CONFIDENTIAL:
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Dear Professor **Pierre-Olivier Parise:**

I acknowledge that I understand the stated conditions below and will take the MATH 302 exam in accordance with these conditions:

CLOSED BOOK, NOTES ALLOWED - (1 dbl-sided sheet), USE OF CALCULATOR (No graphical)

I certify that I will not use any unauthorized materials, written or electronic, and that I have not communicated nor will I communicate with anyone regarding this exam which was administered to me today at the KOKUA Program office. Thank you for working with KOKUA to provide me with appropriate testing accommodations.

Jasmine Carpena

11/1/22
Date

11/1/2022

Dear Professor Pierre-Olivier Parise:

Please find enclosed your **MATH 302** exam taken by **Jasmine Carpena** at the KOKUA Program. In the interest of exam security and validity, we are informing you of service provided to Jasmine.

The student requested:

USE OF KOKUA FACILITIES, TIME EXTENSION

The examination was administered by KOKUA on 11/1/2022 from 11:50 to 1:11.

This envelope and all of its enclosures were returned to **parisepo@hawaii.edu** at **Professor Pierre-Olivier Parise** on by KOKUA staff. Thank you very much for your invaluable cooperation!

KOKUA Program
Office of Student Equity, Excellence and Diversity
(V/TTY) 956-7612 or (V/TTY) 956-7511

Last name: Carpena
First name: Jasmine

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all the questions below and write your answers directly on the questionnaire.
- You have 75 minutes to complete the exam.
- When you are done (or at the end of the 75min period), return your copy.
- No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam.
- **Turn your cellphone off during the exam**.
- You may use a digital calculator (no graphical calculators or symbolic calculators will be allowed).
- You are not allowed to use the lecture notes or the textbook.
- You may bring one 2-sided cheat sheet of handwriting notes.
- You must show ALL your work to have full credit. An answer without justification is worth no point.

Your Signature: Jasmine Carpena

May the Force be with you!

Pierre-Olivier Parisé

UNIVERSITY OF HAWAIʻI

QUESTION 1 (20 pts)

For the given ODE, find the general solution.

(a) (10 points) $y'' + 2y' + y = 0$.

$r^2 + 2r + 1 = 0$

$(r+1)^2 = 0$

$r = -1$ multiplicity 2

$y = c_1 e^{-x} + c_2 x e^{-x}$

(b) (10 points) $y'' + 6y' + 10y = 0$.

$r^2 + 6r + 10 = 0$

$r = \dfrac{-6 \pm \sqrt{(6)^2 - 4(10)}}{2} = \dfrac{-6 \pm \sqrt{-4}}{2} = \dfrac{-6 \pm 2i}{2} = -3 \pm i$

$y = e^{-3x}(c_1 \cos x + c_2 \sin x)$

QUESTION 2 (20 pts)

For the following ODEs, give the form of the particular solution. **Don't solve for the constants.** (My bad, I was thinking $c_1 \cdots$)

(a) (10 points) $y'' + 5y' - 6y = 22 + 18x - 18x^2$.

$\boxed{y_p = Ax^2 + Bx + C} \to y_p' = 2Ax + B \to y_p'' = 2A$

$2A + 5(2Ax + B) - 6(Ax^2 + Bx + c) = 22 + 18x - 18x^2$

$2A + 10Ax + 5B - 6Ax^2 - 6Bx - 6C = 22 + 18x - 18x^2$

$(-6A)x^2 + (10A - 6B)x + (2A + 5B - 6C) = 22 + 18x - 18x^2$

$-6A = -18 \Rightarrow A = 3$

$10(3) - 6B = 18 \Rightarrow B = 2$

$2(3) + 5(2) - 6C = 22 \Rightarrow c = -1$

$y_p = 3x^2 + 2x - 1$

$r^2 + 5r - 6 = 0$

$(r+6)(r-1) \to r = -6, 1$

$r^2 - 2r + 5$

(b) (10 points) $y'' - 2y' + 5y = e^x\big((6 + 8x)\cos(2x) + (6 - 8x)\sin(2x)\big)$.

