Section 5.1 — Problem 3 — 10 points

a) We see that $y_1' = e^x$ and $y_1'' = e^x$ and therefore

$$y_1'' - 2y' + y = e^x - 2e^x + e^x = 0.$$

We see also that $y_2' = e^x + xe^x$ and $y_1'' = 2e^x + xe^x$. Therefore, we have

$$y'' - 2y' + y = 2e^x + xe^x - 2e^x - 2xe^x + xe^x = 0.$$

The functions y_1 and y_2 are solutions to the ODE.

b) This is a consequence of the linearity of the ODE. We have $y(x) = c_1 y_1 + c_2 y_2$.

c) The general solution is $y(x) = c_1 e^x + c_2 xe^x$. Using the initial conditions, we see that

$$y(0) = 7 \quad \Rightarrow \quad c_1 = 7.$$

The derivative is $y'(x) = (c_1 + c_2)e^x + c_2 xe^x$. Using the initial conditions, we see that

$$y'(0) = 4 \quad \Rightarrow \quad c_1 + c_2 = 4 \quad \Rightarrow \quad c_2 = -3.$$

The solution to the IVP is therefore

$$y(x) = 7e^x - 3xe^x.$$

d) In general, if $y(0) = k_0$, then we see that

$$c_1 = k_0.$$

Also, if $y'(0) = k_1$, then we see that

$$c_1 + c_2 = k_1 \quad \Rightarrow \quad c_2 = k_1 - k_0.$$

Therefore, the solution to the IVP is

$$y(x) = k_0 e^x + (k_1 - k_0)xe^x.$$

Section 5.2 — Problem 1 — 5 points

The characteristic equation is

$$\lambda^2 + 5\lambda - 6 = 0.$$

The roots are $r_1 = 6$ and $r_2 = -1$. Therefore, the solution is

$$y(x) = c_1 e^{6x} + c_2 e^{-x}.$$

Section 5.2 — Problem 3 — 5 points

The characteristic equation is

$$\lambda^2 + 8\lambda + 7 = 0.$$

The roots are $r_1 = -7$ and $r_2 = -1$. Therefore, the general solution is

$$y(x) = c_1 e^{-7x} + c_2 e^{-x}.$$

Section 5.2 — Problem 5 — 5 points

The characteristic equation is

$$\lambda^2 + 2\lambda + 10 = 0.$$

Using the quadratic formula, the roots are

$$r_1 = -1 + 2i \quad \text{and} \quad r_2 = -1 - 2i.$$

Therefore, the general solution is

$$y(x) = c_1 e^{-x} \cos(2x) + c_2 e^{-x} \sin(2x).$$

Section 5.2 — Problem 7 — 5 points

The characteristic polynomial is

$$\lambda^2 - 8\lambda + 16 = (\lambda - 4)^2.$$

Therefore, there is only one root and is $\lambda = 4$. It has multiplicity 2. The solution is therefore

$$y(x) = c_1 e^{4x} + c_2 x e^{4x}.$$

Section 5.2 — Problem 9 — 5 points

The characteristic polynomial is

$$\lambda^2 - 2\lambda + 3.$$

Using the quadratic formula, we have

$$r_1 = 1 + i\sqrt{2} \quad \text{and} \quad r_2 = 1 - i\sqrt{2}.$$

Therefore, the solution is

$$y(x) = c_1 e^x \cos(\sqrt{2}x) + c_2 e^x \sin(\sqrt{2}x).$$

Section 5.2 — Problem 11 — 5 points

The characteristic polynomial is

$$4\lambda^2 + 4\lambda + 10.$$

Using the quadratic formula, we get

$$r_1 = -\frac{1}{2} + i\frac{3}{2} \quad \text{and} \quad r_2 = -\frac{1}{2} - i\frac{3}{2}.$$

Therefore, the solution is

$$y(x) = c_1 e^{-x/2} \cos(3x/2) + c_2 e^{-x/2} \sin(3x/2).$$

Section 5.2 — Problem 17 — 10 points

The characteristic polynomial is

$$4\lambda^2 - 12\lambda + 9.$$

Using the quadratic formula, we find the roots:

$$r_1 = r_2 = \frac{3}{2}.$$

There is only one root and it's multiplicity is two. Therefore, the general solution is

$$y(x) = c_1 e^{3x/2} + c_2 x e^{3x/2}.$$

We have that $y'(x) = \frac{3c_1}{2}e^{3x/2} + c_2 e^{3x/2} + \frac{3c_2}{2}xe^{3x/2}$. Therefore, the initial condition becomes concretely

$$c_1 = y(0) = 3 \quad \text{and} \quad \frac{3c_1}{2} + c_2 = y'(0) = \frac{5}{2}.$$

With the value of c_1, we find that

$$c_2 = \frac{5}{2} - \frac{3}{2} = 1.$$

Therefore the solution to the IVP is

$$y(x) = 3e^{3x/2} + xe^{3x/2}.$$

Total (Points): 50.