Created by Pierre-O. Parisé
Fall 2022

Last name: Solution
First name: —
Section: —

Instructions: You must answer all the questions below and give your solutions to the TA at the end of the recitation. Write your solutions directly on the worksheet. Late worksheet will not be accepted.

Question 1 **(10 pts)**

Evaluate the indefinite integral.

(a) (2 points) $\int x^2\sqrt{1+x^3}\,dx$.

(b) (2 points) $\int x(2x+8)^8\,dx$.

(c) (2 points) $\int \sqrt{\cot x}\csc^2 x\,dx$.

(d) (2 points) $\int_0^1 (3t-1)^{50}\,dt$.

(e) (2 points) $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dt$.

(a) Set $u = 1+x^3 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow du = 3x^2 dx$

$\Rightarrow \frac{1}{3} du = x^2 dx.$

So, $\int x^2\sqrt{1+x^3}\,dx = \int \sqrt{u}\,\frac{du}{3} = \frac{1}{3}\,\frac{u^{3/2}}{3/2} + C$

$= 2u^{3/2} + C$

$$\Rightarrow \int x^2\sqrt{1+x^3}\,dx = \boxed{2(1+x^3)^{3/2} + C}$$

(b) Set $u = 2x+8 \Rightarrow \frac{du}{dx} = 2 \Rightarrow du = 2dx \Rightarrow \frac{1}{2}du = dx.$

So, $\int x(2x+8)^8\,dx = \int x\,u^8\,\frac{du}{2} = \frac{1}{2}\int x u^8\,du$

Now, $\frac{u-8}{2} = x \Rightarrow \int x(2x+8)^8\,dx = \frac{1}{4}\int u^9 - u^8\,du$

$$= \boxed{\frac{1}{4}\left(\frac{u^{10}}{10} - \frac{u^9}{9}\right) + C.}$$

(c) Let $u = \cot x \Rightarrow \dfrac{du}{dx} = -\csc^2 x \Rightarrow -du = \csc^2 x\, dx$

So, $\displaystyle\int \sqrt{\cot x}\,\csc^2 x\, dx = -\int \sqrt{u}\, du = -\frac{u^{3/2}}{3/2} + C$

$$= \boxed{-\frac{2}{3}u^{3/2} + C}$$

(d) Let $u = 3t-1 \Rightarrow \dfrac{du}{dx} = 3 \Rightarrow du = 3\,dx.$

$t=0 \longrightarrow u = -1$

$t=1 \longrightarrow u = 2$ $\Rightarrow \displaystyle\int_0^1 (3t-1)^{50}\, dt = \int_{-1}^{2} u^{50}\, du$

$$= \frac{u^{51}}{51}\bigg|_{-1}^{2} = \boxed{\frac{2^{51}+1}{51}}$$

(e) Let $u = x^2 \Rightarrow \dfrac{du}{dx} = 2x \Rightarrow \dfrac{1}{2}du = x\,dx$

So, $\displaystyle\int_0^{\sqrt{\pi}} x\cos(x^2)\, dx = \int_0^{\pi} \cos(u)\,\frac{du}{2} = \frac{1}{2}\sin u\bigg|_0^{\pi}$

$$= \boxed{0}$$

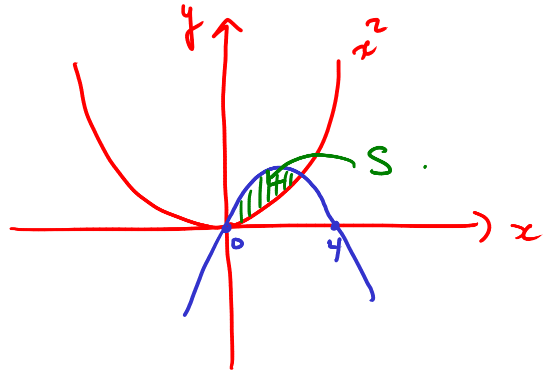
QUESTION 2 **(0 pts)**

Find the area of the region enclosed by the functions $f(x) = x^2$ and $y = 4x - x^2$.

Picture:

$y_1 = x^2$

$y_2 = 4x - x^2$
$= (4-x)x$



Intersections:

$$y_1 = y_2 \iff x^2 = 4x - x^2$$
$$\iff 0 = 4x - 2x^2$$
$$\iff 0 = 2x(2-x) \iff x=0 \text{ or } x=2$$

For $0 < x < 2$, $y_1 < y_2$

Area:

$$\text{Area}(S) = \int_0^2 y_2 - y_1 \, dx = \int_0^2 4x - x^2 - x^2 \, dx$$
$$= \int_0^2 4x - 2x^2 \, dx$$
$$= 2x^2 - \frac{2x^3}{3} \Big|_0^2$$
$$= 8 - \frac{16}{3}$$

$$\Rightarrow \boxed{\text{Area}(S) = \frac{8}{3}}$$

For graders use only:

Question:	1	2	Total
Points:	10	0	10
Score:			