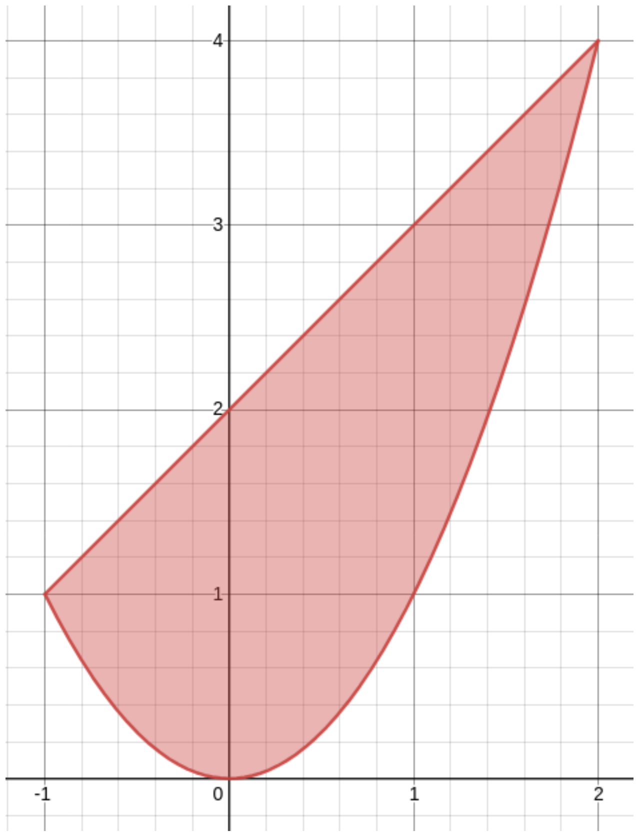
## Problem 8

The shape of the lamina is shown in the picture below. So we get




$$D = \{(x, y) \, : \, -1 \leq x \leq 2 \text{ and } x^2 \leq y \leq x + 2\}.$$

The density is $\phi(x) = kx^2$ for some constant $k$. Then

$$M = \int_{-1}^{2} \int_{x^2}^{x+2} kx^2 \, dydx = 63k/20.$$

The center of mass is given by $(\overline{x}, \overline{y})$. We then compute

$$\overline{x} = \frac{20}{63k} \int_{-1}^{2} \int_{x^2}^{x+2} kx^3 \, dydx = \frac{20}{63} \times \frac{18}{5} = \frac{8}{7}.$$

and

$$\overline{y} = \frac{20}{63} \int_{-1}^{2} \int_{x^2}^{x+2} x^2 y \, dydx = \frac{20}{63} \times \frac{531}{70} = \frac{118}{49}.$$

So the center of mass is $(\overline{x}, \overline{y}) = (8/7, 118/49)$.

## Problem 12 (only the mass)

We have $\rho(x, y) = k(x^2 + y^2)$. In polar coordinate, the disk is described as followed

$$D = \{(r, \theta) \,:\, 0 \leq r \leq 1 \text{ and } 0 \leq \theta \leq \pi/2\}.$$

Setting $x = r\cos\theta$ and $y = r\sin\theta$, we have $dA = r dr d\theta$ and so

$$M = \iint_D \rho(x, y)\, dA = \int_0^{\pi/2} \int_0^1 kr^2 r\, dr d\theta = k\pi/8.$$

## Problem 20

The fan is a suare with sides of length 2 with the lower left corner positioned at the origin, so

$$D = [0, 2] \times [0, 2].$$

We have to compare $I_x$ and $I_y$.

Firstly, we have

$$I_x = \iint_D y^2 \rho(x, y)\, dA = \int_0^2 \int_0^2 y^2(1 + 0.1x)\, dx dy = 88/15.$$

Secondly, we have

$$I_y = \iint_D x^2 \rho(x, y)\, dA = \int_0^2 \int_0^2 x^2(1 + 0.1x)\, dx dy = 92/15.$$

We see that $I_y > I_x$, and so it would be more difficult to rotate the fan blade around the $y$-axis.