Math 302

Chapter 5

Section 5.4: The Method of Undetermined Coefficient I

Contents

When The Force Function Is An Exponential **2**
Case I . . . 2
Case II . . . 3
Case III . . . 5
Recap . . . 7

When The Force Function Is Exponential Times Polynomial **8**
Case I . . . 8
Case II . . . 10
Case III . . . 12
Recap . . . 13

Created by: Pierre-Olivier Parisé
Fall 2022

We consider the following basic case:

$$ay'' + by' + cy = ke^{\alpha x}$$

where a, b, c, α, and k are fixed real numbers.

Case I

When $e^{\alpha x}$ is not a solution to the complementary equation $ay'' + by' + cy = 0$.

Example 1. Find the general solution of

$$y'' - 7y' + 12y = 4e^{2x}.$$

Case II

When $e^{\alpha x}$ is a solution to the complementary equation.

EXAMPLE 2. Find the general solution of

$$y'' - 7y' + 12y = 5e^{4x}.$$

Case III

When $e^{\alpha x}$, and $xe^{\alpha x}$ are solutions to the complementary equation.

EXAMPLE 3. Find the general solution of

$$y'' - 8y' + 16y = 2e^{4x}.$$

Recap

To find a particular solution to

$$ay'' + by' + cy = ke^{\alpha x}$$

where k is a fixed real number, we follow the following tips:

- If $e^{\alpha x}$ is not a solution of the complementary equation, then we take $y_{par}(x) = Ae^{\alpha x}$, where A is a constant.
- If $e^{\alpha x}$ is a solution of the complementary equation, then we take $y_{par}(x) = xAe^{\alpha x}$, where A is a constant.
- If both $e^{\alpha x}$ and $xe^{\alpha x}$ are solutions of the complementary equation, then we take $y_{par}(x) = Ax^2e^{\alpha x}$, where A is a constant.

We now consider a more general case:

$$ay'' + by' + cy = e^{\alpha x} G(x)$$

where a, b, c, α are fixed real numbers and $G(x)$ is a polynomial.

Case I

When $e^{\alpha x}$ is not a solution to the complementary equation $ay'' + by' + cy = 0$.

EXAMPLE 4. Find the general solution to

$$y'' - 3y' + 2y = e^{3x}(x^2 + 2x - 1).$$

Case II

When $e^{\alpha x}$ is a solution to the complementary equation.

Example 5. Find the general solution to

$$y'' - 4y' + 3y = e^{3x}(12x^2 + 8x + 6).$$

Case III

When $e^{\alpha x}$ and $xe^{\alpha x}$ are solutions to the complementary equation.

EXAMPLE 6. Find the general solution to

$$4y'' + 4y' + y = e^{-x/2}(144x^2 + 48x - 8).$$

Recap

To find a particular solution to

$$ay'' + by' + cy = ke^{\alpha x} G(x)$$

where k is a fixed real number and $G(x)$ is a polynomial, we follow the following tips:

- If $e^{\alpha x}$ is not a solution of the complementary equation, then we take $y_{par}(x) = Ae^{\alpha x}Q(x)$, where A is a constant and $Q(x)$ is a polynomial of the same degree as $G(x)$.
- If $e^{\alpha x}$ is a solution of the complementary equation, then we take $y_{par}(x) = Axe^{\alpha x}Q(x)$, where A is a constant and $Q(x)$ is a polynomial of the same degree as $G(x)$.
- If $e^{\alpha x}$ and $xe^{\alpha x}$ are solutions to the complementary equation, then we take $y_{par}(x) = Ax^2e^{\alpha x}Q(x)$, where A is a constant and $Q(x)$ is a polynomial of the same degree as $G(x)$.