3.4 / 5, 11, 21, 30(a)
3.5 7

HW7 $\quad$ 41/50 $\quad$ Très bien !

Shantaro Murase

Section 5.4

5) $y'' + 4y = e^{-x}(7 - 4x + 5x^2)$ $\qquad r^2 + 4r = 0$

$y_{par}(x) = e^{-x}(Ax^2 + Bx + C)$ $\qquad r(r+4) = 0$

$y'_{par} = (2Ax + B)e^{-x} - (Ax^2 + Bx + C)e^{-x}$

$y''_{par} = 2Ae^{-x} - (2Ax + B)e^{-x} - (2Ax + B)e^{-x} + (Ax^2 + Bx + C)e^{-x}$

then,

$2Ae^{-x} - (2Ax + B)e^{-x} - (2Ax + B)e^{-x} + (Ax^2 + Bx + C)e^{-x} + 4e^{-x}(Ax^2 + Bx + C)$

$= 2Ae^{-x} - 2(2Ax + B)e^{-x} + 5e^{-x}(Ax^2 + Bx + C)$

$= 5Ax^2e^{-x} - (4Ax - 5Bx)e^{-x} + (2A - 2B + 5C)e^{-x} = (7 - 4x + 5x^2)e^{-x}$

$5A = 5 \iff A = 1.$

$4A - 5B = 4 \iff B = 0$

$2A - 2B + 5C = 7 \iff C = 1.$

$\therefore \boxed{y_{par}(x) = e^{-x}(x^2 + 1)}$

forgot to solve for homogeneous part ...

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New page

11) $y'' + 2y' + y = e^{-x}(2 + 3x)$ $\qquad r^2 + 2r + 1 = 0$

Let $y_p = x^2e^{-x}(Ax + B)$ $\qquad (r+1)^2 = 0$

$= e^{-x}(Ax^3 + Bx^2)$

$y'_p = (3Ax^2 + 2Bx)e^{-x} - (Ax^3 + Bx^2)e^{-x}$

$y''_p = (6Ax + 2B)e^{-x} - (3Ax^2 + 2Bx)e^{-x} - (3Ax^2 + 2Bx)e^{-x} + (Ax^3 + Bx^2)e^{-x}$

$= (6Ax + 2B)e^{-x} - 2(3Ax^2 + 2Bx)e^{-x} + (Ax^3 + Bx^2)e^{-x}$

then,

$(6Ax + 2B)e^{-x} - 2(3Ax^2 + 2Bx)e^{-x} + (Ax^3 + Bx^2)e^{-x} + (3Ax^2 + 2Bx)e^{-x} - (Ax^3 + Bx^2)e^{-x}$
$+ e^{-x}(Ax^3 + Bx^2)$

$= (6Ax + 2B)e^{-x} - (3Ax^2 + 2Bx)e^{-x} + (Ax^3 + Bx^2)e^{-x}$

$(6Ax + 2B)e^{-x} - 2\cancel{(3Ax^2 + 2Bx)}e^{-x} + \cancel{(Ax^3 + Bx^2)}e^{-x} + 2\cancel{(3Ax^2 + 2Bx)}e^{-x} - 2(Ax^3 + Bx^2)e^{-x}$
$+ \cancel{e^{-x}(Ax^3 + Bx^2)}$

$= (6Ax + 2B)e^{-x} = (2 + 3x)$

$6A = 3 \iff A = \frac{1}{2}$

$2B = 2 \iff B = 1$

$\therefore \boxed{y_p = x^2e^{x}\left(\frac{x}{2} + 1\right)}$ ✓

Homogeneous part ?

