Chapter 1

Functions and Limits

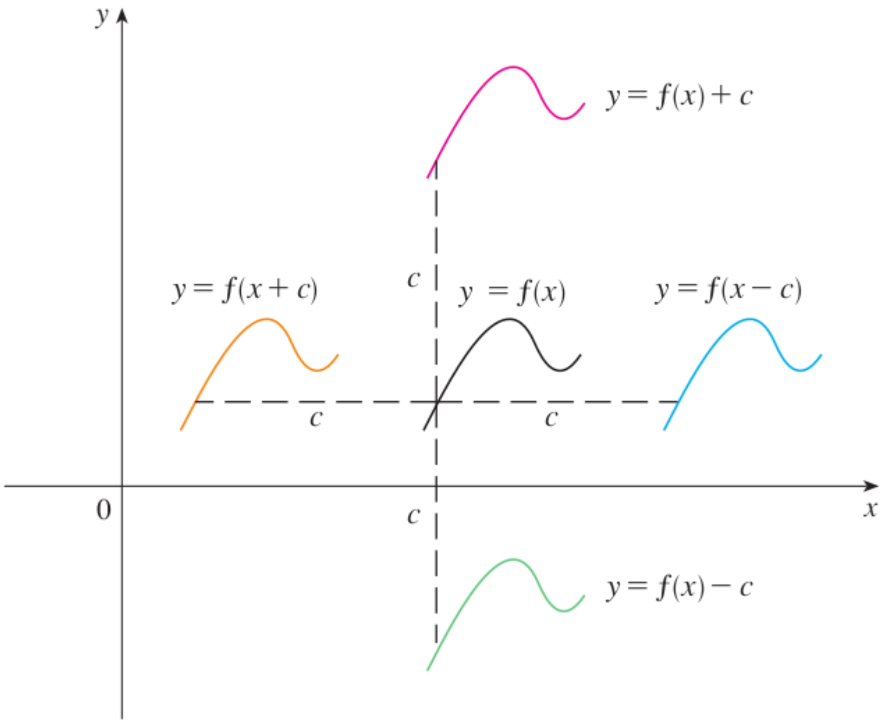
1.3 New Functions from Old Functions

Transformations of Functions.

Translation.

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

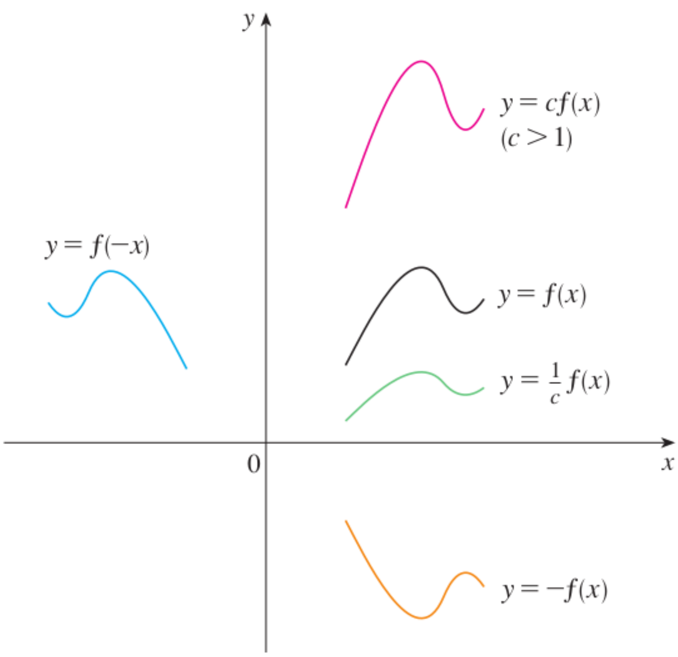
$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left



