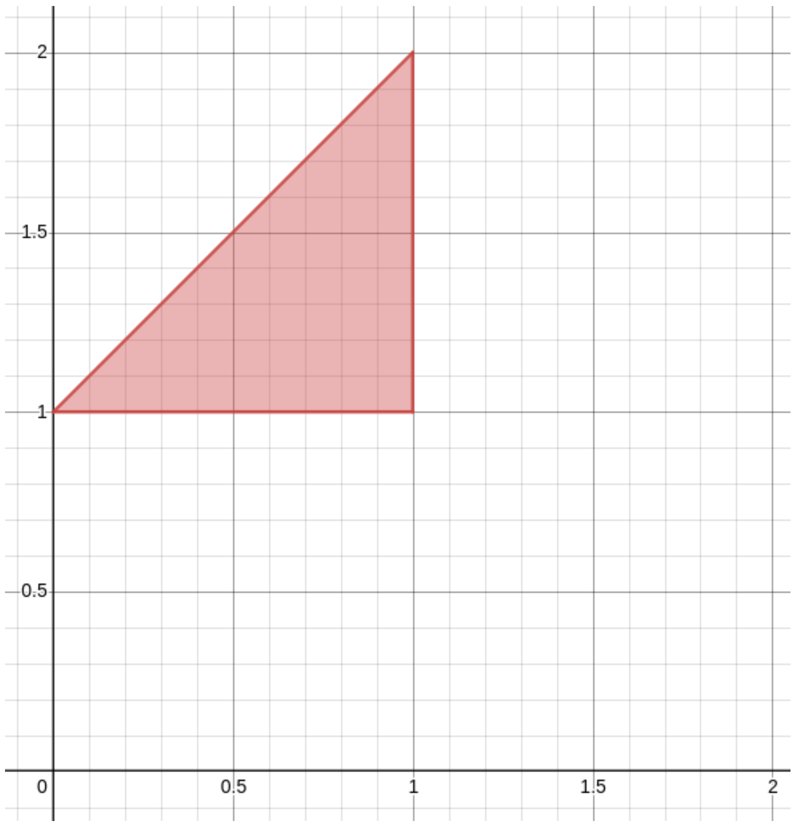
Section 15.2, Problem 8 (5 Pts)

The region D is a type II domain, with $h_1(x) = y - 1$ and $h_2(x) = 1$. See the picture below.



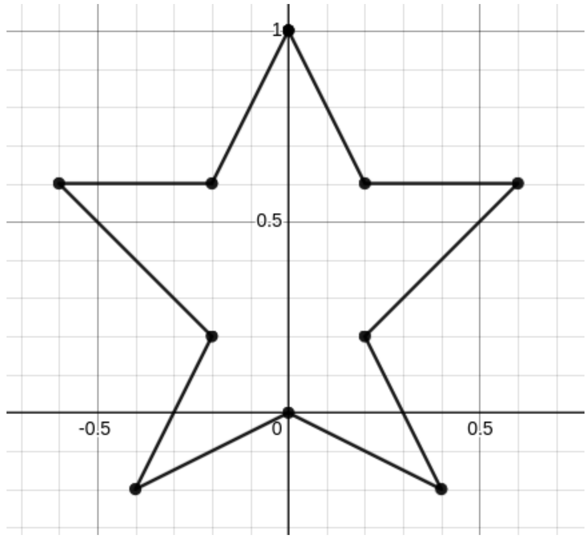
from the formula,

$$\begin{aligned}
\iint_D (2x + y)\, dA &= \int_1^2 \int_{y-1}^1 2x + y\, dx dy \\
&= \int_1^2 (x^2 + xy)\big|_{y-1}^1\, dy \\
&= \int_1^2 1 + y - \Big((y-1)^2 + (y-1)y\Big)\, dy \\
&= \int_1^2 1 + y - y^2 + 2y - 1 - y^2 + y\, dy \\
&= \int_1^2 4y - 2y^2\, dy \\
&= (2y^2 - (2/3)y^3)\big|_1^2 = 4/3.
\end{aligned}$$

The answer should therefore by 4/3.

