Math 302

Chapter 2

Section 2.4: Transformation of Nonlinear Equations Into Separable Equations

Contents

Created by: Pierre-Olivier Parisé
Fall 2022

We were able to solve

$$y' + p(x)y = f(x)$$

by

- finding a solution y_1 to the complementary equation and
- setting $y = uy_1$ where u is the solution to the separable equation

$$u' = \frac{f(x)}{y_1(x)}.$$

Bernoulli Equation

A **Bernoulli equation** is an equation of the form

$$y' + p(x)y = f(x)y^r$$

where r is any real number different from 0 and 1.

Trick to solve it:

EXAMPLE 1. Solve the Bernoulli equation

$$y' - y = xy^2.$$

Homogeneous Nonlinear Equation

The first order ODE

$$y' = f(x, y)$$

is said to be **homogeneous of the second kind** if it takes the form

$$y' = q(y/x)$$

where $q = q(u)$ is a function of a single variable.

Example 2. The following ODEs are homogeneous of the second kind. Explain why.

1. $y' = \dfrac{y + xe^{-y/x}}{x}$.
2. $x^2 y' = y^2 + xy - x^2$.

The trick:

EXAMPLE 3.

1. Solve

$$y' = \frac{y + xe^{-y/x}}{x}.$$

2. Solve the boundary value problem

$$y' = \frac{y + xe^{-y/x}}{x}, \quad y(1) = 0.$$