CONFIDENTIAL:
DO NOT RETURN TO STUDENT.
SHRED TO DISPOSE.

Dear Professor **Pierre-Olivier Parise:**

I acknowledge that I understand the stated conditions below and will take the MATH 302 exam in accordance with these conditions:

CLOSED BOOK, NOTES ALLOWED - (one sheet of 2-sided notes on 8.5x11), USE OF CALCULATOR (Not graphic)

I certify that I will not use any unauthorized materials, written or electronic, and that I have not communicated nor will I communicate with anyone regarding this exam which was administered to me today at the KOKUA Program office. Thank you for working with KOKUA to provide me with appropriate testing accommodations.

Jasmine Carpena

9/27/22
Date

9/27/2022

Dear Professor Pierre-Olivier Parise:

Please find enclosed your **MATH 302** exam taken by **Jasmine Carpena** at the KOKUA Program. In the interest of exam security and validity, we are informing you of service provided to Jasmine.

The student requested:

USE OF KOKUA FACILITIES, TIME EXTENSION

The examination was administered by KOKUA on 9/27/2022 from 11:50 to 1:20.

This envelope and all of its enclosures were returned to **parisepo@hawaii.edu** at **Professor Pierre-Olivier Parise** on by KOKUA staff. Thank you very much for your invaluable cooperation!

KOKUA Program
Office of Student Equity, Excellence and Diversity
(V/TTY) 956-7612 or (V/TTY) 956-7511

MATH-302
Midterm 01

Created by Pierre-O. Parisé
Fall 2022

Last name: Carpena
First name: Jasmine

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							

Instructions: Make sure to write your complete name on your copy. You must answer all the questions below and write your answers directly on the questionnaire. At the end of the 75 minutes, return your copy.

No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam. You are not allowed to use the lecture notes or the textbook. You may bring one 2-sided cheat sheet of handwriting notes. You may use a digital calculator (no graphical calculators or symbolic calculators will be allowed).

You must show ALL your work to have full credit. An answer without justification is worth no point.

May the Force be with you!

Pierre-Olivier Parisé

Your Signature: Jasmine Carpena

UNIVERSITY OF HAWAI'I

QUESTION 1 (20 pts)

Find the solution to the following IVP:

$$y' + \left(\frac{1+x}{x}\right) y = 0 \quad \text{and} \quad y(1) = 1.$$

$$y' = -\left(\frac{1+x}{x}\right)y$$

$$\int \frac{dy}{y} = \int -\left(\frac{1+x}{x}\right)dx$$

$$= -\int \left(\frac{1}{x} + 1\right) dx$$

$$\ln|y| = -\ln|x| - x + k$$

$$y = e^{-\ln|x| - x + k}$$

$$= e^{\ln|x|^{-1}} \cdot e^{-x} \cdot e^{k}$$

$$y = \frac{c}{xe^x}$$

$$1 = \frac{c}{(1)e^1} \Rightarrow c = e$$

$$y = \frac{e}{xe^x} = \frac{e^{1-x}}{x}$$

QUESTION 2 **(20 pts)**

Find the solutions to $3x^2ydx + 2x^3dy = 0$.

$M = 3x^2y$ $N = 2x^3$

$M_y = 3x^2 \neq N_x = 6x^2$ (not exact)

$\frac{N_x - M_y}{M} = \frac{6x^2 - 3x^2}{3x^2y} = \frac{3x^2}{3x^2y} = \frac{1}{y}$

$\mu = e^{\int 1/y \, dy} = e^{\ln|y|} = y$

$\mu M = 3x^2y(y)$ $\mu N = 2x^3y$

$= 3x^2y^2$

$M_{y_1} = 6x^2y = N_{x_1} = 6x^2y$ (exact)

$F_x = \int 3x^2y^2dx$ $F_y = \int 2x^3ydy$

$= x^3y^2$ $= x^3y^2$

$c = x^3y^2$

QUESTION 3 (20 pts)

Solve the Bernoulli's equation $y' - xy = xy^{3/2}$.

