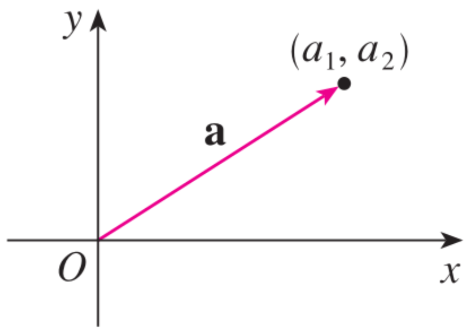
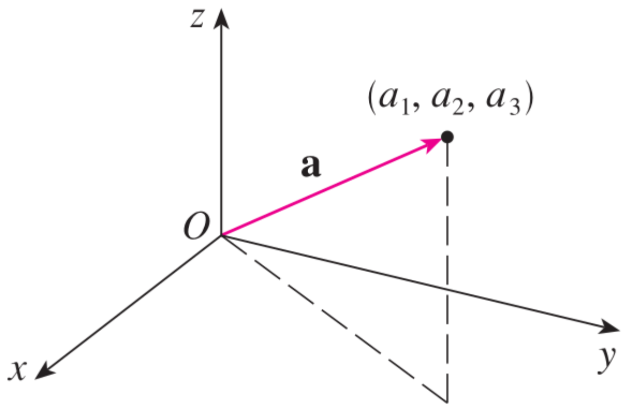
12.5 Equations of Lines and planes.

Vectors.



$\mathbf{a} = \langle a_1, a_2 \rangle$



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

Dot product.

Angle:

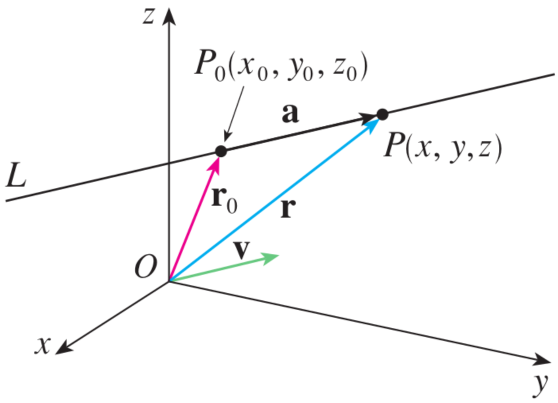
Cross product

Direction given by the right-hand rule

Orthogonal vectors.

Parallel vectors.

Lines.



Vector equation.

$$\boxed{r = r_0 + tv}$$

Parametric equation.

$$\boxed{x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct}$$

Symmetric equations.

$$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

EXAMPLE 1

(a) Find a vector equation and parametric equations for the line that passes through the point $(5, 1, 3)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.
(b) Find two other points on the line.

EXAMPLE 2

(a) Find parametric equations and symmetric equations of the line that passes through the points $A(2, 4, -3)$ and $B(3, -1, 1)$.
(b) At what point does this line intersect the xy-plane?

Line segments. $r(t) = (1-t)r_0 + tr_1$

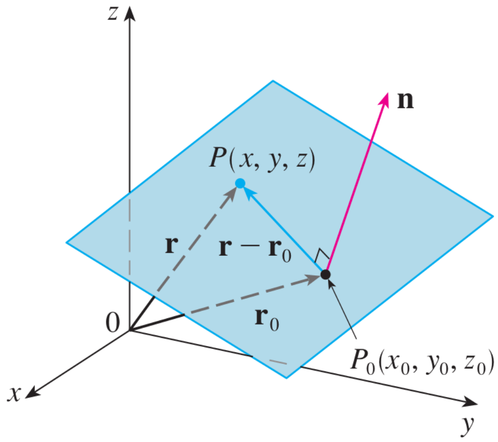
EXAMPLE 3 Show that the lines L_1 and L_2 with parametric equations

$$L_1: \quad x = 1 + t \qquad y = -2 + 3t \qquad z = 4 - t$$

$$L_2: \quad x = 2s \qquad y = 3 + s \qquad z = -3 + 4s$$

are **skew lines**; that is, they do not intersect and are not parallel (and therefore do not lie in the same plane).

Planes.



Vector equation.

$$\boxed{\mathbf{n} \cdot (\mathbf{r} - \mathbf{r_0}) = 0}$$

Scalar equation.

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

Linear equation.

$$\boxed{ax + by + cz + d = 0}$$

EXAMPLE 4 Find an equation of the plane through the point $(2, 4, -1)$ with normal vector $\mathbf{n} = \langle 2, 3, 4 \rangle$. Find the intercepts and sketch the plane.

EXAMPLE 5 Find an equation of the plane that passes through the points $P(1, 3, 2)$, $Q(3, -1, 6)$, and $R(5, 2, 0)$.

EXAMPLE 6 Find the point at which the line with parametric equations $x = 2 + 3t$, $y = -4t$, $z = 5 + t$ intersects the plane $4x + 5y - 2z = 18$.

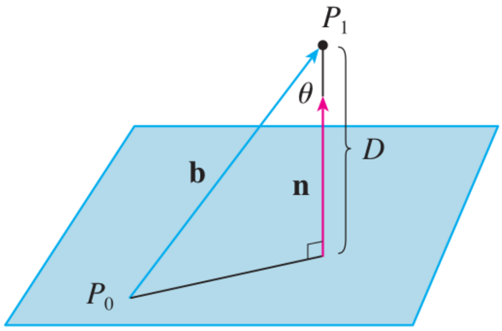
EXAMPLE 7

(a) Find the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

(b) Find symmetric equations for the line of intersection L of these two planes.

Distance.

$$\boxed{D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}}$$



EXAMPLE 9 Find the distance between the parallel planes $10x + 2y - 2z = 5$ and $5x + y - z = 1$.