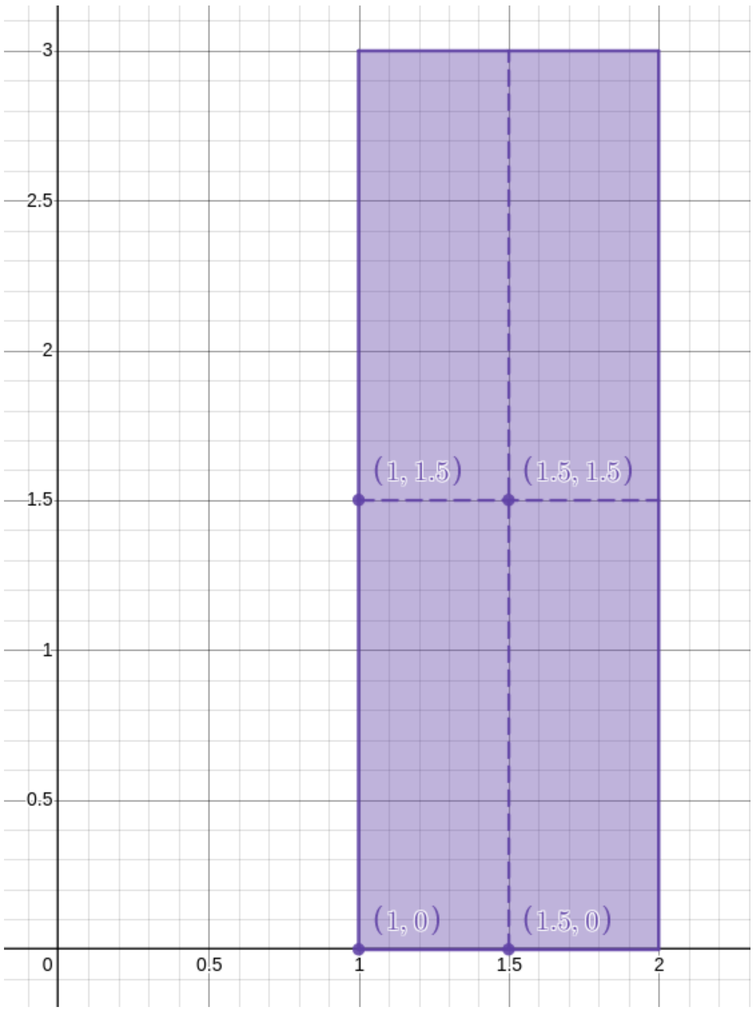
Section 15.1, Problem 4a **(10 Pts)**

We have $\Delta x = (2-1)/2 = 0.5$ and $\Delta y = (3-0)/2 = 1.5$. The rectangle is then divided in the way shown in the figure below:



In the above picture, the coordinates of the bottom left corners of each sub-rectangle are $(x_{11}^*, y_{11}^*) = (1, 0)$, $(x_{12}^*, x_{12}^*) = (1, 1.5)$, $(x_{21}^*, y_{21}^*) = (1.5, 0)$, and $(x_{22}^*, y_{22}^*) = (1.5, 1.5)$. The Area of each sub-rectangle is $0.5 \cdot 1.5 = 0.75$. We first have

$$(1 + 1^2 + (3)(0)) + (1 + 1^2 + (3)(1.5)) + (1 + 1.5^2 + (3)(0)) + (1 + 1.5^2 + (3)(1.5)) = 19.5,$$

and then

$$V \approx 19.5 \cdot 0.75 = 14.625.$$

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Section 15.1, Problem 16 (10 Pts)

Setting $u = x + y$,

$$\int_0^1 (x+y)^2\,dx = \int_y^{1+y} u^2\,du = \frac{(1+y)^3}{3} - \frac{y^3}{3}.$$

Then,

$$\begin{aligned}\int_0^1\int_0^1 (x+y)^2\,dxdy &= \int_0^1 \left(\frac{(1+y)^3}{3} - \frac{y^3}{3}\right)dy \\ &= \int_0^1 \frac{(1+y)^3}{3}\,dy - \int_0^1 \frac{y^3}{3}\,dy.\end{aligned}$$

