MATH-244
Midterm 01

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University of Hawai'i

Last name: Solutions

First name: —

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:	—	—	—	—	—	—

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all 5 questions below and write your answers directly on the questionnaire.
- You have 50 minutes to complete the exam.
- When you are done (or at the end of the 50min period), return your copy.
- Any electronic devices are not aloud during the exam.
- You can use a calculator.
- **Turn off your cellphones during the exam**.
- Lecture notes and the textbook are not allowed during the exam.
- You must show ALL your work to have full credit.
- Draw a square around your final answer.

Your Signature: —

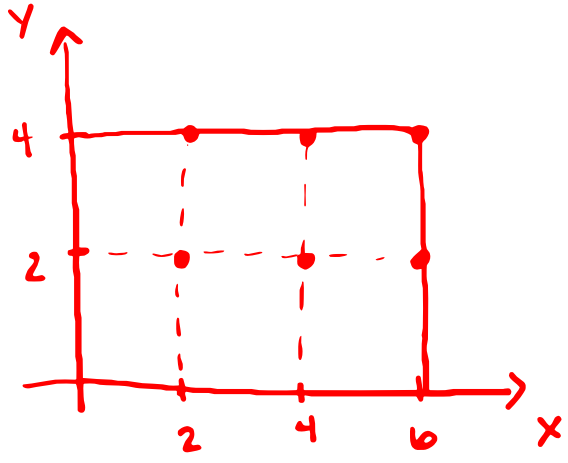
May the Force be with you!
Pierre Parisé

Question 1 **(10 pts)**

Estimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle

$$R = [0, 6] \times [0, 4].$$

Use a Riemann sum with $m = 3$ and $n = 2$, and take the sample point to be the upper right corner of each sub-rectangle.



$m = 3 \rightarrow \Delta x = \frac{6-0}{3} = 2$

$n = 2 \rightarrow \Delta y = \frac{4-0}{2} = 2$

$x_1 = 2, \; x_2 = 4, \; x_3 = 6$

$y_1 = 2, \; y_2 = 4.$

$$V = \iint_R xy \, dA \approx \sum_{i=1}^{3} \sum_{j=1}^{2} x_i y_j \, \Delta A$$

$$= 2 \cdot 2 \cdot 4 + 2 \cdot 4 \cdot 4 + 4 \cdot 2 \cdot 4 + 4 \cdot 4 \cdot 4 + 6 \cdot 2 \cdot 4 + 6 \cdot 4 \cdot 4$$

$$= \boxed{268}$$

Question 2 (10 pts)

Evaluate the following iterated integral:

$$I = \int_0^1 \int_1^2 (x + e^{-y})\, dx dy.$$

$$I = \int_0^1 \frac{x^2}{2} + e^{-y} x \Big|_1^2 dy$$

$$= \int_0^1 \frac{4}{2} + 2e^{-y} - \frac{1}{2} - e^{-y}\, dy$$

$$= \int_0^1 \frac{3}{2} + e^{-y}\, dy$$

$$= \frac{3y}{2} - e^{-y} \Big|_0^1$$

$$= \frac{3}{2} - e^{-1} + 1$$

$$= \boxed{\frac{5}{2} - \frac{1}{e}}$$

Question 3 **(10 pts)**

Evaluate the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = [-1, 2] \times [-1, 1]$.

