# Math 241 Final, Fall 2020

**Name:**

| Question | Points | Score |
|---|---|---|
| 1 | 6 | |
| 2 | 6 | |
| 3 | 8 | |
| 4 | 6 | |
| 5 | 16 | |
| 6 | 6 | |
| 7 | 11 | |
| 8 | 6 | |
| 9 | 10 | |
| 10 | 6 | |
| 11 | 7 | |
| 12 | 4 | |
| 13 | 9 | |
| 14 | 4 | |
| 15 | 4 | |
| 16 | 11 | |
| Total: | 120 | |

- The exam is 2 hours long, plus an extra 15 minutes at each end to download the exam, and to upload your answers.
- You may not use any electronic devices (including calculators) on the test, with one exception: you are welcome to use a tablet or similar to write your solutions if you would like.
- You may use the textbook, and your own personal notes, but no other notes.
- All work must be entirely your own. You cannot discuss the test with anyone else in any way.
- You must show all your work and make clear what your final solution is (for example, by drawing a box around it).
- Unless a question says you do not need to justify your answer, you will get almost no credit for solutions that are not justified.
- You can write your answers on blank paper or on a tablet without printing out the test. If you do this, please make very clear which answer goes with which question, and write your name, and the page number, on each page. You can also print out the test and write on that if you want.
- Good luck!

1. An ‘ilio-holo-i-ka-uaua is fitted with a velocimeter.

She gets in the ocean, and swims in a straight line away from the shore. Her speed is recorded every two minutes for ten minute period. The results are recorded in the following table.

| Time in minutes | Speed in meters per minute |
|---|---|
| 0 | 0 |
| 2 | 50 |
| 4 | 66 |
| 6 | 70 |
| 8 | 60 |
| 10 | 44 |

(a) (1 point) True / false / not enough information: the acceleration of the seal is constant. No justification required.

Not constant because the seal slows down or accelerate.

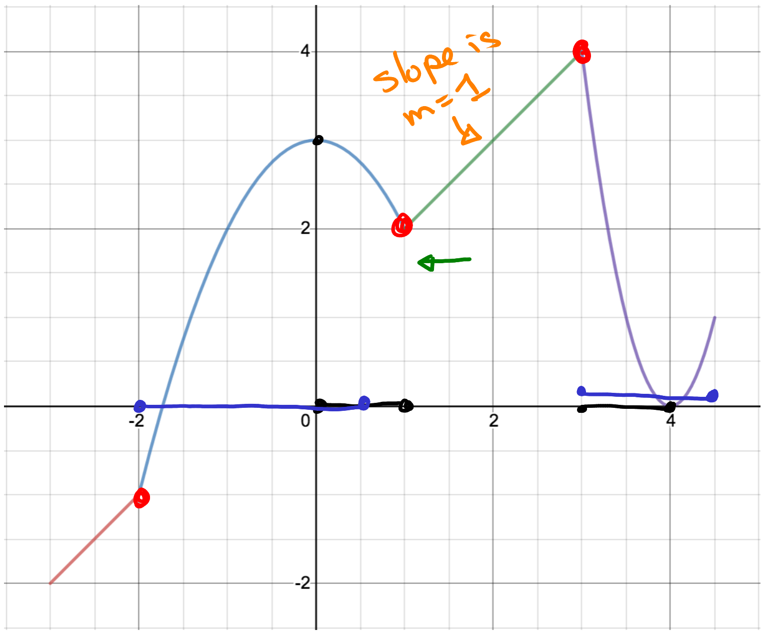
(b) (2 points) What is the seal's average acceleration between minutes 4 and 8? Include units.

$$a_{ave} = \frac{v(8) - v(4)}{8-4} = \frac{60 - 66}{8-4} = \frac{6}{4} = 3/2 \text{ meters/min}^2.$$

(c) (3 points) Estimate the distance swum by the seal in this ten minute period. Include units.

$$\text{distance} = 50 \cdot 2 + 66 \cdot 2 + 70 \cdot 2 + 60 \cdot 2 + 44 \cdot 2$$
$$= 100 + 132 + 140 + 120 + 88$$
$$= 580 \text{ meters.}$$

2. Consider the function $f$ with graph $y = f(x)$ as picture, defined on the interval $[-3, 4.5]$.




You do not need to justify your answers to the following questions.

(a) (1 point) On which interval(s) (if any) is the function decreasing?

on [0,1] & [3,4]

(b) (1 point) On which interval(s) (if any) is the function concave down?

on [-2,0] & [3, 4.5]

(c) (1 point) Where (if anywhere) is the function **not** continuous?

Continuous everywhere.

(d) (1 point) Where (if anywhere) is the function **not** differentiable?

At $x=-2$, $x=1$ & $x=3$.

(e) (1 point) What is $\lim_{x \to 1^+} f(x)$ ('does not exist' is a possible answer)?

