Math 241   Name (Print): _____

Fall 2021   Section Number: _____

Final

15 December 2021

Exam length: 120 minutes

This exam contains 17 pages (including this cover page) and 16 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

Instructions:

- You have 120 minutes for the exam, plus 10 minutes before and after to download the exam and upload your solutions. The exam will be available at 11:50am. **The deadline for uploading is 2:10pm.**

- You are required to **show your work** and justify your answers for all questions *except where explicitly stated.*

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit, if any.

- All work must be your own. **You are not permitted to discuss the test with anyone else.**

- No books or notes are allowed. No electronic devices other than to write the exam, or to upload or download the exam are allowed.

- **To print or not to print:** You may print out the exam and write on it, or you may write your answers on blank paper or on a tablet without printing the exam. If you do this, please clearly indicate which question is being answered, and use a new sheet of paper for a new question (if a question has multiple parts, they may be on the same sheet). Also please write your name and page number on each page.

Academic integrity is expected of all University of Hawaii students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below or on the first page of your answers to indicate that you have read and agree to these instructions.

SIGNATURE OF STUDENT

For official use only:

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
Points:	6	9	4	6	8	8	9	10	10	10	8	10	5	5	7	5	120
Score:																	

Question 1.   (6 points)
A particle moves along a line and its position $s(t)$ in feet after time t seconds is given by the following curve. The dotted straight line is tangent to the curve of $s(t)$ at time $t = 2$ seconds.
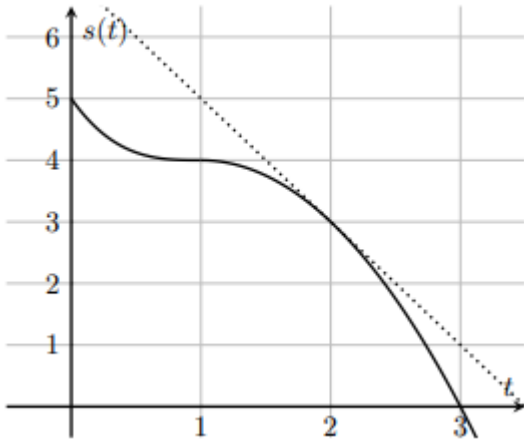
(a) (2 points) Find the average velocity of the particle over the time interval $[0, 2]$. Include units with your answer.

(b) (2 points) Find the instantaneous velocity at time $t = 2$ seconds. Include units with your answer.

(c) (2 points) Find the equation of the dotted line.

Question 2.   (9 points)
   Compute the following limits, or say if they are $+\infty$ or $-\infty$. Do not use L'Hospital's rule.

   (a) (3 points) $\displaystyle\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}}$

   (b) (3 points) $\displaystyle\lim_{x\to 0} \frac{\sin(x^3)}{x}$

   (c) (3 points) $\displaystyle\lim_{x\to -\infty} \frac{x}{\sqrt{1+x^2}}$

Question 3.   (4 points)
Consider the graph of the function $y = f(x)$ below on the interval $[-2, 10]$ where $f(0)$ is undefined. You do not need to justify your answers.
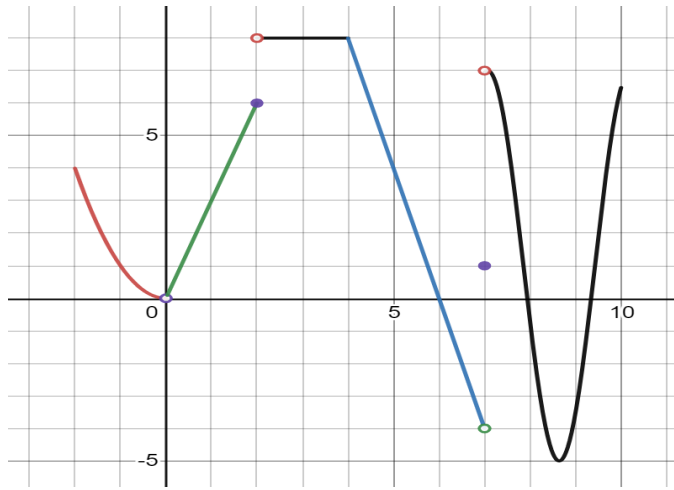
(a) (1 points) What is $\lim\limits_{x \to 0} f(x)$? ('Does not exist' is a possible answer)

(b) (1 points) What is $\lim\limits_{x \to 4} f'(x)$? ('Does not exist' is a possible answer)

(c) (1 points) Where (if anywhere) is the function not continuous?

