MATH-241
Worksheet 02

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Last name: Solutions
First name: —
Section: —

Question:	1	2	Total
Points:	10	10	20
Score:	—	—	—

Instructions: You must answer all the questions below and give your solutions to the TA at the end of the recitation. Write your solutions directly on the worksheet. Late worksheet will not be accepted.

Question 1 (10 pts)

Given that

$$\lim_{x\to 2} f(x) = 5, \quad \lim_{x\to 2} g(x) = 2 \quad \text{and} \quad \lim_{x\to 2} h(x) = -2,$$

find the following limits. If you can't use one of the limit rules, explain why.

(a) (2 points) $\lim_{x\to 2}(f(x) + 5g(x))$.

(b) (2 points) $\lim_{x\to 2}(g(x))^3$.

(c) (2 points) $\lim_{x\to 2} \frac{3f(x)}{g(x)}$.

(d) (2 points) $\lim_{x\to 2}(f(x)g(x))$.

(e) (2 points) $\lim_{x\to 2} \frac{g(x)}{2+h(x)}$.

(a) $\lim_{x\to 2} f(x)+5g(x) = \lim_{x\to 2} f(x) + \lim_{x\to 2} 5g(x)$

$= \lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x) = 5+5\cdot 2 = 15$

(b) $\lim_{x\to 2} (g(x))^3 = \left(\lim_{x\to 2} g(x)\right)^3 = 2^3 = 8$

(c) Since $\lim_{x\to 2} g(x) = 2$ is not zero, we can use quotient rule

$\Rightarrow \lim_{x\to 2} \frac{3f(x)}{g(x)} = \frac{\lim_{x\to 2} 3f(x)}{\lim_{x\to 2} g(x)} = \frac{3\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$

$= \frac{3\cdot 5}{2} = \frac{15}{2}$

(d) $\lim_{x \to 2} \left(f(x) g(x) \right) = \left(\lim_{x \to 2} f(x) \right) \left(\lim_{x \to 2} g(x) \right)$

$$= 5 \cdot 2 = 10$$

(e) $\lim_{x \to 2} 2 + h(x) = \lim_{x \to 2} 2 + \lim_{x \to 2} h(x) = 2 + (-2) = 0$

We can't use the quotient rule and we can't say anything more on the value of

$$\lim_{x \to 2} \frac{g(x)}{2 + h(x)} .$$

Question 2 (10 pts)

Using the limit rules, find the following limits.

(a) (5 points) $\lim_{x\to 2}\sqrt{\frac{2x^2+1}{3x-2}}$.

(b) (5 points) $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}$.

(a) $\lim_{x\to 2}\sqrt{\frac{2x^2+1}{3x-2}} = \sqrt{\lim_{x\to 2}\frac{2x^2+1}{3x-2}}$

Quotient Rule: $\lim_{x\to 2} 3x-2 = 3\cdot 2 - 2 = 4 \neq 0$

↳ okay to use Quotient Rule.

$$\Rightarrow \sqrt{\lim_{x\to 2}\frac{2x^2+1}{3x-2}} = \sqrt{\frac{\lim_{x\to 2} 2x^2+1}{\lim_{x\to 2} 3x-2}}$$

$$= \sqrt{\frac{2\lim_{x\to 2} x^2 + \lim_{x\to 2} 1}{3\lim_{x\to 2} x - \lim_{x\to 2} 2}}$$

$$= \sqrt{\frac{2\cdot 2^2+1}{3\cdot 2-2}}$$

$$= \sqrt{\frac{9}{4}}$$

$$= \frac{3}{2}$$

(b) $\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} = \frac{0}{0}$ not good! Can't use any rule!

We have

$$\frac{\sqrt{9+h}-3}{h} = \frac{\sqrt{9+h}-3}{h} \cdot \frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}$$

$$= \frac{9+h-9}{h\left(\sqrt{9+h}+3\right)}$$

$$= \frac{\cancel{h}}{\cancel{h}\left(\sqrt{9+h}+3\right)} = \frac{1}{\sqrt{9+h}+3} \quad (h \neq 0)$$

Therefore,

$$\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} = \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} \longrightarrow \text{limit is not zero, Quot. Rule}$$

$$= \frac{\lim_{h\to 0} 1}{\lim_{h\to 0} \sqrt{9+h}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{6}$$