Math 302

Chapter 5

Section 5.7: Variation Of Parameters

Contents

Created by: Pierre-Olivier Parisé
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What Is The Method Of Variation Of Parameters

Our goal in this section is to find the solutions to

$$P_0(x)y'' + P_1(x)y' + P_2(x)y = F(x)$$

using the method **variation of parameters**. Our assumption is

- We know at least two solutions to the complementary equation $P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$.

Example 1. Find the general solution to

$$x^2y'' - 2xy' + 2y = x^{9/2}$$

given that $y_1(x) = x$ and $y_2(x) = x^2$ are solutions to the complementary equation.

General Procedure

To find a particular solution to

$$P_0(x)y'' + P_1(x)y' + P_0(x)y = F(x)$$

knowing two solutions $y_1(x)$ and $y_2(x)$ to the complementary equation, we follow these steps:

- Write $y_{par}(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$.
- Write the system

$$\begin{aligned} u_1'y_1 + u_2'y_2 &= 0 \\ u_1'y_1' + u_2'y_2' &= \frac{F}{P_0}. \end{aligned}$$

- Solve the system for u_1' and u_2':

$$u_1' = -\frac{Fy_2}{P_0(y_1y_2' - y_1'y_2)} \quad \text{and} \quad u_2' = \frac{Fy_1}{P_0(y_1y_2' - y_1'y_2)}.$$

- Obtain u_1 and u_2 by integrating u_1' and u_2' respectively.
- Substitute u_1 and u_2 in $y_{par}(x)$ to obtain the particular solution.

EXAMPLE 2. Find a particular solution to

$$y'' + 3y' + 2y = \frac{1}{1 + e^x}.$$