MATH-244 | Created by Pierre-O. Parisé
Final 120min (2h) | December, Fall 2023

University of Hawai'i

Last name: ______________________________

First name: ______________________________

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							

Instructions:

- Write your complete name on your copy.
- Answer all 6 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 2h period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam**.
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: ______________________________

May the Force be with you!
Pierre Parisé

Question 1 **(20 pts)**

Here is a parametrization of a surface S:

$$\vec{r}(u,v) = \left\langle u^3 - u, v^2, u^2 \right\rangle$$

for $-1 \leq u \leq 1$ and $-1 \leq v \leq 1$.

(a) (5 points) Is the point $P = (0,0,1)$ lie on the surface S?

(b) (5 points) Is the point $Q = (0,0,1/4)$ lie on the surface S?

(c) (5 points) Find the equation of the tangent plane to the surface at $u = 1/3$, $v = 1/2$.

(d) (5 points) Find an expression of $\vec{r}_u \times \vec{r}_v$.

QUESTION 2 **(20 pts)**

Evaluate the following surface integrals using only the definition. Recall that

$$dS = |\vec{r}_u \times \vec{r}_v|\, dA \quad \text{and} \quad d\vec{S} = \vec{r}_u \times \vec{r}_v\, dA.$$

Let S be the part of the plane $-2x - 3y + z = 1$ that lies above the rectangle $[0, 3] \times [0, 2]$.

(a) (10 points) $\displaystyle\iint_S z\, dS$.

(b) (10 points) $\displaystyle\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F}(x, y, z) = \langle -2x - 3y, 0, -2z \rangle$.

QUESTION 3 **(20 pts)**

Recall that the *curl* of a vector field $\vec{F} = \langle P, Q, R\rangle$ is given by

$$\operatorname{curl} \vec{F} = \vec{\nabla} \times \vec{F}.$$

Using the *curl*, determine wheter or not the following vector fields are conservative. **If it is conservative, find a function f such that $\vec{F} = \vec{\nabla} f$.**

(a) (10 points) $\vec{F}(x, y, z) = \langle z\cos y, xz\sin y, x\cos y\rangle$.

(b) (10 points) $\vec{F}(x, y, z) = \langle 1, \sin z, y\cos z\rangle$.

QUESTION 4 **(20 pts)**

Recall the identity in Stoke's Theorem:

$$\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r},$$

where S is a surface and C is the boundary (the "edge") of the surface.

(a) (5 points) Let $\vec{F}$ be a generic vector field. Let S_1 be the surface $x^2 + y^2 + z^2 = 1$, with $z \geq 0$ and let S_2 be the paraboloid $z = 2(1 - x^2 - y^2)$. Explain why

$$\iint_{S_1} \operatorname{curl} \vec{F} \cdot d\vec{S} = \iint_{S_2} \operatorname{curl} \vec{F} \cdot d\vec{S}.$$

(b) (15 points) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = \langle x + y^2, y + z^2, z + x^2 \rangle$ and C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

QUESTION 5 **(10 pts)**

Recall the Divergence Theorem:

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F} \, dV,$$

where S is a closed surface with the outward orientation and E is the solid enclosed within S. Recall that $\operatorname{div}\vec{F} = \vec{\nabla} \cdot \vec{F}$.

Use the Divergence Theorem to compute the flux of $\vec{F} = \langle xye^z, xy^2z^3, -ye^z \rangle$ through the surface S of the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, and $z = 1$.

Question 6 **(10 pts)**

Let C be a generic loop that lies in the plane $x + y + z = 1$ and let S be the surface enclosed by the curve in the plane $x + y + z = 1$. Show that the line integral

$$\int_C z dx + 2x dy - 3y dz$$

is equal to zero.