$(5)\ \boxed{y_p = xe^x\left[(Ax+B)\cos(2x) + (Cx+D)\sin(2x)\right]}$

$(-2)\ y_p' = e^x\left[(Ax+B)\cos(2x) + (Cx+D)\sin(2x)\right]$

$+ e^x\left[A\cos(2x) - 2(Ax+B)\sin(2x) + C\sin(2x) + 2(Cx+D)\cos(2x)\right]$

$= e^x\left[((A+2C)x + A+B+2D)\cos(2x) + ((-2A+C)x - 2B + C + D)\sin(2x)\right]$

$y_p'' = e^x\left[((A+2C)x + A + B + 2D)\cos(2x) + ((-2A+C)x - 2B + C + D)\sin(2x)\right]$

$+ e^x\left[(A+2C)\cos(2x) - 2((A+2C)x + A + B + 2D)\sin(2x) + (-2A+C)\sin(2x)\right.$

$\left.+ 2((-2A+C)x - 2B + C + D)\cos(2x)\right)$

$= e^x\left[((-3A+4C)x + (2A-3B+3C+2D))\cos(2x) + ((-4A-3C)x + (-A-4B+C-3D))\right]$

$A = 1 \quad B = 1 \quad C = 1 \quad D = -1$

$y_p = e^x\left[(x+1)\cos 2x + (x-1)\sin 2x\right]$

$r = \dfrac{2 \pm \sqrt{(-2)^2 - 4(5)}}{2} = \dfrac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i \qquad y_h = e^x(c_1\cos 2x + c_2 \sin 2x)$

QUESTION 3 **(20 pts)**

Find the general solution to the following ODE:

$$y'' - 4y' - 5y = -6e^{-x}.$$

$r^2 - 4r - 5 = 0$

$(r-5)(r+1) = 0 \Rightarrow r = 5, -1$

$y_h = c_1 e^{5x} + c_2 e^{-x}$

$y_p = Axe^{-x} \rightarrow y_p' = Ae^{-x} - Axe^{-x} \rightarrow y_p'' = -Ae^{-x} - Ae^{-x} + Axe^{-x}$

$= -2Ae^{-x} + Axe^{-x}$

$-2Ae^{-x} + Axe^{-x} - 4(Ae^{-x} - Axe^{-x}) - 5(Axe^{-x}) = -6e^{-x}$

$-2Ae^{-x} + \cancel{Axe^{-x}} - 4Ae^{-x} + 4\cancel{Ax}e^{-x} - 5\cancel{Ax}e^{-x} = -6e^{-x}$

$-6Ae^{-x} = -6e^{-x}$

$-6A = -6 \Rightarrow A = 1$

$y_p = xe^{-x}$

$y = c_1 e^{5x} + c_2 e^{-x} + xe^{-x}$

QUESTION 4 **(20 pts)**

Find the general solution to the following ODE:

$$x^2y'' + xy' - 4y = -6x - 4$$

knowing that $y_1(x) = x^2$ is a solution to the complementary equation.

$y = ux^2 \to y' = u'x^2 + 2ux \to y'' = u''x^2 + 2u'x + 2u'x + 2u$

$= u''x^2 + 4u'x + 2u$

$x^2(u''x^2 + 4u'x + 2u) + x(u'x^2 + 2ux) - 4(ux^2)$

$u''x^4 + 4u'x^3 + 2ux^2 + u'x^3 + 2ux^2 - 4ux^2$

$u''x^4 + 5u'x^3$

$z = u'$

$z'x^4 + 5zx^3 = -6x - 4$

$z'x^4 + 5zx^3 = 0$

$z'/z = -5x^3/x^4$

$\ln|z| = -5\ln|x| + c$

$z = k/x^5$

$z = w(1/x^5) \to z' = w'(1/x^5) + w(-5/x^6)$

$[w'(1/x^5) + w(-5/x^6)]x^4 + 5(w(1/x^5))x^3 = -6x - 4$

$w'\left(\frac{1}{x}\right) + \cancel{w\left(\frac{-5}{x^2}\right) + w\left(\frac{5}{x^2}\right)} = -6x - 4$

