Section 16.6, Problem 2 (5 Pts)

For the point P, we have $x = 1$, $y = 2$, $z = 1$, so that

$$\vec{r}(u,v) = \langle 1,2,1\rangle \iff \begin{cases} 1+u-v = 1 \\ u+v^2 = 2 \\ u^2-v^2 = 1. \end{cases} \iff \begin{cases} u-v = 0 \\ u+v^2 = 2 \\ u^2-v^2 = 1. \end{cases}$$

Notice that the third equation can be written as $(u-v)(u+v) = 1$. Using the first equation $u-v = 0$ and plugging that equation in the third, we see that for $\vec{r}(u,v) = \langle 1,2,1\rangle$, then $0(u+v) = 1$ which is equivalent to $0 = 1$. This is impossible and therefore the point P is not on the surface.
For the point Q, we have $x = 2$, $y = 3$ and $z = 3$, so that

$$\vec{r}(u,v) = \langle 2,3,3\rangle \iff \begin{cases} 1+u-v = 2 \\ u+v^2 = 3 \\ u^2-v^2 = 3. \end{cases} \iff \begin{cases} u-v = 1 \\ u+v^2 = 3 \\ (u-v)(u+v) = 3. \end{cases} \iff \begin{cases} u+v^2 = 3 \\ u+v = 3. \end{cases}$$

Subtracting $u+v = 3$ from $u+v^2 = 3$, we get $v^2 - v = 0$. The solutions are $v = 0$ and $v = 1$. Therefore, $u = 3$ if $v = 0$ and $u = 2$ if $v = 1$. In this situation, we get

$$\vec{r}(3,0) = \langle 4,3,3\rangle \neq \vec{OQ} \quad \text{and} \quad \vec{r}(2,1) = \langle 1,2,1\rangle = \vec{OQ}.$$

Therefore, since $\vec{r}(2,1) = \vec{OQ}$, the point Q lies on the surface.

Section 16.6, Problem 4 (5 Pts)

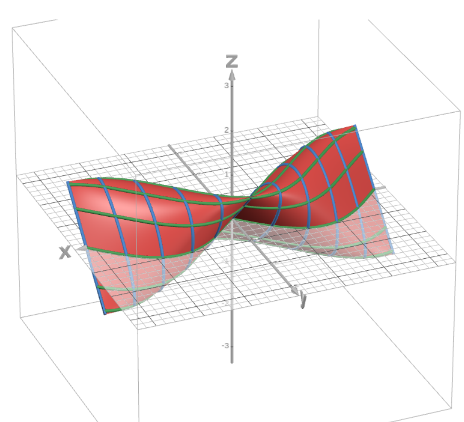
We have $y = u\cos v$ and $z = u\sin v$. Hence

$$y^2 + z^2 = u^2\cos^2(v) + u^2\sin^2(v) = u^2 = x.$$

This is a paraboloid with the x-axis as its main axis.

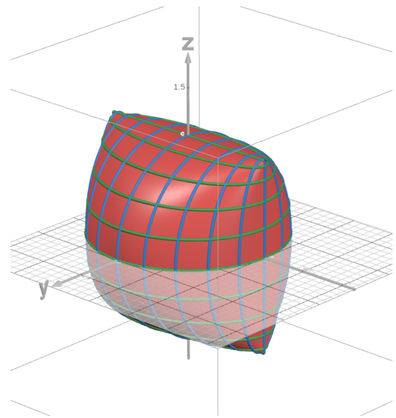
Section 16.6, Problem 10 (5 Pts)

In the picture on the right, the line in green represents $u =$ constant and the lines in blue, $v =$ constant. Here is the link to the Desmos app: `https://www.desmos.com/3d/8bb09174fc`.



Section 16.6, Problem 12 **(5 Pts)**

In the picture on the right, the line in green represents u = constant and the lines in blue, v = constant. Here is the link to the Desmos app: `https://www.desmos.com/3d/67e0b4d314`.



Section 16.6, Problem 22 **(5 Pts)**

We put $x = v\cos u$ and $z = \frac{v}{\sqrt{3}}\sin u$. Since we want the part of the ellipsoid on the left of the xz-plane, we get from the equation of the ellipsoid:

$$x^2 + 2y^2 + 3z^2 = 1 \iff y = -\sqrt{\frac{1 - x^2 - 3z^2}{2}} = -\sqrt{\frac{1 - v^2}{2}}$$

Hence,

$$\vec{r}(u, v) = (v\cos u, -(1/2)\sqrt{1 - v^2}, (v/\sqrt{3})\sin(u)),$$

with $0 \le u \le 2\pi$ and $0 \le r \le 1$.

Section 16.6, Problem 24 **(5 Pts)**

Let $x = 3\cos(u)$ and $z = 3\sin(u)$. To make sure we get the part of the cylinder above the xy-plane, we let $u \in [0, \pi]$. We also let $y = v$, with $-4 \le v \le 4$. Hence,

$$\vec{r}(u, v) = (3\cos(u), v, 3\sin(u)),$$

with $0 \le u \le \pi$ and $-4 \le v \le 4$.

Section 16.6, Problem 42 **(10 Pts)**

Here, we parametrize the cone using cartesian coordinates:

$$\vec{r}(x, y) = \left\langle x, y, \sqrt{x^2 + y^2} \right\rangle.$$

The region D of interest is

$$D = \{(x, y) \,:\, 0 \le x \le 1, x^2 \le y \le x\}.$$

We have

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & \frac{x}{\sqrt{x^2+y^2}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2}} \end{vmatrix} = \left\langle \frac{-x}{\sqrt{x^2 + y^2}}, \frac{-y}{\sqrt{x^2 + y^2}}, 1 \right\rangle.$$

Therefore,

$$dS = \left(\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1\right)^{1/2} = \sqrt{2}.$$

Hence,

$$\text{Area}(S) = \iint_S dS = \iint_D \sqrt{2}\, dA = \int_0^1 \int_{x^2}^{x} \sqrt{2}\, dydx = \sqrt{2}\int_0^1 x - x^2\, dx = \frac{\sqrt{2}}{6}.$$

Section 16.6, Problem 44 (10 Pts)

A parametrization of the surface is

$$\vec{r}(x,y) = \left\langle x, y, 4 - 2x^2 + y \right\rangle$$

where $0 \le x \le 1$ and $0 \le y \le x$. We then get

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -4x \\ 0 & 1 & 1 \end{vmatrix} = \langle 4x, -1, 1 \rangle$$

and so

$$dS = \sqrt{2 + 16x^2}\, dA.$$

Hence,

$$\begin{aligned} \text{Area}(S) = \iint_D \sqrt{2 + 8x^2}\, dA &= \int_0^1 \int_0^x \sqrt{2 + 16x^2}\, dydx \\ &= \int_0^1 x\sqrt{2 + 16x^2}\, dx \\ &= \frac{1}{16}\int_2^{18} \sqrt{u}\, du \\ &= \frac{1}{48}\left(18^{3/2} - 2^{3/2}\right). \end{aligned}$$