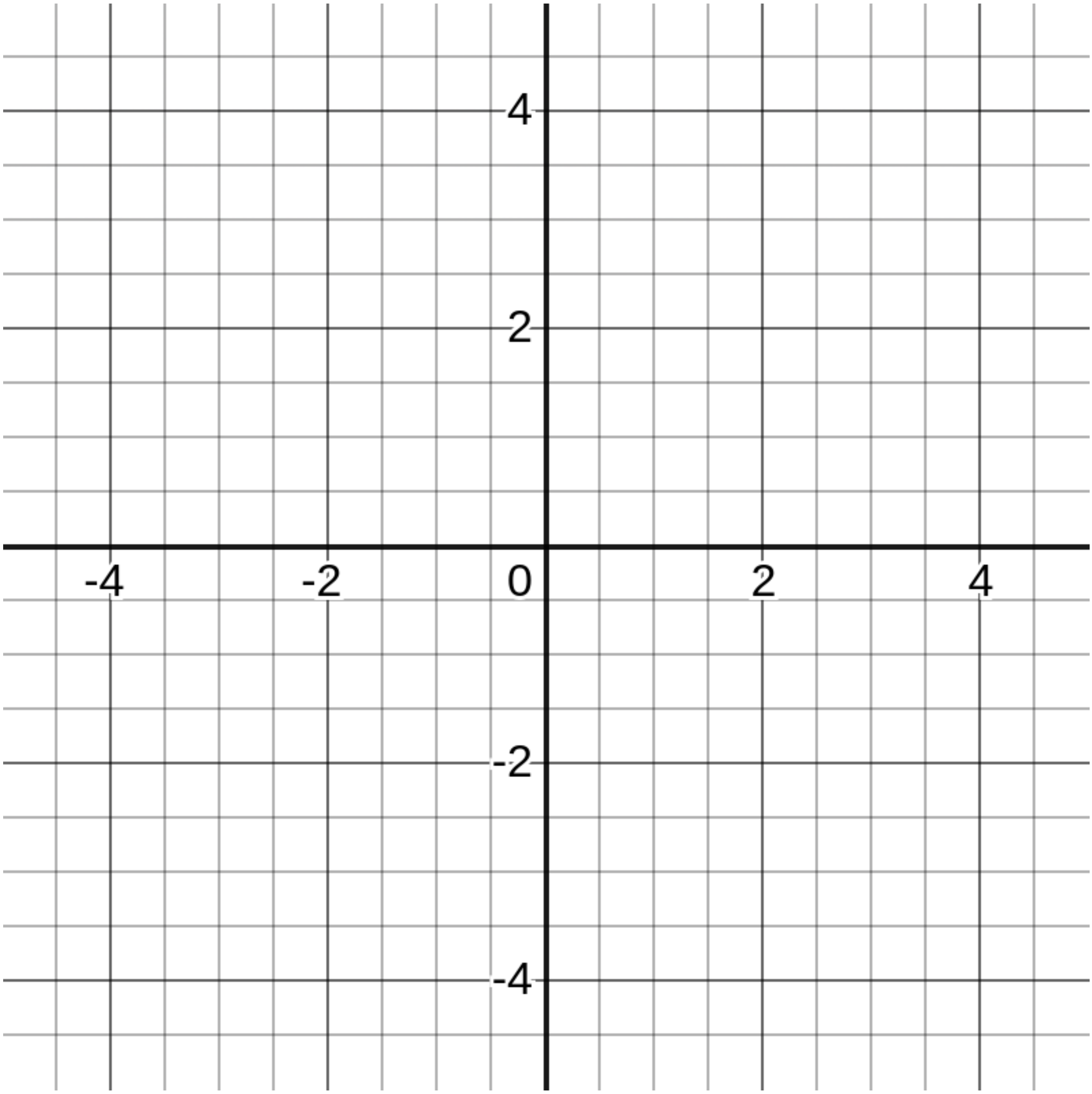
Stretching and reflecting.

