MATH-241 Created by Pierre-O. Parisé
Worksheet 07 Fall 2022

Last name: Solutions
First name: —
Section: —

Question:	1	2	Total
Points:	10	10	20
Score:	—	—	—

Instructions: You must answer all the questions below and give your solutions to the TA at the end of the recitation. Write your solutions directly on the worksheet. Late worksheet will not be accepted.

Question 1 **(10 pts)**

Let $y = f(x)$ be the function defined by

$$f(x) = x\sqrt{6-x}.$$

An answer without using the derivative or the second derivative won't be credited.

(a) (2 points) Find the critical numbers of f.

(b) (3 points) Find the interval of increase and decrease of the function.

(c) (3 points) Find the inflection points and the interval of concavity.

(d) (2 points) Find the local maximum(s) and local minimum(s).

(a) $f'(x) = \sqrt{6-x} - \dfrac{x}{2\sqrt{6-x}} = \dfrac{12-2x-x}{2\sqrt{6-x}} = \dfrac{12-3x}{2\sqrt{6-x}}.$

$\Rightarrow$ (1) $f'(x) = 0 \iff 12-3x=0 \iff x=4$

(2) $f'(x) \not\exists \iff x = 6.$

C.N: 4 & 6.

(b) We have $f'(x) = \left(\dfrac{3}{2}\right)\dfrac{(4-x)}{\sqrt{6-x}}$

We will use a table.

factors	$x < 4$	4	$4 < x < 6$	6
$3/2$	+		+	
$4-x$	+		−	
$\sqrt{6-x}$	+		+	
$f'(x)$	+	0	−	$\nexists$
$f(x)$	↗	loc. min.	↘	

$f \nearrow$ on $(-\infty, 4)$ & $f \searrow$ on $(4, 6)$.

(c) $$f''(x) = \frac{\frac{3}{2}\left(-\sqrt{6-x} + \frac{(4-x)}{2\sqrt{6-x}}\right)}{6-x} = \frac{-12+2x+4-x}{2(6-x)^{3/2}}$$

$$= \frac{x-8}{2(6-x)^{3/2}}$$

$\rightarrow f''(x) = 0$ if $x = 8$ (No zero because $x < 6$) reject

$\rightarrow f''(x) \nexists$ if $x = 6$.

factors	$x < 6$	6
$x-8$	−	
$(6-x)^{3/2}$	+	
$f''(x)$	−	
$f(x)$	⌢	

f is concave down on $(-\infty, 6)$.

(d) At $x=4$, $f''(4) = \dfrac{-4}{2\cdot\sqrt{8}} < 0$

$\Rightarrow f(4) = 4\sqrt{2}$ is a maximum value.

Question 2 **(10 pts)**

Find the value of the following limit:

$$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3}.$$

We have

$$\sqrt{1+4x^6} = \sqrt{1/x^6 + 4}\ \sqrt{x^6}$$

$$= \sqrt{4 + 1/x^6}\ |x|^3$$

Since $x \to -\infty$, $|x| = -x$

$$\Rightarrow \lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \lim_{x\to-\infty} \frac{(-x)^3\sqrt{4+1/x^6}}{x^3\left(\frac{2}{x^3}-1\right)}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{4+1/x^6}}{\frac{2}{x^3}-1}$$

Since $\lim_{x\to-\infty} 4+\frac{1}{x^6} = 4$ &

$\lim_{x\to-\infty} \frac{2}{x^3} - 1 = -1$

$$\Rightarrow \lim_{x\to-\infty} \frac{-\sqrt{4+1/x^6}}{2/x^3-1} = \frac{-2}{-1} = \boxed{2}$$