## 12.6 Cylinders and Quadric Surfaces.

### Parabolic cylinders

**EXAMPLE 1** Sketch the graph of the surface $z = x^2$.
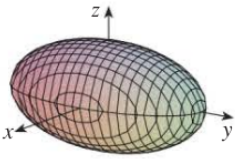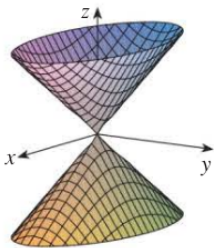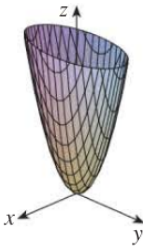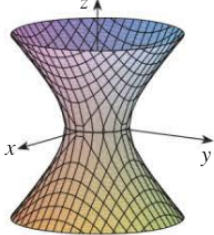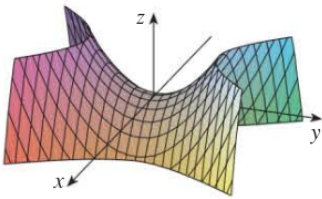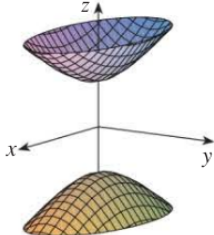
### Cylinders.

**EXAMPLE 2** Identify and sketch the surfaces.
(a) $x^2 + y^2 = 1$          (b) $y^2 + z^2 = 1$

Quadric surfaces.

| Surface | Equation | Surface | Equation |
|---|---|---|---|
| Ellipsoid  | $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$  All traces are ellipses.  If $a = b = c$, the ellipsoid is a sphere. | Cone  | $\dfrac{z^2}{c^2} = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$  Horizontal traces are ellipses.  Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$. |
| Elliptic Paraboloid  | $\dfrac{z}{c} = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$  Horizontal traces are ellipses.  Vertical traces are parabolas.  The variable raised to the first power indicates the axis of the paraboloid. | Hyperboloid of One Sheet  | $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$  Horizontal traces are ellipses.  Vertical traces are hyperbolas.  The axis of symmetry corresponds to the variable whose coefficient is negative. |
| Hyperbolic Paraboloid  | $\dfrac{z}{c} = \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}$  Horizontal traces are hyperbolas.  Vertical traces are parabolas.  The case where $c < 0$ is illustrated. | Hyperboloid of Two Sheets  | $-\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$  Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$.  Vertical traces are hyperbolas.  The two minus signs indicate two sheets. |

**EXAMPLE 3**  Use traces to sketch the quadric surface with equation

$$x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

**EXAMPLE 4**  Use traces to sketch the surface $z = 4x^2 + y^2$.

**EXAMPLE 5**  Sketch the surface $z = y^2 - x^2$.

**EXAMPLE 6** Sketch the surface $\dfrac{x^2}{4} + y^2 - \dfrac{z^2}{4} = 1$.

**EXAMPLE 7** Identify and sketch the surface $4x^2 - y^2 + 2z^2 + 4 = 0$.

**EXAMPLE 8** Classify the quadric surface $x^2 + 2z^2 - 6x - y + 10 = 0$.