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

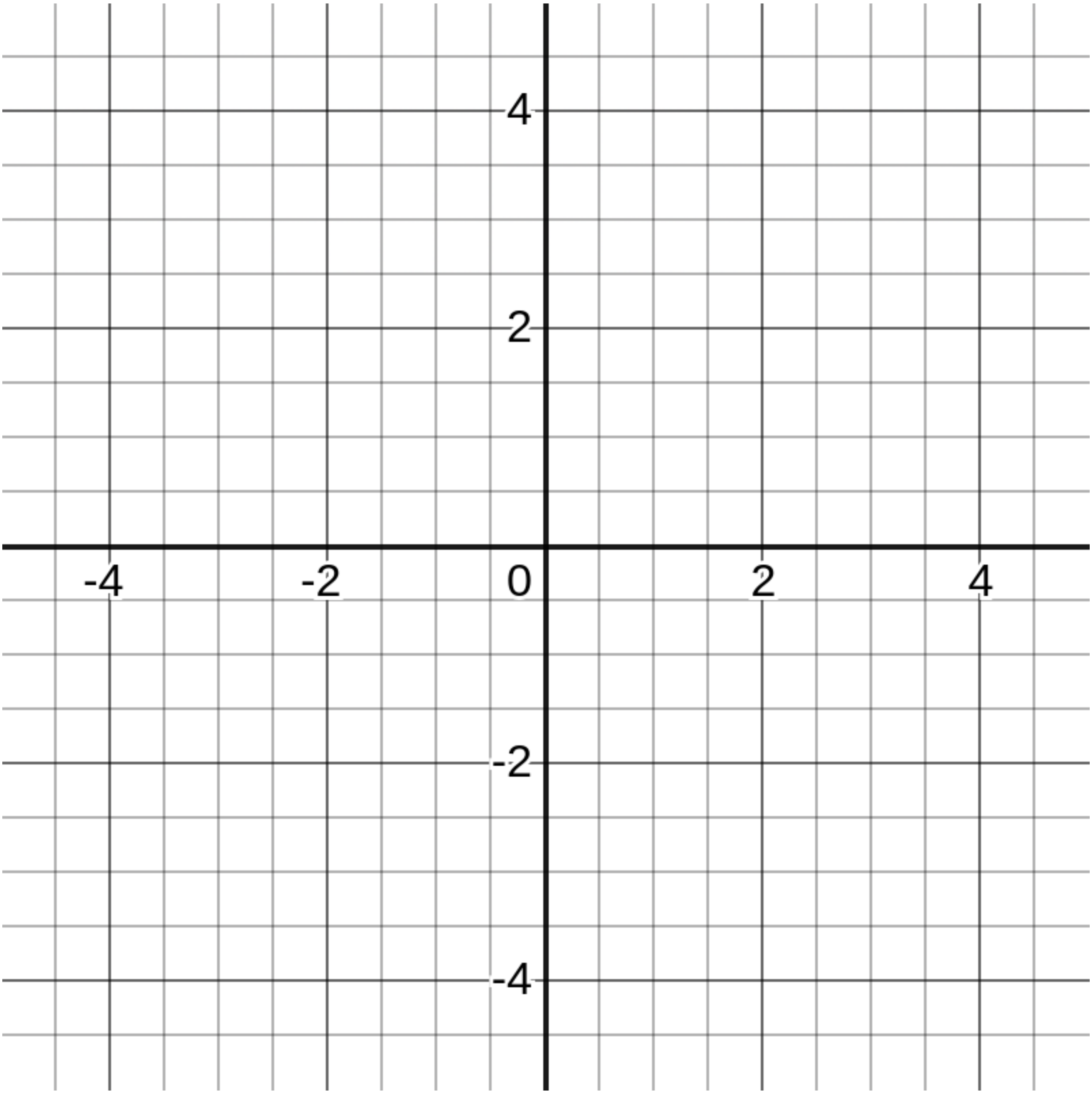
$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c
$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c
$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c
$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c
$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis



EXAMPLE 1 Given the graph of $y = \sqrt{x}$, use transformations to graph $y = \sqrt{x} - 2$, $y = \sqrt{x - 2}$, $y = -\sqrt{x}$, $y = 2\sqrt{x}$, and $y = \sqrt{-x}$.