(d) (1 points) Where (if anywhere) is the function not differentiable?

Question 4. (6 points)

Use the limit definition of the derivative to compute $f'(x)$ when $f(x) = 2 + \frac{1}{x}$.

You will get no credit for computing the derivative without using the definition.

Question 5.   (8 points)

Consider the graph of the function $y = f(x)$ below. For t in the interval $[0, 7]$ we define $g(t) = \int_0^t f(x)dx$.
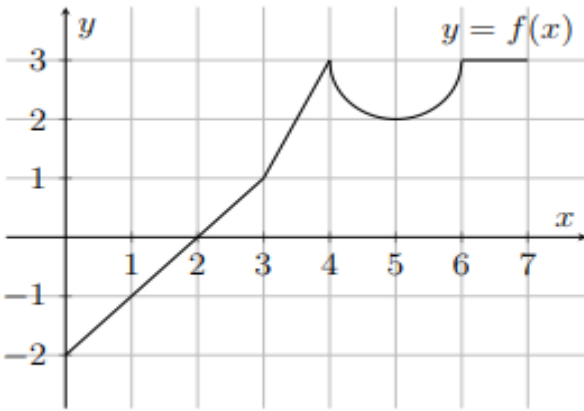
(a) (3 points) Find $g(0)$ and $g(3)$.

(b) (2 points) Find all critical points of the function g on the interval $(0, 7)$.

(c) (3 points) Find the absolute maximum value of g on the interval $[0, 3]$.

Question 6.   (8 points)

For this question, let g be a differentiable function such that g and g' take the following values:

$$g(\pi/3) = 1, \quad g'(\pi/3) = 0, \quad g(2) = 2, \quad g'(2) = 4,$$

(a) (4 points) Compute $f'(\pi/3)$ if $f(x) = \dfrac{g(x)}{\sin(x)}$.

(b) (4 points) Compute $h'(2)$ if $h(x) = \sqrt{1 + 2\,g(x)}$.

Question 7.　　(9 points)

Let y be defined implicitly as $4\sqrt{3x - y} = xy + 9$.

(a) (5 points) Find $\dfrac{dy}{dx}$ in terms of x and y.

(b) (4 points) Find the equation of the tangent line to the curve at the point $(1, -1)$.

Question 8. (10 points)

The volume inside of a sphere expands at a rate of $100 \text{cm}^3/s$. How fast is the radius growing when the radius is $25\,\text{cm}$? Do not simplify your answer.

Hint: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

Question 9.   (10 points)

Alice is making a large rectangular base aquarium tank without a lid of volume 18ft^3. The length of the base of the tank is three times its width. In order to minimize the cost she needs to minimize the amount of material used. Find the dimensions of the tank that minimize the amount of material used.

Question 10.   (10 points)
   Consider the function F defined on the interval $[-1, 6]$ by

$$F(s) = s^3 + 6s^2 - 1.$$

(a) (3 points) On what subintervals (if any) of $[-1, 6]$ is F increasing and on which is it decreasing?

(b) (3 points) Where (if anywhere) does F have a local minimum or a local maximum?

(c) (4 points) On what subintervals (if any) of $[-1, 6]$ is F concave up or concave down, and where (if anywhere) does F have an inflection point?

Question 11.   (8 points)
For this question, consider the equation $x^3 + 2x - 2 = 0$

(a) (4 points) Show that the given equation has a solution in the interval $[-1, 1]$. [*Hint:* Use Intermediate Value Theorem]

(b) (4 points) Show that the given equation cannot have more than one solution in the interval $[-1, 1]$. [*Hint:* You may consider using Rolle's theorem or Mean Value Theorem]

Question 12.   (10 points)
    Consider the function $f(x) = 9 - x^2$ on the interval $[0, 3]$

(a) (4 points) Estimate $\int_0^3 f(x)dx$ with a Riemann sum using $n = 3$ subintervals of equal width and right endpoints.