$f(x,y) = z = 2x + 3y + \frac{15}{2}$

So,

$$V = \iint_R 2x + 3y + \frac{15}{2}\, dA$$

$$= \int_{-1}^{2}\int_{-1}^{1} 2x + 3y + \frac{15}{2}\, dy\, dx$$

$$= \int_{-1}^{2} 2xy + \frac{3y^2}{2} + \frac{15y}{2}\Big|_{-1}^{1} dx$$

$$= \int_{-1}^{2} \left(2x + \cancel{\frac{3}{2}} + \frac{15}{2} + 2x - \cancel{\frac{3}{2}} + \frac{15}{2}\right) dx$$

$$= \int_{-1}^{2} 4x + 15\, dx$$

$$= x^2 + 15x\Big|_{-1}^{2}$$

$$= 4 + 30 - (1 - 15)$$

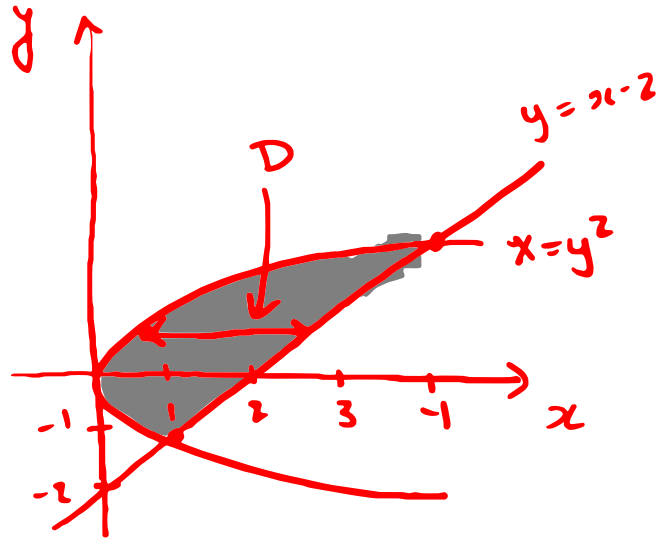
$$= \boxed{48}$$

QUESTION 4 (10 pts)

Setup the integral by taking the following order: $dA = dxdy$. **Do not evaluate the integrals.**

$$\iint_D y\,dA, \quad D \text{ is bounded by } y = x - 2,\ x = y^2.$$

① Picture



$y = y^2 - 2$

$\Leftrightarrow (y-2)(y+1) = 0$

$\Leftrightarrow y = 2$ or $y = -1$

TYPE II: $-1 \le y \le 2$, $y^2 \le x \le y+2$.

$\Rightarrow D = \{(x,y) : -1 \le y \le 2,\ y^2 \le x \le y+2\}$.

So,

$$\iint_D y\,dA = \int_{-1}^{2} \int_{y^2}^{y+2} y\,dx\,dy$$

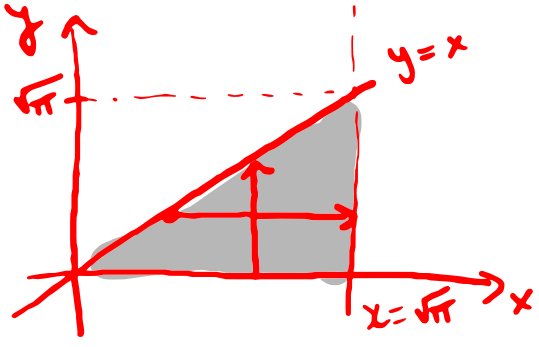
Question 5 (10 pts)

Evaluate the following integral.

$$I = \int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(x^2)\, dx dy.$$

Change the order:

$D = \{(x,y): \quad 0 \le y \le \sqrt{\pi}, \quad y \le x \le \sqrt{\pi}\}.$



Type II.

Switch to type I: $D = \{(x,y): 0 \le x \le \sqrt{\pi}, \; 0 \le y \le x\}$.

So,

$$I = \iint_D \sin x^2 \, dA = \int_0^{\sqrt{\pi}} \int_0^x \sin x^2 \, dy \, dx$$

$$= \int_0^{\sqrt{\pi}} x \sin x^2 \, dx \qquad u = x^2, \; du = 2x\,dx$$

$$= \frac{1}{2}\int_0^{\pi} \sin(u)\, du$$

$$= \frac{1}{2}\left(-\cos(u)\right)\Big|_0^{\pi} = \boxed{-1}$$