The value is 2.

(f) (1 point) What is $\lim_{x \to 1^+} f'(x)$ ('does not exist' is a possible answer)?

The value is 1

3. Say $f(x)$ is a function that satisfies $\lim_{x\to 2^+} f(x) = -1$, $\lim_{x\to 2^-} f(x) = 2$, $\lim_{x\to 1} f(x) = -2$, and $\lim_{x\to\infty} f(x) = 4$. Compute the following limits, or say if they are $+\infty$, or $-\infty$. Do **not** use L'Hospital's rule.

(a) (3 points) $\lim_{x\to 2^-} \dfrac{f(x)x^2 - 5xf(x) + 6f(x)}{x^2 - 4x + 4}$. $= L$

$2^2 - 4\cdot 2 + 4 = 4 - 8 + 4 = 0$

$\lim_{x\to 2^-} f(x) \cdot \lim_{x\to 2^-} x^2 - 5 \lim_{x\to 2^-} x \lim_{x\to 2^-} f(x) + 6 \lim_{x\to 2^-} f(x) = 2\cdot 4 - 5\cdot 2\cdot 2 - 6\cdot 2$
$= 8 - 20 - 12$
$= -24.$

Since $x^2 - 4x + 4 = (x-2)(x-2) = (x-2)^2$, then

$\Rightarrow L = \dfrac{-24}{(0^-)^2} = \dfrac{-24}{0^+} = -\infty.$

(b) (3 points) $\lim_{x\to 1} \left( \dfrac{\sqrt{x^2+3} - 2}{x-1} + f(x) \right)$. $= L$

$A = \lim_{x\to 1} \dfrac{\sqrt{x^2+3} - 2}{x-1}$

$B = \lim_{x\to 1} f(x) = -2$

A is of $\frac{0}{0}$.

$A = \lim_{x\to 1} \left(\dfrac{\sqrt{x^2+3} - 2}{x-1}\right)\left(\dfrac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}\right)$

$= \lim_{x\to 1} \dfrac{x^2+3-4}{(x-1)(\sqrt{x^2+3}+2)}$

$= \lim_{x\to 1} \dfrac{x^2-1}{(x-1)(\sqrt{x^2+3}+2)}$ $\left(x^2 - 1 = (x+1)(x-1)\right)$

$= \lim_{x\to 1} \dfrac{x+1}{(\sqrt{x^2+3}+2)} = \dfrac{2}{4} = \dfrac{1}{2}$

So, $L = A + B = \frac{1}{2} + (-2) = -\frac{3}{4}$

(c) (2 points) $\lim_{x\to\infty} \dfrac{x^2 + f(x) - 4x^3}{1 + 2x^3}$. $= L$

$A = \lim_{x\to\infty} \dfrac{x^2 - 4x^3}{1 + 2x^3}$

$B = \lim_{x\to\infty} \dfrac{f(x)}{1+2x^3}$

$A = \lim_{x\to\infty} \dfrac{x^3}{x^3} \dfrac{\left(\frac{1}{x} - 4\right)}{(1/x^3 + 2)}$

$= \lim_{x\to\infty} \dfrac{\frac{1}{x} - 4}{1/x^3 + 2}$

$= \dfrac{0-4}{0+2} = -2.$

$B = \lim_{x\to\infty} \dfrac{f(x)}{1+2x^3} = \dfrac{4}{\infty} = 0$

So, $L = A + B = -2 + 0 = -2.$

4. (6 points) *Using the definition of the derivative*, compute the derivative of the function $f(x) = \dfrac{1}{x+2}$.

You will get no credit for computing the derivative without using the definition!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+2 - (x+h+2)}{(x+h+2)(x+2)}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h+2)(x+2)} = -\frac{1}{(x+2)^2}$$

<u>Verification.</u> $f'(x) = \left((x+1)^{-1}\right)' = -(x+1)^{-2}\,(x+1)'$

$$= \frac{-1}{(x+1)^2} \cdot (1)$$

$$= -\frac{1}{(x+1)^2}.$$

5. For the questions on this page, assume that $f$ is a differentiable function such that $f(2) = 3$, $f'(2) = -1$, and $f''(2) = 4$. Simplify your answers to the following as much as you can without using a calculator.

