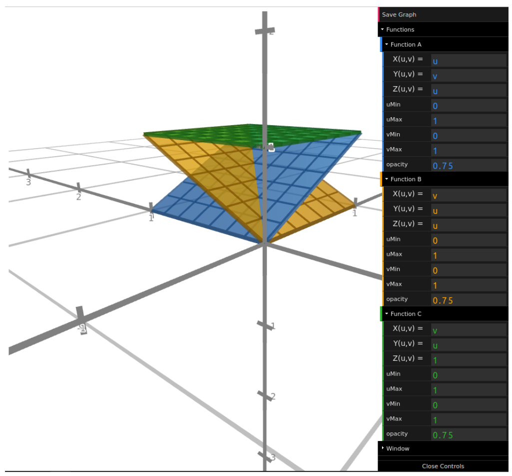
Section 15.6, Problem 36 (10 Pts)

Based on the bounds in each integral, we can describe E as followed:

$$E = \{(x, y, z) : 0 \leq x \leq z,\, 0 \leq y \leq 1,\, y \leq z \leq 1\}$$

The solid is described as a Type 2 and is illustrated in the picture below. The solid is enclosed by the blue, yellow, and green planes.



We will describe it as a Type 3, so we need to bound the y values. The y values will be bounded by $y = 0$ and $y = z$ (the plane in yellow in the picture). Then the shadow of the solid in the XZ-plane will be a triangular region that can be described as followed:

$$D = \{(x, z) : 0 \leq x \leq z,\, 0 \leq z \leq 1\}.$$

Therefore, the integral can be rewritten as

$$\int_0^1 \int_0^z \int_0^z f(x, y, z)\, dydxdz.$$

We can also describe is as a Type 1. But we have to split into two integrals because the bounds for the z values will change (from the blue plane to the yellow plane). The shadow of the object in the XY-plane is a square $[0, 1] \times [0, 1]$. The two planes $x = z$ and $y = z$ meets exactly when $y = x$. We will therefore divide the square $[0, 1] \times [0, 1]$ along the line $y = x$ into two rectangular regions, call them R_1, and R_2. We have

$$R_1 = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x\} \quad \text{and} \quad R_2 = \{(x, y) : 0 \leq x \leq 1, x \leq y \leq 1\}.$$

On R_1, we have $x \leq z \leq 1$. On R_2, we have $y \leq z \leq 1$. Therefore, the integral takes the following form:

$$\int_0^1 \int_0^x \int_x^1 f(x, y, z)\, dzdydx + \int_0^1 \int_x^1 \int_y^1 f(x, y, z)\, dzdydx.$$

Notice that we could change the order of $dxdz$ to $dzdx$ and $dydx$ to $dxdy$ respectively.

Section 15.7, Problem 10 **(10 Pts)**

a) We set $x = r\cos\theta$ and $y = r\sin\theta$ and $z = z$. Therefore,
$$2x^2 + 2y^2 - z^2 = 2r^2\cos^2(\theta) + 2r^2\sin^2(\theta) - z^2 = 2r^2 - z^2$$
and the equation is $2r^2 - z^2 = 4$.

b) We set $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$. Therefore,
$$2x - y + z = 2r\cos\theta - r\sin\theta + z = r(2\cos\theta - \sin\theta) + z.$$
The equation becomes $r(2\cos\theta - \sin\theta) + z = 1$.

Section 15.7, Problem 18 **(10 Pts)**

The solid E can be described easily as a type 1. The z-values are bounded below by $z = x^2 + y^2$ and above by $z = 4$. The shadow created in the xy-plane is a circular region bounded by the circle $x^2 + y^2 = 4$ of radius 2. Therefore, using polar coordinates in the xy-plane, we have
$$\begin{aligned}\iiint_E z\,dV = \int_0^{2\pi}\int_0^2\int_{r^2}^4 z\,dz\,r\,dr\,d\theta &= \int_0^{2\pi}\int_0^2 \frac{16 - r^4}{2}\,r\,dr\,d\theta \\ &= \Big(\int_0^2 8r - \frac{r^5}{2}\,dr\Big)\Big(\int_0^{2\pi} d\theta\Big) \\ &= (32/3)(2\pi) \\ &= \frac{64\pi}{3}.\end{aligned}$$
△

Section 15.7, Problem 22 **(10 Pts)**

We will describe the solid enclosed by the sphere and the cylinder as a type 1. This is because the z-values are restricted by the sphere in the following way:
$$-\sqrt{4 - x^2 - y^2} \le z \le \sqrt{4 - x^2 - y^2}.$$
The shadow in the xy-plane will simply be the inside of the circle, that is a circular shape bounded by the circle $x^2 + y^2 = 1$. Therefore, in cylindrical coordinates:
$$E = \{(r, \theta, z) : 0 \le r \le 1, 0 \le \theta \le 2\pi, -\sqrt{4 - x^2 - y^2} \le z \le \sqrt{4 - x^2 - y^2}\}.$$
The volume is given by the triple integral of 1 over the solid E. Therefore,
$$\begin{aligned}\mathrm{Vol}(E) = \iiint_E 1\,dV &= \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} dz\,r\,dr\,d\theta \\ &= \int_0^{2\pi}\int_0^1 2r\sqrt{4 - r^2}\,dr\,d\theta \\ &= \Big(\int_0^1 2r\sqrt{4 - r^2}\,dr\Big)\Big(\int_0^{2\pi} d\theta\Big) \\ &= \Big(\int_3^4 \sqrt{u}\,du\Big)2\pi \\ &= \Big(\frac{16}{3} - 2\sqrt{3}\Big)2\pi.\end{aligned}$$
△

Section 15.7, Problem 30 **(10 Pts)**

From the bounds in the integrals, we see that

$$E = \{(x, y, z) \,:\, -3 \leq x \leq 3,\, 0 \leq y \leq \sqrt{9 - x^2},\, 0 \leq z \leq 9 - x^2 - y^2\}.$$

The region D described by (x, y) such that $-3 \leq x \leq 3$ and $0 \leq y \leq \sqrt{9 - x^2}$ is in fact a circular region bounded by the circle $x^2 + y^2 = 9$. With this observation and by using cylindrical coordinates, we see that

$$E = \{(r, \theta, z) \,:\, 0 \leq r \leq 3, 0 \leq \theta \leq \pi, 0 \leq z \leq 9 - r^2\}.$$

Therefore,

$$\begin{aligned} \iiint_E \sqrt{x^2 + y^2}\, dV = \int_0^{2\pi} \int_0^3 \int_0^{9-r^2} r\, dz r dr d\theta &= \int_0^{2\pi} \int_0^3 r^2(9 - r^2)\, dr d\theta \\ &= \left(\int_0^3 9r^2 - r^4\, dr \right) \left(\int_0^{2\pi} d\theta \right) \\ &= \frac{162\pi}{5} \end{aligned}$$

$\triangle$