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Lossless source coding

087204 – Multimedia Signal Processing
2nd Module

Politecnico di Milano – Polo regionale di Como

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Summary

- Lossless source coding:
  - Ex1: Audio signal encoding
  - Ex2: Image signal encoding
  - Ex3: Discrete memoryless source coding
  - Ex4: Discrete memoryless source coding (taken from past exam test)

Exercise 1

1. Load file '*pf.wav*' and plot it. Check that its values are included between [-1,1]
2. Take only the first 60 sec. of the file and rescale its values between [0,255]
3. Convert each value into its binary representation over 8 bit (for this purpose use the provided function dec2binary(…))
4. Find the entropy of the **binary** source that has generated the above audio file. What can be inferred from results?

Exercise 1

5. Now consider the file as generated by a finite source whose alphabet is [0:255]. Plot the normalized histogram of the file and find the entropy of the above source

Hint:

While computing entropy be careful with zero values of the normalized histogram. They force entropy value to NaN. In order to avoid this situation, these values need to be discarded

Exercise 1

6. Consider now the audio file as generated by a source with memory (let us suppose memory = 1). Find the conditional entropy:

$$H\left(X_k \middle| X_{k-1}\right) = \sum_{X_k, X_{k-1}} P\left(X_k, X_{k-1}\right) \log_2\left(\frac{1}{P\left(X_k \middle| X_{k-1}\right)}\right)$$

Verify that $H(X) > H(X_k \mid X_{k-1})$.

For this particular case, how can be explained this entropy reduction?

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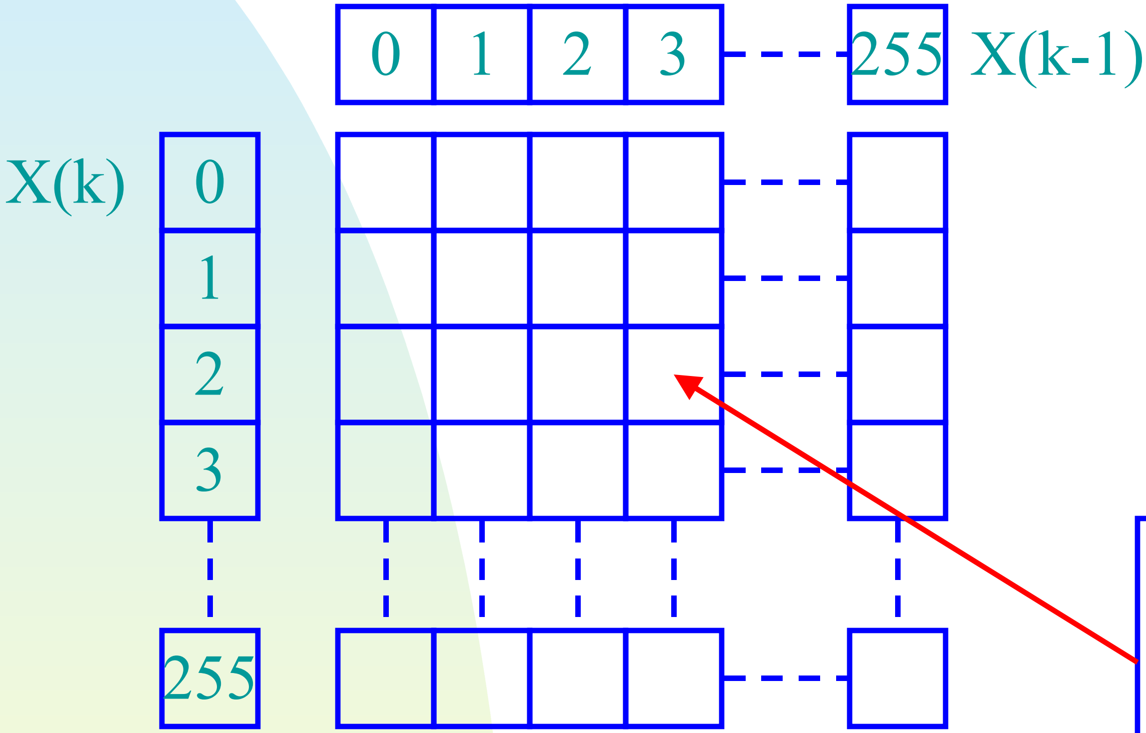
Exercise 1

- **Hint (1/3):**
  - **To calculate conditional probability $P(X_k | X_{k-1})$ use the definition:**

$$P\left(X_k | X_{k-1}\right) = \frac{P\left(X_k, X_{k-1}\right)}{P\left(X_{k-1}\right)}$$