(a) (4 points) Compute $g'(2)$ if $g(x) = x\cot(f(x))$.

$$g'(x) = (x)'\cot(f(x)) + x\left(\cot(f(x))\right)'$$
$$= \cot(f(x)) + x(-\operatorname{cosec}^2(f(x)))\cdot f'(x)$$
$$= \cot(f(x)) - x\operatorname{cosec}^2(f(x))\cdot f'(x)$$

$$\Rightarrow g'(2) = \cot(3) - 2\cdot\operatorname{cosec}^2(3)\cdot -1$$
$$= \boxed{\cot(3) + 2\operatorname{cosec}^2(3)}$$

(b) (4 points) Compute $h''(2)$ if $h(x) = \dfrac{f(x)}{x+1}$.

$$h'(x) = \frac{f'(x)(x+1) - f(x)}{(x+1)^2}$$

$$h''(x) = \frac{[f''(x)(x+1) + f'(x) - f'(x)](x+1)^2 - 2(f'(x)(x+1) - f(x))(x+1)}{(x+1)^4}$$

$$h''(2) = \frac{(4\cdot 3)(3^2) - 2((-1)(3) - 3)(3)}{3^4} = \frac{108 + 36}{81} = \boxed{\frac{144}{81}}$$

(c) (4 points) Compute $y'$ at $(2, 1)$ if $xy + y^2 = y^3 + 2$.

Implicit diff.

$$y + xy' + 2yy' = 3y^2y'$$
$$\Rightarrow xy' - 3y^2y' + 2yy' = -y$$
$$\Rightarrow (x - 3y^2 + 2y)y' = -y.$$
$$\Rightarrow y' = -y / (x - 3y^2 + 2y).$$

So, $y' = -1/(2-3+2) = \boxed{-1}$.

(d) (4 points) Compute the tangent line to the graph $y = \sqrt{3-x} + f(x^2 - 2)$ when $x = 2$.

$y_t = mx + b$

① $$y' = \frac{-1}{\sqrt{3-x}} + f'(x^2-2)\cdot(2x)$$

$$\Rightarrow y' = -1 + f'(2)\cdot 4$$
$$= -1 - 4$$
$$= -5$$

② $y_t = -5x + b$ & $(2,4)$ belongs to $y_t$

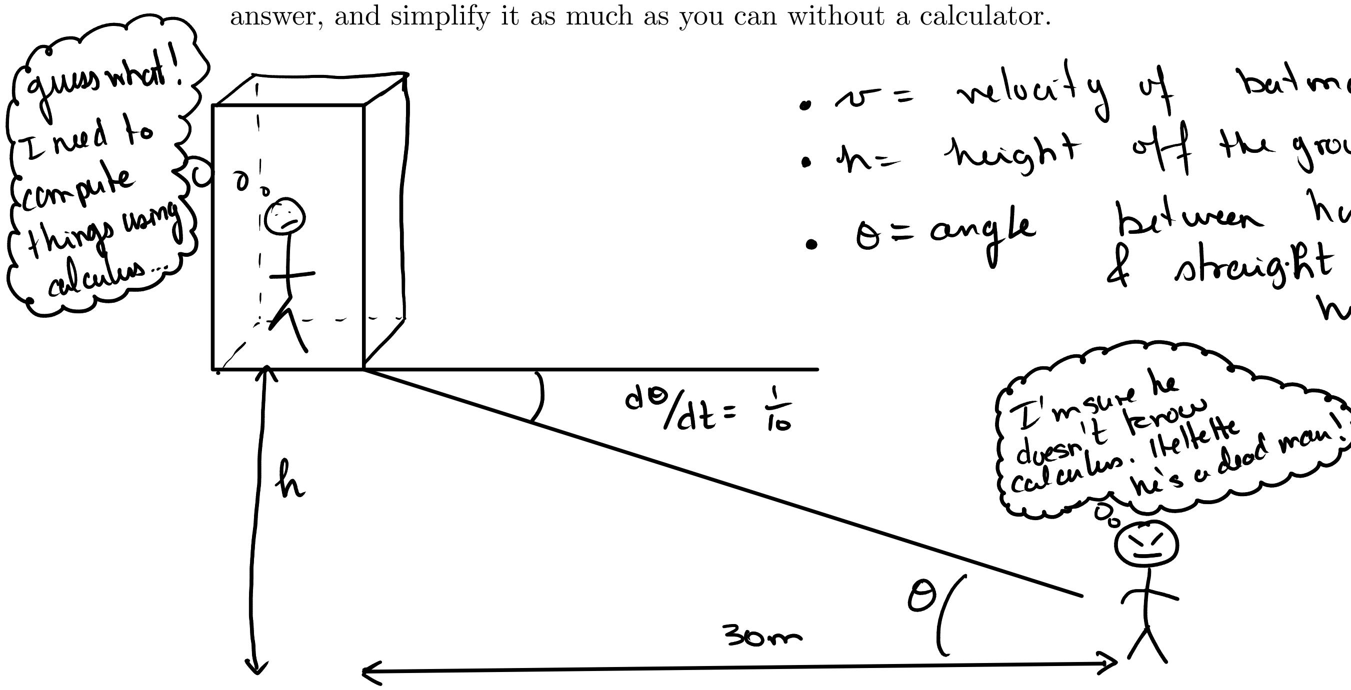
$$\Rightarrow 4 = -5\cdot 2 + b$$
$$\Rightarrow 14 = b.$$

$$\boxed{y_t = -5x + 14}$$

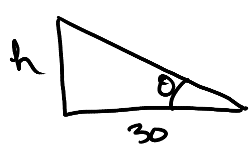
6. (6 points) Batman is trapped in a glass elevator, and needs to know how fast he is going to formulate an escape plan. The Riddler stands on the ground 30 meters away from the base of the elevator. Batman measures the angle between a horizontal line and the straight line between him and the Riddler: he finds the angle is changing at a constant rate of 1/10 radians / second.