complementary: $y' - xy = 0$

$$e^{-\int -x\,dx} = e^{0.5x^2 + k}$$

$$ce^{0.5x^2}$$

$$y(x) = u(x)e^{0.5x^2}$$

$$y'(x) = u'e^{0.5x^2} + ue^{0.5x^2}x$$

$$u'e^{\frac{1}{2}x^2} + \cancel{uxe^{\frac{1}{2}x^2}} - x\left[\cancel{ue^{\frac{1}{2}x^2}}\right] = x(ue^{1/2x^2})^{3/2}$$

$$u'e^{1/2x^2} = u^{3/2}xe^{3/4x^2}$$

$$u' = u^{3/2}xe^{1/4x^2}$$

$$\int \frac{du}{u^{3/2}} = \int e^{\frac{1}{4}x^2}(x)\,dx$$

$u^{-3/2}$ $\qquad v = \frac{1}{4}x^2$

$$dv = \frac{1}{2}x\,dx$$

$$2dv = x\,dx$$

$$-2u^{-1/2} = 2\int e^v\,dv$$

$$-2u^{-1/2} = 2e^{1/4x^2} + k$$

$$u^{-1/2} = e^{1/4x^2} + c$$

$$u = (e^{1/4x^2} + c)^{-2}$$

$$y = (e^{\frac{1}{4}x^2} + c)^{-2}e^{\frac{1}{2}x^2}$$

QUESTION 4 (20 pts)

A tank with a maximal capacity of 1200 gallons initially contains 40 pounds of salt dissolved in 600 gallons of water. Starting at $t_0 = 0$, water that contains 1/2 pound of salt per gallon is added to the tank at the rate of 6 gal/min and the resulting mixture is drained from the tank at 6gal/min.

(lb)

(a) (5 points) Find a differential equation for $Q(t)$, the quantity of salt in the tank at time t (time is in minutes).

(b) (12 points) Solve the equation obtained from part (a).

(c) (3 points) Compute $\lim_{t\to\infty} Q(t)$. What does it represent physically?

a) (rate in) - (rate out) = Q'

rate in $= \frac{1}{2}\ \text{lb/gal} \cdot 6\ \text{gal/min} = 3\ \text{lb/min}$

rate out $= \dfrac{Q(t)}{100+3t}$?

$$\frac{dQ}{dt} = 3 - \frac{Q}{100+3t}$$

b) $Q' + \left(\dfrac{1}{100+3t}\right) Q = 3$

$$e^{-\int \frac{1}{100+3t}dt} = e^{-\frac{1}{3}\ln|100+3t|+k} = (100+3t)^{-1/3}\cdot c$$

$$Q(t) = u(t)(100+3t)^{-1/3}$$

$$Q' = u'(100+3t)^{-1/3} - u(100+3t)^{-4/3}$$

$$u'(100+3t)^{-1/3} - \cancel{u(100+3t)^{-4/3}} + \cancel{(100+3t)^{-1}(u(100+3t)^{-1/3})} = 3$$

$$u'(100+3t)^{-1/3} = 3$$

$$u' = 3(100+3t)^{1/3}$$

$$u = \int 3(100+3t)^{1/3}\,dt$$

$v = 100+3t$

$dv = 3dt$

$$= \int v^{1/3}$$

$$= \frac{3}{4}v^{4/3} + c$$

$$u = \frac{3}{4}(100+3t)^{4/3} + c$$

$$Q = \left[\frac{3}{4}(100+3t)^{4/3} + c\right]\left[(100+3t)^{-1/3}\right]$$

$$= \frac{3}{4}(100+3t) + c(100+3t)^{-1/3}$$

$$= 75 + \frac{9}{4}t + c(100+3t)^{-1/3}$$

c) $\lim_{t\to\infty} Q(t) = \infty$

As time goes on, there is too much salt in the tank.

QUESTION 5 (10 pts)

Answer the following questions.

(a) (5 points) Find **all** functions M such that the following ODE is exact:

$$M(x, y)dx + (x^2 - y^2)dy = 0.$$

$N_x = 2x = M_y$

$M = \int 2x\,dy$

$M(x,y) = 2xy + c(y)$

(b) (5 points) The picture below represents a direction field for a certain differential equation. Draw five different integral curves on the picture below. Explain, with a short paragraph, how you drew the integral curves.

