Problem A

Find an expression of the following functions involving only unit step functions.

1) $f(t) = \begin{cases} 0 & t < 2 \\ (t-2)^2 & t > 2. \end{cases}$

2) $f(t) = \begin{cases} 0 & t < \pi \\ t - \pi & t \in [\pi, 2\pi] \\ 0 & t > 2\pi. \end{cases}$

Problem B

Find the Laplace transform of the following functions.

1) $f(t) := tu(t-1)$.
2) $f(t) := te^{2t}u(t-1)$.
3) $f(t) := (t^2-1)u(t-1)$.
4) $f(t) := t\sin tu(t-2\pi)$.
5) $f(t) := \cos tu(t-a)$.

Problem C

Find the inverse Laplace transform of the following functions.

1) $F(s) = \frac{3!}{(s-2)^4}$.
2) $F(s) = \frac{2(s-1)e^{-2s}}{s^2-2s+2}$.
3) $F(s) = \frac{(s-2)e^{-s}}{s^2-4s+3}$.
4) $F(s) = \frac{e^{-2s}}{s^2+s-2}$.
5) $F(s) = \frac{2e^{-2s}}{s^2-4}$.
6) $F(s) = \frac{e^{-s}+e^{-2s}-e^{-3s}-e^{-4s}}{s}$.