Setting $v = 1 + y$,

$$\begin{aligned}\int_0^1 \frac{(1+y)^3}{3}\,dy &= \int_1^2 \frac{v^3}{3}\,dv \\ &= \frac{2^4}{12} - \frac{1}{12} = \frac{15}{12}.\end{aligned}$$

Also,

$$\int_0^1 \frac{y^3}{3}\,dy = \frac{1}{12} - 0 = \frac{1}{12}.$$

So,

$$\int_0^1\int_0^1 (x+y)^3\,dxdy = \frac{15}{12} - \frac{1}{12} = \frac{7}{6} \approx 1.1667. \qquad \triangle$$

Section 15.1, Problem 20 (10 Pts)

With $u = \ln y$, we have $du = \frac{dy}{y}$ and so

$$\int_1^5 \frac{\ln y}{xy}\,dy = \int_0^{\ln 5} \frac{u}{x}\,du = \frac{(\ln 5)^2}{2x}.$$

Then,

$$\begin{aligned}\int_1^3\int_1^5 \frac{\ln y}{xy}\,dydx &= \int_1^3 \frac{(\ln 5)^2}{2x}\,dx \\ &= \frac{(\ln 5)^2}{2}(\ln 3 - \ln 1) \\ &= \frac{(\ln 5)^2(\ln 3)}{2} \approx 1.4228. \qquad \triangle\end{aligned}$$

Section 15.1, Problem 22 (10 Pts)

Using the fact that $e^{x-y} = e^x e^{-y}$,

$$\begin{aligned}\int_0^1 \int_0^2 ye^{x-y}\,dxdy &= \int_0^1 \int_0^2 e^x ye^{-y}\,dxdy \\ &= \Big(\int_0^2 e^x\,dx\Big)\Big(\int_0^1 ye^{-y}\,dy\Big).\end{aligned}$$

We compute

$$\int_0^2 e^x\,dx = e^2 - 1.$$

Then, from an integration by parts,

$$\begin{aligned}\int_0^1 ye^{-y}\,dy &= (-ye^{-y})\big|_0^1 + \int_0^1 e^{-y}\,dy \\ &= -e^{-1} + (1 - e^{-1}).\end{aligned}$$

Therefore,

$$\begin{aligned}\int_0^1 \int_0^2 ye^{x-y}\,dxdy &= (e^2-1)(-e^{-1}+1-e^{-1}) \\ &= -e + e^{-1} + e^2 - 1 - e + e^{-1} \\ &= -1 + 2e^{-1} + (e-2)e \\ &\approx 1.6883.\end{aligned}$$

△

Section 15.1, Problem 34 (10 Pts)

Using Fubini's Theorem,

$$\iint_R \frac{1}{1+x+y}\,dA = \int_1^2 \int_1^3 \frac{1}{1+x+y}\,dxdy.$$

Letting $u = 1+x+y$, we have $du = dx$, so that

$$\begin{aligned}\int_1^3 \frac{1}{1+x+y}\,dx &= \int_{2+y}^{4+y} \frac{1}{u}\,du \\ &= \ln(4+y) - \ln(2+y).\end{aligned}$$

Then,

$$\begin{aligned}\int_1^2 \int_1^3 \frac{1}{1+x+y}\,dxdy &= \int_1^2 \ln(4+y) - \ln(2+y)\,dy \\ &= \int_1^2 \ln(4+y)\,dy - \int_1^2 \ln(2+y)\,dy.\end{aligned}$$

From an integration by part with $u = \ln(4+y)$ and $dv = dx$, we obtain

$$\begin{aligned}\int_1^2 \ln(4+y)\,dy &= \Big[(4+y)\ln(4+y) - y\Big]\Big|_1^2 \\ &= 6\ln(6) - 5\ln(5) - 1\end{aligned}$$

and similarly, we obtain

$$\int_1^2 \ln(2+y)\,dy = 4\ln(4) - 3\ln(3) - 1.$$

Therefore,

$$\begin{aligned}\int_1^2 \int_1^3 \frac{1}{1+x+y}\,dxdy &= 6\ln(6) - 5\ln(5) - 1 - 4\ln(4) + 3\ln(3) + 1 \\ &= 9\ln(3) - 2\ln(2) - 5\ln(5) \\ &\approx 0.4540.\end{aligned}$$

△

Total: 50 Pts.