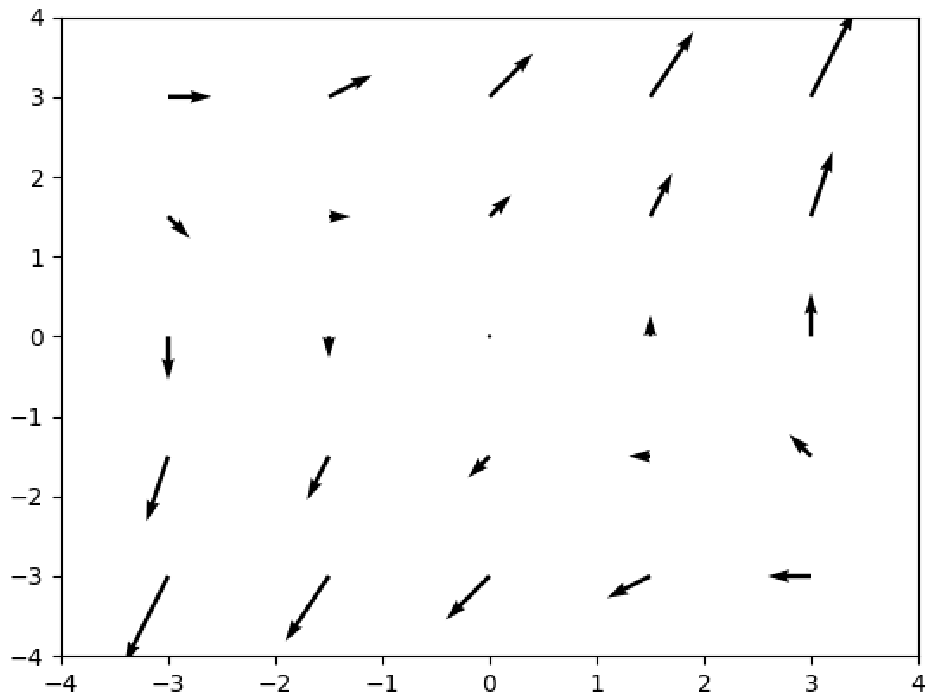
Section 16.1, Problem 4 (5 Pts)

We can create a table with five values for x and five values for y along the sides of a square $[-3, 3] \times [-3, 3]$.

y, x	-3	-1.5	0	1.5	3
-3	$\langle -3, -6\rangle$	$\langle -1.5, -4.5\rangle$	$\langle 0, -3\rangle$	$\langle 1.5, -1.5\rangle$	$\langle 3, 0\rangle$
-1.5	$\langle -3, -4.5\rangle$	$\langle -1.5, -3.0\rangle$	$\langle 0, -1.5\rangle$	$\langle 1.5, 0.0\rangle$	$\langle 3, 1.5\rangle$
0	$\langle -3, -3\rangle$	$\langle -1.5, -1.5\rangle$	$\langle 0, 0\rangle$	$\langle 1.5, 1.5\rangle$	$\langle 3, 3\rangle$
1.5	$\langle -3, -1.5\rangle$	$\langle -1.5, 0.0\rangle$	$\langle 0, 1.5\rangle$	$\langle 1.5, 3.0\rangle$	$\langle 3, 4.5\rangle$
3	$\langle -3, 0\rangle$	$\langle -1.5, 1.5\rangle$	$\langle 0, 3\rangle$	$\langle 1.5, 4.5\rangle$	$\langle 3, 6\rangle$

Here is a picture of the vector field:

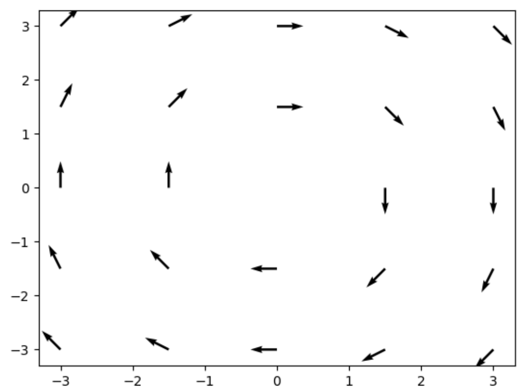


Section 16.1, Problem 6 (5 Pts)

We can create a table with five values for x and five values for y along the sides of a square $[-3, 3] \times [-3, 3]$.

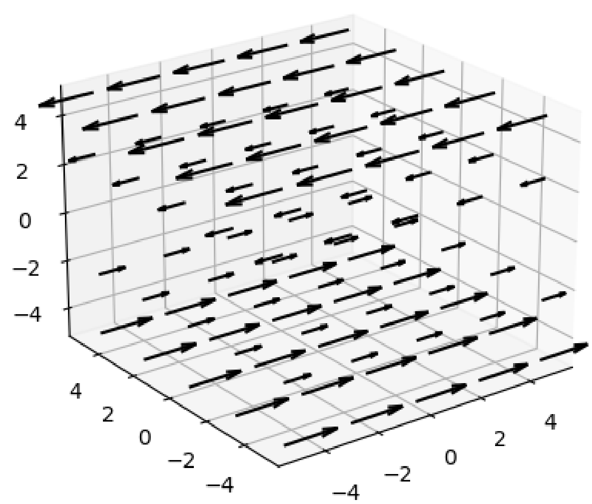
y, x	-3	-1.5	0	1.5	3
-3	$\langle -0.71, 0.71\rangle$	$\langle -0.45, 0.89\rangle$	$\langle 0.00, 1.00\rangle$	$\langle 0.45, 0.89\rangle$	$\langle 0.71, 0.71\rangle$
-1.5	$\langle -0.89, 0.45\rangle$	$\langle -0.71, 0.71\rangle$	$\langle 0.00, 1.00\rangle$	$\langle 0.71, 0.71\rangle$	$\langle 0.89, 0.45\rangle$
0	$\langle -1.00, 0.00\rangle$	$\langle -1.00, 0.00\rangle$	$\nexists$	$\langle 1.00, 0.00\rangle$	$\langle 1.00, 0.00\rangle$
1.5	$\langle -0.89, -0.45\rangle$	$\langle -0.71, -0.71\rangle$	$\langle 0.00, -1.00\rangle$	$\langle 0.71, -0.71\rangle$	$\langle 0.89, -0.45\rangle$
3	$\langle -0.71, -0.71\rangle$	$\langle -0.45, -0.89\rangle$	$\langle 0.00, -1.00\rangle$	$\langle 0.45, -0.89\rangle$	$\langle 0.71, -0.71\rangle$

