Section 8.2 — Problem A **25 Points**

Solve the following IVP using the Laplace transform:

$$y'' + 3y' + 2y = 2e^t, \quad y(0) = 0,\, y'(0) = -1.$$

Section 8.2 — Problem B **25 Points**

Solve the following IVP using the Laplace transform:

$$y'' - 2y' + y = t^2 e^t, \quad y(0) = 1,\, y'(0) = 2.$$

Total (Points): 50.

Complete Solutions

Section 8.2 — Problem A — 25 Points

Apply the Laplace transform to the ODE to get

$$s^2Y - sy(0) - y'(0) + 3(sY - f(0)) + 2Y = \frac{2}{s-1}.$$

Using the initial condition and collecting the terms, we obtain

$$(s^2 + 3s + 2)Y = \frac{2}{s-1} - 1$$

Using the fact that $s^2 + 3s + 2 = (s+1)(s+2)$, we obtain

$$Y(s) = \frac{2}{(s-1)(s+1)(s+2)} - \frac{1}{(s+1)(s+2)}$$

We can rewrite each term in the RHS using the partial fraction decomposition:

$$\frac{2}{(s-1)(s+1)(s+2)} = \frac{1/3}{s-1} + \frac{-1}{s+1} + \frac{2/3}{s+2}$$

and

$$\frac{1}{(s+1)(s+2)} = \frac{1}{s+1} + \frac{-1}{s+2}.$$

Therefore, the expression of $Y(s)$ becomes

$$Y(s) = \frac{1/3}{s-1} - \frac{1/3}{s+2}.$$

Taking the inverse Laplace transform, we obtain

$$y(t) = \frac{1}{3}e^t - \frac{1}{3}e^{-2t}.$$

Section 8.2 — Problem B   25 Points

After applying the Laplace transform, we get

$$s^2Y - sy(0) - y'(0) - 2sY + 2y(0) + Y = \frac{2}{(s-1)^3}$$

Using the initial condition, we then get

$$(s^2 - 2s + 1)Y - s = \frac{2}{(s-1)^3}.$$

Therefore, we obtain

$$(s^2 - 2s + 1)Y = s + \frac{2}{(s-1)^3}.$$

Using the fact that $s^2 - 2s + 1 = (s-1)^2$, we get

$$Y = \frac{s}{(s-1)^2} + \frac{2}{(s-1)^5}.$$

We notice that

$$\frac{s}{(s-1)^2} = sF(s).$$

The inverse of $F(s)$ is te^t and recall that

$$L(f'(t)) = sF - f(0).$$

Since at $t = 0$, the function te^t is 0, we get

$$\frac{s}{(s-1)^2} \longrightarrow (te^t)' = e^t + te^t.$$

Also, since

$$L(t^4 e^t) = \frac{4!}{(s-1)^5},$$

we have

$$\frac{1}{(s-1)^5} \longrightarrow \frac{1}{4!} t^4 e^t.$$

Taking the inverse Laplace transform, we obtain

$$y(t) = e^t + te^t + \frac{1}{12} t^4 e^t.$$

Total (Points): 50.