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Section 5.4 $y'' + 3y' - 4y = e^{2x}(7+6x)$ $y(0)=2$ $y'(0)=8$

21) $r^2 + 3r - 4 = 0$

$(r-1)(r+4) = 0$

$r = -4, 1$ $\therefore$ $y_h(x) = C_1 e^{-4x} + C_2 e^{x}$ ✓

$y_p = e^{2x}(Ax+B)$

$y_p' = Ae^{2x} + 2(Ax+B)e^{2x}$

$y_p'' = 2Ae^{2x} + 2Ae^{2x} + 4(Ax+B)e^{2x}$

$= 4Ae^{2x} + 4(Ax+B)e^{2x}$

$4Ae^{2x} + 4(Ax+B)e^{2x} + 3Ae^{2x} + 6(Ax+B)e^{2x} - 4(Ax+B)e^{2x}$

$= 7Ae^{2x} + 6(Ax+B)e^{2x} = 7+6x$

$6A = 6 \Leftrightarrow A = 1$

$7A + 6B = 7$

$B = 0.$

$y_p = xe^{2x}$

So. $y(x) = xe^{2x} + C_1 e^{-4x} + C_2 e^{x}$ $\Rightarrow$ $y'(x) = 2xe^{2x} + e^{2x} - 4C_1 e^{-4x} + C_2 e^{x}$

$y(0) = C_1 + C_2 = 2$ $-①$ $\qquad$ $y'(0) = 1 - 4C_1 + C_2 = 8$

$-4C_1 + C_2 = 7.$ $-②$

① - ②

$5C_1 = -5$

$C_1 = -1$

$C_2 = 3$

$\therefore$ $\boxed{y(x) = xe^{2x} + 3e^{x} - e^{-4x}}$ ✓

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Section 5.4

3(a)

$'' + p'(\alpha)u' + p(\alpha)u = G(x)$

$ay'' + by' + cy = e^{\alpha x} G(x) \quad -(A)$

$P(r) = ar^2 + br + c = 0.$

$p'(r) = 2ar + b$

① … $\begin{cases} y = ue^{\alpha x} \\ y' = u'e^{\alpha x} + \alpha u e^{\alpha x} \\ y'' = u''e^{\alpha x} + \alpha u' e^{\alpha x} + \alpha u' e^{\alpha x} + \alpha^2 u e^{\alpha x} \\ \quad = u'' e^{\alpha x} + 2\alpha u' e^{\alpha x} + \alpha^2 u e^{\alpha x} \end{cases}$

Substitute ① into (A)

$ae^{\alpha x}(u'' + 2\alpha u' + \alpha^2 u) + be^{\alpha x}(u' + \alpha u) + cue^{\alpha x} = e^{\alpha x} G(x)$

$au'' + 2a\alpha u' + a\alpha^2 u + bu' + b\alpha u + cu = G(x)$

$au'' + (2a\alpha + b)u' + (a\alpha^2 + b\alpha + c)u = G(x)$ ✓ 5/5

$au'' + P'(\alpha)u' + P(\alpha)u = G(x) = (B)$

$\therefore$ y is a Solution of the Constant coefficient equation (A) iff $y = ue^{\alpha x}$, where u satisfies (B)

Q.E.D

Sections 5.5

7) $y'' + 4y = -12\cos 2x - 4\sin 2x$

$r^2 + 4r = 0$

$r(r+4) = 0 \rightarrow r = 0, -4.$

$y_p = x(A\cos 2x + B\sin 2x)$

$y_p' = A\cos 2x - 2Ax\sin 2x + B\sin 2x + 2Bx\cos 2x$

$y_p'' = -2A\sin 2x - 2A\sin 2x - 4Ax\cos 2x + 2B\cos 2x + 2B\cos 2x - 4Bx\sin 2x$

$= -4A\sin 2x - 4Ax\cos 2x + 4B\cos 2x - 4Bx\sin 2x$

$-4A\sin 2x - 4Ax\cos 2x + 4B\cos 2x - 4Bx\sin 2x + 4(Ax\cos 2x + Bx\sin 2x)$

$= -4A\sin 2x + 4B\cos 2x = -12\cos 2x - 4\sin 2x$

then, $-4A = -4$, $4B = -12$

$A = 1$ $\quad B = -3$

$\boxed{y_p = x(\cos 2x - 3\sin 2x)}$ ✓

General solution?

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