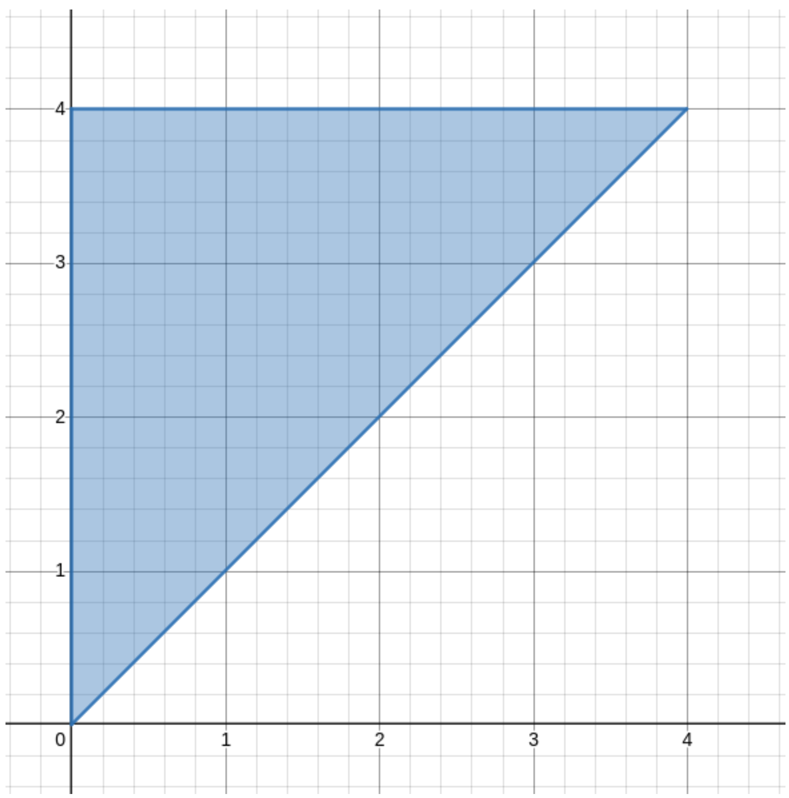
Section 15.2, Problem 12b **(5 Pts)**

Here is an example of a region D which is neither of Type I nor of Type II.



Section 15.2, Problem 16 **(10 Pts)**

The region is illustrated below.



Type I. To integrate firstly in y (inner integral) and secondly in x (outer integral), we need to give a description of D. We have

$$D = \{(x, y) : 0 \leq x \leq 4 \text{ and } x \leq y \leq 4\}.$$

Therefore,

$$\iint_D y^2 e^{xy}\, dA = \int_0^4 \int_x^4 y^2 e^{xy}\, dydx.$$

But this integral is hard because, after integrating by parts two times, we get

$$\iint_D y^2 e^{xy}\, dA = \int_0^4 \left(\frac{y^2 e^{xy}}{x} - \frac{2ye^{xy}}{x^2} + \frac{2e^{xy}}{x^3}\right)\Bigg|_x^4 dx.$$

This is really hard to integrate! We instead consider the region as a Type II.

Type II. The description of the region as a type II is

$$D = \{(x, y) \,:\, 0 \le x \le y \text{ and } 0 \le y \le 4\}.$$

Therefore,

$$\begin{aligned}
\iint_D y^2 e^{xy}\, dA &= \int_0^4 \int_0^y y^2 e^{xy}\, dxdy \\
&= \int_0^4 ye^{xy}|_0^y\, dy \\
&= \int_0^4 ye^{y^2} - y\, dy \\
&= \left(\frac{e^{y^2}}{2} - \frac{y^2}{2}\right)\Bigg|_0^4 \\
&= \frac{e^{16} - 17}{2}.
\end{aligned}$$

Section 15.2, Problem 26 **(10 Pts)**

The domain D is

$$D = \{(x, y) \,:\, x \ge 0, y \ge 0, x + y = 2\}.$$

We will describe D as a type I domain:

$$D = \{(x, y) \,:\, 0 \le x \le 2 \text{ and } 0 \le y \le 2 - x\}.$$

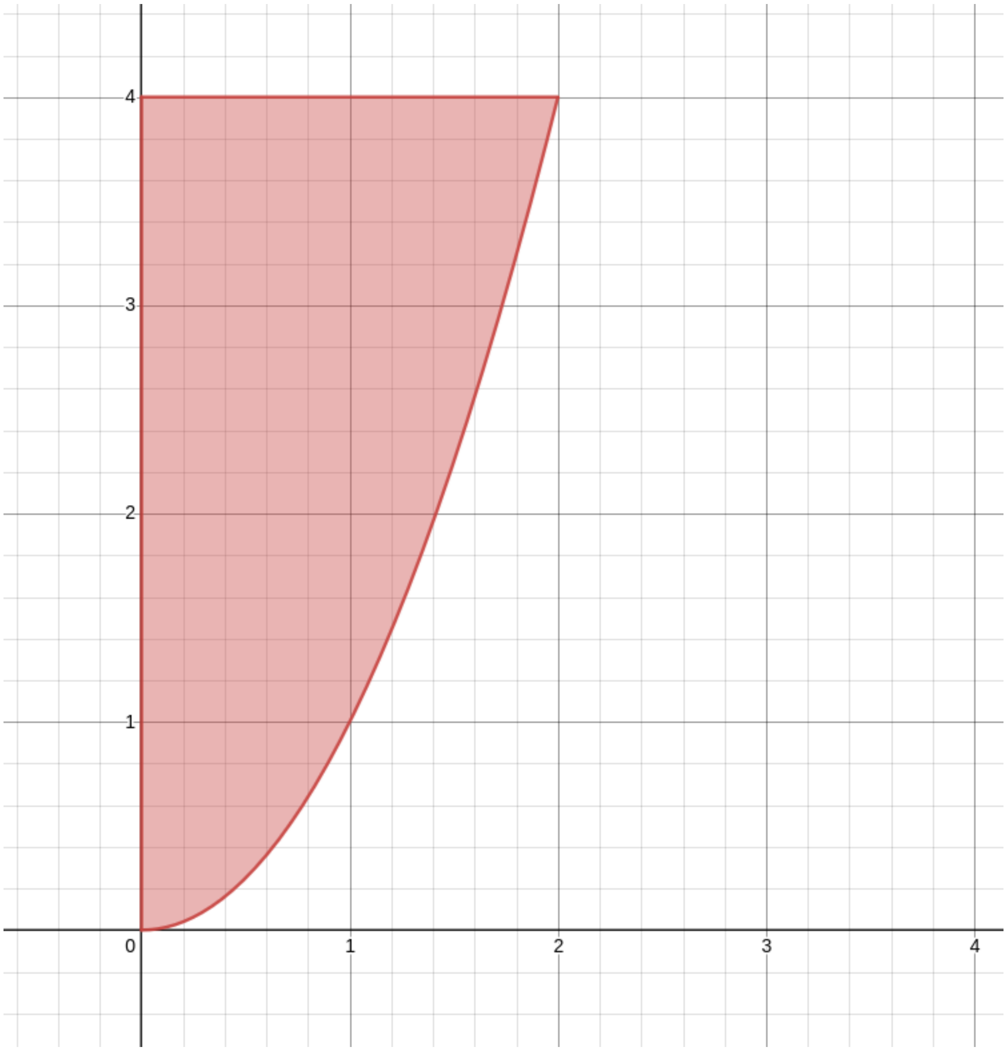
Therefore,

$$\begin{aligned}
Vol = \iint_D x^2 + y^2 + 1\, dA &= \int_0^2 \int_0^{2-x} x^2 + y^2 + 1\, dydx \\
&= \int_0^2 \left(x^2 y + \frac{y^3}{3} + y\right)\Bigg|_0^{2-x} dx \\
&= \int_0^2 x^2(2 - x) + \frac{(2 - x)^3}{3} + 2 - x\, dx \\
&= \left(\frac{2x^3}{3} - \frac{x^4}{4} - \frac{(2 - x)^4}{12} + 2x - \frac{x^2}{2}\right)\Bigg|_0^2 \\
&= \frac{14}{3} \approx 4.6667.
\end{aligned}$$

Section 15.2, Problem 46 (10 Pts)

From the bounds in the iterated integral, we have

$$D = \{(x, y) : 0 \leq x \leq 2 \text{ and } x^2 \leq y \leq 4\}.$$



From the picture, we see that $0 \leq y \leq 4$. The function acting as a lower-bound for the region is $y = x^2$, so that $\sqrt{y} = x$ because x is positive when restricted to the region D. Therefore,

$$D = \{(x, y) : 0 \leq y \leq 4 \text{ and } 0 \leq x \leq \sqrt{y}\}.$$

The integral then become

$$\int_0^2 \int_{x^2}^4 f(x, y)\, dydx = \iint_D f(x, y)\, dA = \int_0^4 \int_0^{\sqrt{y}} f(x, y)\, dxdy.$$

Section 15.2, Problem 56 (10 Pts)

From the bounds in the iterated integral, we have

$$D = \{(x, y) : 0 \leq y \leq 8 \text{ and } \sqrt[3]{y} \leq x \leq 2\}.$$

The lowerbound for x is the function $x = \sqrt[3]{y}$, so that $x^3 = y$. So, when $y = 0$, we get $x = 0$ and when $y = 8$, we get $x = 2$. The description of D as a type I is therefore

$$D = \{(x, y) \,:\, 0 \le x \le 2 \text{ and } 0 \le y \le x^3\}.$$

The integral then becomes

$$\begin{aligned}
\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\, dxdy = \iint_D e^{x^4}\, dA &= \int_0^2 \int_0^{x^3} e^{x^4}\, dydx \\
&= \int_0^2 (x^3 - 0)e^{x^4}\, dx \\
&= \left(\frac{e^{x^4}}{4}\right)\Bigg|_0^2 \\
&= \frac{e^{16} - 1}{4}.
\end{aligned}$$