MATH-241 | Created by Pierre-O. Parisé
Worksheet 05 | Fall 2022

Last name: ______________________
First name: ______________________
Section: ______________________

Question:	1	2	Total
Points:	10	10	20
Score:			

Instructions: You must answer all the questions below and give your solutions to the TA at the end of the recitation. Write your solutions directly on the worksheet. Late worksheet will not be accepted.

Question 1 **(10 pts)**

Compute the derivative of the following functions:

(a) (5 points) $f(x) = x^2 \sec(x)$.

Solution: Using the product rule, we have

$$f'(x) = (x^2)' \sec(x) + x^2 (\sec(x))' = 2x \sec(x) + x^2 \sec(x) \tan(x)$$
$$= x \sec(x) \left(2 + x \tan(x)\right).$$

(b) (5 points) $f(x) = \dfrac{\sqrt{x^2 + x}}{\sin(x)}$.

Solution: Using the quotient rule, we have

$$f'(x) = \frac{(\sqrt{x^2+x})' \sin(x) - \sqrt{x^2+x}(\sin(x))'}{\sin^2(x)}.$$

Now, we have

$$\frac{d}{dx}\left(\sqrt{x^2+x}\right) = \frac{1}{2\sqrt{x^2+x}} \frac{d}{dx}\left(x^2+x\right) = \frac{1}{2\sqrt{x^2+x}}(2x+1) = \frac{2x+1}{2\sqrt{x^2+x}}.$$

Therefore, we get

$$f'(x) = \frac{\frac{2x+1}{2\sqrt{x^2+x}} \sin(x) - \cos(x)\sqrt{x^2+x}}{\sin^2(x)}$$
$$= \frac{(2x+1)\sin(x) - 2(x^2+x)\cos(x)}{2\sqrt{x^2+x}\,\sin^2(x)}.$$

Question 2 **(10 pts)**

Isolate y and find an expression for y' if

$$x^2 + y^2 = 1.$$

Solution: We have

$$y^2 = 1 - x^2 \Rightarrow y = \pm\sqrt{1-x^2}.$$

For the plus sign, we have

$$y' = \frac{d}{dx}\left(\sqrt{1-x^2}\right) = \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right) = \frac{1}{2\sqrt{1-x^2}}(-2x) = -\frac{x}{\sqrt{1-x^2}}.$$

For the minus sign, we have

$$y' = \frac{d}{dx}\left(-\sqrt{1-x^2}\right) = -\frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right) = -\frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{x}{\sqrt{1-x^2}}.$$