MATH-244
Midterm 03

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November, Fall 2023

University of Hawai'i

Last name: _____

First name: _____

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all 5 questions below and write your answers directly on the questionnaire.
- You have 50 minutes to complete the exam.
- When you are done (or at the end of the 50min period), return your copy.
- Any electronic devices are not allowed during the exam.
- You can use a scientific calculator (not a graphical).
- **Turn off your cellphone(s) during the exam**.
- Lecture notes and the textbook are not allowed during the exam.
- You must show ALL your work to have full credit.
- Draw a square around your final answer.

Your Signature: _____

May the Force be with you!
Pierre Parisé

Question 1 **(10 pts)**

Consider the transformation

$$T(u, v) = (2u \cos v, 3u \sin v).$$

(a) (6 points) Find the Jacobian of the transformation T.

(b) (4 points) Using the transformation T, find the area of the region bounded by the ellipse of equation $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.

Question 2 **(10 pts)**

Let $\vec{F}(x, y) = \langle x - y, x \rangle$.

(a) (5 points) Match this vector field with its plot. Explain you choice.

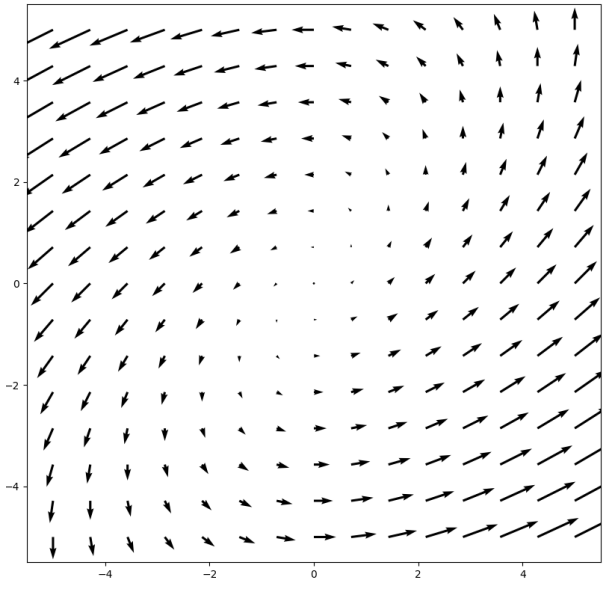

Figure I

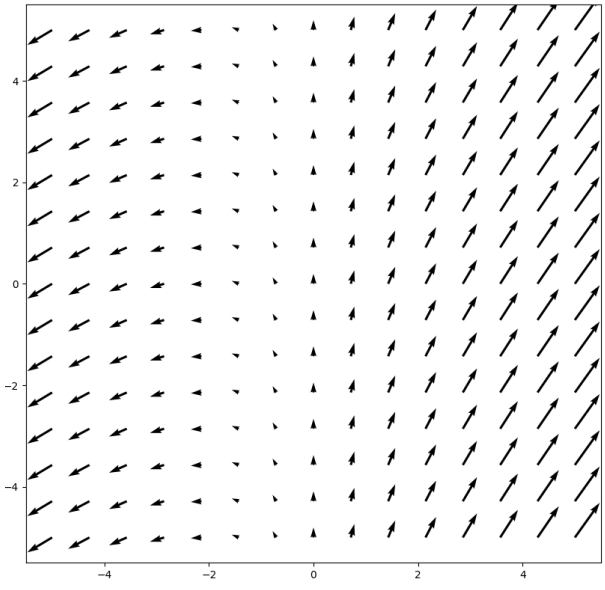

Figure II

(b) (5 points) Is it a conservative vector field?

Question 3 **(10 pts)**

Evaluate the following integrals.

(a) (5 points) $\int_C x\,ds$, where C is the line segment from $(0,0)$ to $(2,4)$.

(b) (5 points) $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y) = \langle xy^2, -x^2\rangle$ and $\vec{r}(t) = \langle t^3, t^2\rangle$, $0 \le t \le 1$.

QUESTION 4 **(10 pts)**

Consider the following vector field:

$$\vec{F}(x,y) = \left\langle 2xy + \frac{y^3}{3}, x^2 + xy^2 \right\rangle.$$

(a) Is $\vec{F}$ conservative? If so, find a function f such that $\vec{\nabla} f = \vec{F}$.

(b) Evaluate the integral

$$\int_C \vec{F} \cdot d\vec{r}$$

along the path C parametrized by $\vec{r}(t) = \left\langle \left(\frac{t}{\pi}\right)^2 - \sin(t), \frac{t}{\pi} + \left(\frac{t}{\pi}\right)^2 \cos(t+\pi) \right\rangle$, where $0 \leq t \leq \pi$.

Question 5 **(10 pts)**

Evaluate the integral

$$\int_C (y\cos x - xy\sin x)\,dx + (xy + x\cos x)\,dy,$$

where C is the rectangle with vertices $(0,0)$, $(0,4)$, $(2,4)$ and $(2,0)$.

Bonus Question

Let C be a closed path surrounding the origin and parametrized by $\vec{r}(t)$. Let $\vec{F}$ be the vector field

$$\vec{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle.$$

What is the value of $\int_C \vec{F} \cdot d\vec{r}$?