$w'\left(\frac{1}{x}\right) = -6x - 4$

$w' = -6x^2 - 4x$

$w = \int(-6x^2 - 4x)\,dx = -2x^3 - 2x^2 + k_1$

$z = (-2x^3 - 2x^2 + k_1)(1/x^5) = -2/x^2 - 2/x^3 + k_1/x^5$

$u = \int(-2/x^2 - 2/x^3 + k_1/x^5)\,dx = 2/x + 1/x^2 + c_1/x^4 + c_2$

$y = x^2\left(\frac{2}{x} + \frac{1}{x^2} + \frac{c_1}{x^4} + c_2\right)$

QUESTION 5 (10 pts)

(a) (5 points) If y_1 and y_2 are two differentiable functions not identically zero, the Wronkians W of $\{y_1, y_2\}$ is

$$W = y_1 y_2' - y_1' y_2.$$

Show that if $\{y_1, y_2\}$ is **not** a set of fundamental solutions for a second order differential equation, then $W = 0$.

$$\det \begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix} = y_1 y_2' - y_1' y_2 = 0$$

$$y_1 y_2' = y_1' y_2$$

(b) (5 points) Solve the following IVP:

$$y'' + y = 0, \quad y(0) = 0, y'(0) = 1.$$

$r^2 + 1 = 0 = r = \pm i$

$y = c_1 \cos x + c_2 \sin x$

$y' = -c_1 \sin x + c_2 \cos x$

$y(0) = c_1 \cos(0) + c_2 \sin(0) = 0 \Rightarrow c_1 = 0$

$y'(0) = -(0)\sin(0) + c_2 \cos(0) = 1 \Rightarrow c_2 = 1$

$y = \sin x$

QUESTION 6 (10 pts)

Answer the following statements with **True** or **False**. Write your answer on the horizontal line at the end of each statement. Justify your answer in the white space underneath each statement.

(a) (/ 2) $\{x, 1\}$ is a fundamental of solutions to $y'' = 0$.

$(x)' = 1 \to (1)' = 0$

$(1)' = 0 \to (0)' = 0$

(a) true

(b) (/ 2) If $y_1(x) = \cos(2x) + \sin(2x)$ and $y_2(x) = 2\cos(2x) + 2\sin(2x)$ are solutions to $y'' + 4y = 0$, then $y(x) = 3\cos(2x) + 3\sin(2x)$ is a solution to $y'' + 4y = 0$.

$y' = -6\sin 2x + 6\cos 2x \to y'' = -12\cos 2x - 12\sin 2x$

$-12\cos 2x - 12\sin 2x + 4(3\cos(2x) + 3\sin(2x)) = 0$ ✓

(b) true

(c) (/ 2) In the Spring-mass system model $y'' + (k/m)y = \frac{F_0}{m}\cos(\omega t)$, a resonance occurs when $\sqrt{k/m} = \omega$.

$y'' + \omega^2 y = \frac{F_0}{m}\cos(\omega t)$

(c) false

(d) (/ 2) If $y_1 = x$ and $y_2 = e^x$ are solutions to the complementary equation $(x-1)y'' - xy' + y = (x-1)^2$, then the solution should have the form $y(x) = xu_1(x) + e^x u_2(x)$.

(d) false

(e) (/ 2) The function $y(x) = \sin(x) + \cos(x)$ is a solution to the following IVP: $y'' + y = 0$, ✓ $y(0) = 1$, $y'(0) = 1$.

$y' = \cos x - \sin x \to y'' = -\sin x - \cos x$

(e) true

Do not write on this page.

For officials use only:

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							