(b) (4 points) Sketch the graph of $f(x)$ and the rectangles that you used in part (a) on the same axis.
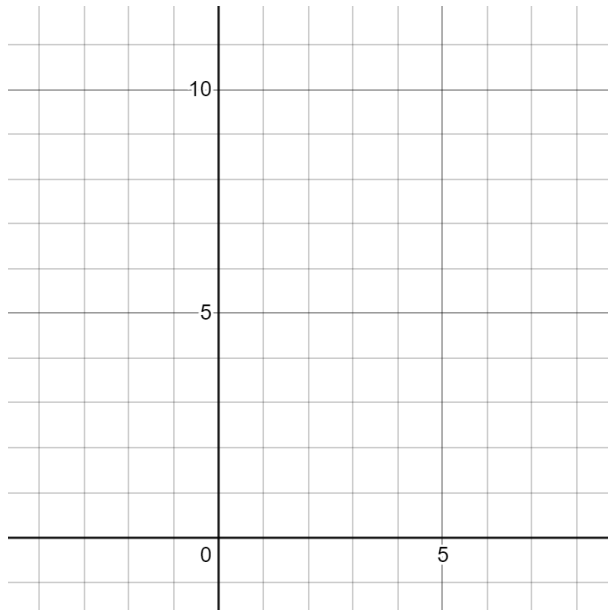
(c) (2 points) Is this estimate over or under the actual answer? Or is it impossible to tell? Explain your answer.

Question 13.   (5 points)

A function $f(x)$ is defined by $f(x) = \int_2^{x^2} \sqrt{1 + t^3} \, dt$. Find $f'(x)$ by using the Fundamental Theorem of Calculus.
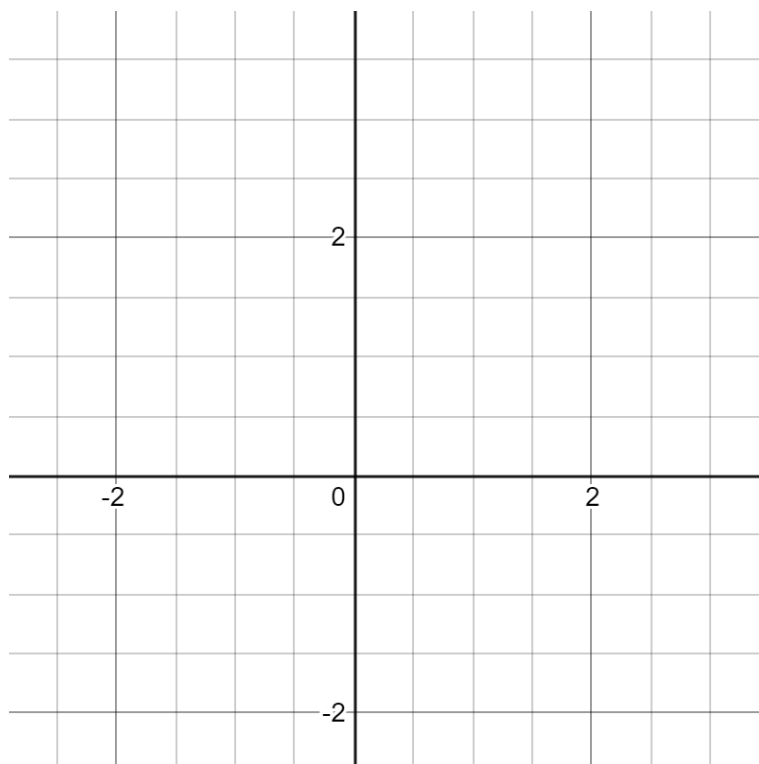
Question 14. (5 points)

Find the function $f(x)$ that satisfies the conditions $f(1) = 1$ and $f'(x) = \dfrac{2x+1}{(x^2+x+1)^3}$.

Question 15.   (7 points)

    (a) (4 points) Graph and shade the area bounded by the following. Label points of intersection points of the graph.

$$y = x^2, \quad y = 2x - 1, \quad \text{and the x-axis.}$$

    (b) (3 points) Set up the integral(s) that gives the area of the shaded region. **DO NOT EVALUATE** the integral.

Question 16.   (5 points)

Find the volume of the solid generated using the area bounded by $y = 0$, $y = \sqrt{x^2 - 1}$, $1 \le x \le 4$ rotated about the $x-$axis.