Problem 6

The cylindrical coordinates are (r, θ, z) and the transformations to the cartesian coordinates (x, y, z) are $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$. So if $\theta = \pi/6$, then $x = r(\sqrt{2}/2)$ and $y = r(1/2)$. So

$$2x/\sqrt{2} = r = 2y \quad \Rightarrow \quad x/\sqrt{2} = y.$$

So the surface described by the equation $\theta = \pi/6$ is the plane $x/\sqrt{2} - y = 0$.

Problem 8

The transformation from cylindrical coordinates to the cartesian coordinates are $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$. Since $\sin\theta = y/r$, the equation $r = 2\sin\theta$ becomes

$$r = 2y/r \quad \Rightarrow \quad r^2 = 2y.$$

But $r^2 = x^2 + y^2$, and so replacing in the last equation, we obtain

$$x^2 + y^2 = 2y \quad \iff \quad x^2 + (y-1)^2 = 1.$$

This last equation represents a cylinder of radius 1 with center at $(0, 1)$.

Problem 12

The inequalities $0 \leq \theta \leq \pi/2$ means that we are in the first octant and the eigth octant. The inequalities $r \leq z \leq 2$ means that the value for z is positive and it lies between the equations $z = r$ and $z = 2$. Since $r = \sqrt{x^2 + y^2}$, then z lies above the cone $z = \sqrt{x^2 + y^2}$. Thus the solid is a quarter of a cone.

Problem 20

In cylindrical coordinate, the solid E has the following description:

$$E = \{(r, \theta, z) \,:\, 1 \leq r \leq 4,\, 0 \leq \theta \leq 2\pi,\, 0 \leq z \leq y + 4\}.$$

So the triple integral can be rewritten as

$$\begin{aligned}
\iiint_E (x - y)\, dV &= \int_0^{2\pi} \int_1^4 \int_0^{r\sin\theta + 4} r\cos\theta - r\sin\theta \, dz r dr d\theta \\
&= \int_0^{2\pi} \int_1^4 (\cos\theta - \sin\theta)(r\sin\theta + 4) r^2 \, dr d\theta \\
&= \int_0^{2\pi} \int_1^4 r^3 \cos\theta \sin\theta + 4r^2\cos\theta - r^3\sin^2\theta - 4r^2\sin\theta \, dr d\theta \\
&= -255\pi/4.
\end{aligned}$$