How fast is Batman going up when he is $10\sqrt{3}$ meters off the ground? Include units in your answer, and simplify it as much as you can without a calculator.

- $v$ = velocity of batman
- $h$ = height off the ground
- $\theta$ = angle between horizontal & straight line between him & Riddler




We have $\frac{dh}{dt} = v$. From triangle identity:




$$\tan\theta = \frac{h}{30} \Rightarrow h = 30\tan\theta.$$

$\frac{3}{9} + \frac{6}{9}$ $\quad \frac{2}{3}$

Thus, $$\frac{dh}{dt} = 30\sec^2\theta \cdot \frac{d\theta}{dt}$$

When $h = 10\sqrt{3} \Rightarrow \tan\theta = \frac{10\sqrt{3}}{30} = \frac{\sqrt{3}}{3} \rightarrow \theta = \frac{\pi}{6}$

So, $$v = \frac{dh}{dt} = 30 \cdot \sec^2(\pi/6) \cdot \left(\frac{1}{10}\right) = 3 \cdot \frac{1}{(\cos\frac{\pi}{6})^2}$$

$$= 3 \cdot \frac{1}{(1/2)^2} = 3 \cdot 4 = 12$$

$$\Rightarrow \boxed{v = 12 \text{ meters/sec.}}$$

7. A function $f$ defined on the interval $[-3, 3]$ has *first derivative* $f'(x) = (x+1)\sqrt{4-x}$.

(a) (2 points) On what subintervals (if any) of $[-3, 3]$ is the function $f$ increasing, and on which is it decreasing?

$f'(x) = 0$ if $(x+1) = 0$ or $4-x=0$ if $x=-1$ & $\cancel{x=4}$.

| $x$ | $-3$ | | $-1$ | | $3$ |
|---|---|---|---|---|---|
| $x+1$ | | $-$ | $0$ | $+$ | |
| $\sqrt{4-x}$ | | $+$ | $+$ | $+$ | |
| $f'(x)$ | | $-$ | $0$ | $+$ | |

$f \nearrow$ on $[-1, 3)$

$f \searrow$ on $[3, -1]$

(b) (2 points) Where (if anywhere) does $f$ have a local minimum or a local maximum?

$f$ has a local min. at $x=-1$

(because $\searrow$ left of $x=-1$
$\nearrow$ right of $x=-1$)

(c) (5 points) On which subintervals (if any) of $[-3, 3]$ is the function $f$ concave up or concave down, and where (if anywhere) does it have an inflection point?

$$f''(x) = \sqrt{4-x} - \frac{(x+1)}{\sqrt{4-x}} = \frac{4-x-(x+1)}{\sqrt{4-x}}$$

$$= \frac{3-2x}{\sqrt{4-x}}$$

| $x$ | $-3$ | | $3/2$ | | $3$ |
|---|---|---|---|---|---|
| $3-2x$ | | $+$ | $0$ | $-$ | |
| $\sqrt{4-x}$ | | $+$ | $+$ | $+$ | |
| $f''(x)$ | | $+$ | $0$ | $-$ | |

