Section 16.9, Problem 8 (20 Pts)

We have div $\vec{F} = 3x^2 + 3y^2 + 3z^2$ and

$$E = \{(x, y, z) \,:\, x^2 + y^2 + z^2 \leq 4\}.$$

By the Divergence Theorem,

$$\begin{aligned}
\iint_S \vec{F}\, d\vec{S} = \iiint_E \operatorname{div} \vec{F}\, dV &= 3 \iiint_E x^2 + y^2 + z^2\, dV \\
&= 3 \int_0^{\pi} \int_0^{2\pi} \int_0^2 \rho^2 \rho \sin^2(\phi)\, d\rho d\theta d\phi \\
&= 3 \Big(\int_0^2 \rho^3\, d\rho \Big) \Big(\int_0^{2\pi} d\theta \Big) \Big(\int_0^{\pi} \sin^2(\phi)\, d\phi \Big) \\
&= 12\pi^2.
\end{aligned}$$

Section 16.9, Problem 24 (10 Pts)

Notice that

$$2x + 2y + z^2 = \langle 2, 2, z \rangle \cdot \langle x, y, z \rangle = \vec{F} \cdot \vec{n}$$

and $\langle x, y, z \rangle = \vec{n}$ is a normal vector to the sphere because $x^2 + y^2 + z^2 = 1$. Therefore,

$$\iint_S 2x + 2y + z^2\, dS = \iint_S \vec{F} \cdot \vec{n} dS = \iint_S \vec{F} \cdot d\vec{S}$$

because $d\vec{S} = \vec{n} dS$. Using the Divergence Theorem,

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} \vec{F}\, dV$$

where $E = \{(x, y, z) \,:\, x^2 + y^2 + z^2 \leq 1\}$. Since div $\vec{F} = 1$, we get

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E 1\, dV = \operatorname{Vol}(E) = \frac{4\pi}{3}.$$

Section 16.9, Problem 27 (20 Pts)

By the Divergence Theorem, we have

$$\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div} (\operatorname{curl} \vec{F})\, dV.$$

Now we know that div $(\text{curl}\vec{F}) = 0$ and therefore

$$\iiint_E \operatorname{div} (\operatorname{curl} \vec{F})\, dV = 0.$$