# MATH 302

# CHAPTER 1

### SECTION 1.2: BASIC CONCEPTS

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# What's a DE?

- A differential equation (abbreviated by DE) is an equation that contains one or more derivatives of an unknown function.
  - Examples:  $T' = -k(T T_m), y' = x^2, x^2y'' + xy' + 2 = 0.$
- The **order** of a DE is the order of the highest derivatives that it contains.
  - Example:  $y' = x^2$  is of order \_\_\_\_\_.
  - Example:  $x^2y'' + xy' + 2 = 0$  is of order \_\_\_\_\_.
- An **Ordinary Differential Equation** (abbreviated ODE) is a DE involving an unknown function of only one variable.
- An Partial Differential Equation (abbreviated PDE) is a DE involving an unknown function of more than one variable.

The simplest ODE is of the form

$$y' = f(x)$$
 or  $y^{(n)} = f(x)$ 

where f is a known function of x.

**EXAMPLE 1.** Find functions y = y(x) satisfying

- 1.  $y' = x^2$ .
- $2. \ y'' = \cos(x).$

Our goal is to study general ODEs of the form

$$y^{(n)} = f(x, y, y', \dots, y^{(n-1)}).$$

# WHAT IS A SOLUTION TO AN ODE?

#### A solution to the ODE

$$y^{(n)}(x) = f(x, y(x), y'(x), \dots, y^{(n-1)}(x))$$

is a function y = y(x) that verifies the ODE for any x in some open interval (a, b).

#### Remark:

• Functions that satisfy an ODE at isolated points are not considered solutions.

### **EXAMPLE 2.** Verify that

$$y = \frac{x^2}{3} + \frac{1}{x}$$

is a solution of

$$xy' + y = x^2$$

on  $(-\infty, 0)$  and  $(0, \infty)$ .

## Solution and Integral Curves

- The graph of a solution of an ODE is a solution curve.
- More generally, a curve C in the plane is said to be an **integral curve** of an ODE if every function y = y(x) whose graph is a segment of C is a solution of the ODE.

**EXAMPLE 3.** Plot the solutions obtained in Example 2. Are they solution curves of the ODE?

**EXAMPLE 4.** If a is any positive constant, check that the circle

$$x^2 + y^2 = a^2$$

is an integral curve of y' = -x/y.

# Initial Value Problems

## **EXAMPLE 5.** Find a solution of

$$y' = x^3$$

satisfying the additional condition y(1) = 2.

## **EXAMPLE 6.** All the solutions to

$$y'' - 2y' + 3y = 0$$

are the functions

$$y(x) = c_1 e^x + c_2 e^{-3x}$$

where  $c_1$ ,  $c_2$  are arbitrary constants. Find the solution that satisfies y(0) = 1 and y'(0) = 0.

An **Initial Value Problem** (abbreviated by IVP) is an ODE with additional **Initial conditions**. The general form of an IVP is

$$y^{(n)}(x) = f(x, y(x), y'(x), \dots, y^{(n-1)}(x)), \quad y(x_0) = k_0, y'(x_0) = k_1, \dots, y^{(n-1)}(x_0) = k_{n-1}.$$

• The largest open interval that contains  $x_0$  on which y(x) is defined and satisfies the ODE is called the **interval of validity** of y.

**EXAMPLE 7.** Find the interval of validity of the solution to

$$y' = x^3, y(1) = 2.$$

**EXAMPLE 8.** Find the interval of validity of the solution to the following IVPs:

- 1.  $xy' + y = x^2$ , y(1) = 4/3.
- 2.  $xy' + y = x^2$ , y(-1) = -2/3.