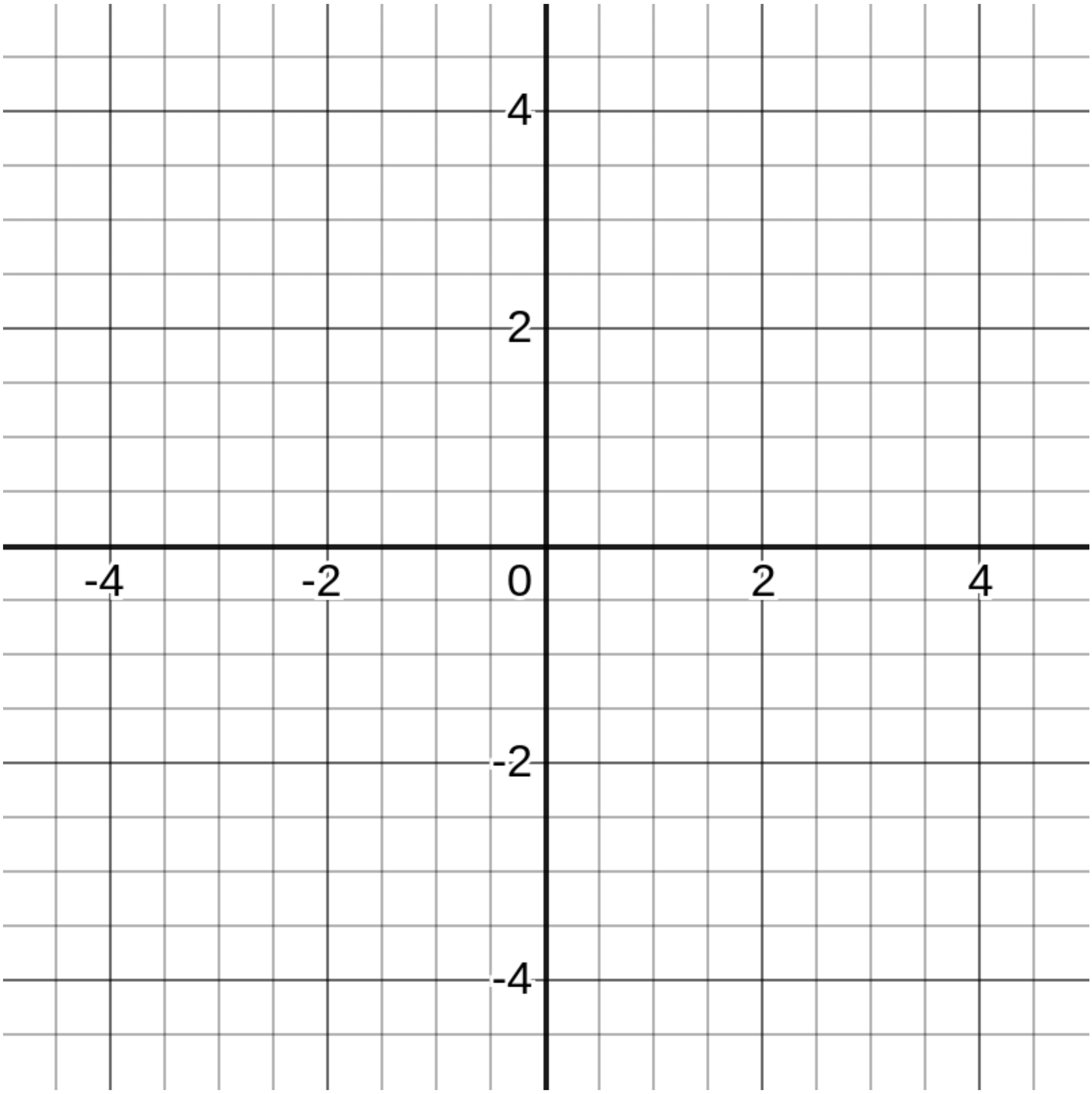


EXAMPLE 2 Sketch the graph of the function $f(x) = x^2 + 6x + 10$.

