}=(p_1z^{-1}-1)(p_2z^{-1}-1)$$

$$1-az^{-1}-bz^{-2}=1-(p_1+p_2)z^{-1}+p_1p_2z^{-2}$$

Agenda

- Ex8: Allpass filter

Exercise 8 (1/3)

- GOAL: Implement an all pass filter that has two complex poles in $\rho e^{j\omega_0}$ and $\rho e^{-j\omega_0}$.

$$A(z) = (1 - \rho e^{j\omega_0} z^{-1})(1 - \rho e^{-j\omega_0} z^{-1})$$

- The allpass filter passes all frequencies with equal gain. The only requirement is that its amplitude response be constant:

$$| H(\omega) | = 1$$

- An allpass filter must have zero at $z = 1/\bar{p}$ for each pole at z=p.

Exercise 8 (2/3)

- Solution: The transfer function of every finite-order, causal, allpass IIR digital filter can be written as:

$$H(z) = e^{j\phi} z^{-K} \frac{\tilde{A}(z)}{A(z)} \qquad K \geq 0$$

- Where

$$\tilde{A}(z) = z^{-N} \overline{A}(z^{-1}) = \overline{a}_N + \overline{a}_{N-1} z^{-1} + \ldots \overline{a}_1 z^{-(N-1)} + z^{-N}$$

Exercise 8 (3/3)

```
rho = 0.99;
omega0 = 0.3*pi;
p1 = rho*exp(j*omega0);
p2 = rho*exp(-j*omega0);

a = poly([ p1; p2]);  % denominator

b = conj(fliplr(a));

[H, w] = freqz(b,a,1024);
```

Agenda

- Ex9: Minimum phase/allpass decomposition

Exercise 9 (1/3)

- Minimum phase/allpass decomposition.
- GOAL: Compute the MP/AP decomposition of the causale stable filter H(z):

$$H(z) = H_{mp}(z)\,S(z)$$

- Where: $$S(z) = \frac{\bar{s}_L + \bar{s}_{L-1} z^{-1} + \ldots + z^{-L}}{1 + s_1 z^{-1} + \ldots + s_L z^{-L}}$$

and L is the number of non-minimum phase zeros of H(z).

Exercise 9 (2/3)

- Pseudocode:
  - Define the polynomial at the numerator of the transfer function: b = poly(z);
  - Define the polynomial at the denominator of the transfer function: a = poly(p);
  - Find the minimum phase zeros: z_minp = z(abs(z) < 1)
  - Find the non-minimum phase zeros:

    z_maxp = z(abs(z) >= 1)
  - Compute the minimum phase filter: b_minp = poly(z_minp)

    a_minp = poly(p) ;
  - Compute the allpass filter:

    b_allpass = poly(z_maxp);

    a_allpass = conj(fliplr(b_allpass));

Exercise 9 (3/3)

- Pseudocode (continued):
  - Compute the frequency response: [H, w] = freqz(b,a,1024);
  - Compute the frequency response of the minimum phase filter: [H_minp, w] = freqz(b_minp,a_minp,1024);
  - Compute the frequency response of the allpass filter: [H_allpass , w] = freqz(b_allpass,a_allpass,1024);

- The original filter is the cascade of H_minp and H_allpass.

Agenda

- Ex10: Non linear filter

Exercise 10 (1/1)

- **Goal**: given the input sequence (sum of two sinusoids at frequencies 0.1 and 0.125), show the spectrum of both the input and output signal for a non-lineal filter defined by:

$$y(n) = T\{x(n)\} = x^2(n)$$

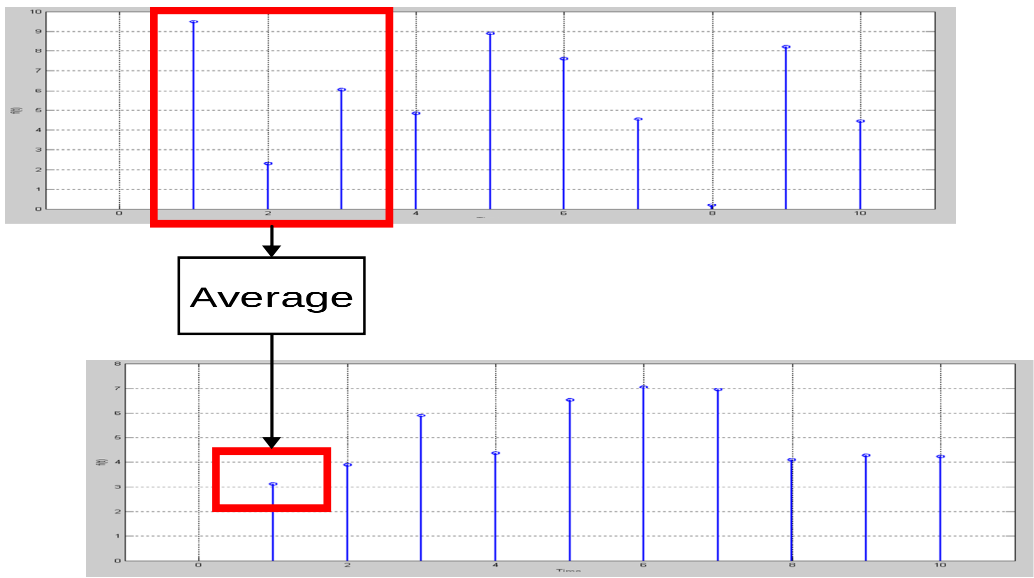
- What is the effect of the non-linearity?

Agenda

- Ex11: Pass Bass FIR filter

Exercise 11 (1/4)

- **Goal**: implement a lowpass filter that performs the average of the last M samples (Moving Average (MA))
- **Example (M = 3)**:

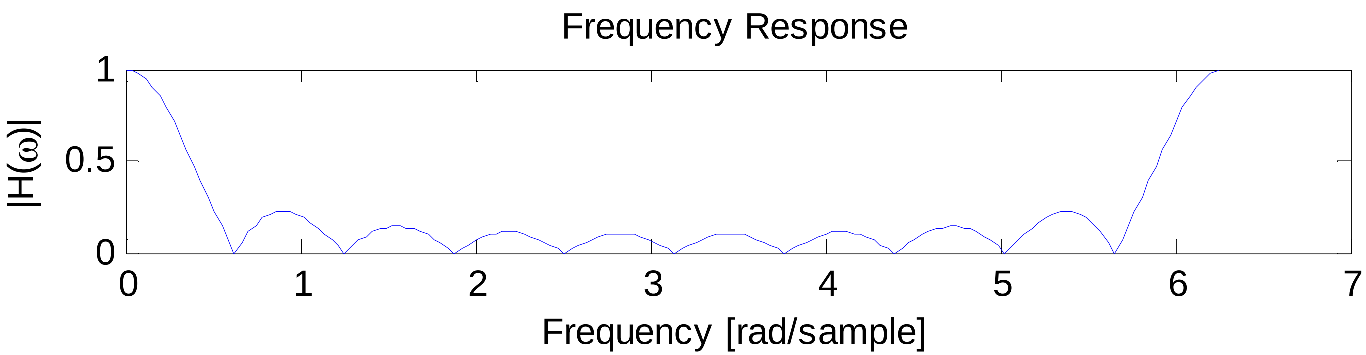


Exercise 11 (2/4)

- **Input**: sum of a low frequency sinusoid (0.05*Fs) and a high frequency sinusoid (0.44*Fs).
- **Fs:=** 16000 Hz.
- Pseudocode:
  - Generate the input signal as summation of the two sinusoids
  - Require to the user the filter length
  - Filter the input signal
  - Plot both the input and the filtered signals (both in time domain and in frequency domain).

Exercise 11 (3/4)

- Hint:



- The frequency response is a sinc that has zero-crossings at integer multiples of the radian frequency:

$$\omega = \frac{2\pi}{M} \quad {rad}/{sample}$$

Exercise 11 (4/4)

- **EX11b.Goal**: Use this filter on a sinusoidal signal with AWGN
- **Input**: consider the low frequency sinusoid (0.05*Fs) with a additive white gaussian noise
  - **Fs:=** 16000 Hz.
  - t = 0:1/Fs:0.01-(1/Fs);
  - xl = cos(2*pi*0.05*Fs*t);

  - sigma2 = 0.05;
  - xn = sqrt(sigma2)*randn(1,N);