$f \cup$ on $[-3, 3/2]$

$f \cap$ on $[3/2, 3]$

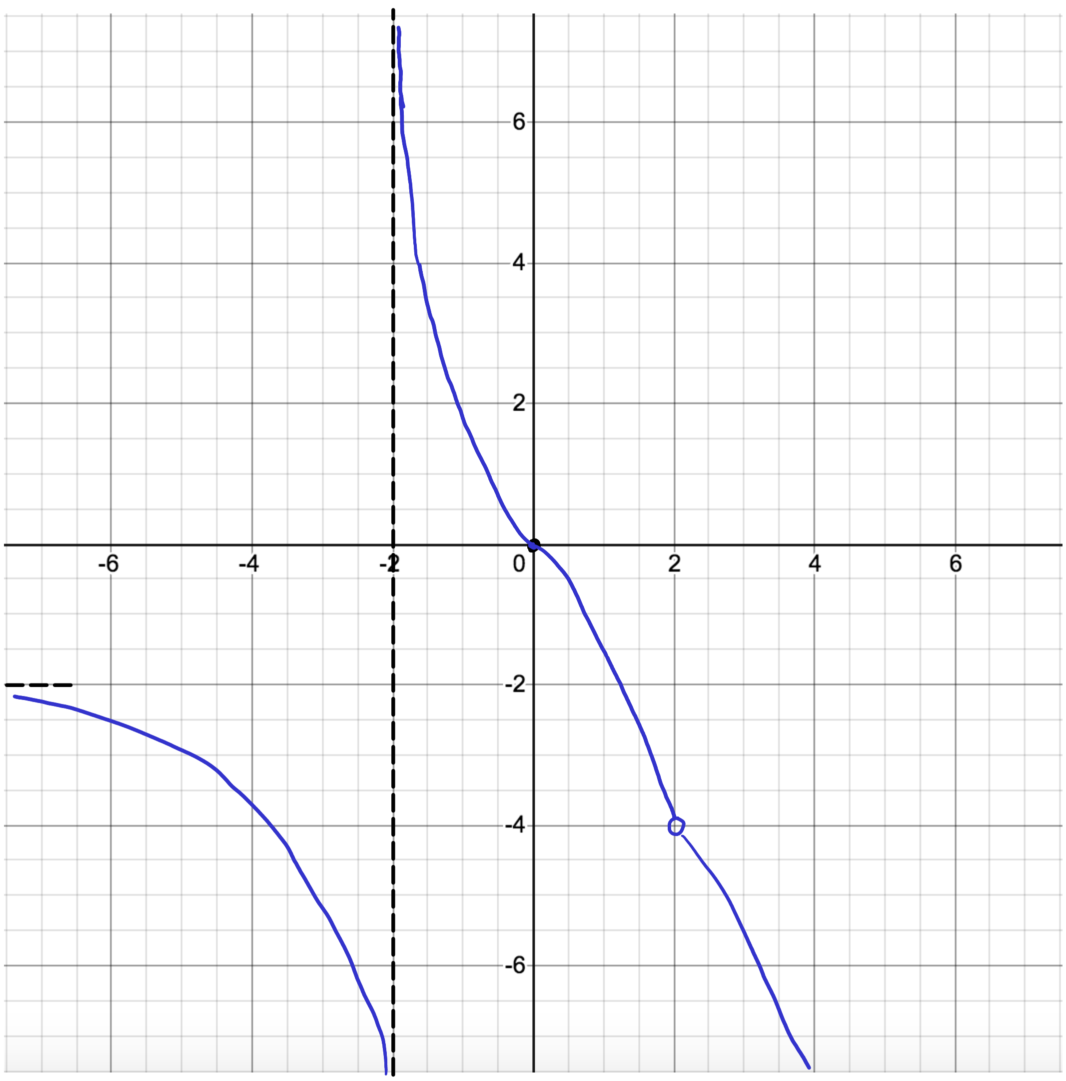
(d) (2 points) Where in $[-3, 3]$ does the absolute maximum of $f'$ occur?

It occurs at either $-3$ or $3$.

8. (6 points) Either using the axes below, or copying these axes[1], draw the graph $y = f(x)$ of a function $f$ that satisfies the following conditions:

- $f$ is an odd function.
- $f$ has a zero at $x = 0$.
- $f$ is defined and differentiable everywhere except $x = -2$ and $x = 2$.
- $f'(x) < 0$ everywhere it is defined.
- $\lim_{x \to -\infty} f(x) = -2$.
- $\lim_{x \to -2^-} f(x) = -\infty$ and $\lim_{x \to -2^+} f(x) = \infty$.
- $f''(x) > 0$ on $(-2, 0)$ and $(2, \infty)$, and $f''(x) < 0$ on $(-\infty, -2)$ and $(0, 2)$.

Your graph must be careful and accurate, and must unambiguously satisfy the conditions above. Include any asymptotes explicitly in the picture using dashed lines.




[1] If you copy the axes, make sure you label them with numbers as in the picture below.

9. You know that a differentiable function $g$ has derivatives and values as in the following table.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $g(x)$ | 7 | 5 | 3 | -1 |
| $g'(x)$ | -2 | -2 | -4 | 1 |

(a) (3 points) Use a linear approximation or differential to estimate how much $g$ changes when $x$ changes by $-0.1$ relative to $x = 2$.

$g(2) = 3$

$g(x) \approx g'(2)(x-2) + g(2)$

So, $g(1.9) \approx -4(-0.1) + 3 = 0.4 + 3 = 3.4$.

So, $g$ changes by $0.4$.

(b) (2 points) The function $g$ must have at least one zero in the interval $(2, 3)$. Explain why, making explicit which theorem(s), and which assumption(s) on $g$, you are using.

Use the IVT.

$g(2) = 3$

$g(3) = -1$

$g$ diff. $\Rightarrow$ $g$ continuous.

So, by IVT, there is a $c \in (2,3)$ s.t. $g(c) = 0$

(c) (2 points) The function $g'$ must equal $-3$ at least once in the interval $(1, 3)$. Explain why, making explicit which theorem(s), and which assumption(s) on $g$, you are using.