Exercise 1

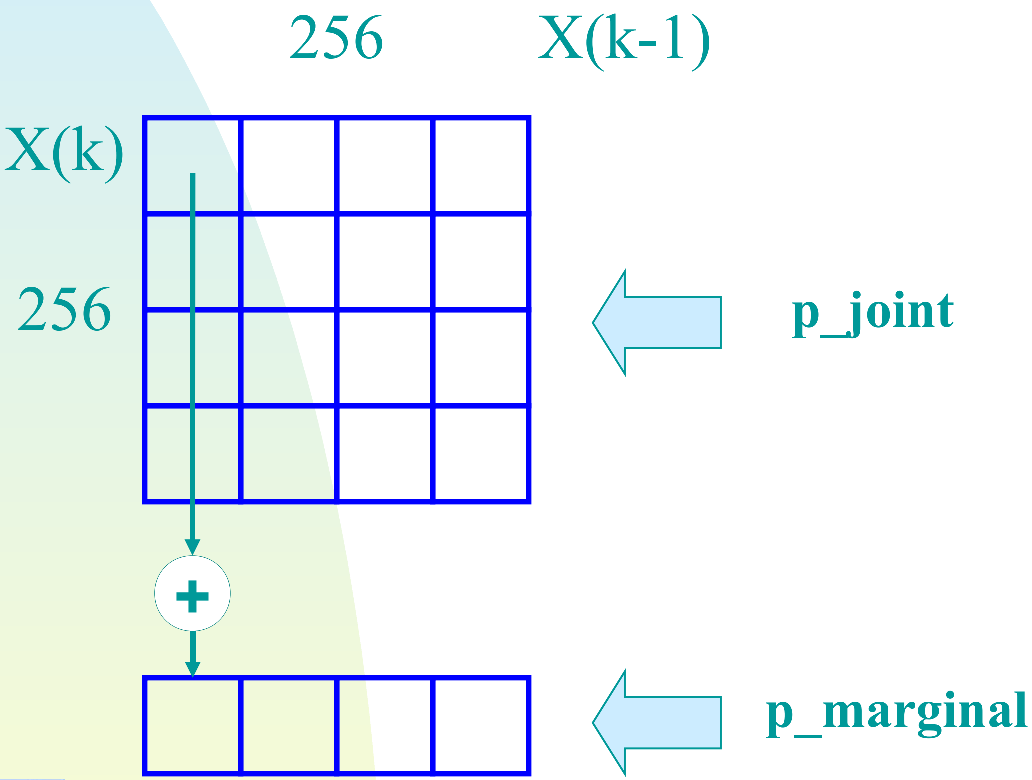
- Hint (2/3):
  - For joint probability you can use this structure:



Number of times that X_k and X_{k-1} assume values 2 and 3, respectively

Exercise 1

- **Hint (3/3):**
  Joint and marginal probabilities structures:



Exercise 2

1. Load the image ‘lena512color.tiff’ (hint: use ‘imread(…)’, see matlab help especially to gather information about the format of returned data). For each image component (R, G, B) display the histogram
2. Approximate the pdf of each channel as the normalized histogram. Compute the entropy (in bpp) of each channel
3. Let X be the source represented by the red channel and Y the source represented by the green channel. Compute and plot the joint pdf, p(X,Y) (You can also use hist3 function for joint histogram calculation)
4. Compute the joint entropy H(X,Y) and verify that H(X,Y) ≤ H(X) + H(Y). Why the equality is not satisfied?

Exercise 2

5. Suppose to encode Y with H(Y) bits and to send N = aX+b – Y instead of X, where a and b are obtained by linear regression (least squares). Compute the entropy of N and compare it with the conditional entropy H(X|Y) = H(X,Y) – H(Y).

   Why H(N) > H(X|Y) ?

Hint:

- Linear regression formula?

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Exercise 3

Consider a discrete random sequence described by the following equation:

$$x(n) = \min\left[\max\left[0, round\left[\rho x(n-1) + z(n)\right]\right], 15\right]$$

Where z(n) is a Gaussian white noise of variance equal to 1 and ρ=0.95

1. Generate one realization of the process of length = 1,000,000
2. Determine the size of the alphabet of the source
3. Find the entropy *H(X)* assuming that *x(n)* is a discrete memoryless source

Exercise 3

4. Let X = x(n) and Y = ρx(n-1). Compute the joint PDF $p_{XY}(x,y)$ and the joint entropy H(X,Y).
5. Compute the conditional entropy H(X|Y). Compare it with H(X). How many bps are needed to represent the source exploiting inter-symbol redundancy?

Exercise 4 (Exam of 19th June 2006)

- Generate N = 10000 samples of a AR(1) stationary random process:

$$x_n = \rho \cdot x_{n-1} + z_n$$

- With $\rho = 0.99$, $\sigma_x = 1$, $\mu_z = 0$ and $E[x \cdot z] = 0$
- Clip the sample values in the range [-20,+20] and round them to the nearest integer.
- Compute the entropy H(x) of the source assuming that there is no memory.
- Compare H(x) with the maximum entropy of a source having the same alphabet.

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