Multimedia Signal Processing
2^{nd} Module

Concluding Remarks about the first Laboratory

(Matteo Naccari and Michele Cossalter)

Exercise 1:

- **Question:** "*Find the entropy of the binary source that has generated above audio file. What can be inferred from results?*"

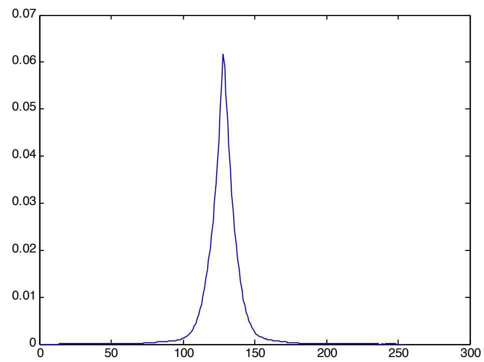
  Answer:

  As you have seen the entropy of this binary source is 1, thus the two symbols (0 and 1) have the same probability. Furthermore it is clear that at the bit level we loose the positional meaning of each bit inside each byte. Therefore, at the bit level, we can just "count" the occurrences of 1 and 0. Since we are processing a musical sequence, which contains certain regularity, it is quite obvious to find again this regularity at the bit level. This regularity is translated in the same probability of the symbol 0 and 1.

- **Question:** *"Now consider the file as generated by a finite source whose alphabet is [0:255]. Plot the normalized histogram of the file and find the entropy of the above source"*

  Answer:

  The normalized histogram of this file is shown below:



We can notice that the symbols do not have the same probability but the symbol (i.e. the intensity level) '128' has the maximum probability. This turns into a reduction of the entropy of the source.

Exercise 2:

- **Question:** "*Suppose to encode Y with H(Y) bits and to send N = aX+b – Y instead of X, where a and b are obtained by linear regression (least squares). Compute the entropy of N and compare it with the conditional entropy H(X|Y) = H(X,Y) – H(Y). Why H(N) > H(X|Y) ?*"

  Answer:

  We can proof that *H(N) > H(X|Y)* in two ways: by the definitions or by considering what *N* and *X|Y* represent.

  1. (By the definitions): without loss of generality we can see $N = X - Y$ instead of ($N = aX+b - Y$). Hence $P(N = n|Y = y) = P(N = n + y|Y = y)$; by the definition we derive that:

  $$\begin{aligned} H(N|Y) &= \sum_y P(y) H(N|Y=y) = \\ &= -\sum_y P(y) \sum_z P(N=n|Y=y)\log(P(N=n|Y=y)) = \\ &= -\sum_y P(y) \sum_{x=n+y\in X} P(X=n+y|Y=y)\log(P(X=n+y|Y=y)) = \\ &= \sum_y P(y) H(X|Y=y) = H(X|Y) \end{aligned}$$

     Now remembering that $I(Z,Y) \geq 0$ we obtain:

  $$I(N,Y) = H(N) - H(N|Y) = H(N) - H(N|Y) \geq 0 \Rightarrow H(N) \geq H(X|Y)$$

  2. (Considering what *N* and *X|Y* represent): the source $N = aX+b - Y$ measures the linear dependency between X and Y. The source $\tilde{N} = X|Y$ measures the information in X which can be inferred knowing Y, that is to say the statistical dependency between X and Y. Linear predictors can only measure the statistical dependency until the second-order, while non-linear predictors (such as H|Y) can measure higher order statistical dependencies. Hence the remaining information of N is higher than that in $\tilde{N}$.