If we suppose that $g'$ is continuous, then we see that $g'(1) = -2$ and $g'(2) = -4$

Since $-3$ is between $-2$ & $-4$, by IVT, there must be a $c$ between $1$ & $2$ s.t. $g'(c) = -3$.

So, since $2 < 3$, there is a $c$ between $1$ & $3$ with $g'(c) = -3$.

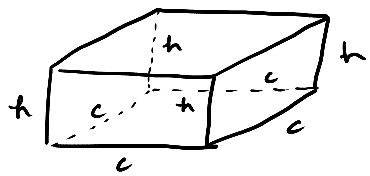
(d) (3 points) Use Newton's method to give an estimate $x_1$ for a zero of $g$ in the interval $(2, 3)$ starting from $x_0 = 2$.

$x_1 = x_0 - \dfrac{g(x_0)}{g'(x_0)} = 2 - \dfrac{g(2)}{g'(2)} = 2 - \dfrac{3}{-4} = 2 + \dfrac{3}{4}$

$= \dfrac{11}{4}$

So, $x_1 = \dfrac{11}{4}$.

10. (6 points) A cardboard box with a square bottom, rectangular sides, and no top, needs to be constructed to have a volume of 4 cubic feet. What is the least area of cardboard that can be used? Include units in your answer, and make sure you justify everything.




Total Volume $= 4 = c^2 \cdot h$.

$c =$ length side of the base
$h =$ height of the box.
$A =$ area of cardboard.

$$A = c^2 + 4ch$$

Since $V = 4 = c^2 h \rightarrow h = \frac{4}{c^2}$.

So, $A = c^2 + 4c \cdot \frac{4}{c^2} = c^2 + \frac{16}{c}$.

$\frac{dA}{dt} = 2c - \frac{16}{c^2} = 0$ iff $2c - \frac{16}{c^2} = 0$

iff * $\frac{2c^3 - 16}{c^2} = 0$

iff $2c^3 - 16 = 0$

iff $c^3 = 8$

iff $c = 2$

* $\frac{2c^3-16}{c^2} = \frac{2(c^3-8)}{c^2} = \frac{2(c-2)(c^2+2c+4)}{c^2}$

| $x$ | 0 | | 2 | |
|---|---|---|---|---|
| $2(c^3-8)$ | | $-$ | 0 | $+$ |
| $c^2$ | | $+$ | $+$ | $+$ |
| $A'$ | | $\searrow$ | 0 | $\nearrow$ |

By the 1st test, $c = 2$ is an absolute min.

So, $A = 2^2 + \frac{16}{2} = 4 + 8 = \boxed{12 \text{ ft}^2}$.

11. We use right endpoints and evenly spaced intervals to write an integral according to the definition, and get the following result.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\Big(\frac{i}{n}-1\Big)^2. \tag{1}$$

(a) (1 point) Which of the following does the limit above equal?

$$(i)\ \int_1^2 (x^2-1)dx, \quad (ii)\ \int_{-1}^1 x^2dx, \quad (iii)\ \int_{-1}^0 x^2dx.$$

(b) (4 points) Using the formulas $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, compute the limit in line (1) above.

$$\sum_{i=1}^{n}\frac{1}{n}\Big(\frac{i}{n}-1\Big)^2 = \sum_{i=1}^{n}\frac{1}{n}\Big(1-\frac{i}{n}\Big)^2 = \sum_{i=1}^{n}\frac{1}{n}\Big(\frac{n-i}{n}\Big)^2 = \frac{1}{n^3}\sum_{i=1}^{n}(n-i)^2$$

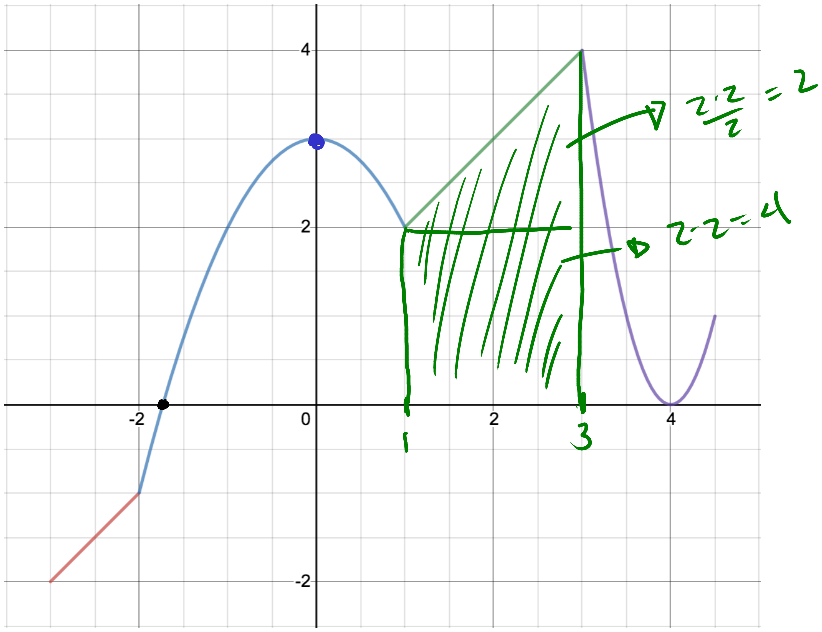
Here, $\sum_{i=1}^{n}(n-i)^2 = (n-1)^2+(n-2)^2+\cdots+2+1 = \sum_{i=1}^{n-1} i^2 = \frac{(n-1)n(2n-1)}{6}$

