**Last name:** _____

**First name:** _____

**Instructions:**

- Make sure to write your complete name on your copy.

- You must answer all the questions below and write your answers directly on the questionnaire.

- You have 120 minutes (2 hours) to complete the exam.

- When you are done (or at the end of the 120min period), return your copy.

- No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam.

- **Turn off your cellphone during the exam**.

- You may use a digital calculator (no graphical calculators or symbolic calculators will be allowed).

- You are not allowed to use the lecture notes or the textbook.

- You may bring one 2-sided cheat sheet of handwriting notes.

- You must show ALL your work to have full credit. An answer without justification is worth no point.

**Your Signature:** _____

May the Force be with you!                                Pierre-Olivier Parisé

UNIVERSITY
OF HAWAI'I

Find the solution of the following ODE using the power series method.

$$(1 - x^2)y'' + xy' + y = 0, \quad y(0) = 2, \, y'(0) = -1.$$

Give only the first five coefficients of the power series solution.

Answer the following questions.

(a) (10 points) Find the Laplace transform of $f(t) = t\cos(2t) + e^t \sin(t)$.

(b) (10 points) Find the inverse Laplace transform of $F(s) = \dfrac{1}{(s+2)(s-10)}$.

Answer the following questions.

(a) (10 points) Find the Laplace transform of the function

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 2t + 1 & 1 \le t. \end{cases}$$

(b) (10 points) Find the inverse Laplace transform of the function $F(s) = \dfrac{e^{-2s}}{(s-1)^2}$.

Find the solution to the following IVP using the Laplace transform:

$$y'' - y' - 6y = 0, \quad y(0) = 1,\, y'(0) = 0.$$

(a) (5 points) Denote by $F(s)$ the Laplace transform of $f(t)$. Show that if $h(t) = (tf(t))'$, then $L(h(t)) = -sF'$.

(b) (5 points) Find the solution $y(t)$ of the following integral equation:

$$\int_0^t y(x)(t-x)^3 \, dx = t^5.$$

Answer the following statements with **True** or **False**. Write your answer on the horizontal line at the end of each statement. Justify your answer in the white space underneath each statement.

(a) ( / 2) The radius of convergence of a power series solution $\sum_{n=0}^{\infty} a_n(x-1)^n$ of the ODE $(1+x^2)y'' + xy' + y = 0$ is $\sqrt{2}$.

(a) ————

(b) ( / 2) If $f(t) = t$ and $g(t) = \sin(t)$, then $L(f(t)g(t)) = \dfrac{1}{s^2(s^2+1)}$.

(b) ————

(c) ( / 2) If $f(t) = u(t-1)u(t-2)$, then $f(t) = u(t-2)$.

(c) ————

(d) ( / 2) If $f(t) = t$ and $g(t) = t$, then $f(t) * g(t) = \dfrac{t^3}{6}$.

(d) ————

(e) ( / 2) The number $x = 0$ is a singular point of the ODE $(x^2+x)y'' + xy' + y = 0$.

(e) ————

*For officials use only:*

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|-----------|-----|-----|-----|-----|-----|-----|-------|
| Points:   | 20  | 20  | 20  | 20  | 10  | 10  | 100   |
| Score:    |     |     |     |     |     |     |       |