MATH-241 Calculus I | Created by Pierre-O. Parisé
Worksheet 08 | Fall 2022

Last name: ____________________

First name: ____________________

Section: ____________________

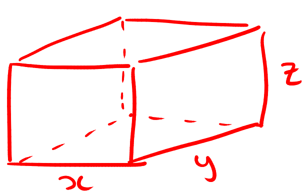
Question:	1	Total
Points:	20	20
Score:		

Instructions: You must answer all the questions below and give your solutions to the TA at the end of the recitation. Write your solutions directly on the worksheet. Late worksheet will not be accepted.

Question 1 **(20 pts)**

A rectangular storage container with an open top is to have a volume of 10m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of the materials for the cheapest such container.

① Sketch.



x: width in meter.
y: length in meter.
z: heigth in meter.
V: volume. in m^3
A: surface area. in m^2
C: cost of the material is \$

Goal: minimize C.

② Equations.

surface area : $2 \times \underset{x}{\square} z + 2 \times \underset{y}{\square} z + \overbrace{1 \times \underset{x}{\square} y}^{\text{base}}$.

Cost for the base $= 10xy$

Cost for sides $= 6(2xz + 2yz) = 12xz + 12yz$

Thus,

$$C = 10xy + 12xz + 12yz.$$

We have $V = 10 \Rightarrow xyz = 10$

Also, $y = 2x$

so, $x \cdot (2x) \cdot z = 10 \Rightarrow z = 5/x^2$.

Replacing in C:

$$C(x) = 20x^2 + \frac{60}{x} + \frac{120}{x} = 20x^2 + \frac{180}{x}.$$

(3) Optimize.

$$C'(x) = 40x - \frac{180}{x^2} = 0$$

$$\Leftrightarrow 4x^3 = 18$$

$$\Leftrightarrow x^3 = \frac{9}{2}$$

$$\Leftrightarrow x = \sqrt[3]{\frac{9}{2}}$$

If $x < \sqrt[3]{9/2}$, then $C'(x) < 0$

If $x > \sqrt[3]{9/2}$, then $C'(x) > 0$

$\longrightarrow$ 1st derivative test $x = \sqrt[3]{9/2}$ corresponds to an abs. minimum.

Answer

$$\boxed{\begin{aligned} x &= \sqrt[3]{9/2} \approx 1.6509 \text{ m}. \\ y &= 3.3019 \text{ m}. \\ z &= 1.8344 \text{ m}. \end{aligned}}$$

with a cost of $C \approx 163.5408$ \$.