So, $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\Big(\frac{i}{n}-1\Big)^2 = \lim_{n\to\infty}\frac{n(2n^2-3n+1)}{6n^3} = \lim_{n\to\infty}\frac{n^3}{n^3}\Big(\frac{2-3/n+1/n^2}{6}\Big) = \frac{2}{6} = \boxed{\frac{1}{3}}$

(c) (2 points) Use your answer to part (a) to check your answer to part (b).

$$\int_{-1}^{0} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{0} = 0-\Big(-\frac{1}{3}\Big) = \frac{1}{3}.$$

12. Consider the graph of a function $f$ again.




For $x$ in $[-3, 4.5]$, define

$$g(x) = \int_{-3}^{x} f(t)dt.$$

(a) (1 point) On what interval(s) is $g$ decreasing? No justification needed, and a reasonably close estimate is fine.

$g'(x) = f(x)$. So $g$ decreasing if $f(x) < 0$.

So, $g$ decreasing on $[3, -1.8]$ (approx.)

(b) (1 point) What is $g'(0)$? No justification needed, and a reasonably close estimate is fine.

$g'(0) = f(0) = 3$

(c) (2 points) What is $g(3) - g(1)$? Justify your answer.

$g(3) - g(1) = \int_{-3}^{3} f(t)\,dt - \int_{-3}^{1} f(t)\,dt = \int_{-3}^{3} f(t) + \int_{1}^{-3} f(t)$

$= \int_{1}^{3} f(t)\,dt.$

= Area under curve on $[1,3]$

$= 4+2$

$= 6.$

13. Say $f$ is a differentiable function that satisfies $\int_1^2 f(x) = 1$ Compute the following. Simplify as much as you can.

(a) (3 points) $\int_1^2 (x^{-1/3} + f(x) + x^{1/3})dx$.

$$\int_1^2 x^{-1/3} + f(x) + x^{1/3}\,dx = \frac{x^{2/3}}{2/3}\Big|_1^2 + \int_1^2 f(x)dx + \frac{x^{4/3}}{4/3}\Big|_1^2$$

$$= \frac{3}{2}\left(4^{1/3} - 1\right) + 1 + \frac{16^{1/3} - 1}{4/3}$$

$$= \boxed{\frac{3}{2}\left(4^{1/3}-1\right) + \frac{3}{4}\left(16^{1/3}-1\right) + 1}$$

(b) (3 points) $\int (f'(x) + \sec^2(x))dx$ (leave your answer in terms of $f$ and related functions).

$$\int f'(x) + \sec^2(x)\,dx = \int f'(x)\,dx + \int \sec^2 x\,dx$$

$$\boxed{= f(x) + \tan x + C}$$

(c) (3 points) $\frac{d}{dx}\underbrace{\boxed{\int_{-x^3}^4 f(t)dt}}_{g(x)}$ (leave your answer in terms of $f$ and related functions).

$F(x) = \int_4^x f(t)\,dt$ . So

$$g(x) = -\int_4^{-x^3} f(t)\,dt = -F(-x^3).$$

$$\Rightarrow g'(x) = -F'(-x^3)\cdot(-x^3)'$$

$$= -f(-x^3)\cdot(-3x^2) = 3x^2 f(x^3).$$

14. (4 points) Consider the integral

$$I = \int_0^2 x^4 \tan(x^4)dx$$

Use the substitution to $u = x^4$ to transform this into an integral in terms of $u$ (do not solve the new integral).

$u = x^4 \rightarrow \frac{du}{dx} = 4x^3 \rightarrow du = 4x^3\,dx$

$\rightarrow \frac{x}{4}\,du = x^4\,dx$

So,

$$\int_0^2 x^4 \tan(x^4)\,dx = \int_0^{16} \tan(u)\,\frac{x}{4}\,du$$

But $u = x^4 \rightarrow x = u^{1/4}$ (because $u \geq 0$, don't have to take $-u^{1/4}$)

$$\rightarrow I = \int_0^{16} \tan(u)\,\frac{u^{1/4}}{4}\,du$$

15. (4 points) A bike accelerates according to the formula $a(t) = 2t - t^2$ m/s$^2$, for $0 \leq t \leq 3$ in terms of seconds. Find the displacement of the bike from its initial position after 3 seconds if the initial velocity is 4 m/s.