Here is a picture of the vector field:



Section 16.1, Problem 8 (5 Pts)

This vector field is simple to visualize. It consists of parallel vectors to the x-axis point in the same direction as the x-axis if $z > 0$ and in the opposite direction as the x-axis if $z < 0$.
Here is a picture of the vector field:



Section 16.1, Problem 12 (5 Pts)

We see that, when $y = x$, $\vec{F}(x, y) = \langle x, 0\rangle$. Therefore, along the line $y = x$, we should see vectors pointing only in the direction (or the opposite direction) of the x-axis. We can see this property in the plot label III.

Section 16.1, Problem 14 (5 Pts)

Fixing $x = x_0$ to be constant, we see that $\vec{F}(x_0, y) = \langle \cos(x_0 + y), x_0\rangle$. Therefore, when moving along the vectical line $x = x_0$, the x-component oscillates like the function $\cos(x_0 + y)$. We observe this in the plot labeled II.

Section 16.1, Problem 16 (5 Pts)

When $z = 0$, we see that $\vec{F}(x, y, 0) = \langle 1, 2, 0\rangle$. In other words, the is no z-component and this is observed in the plot labeled I.

Section 16.1, Problem 24 (10 Pts)

We have $f_x = 2xye^{y/z}$, $f_z = -\frac{x^2y^2}{z^2}e^{y/z}$, and

$$f_y = x^2e^{y/z} + \frac{x^2y}{z}e^{y/z}.$$

Therefore,

$$\vec{\nabla} f = \left\langle 2xye^{y/z}, \left(x^2 + \frac{x^2y}{z}\right)e^{y/z}, -\frac{x^2y^2}{z^2}e^{y/z}\right\rangle.$$

Section 16.1, Problem 26 (5 Pts)

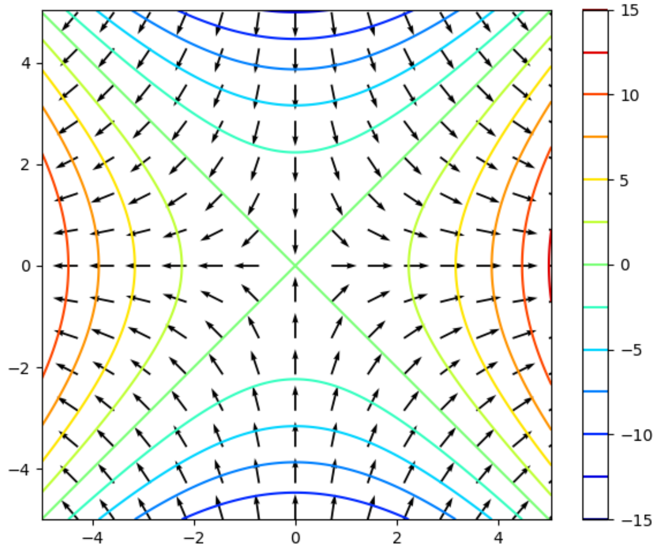
The gradient is given by $\vec{\nabla} f = \langle f_x, f_y\rangle$. We have

$$f_x = \frac{\partial}{\partial x}\left(\frac{1}{2}(x^2 - y^2)\right) = x$$

and

$$f_y = \frac{\partial}{\partial y}\left(\frac{1}{2}(x^2 - y^2)\right) = -y.$$

Therefore, $\vec{\nabla} f = \langle x, -y\rangle$.
Here is a sketch of the gradient of f and some level curves of f:



Section 16.1, Problem 32 (5 Pts)

We will restrict the points (x, y) to be on certain curves. If we assume that $x^2 + y^2 = c$ is constant, so that (x, y) lies on a circle of radius $\sqrt{c}$, then

$$\vec{\nabla} f = \left\langle \frac{\cos\sqrt{c}}{\sqrt{c}} x, \frac{\cos\sqrt{c}}{\sqrt{c}} y \right\rangle = \frac{\cos\sqrt{c}}{\sqrt{c}} \langle x, y\rangle .$$

Therefore, the gradient points in the same direction as the vector $\langle x, y\rangle$, scaled by the fact $\cos\sqrt{x}/\sqrt{x}$. If $c = \pi^2/4$, then $\cos\sqrt{c} = 0$ and therefore $\vec{F}(x, y) = \langle 0, 0\rangle$ on the circle $x^2 + y^2 = c$. Now the radius of the circle is approximately 1.57 and we can see that this feature is present in the plot I.

TOTAL: 50 Pts.