MATH-241 Created by Pierre-O. Parisé
Worksheet 06 Fall 2022

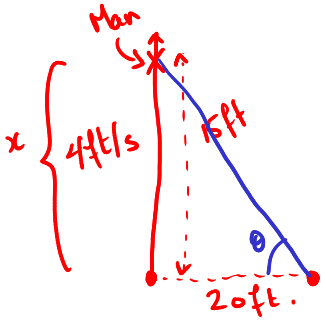
Last name: Solution.
First name: —
Section: —

Question:	1	Total
Points:	20	20
Score:	—	—

Instructions: You must answer all the questions below and give your solutions to the TA at the end of the recitation. Write your solutions directly on the worksheet. Late worksheet will not be accepted.

Question 1 (20 pts)

A man walks along a straight path at a speed of 4ft/s. A searchlight is located on the ground 20ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15ft from th epoint on the path closest to the searchlight.



θ: angle determined by the man.
x: distance traveled by the man.

Goal: $\left.\frac{d\theta}{dt}\right|_{x=15}$

We have $\tan\theta = \frac{x}{20}$

Taking derivative:

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$$

$$\Rightarrow \left.\frac{d\theta}{dt}\right|_{x=15} = \frac{1}{20}\,\frac{1}{\sec^2\theta}\left.\frac{dx}{dt}\right|_{x=15}.$$

We know that $\left.\frac{dx}{dt}\right|_{x=15} = 4$

and

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{20}{\sqrt{15^2+20^2}}\right)} = \frac{1}{\frac{20}{25}} = \frac{5}{4}.$$

Therefore

$$\left.\frac{d\theta}{dt}\right|_{x=15} = \frac{1}{20}\cdot\frac{16}{25}\cdot 4 = \frac{16}{125}\ \text{rad/s}$$

$$\approx \boxed{0.128\ \text{rad/s}}$$