Anti-derivative of $a(t) \rightarrow t^2 - \frac{t^3}{3} + C = v(t)$

So, $v(0) = 3 \rightarrow 0 - \frac{0^3}{3} + C = 3$

$\rightarrow C = 3.$

So, $v(t) = t^2 - \frac{t^3}{3} + 3$

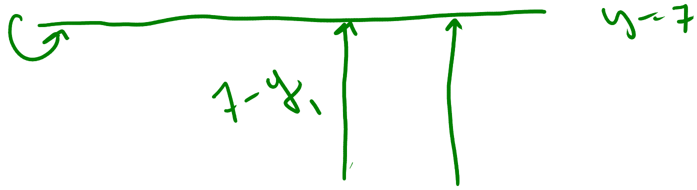
$$\text{displacement} = \int_0^3 v(t)\,dt = \left.\frac{t^3}{3} - \frac{t^4}{12} + 3t\right|_0^3$$

$$= \frac{27}{3} - \frac{81}{12} + 9$$

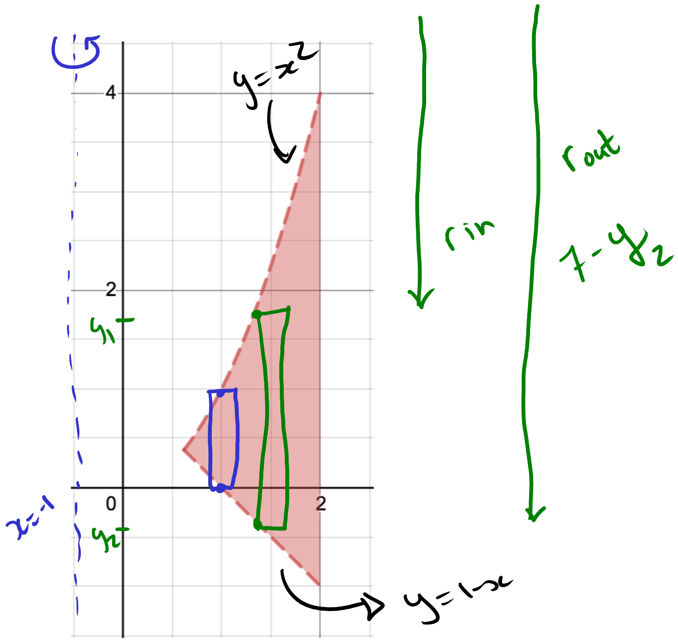
$$= 9 - \frac{27}{4} + 9$$

$$= \frac{18\cdot 4 - 27}{4} = \frac{72-27}{4}$$

$\boxed{\text{displ.} = \frac{45}{4} \text{ meters.}}$




16. The shaded region below is between the graphs of $y = 1-x$ and $y = x^2$ for $x$ positive, and $x \leq 2$.




(a) (3 points) Set up (do not solve) an integral for the area of the shaded region. Algebraically justify the bounds of integration you are using.

$x^2 = 1-x$

iff $x^2 + x - 1 = 0$

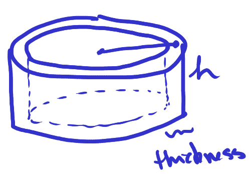
iff $x = \frac{-1 \pm \sqrt{1+4}}{2}$

iff $x = -1 \pm \sqrt{5} / 2$

we have to take $x \geq 0$, so $x = \frac{-1+\sqrt{5}}{2}$.

$$\boxed{A = \int_{\frac{-1+\sqrt{5}}{2}}^{2} x^2 - (1-x)\, dx}$$

(b) (4 points) Set up (do not solve) an integral for the volume of the region arrived at by rotating this region around the line $x = -1$.




thickness $= \Delta x \to dx$

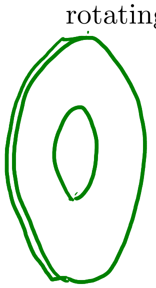
radius $= 1+x$

height $= x^2 - (1-x)$

so,

$$V = \int_{\frac{-1+\sqrt{5}}{2}}^{2} 2\pi(1+x)\left(x^2 - (1-x)\right) dx$$

(c) (4 points) Set up (do not solve) an integral for the volume of the region arrived at by rotating this region around the line $y = 7$.



$r_{in} = 7 - y_1 = 7 - x^2$

$r_{out} = 7 - y_2 = 7-(1-x) = 6-x$

thickness $= \Delta x \to dx$

$$V = \int_{\frac{-1+\sqrt{5}}{2}}^{2} \pi\left(r_{out}^2 - r_{in}^2\right) dx$$

$$\to \boxed{V = \int_{\frac{-1+\sqrt{5}}{2}}^{2} \pi\left((6-x)^2 - (7-x^2)^2\right) dx}$$