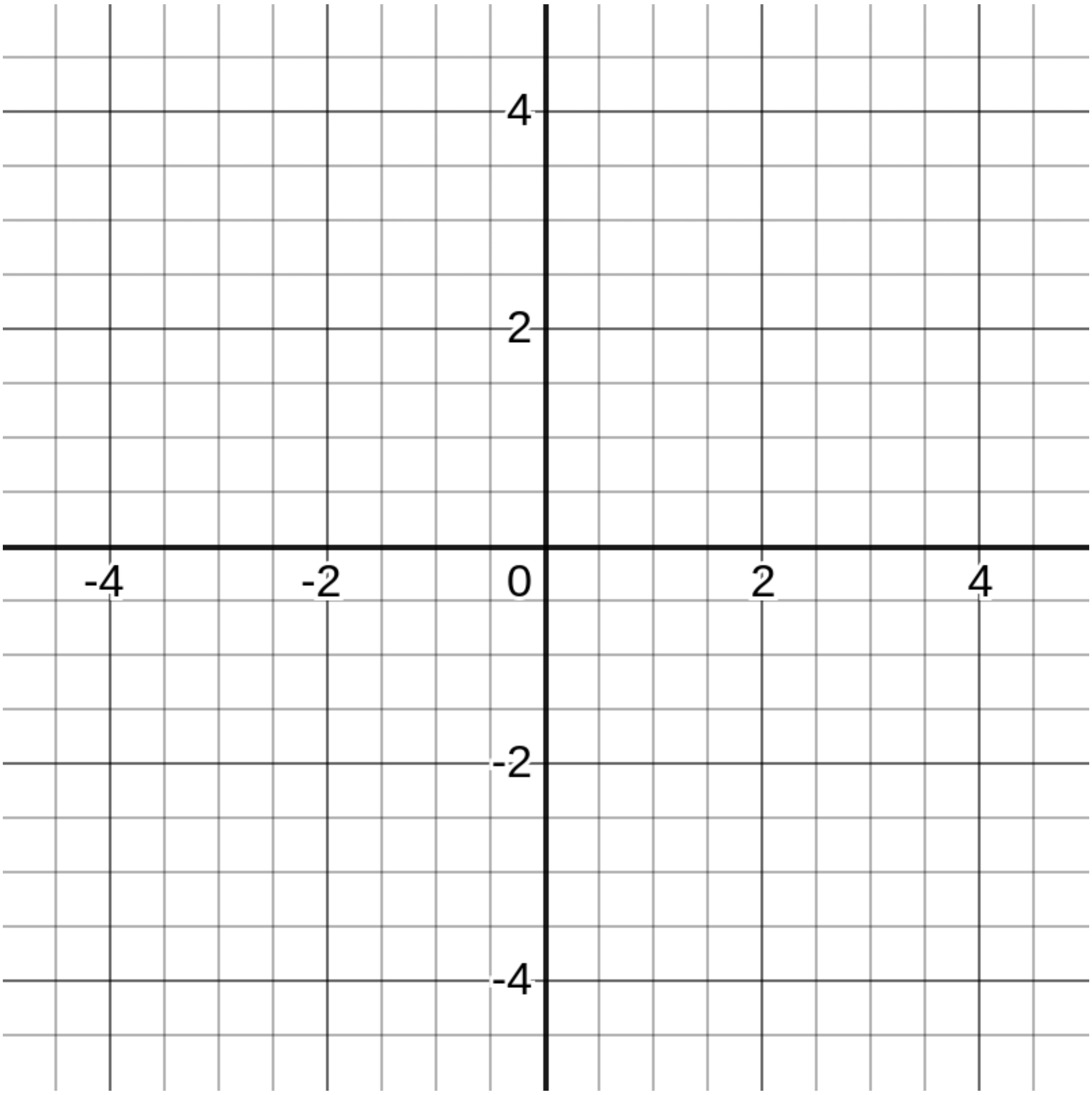


EXAMPLE 3 Sketch the graphs of the following functions.

(a) $y = \sin 2x$ (b) $y = 1 - \sin x$



EXAMPLE 5 Sketch the graph of the function $y = |x^2 - 1|$.



Combinaisons of Functions.

f & g two functions.

Adding.

$$(f+g)(x) = f(x) + g(x)$$

Domain: commun to Dom(f) & Dom(g)

$\mathrm{Dom}(f) \cap \mathrm{Dom}(g)$

Substracting.

$$(f-g)(x) = f(x) - g(x)$$

Domain: commun to Dom(f) & Dom(g)

$\mathrm{Dom}(f) \cap \mathrm{Dom}(g)$

Multiplying.

$$(fg)(x) = f(x)\, g(x)$$

Domain: $\mathrm{Dom}(f) \cap \mathrm{Dom}(g)$

Dividing.

$$\left(f/g\right)(x) = \frac{f(x)}{g(x)}$$

Domain: $\mathrm{Dom}(f) \cap \mathrm{Dom}(g) \setminus \{x : g(x) = 0\}$.

remove zero of g.

Example. Find the domain of the function

$$f(x) = \sqrt{x} + \sqrt{2-x}\ .$$

$[0,\infty)$ $\quad$ $(-\infty, 2]$

$\mathrm{Dom}(f) = [0, 2]$

Example Find the domain of the function $f(x) = \dfrac{x^2}{x-1}$.

$x = 1$

$x - 1 = 0$

$\mathrm{Dom}(f) = (-\infty, \infty) \setminus \{1\}$

$= (-\infty, \infty)$ except $\{1\}$.

Composite of two functions (Composition).

Definition Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(\underline{g(x)})$$

$\sqrt{x-2} \to f(x)$

$x - 2 \to g(x)$

Domain: $\mathrm{Dom}(f \circ g) = \mathrm{Dom}\, g$ (remove values of $g(x)$ which don't go in $f(x)$)

EXAMPLE 6 If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$.

EXAMPLE 7 If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$, find each of the following functions and their domains.

(a) $f \circ g$ (b) $g \circ f$ (c) $f \circ f$ (d) $g \circ g$

EXAMPLE 8 Find $f \circ g \circ h$ if $f(x) = x/(x + 1)$, $g(x) = x^{10}$, and $h(x) = x + 3$.

EXAMPLE 9 Given $F(x) = \cos^2(x + 9)$, find functions f, g, and h such that $F = f \circ g \circ h$.