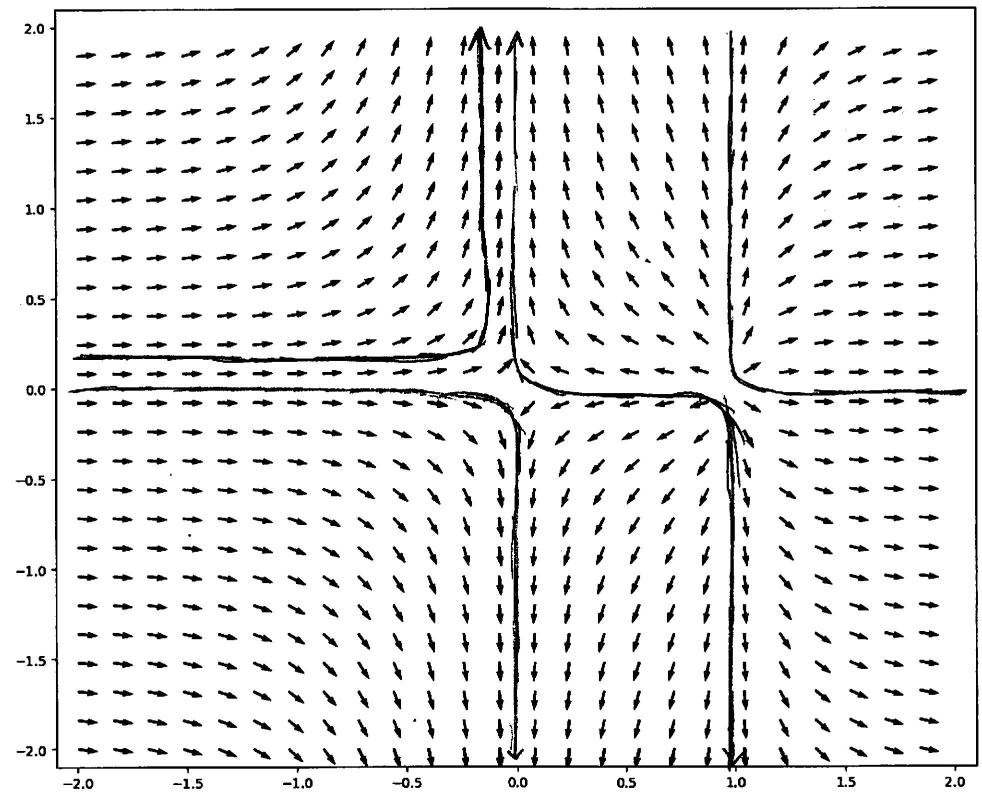

Figure 1: Direction field of some ODE

When drawing the integral curve, I followed the arrows of the direction field from tail to tip. I noticed that around $y=0$, the curve starts to point away from that value near $x=0$ and $x=1$.

QUESTION 6 (10 pts)

Answer the following statements with **True** or **False**. Write your answer on the horizontal line at the end of each statement. Justify your answer in the white space underneath each statement.

(a) (/ 2) The DE $(x^{\cancel{2}} + xy)dx + (y^3 + \cancel{x})dy = 0$ is exact.

$M_y = x$

$N_x = 1$

(a) False

(b) (/ 2) The order of the DE $x^2y^{(3)} + y^4 = \tan(x)$ is four.

(b) False

(c) (/ 2) If $y_1(x)$ is a solution to $y^2 + y' = 0$, then $2y_1$ is a solution to $y^2 + y' = 0$.

$y = \frac{1}{x}$; $y' = -\frac{1}{x^2}$

$2y_1 = \frac{2}{x}$

$\frac{4}{x^2} +$

(c) False

(d) (/ 2) Any solution to $y' + p(x)y = 0$ is of the form $y(x) = ce^{-\int p(x)\,dx}$.

(d) True

(e) (/ 2) The function $y(x) = \frac{1}{x-1}$ is a solution to the following IVP: $y' + y^2 = 0$ and $y(0) = -1$.

$= (x-1)^{-1}$ $y(0) = (0-1)^{-1}$

$y'(x) = -\frac{1}{(x-1)^2}$

$-\frac{1}{(x-1)^2} + \frac{1}{(x-1)^2} = 0$ ✓

(e) True