

Last name: \_\_\_\_\_  
First name: \_\_\_\_\_

**Instructions:**

- Make sure to write your complete name on your copy.
- You must answer all the questions below and write your answers directly on the questionnaire.
- You have 120 minutes (2 hours) to complete the exam.
- When you are done (or at the end of the 120min period), return your copy.
- No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam.
- **Turn off your cellphone during the exam.**
- You may use a digital calculator (no graphical calculators or symbolic calculators will be allowed).
- You are not allowed to use the lecture notes or the textbook.
- You may bring one 2-sided cheat sheet of handwriting notes.
- You must show ALL your work to have full credit. An answer without justification is worth no point.

**Your Signature:** \_\_\_\_\_

May the Force be with you!

Pierre-Olivier Parisé

UNIVERSITY  
OF HAWAI'I



---

QUESTION 1

---

(20 pts)

Find the solution of the following ODE using the power series method.

$$(1 + x^2)y'' + xy' + y = 0, \quad y(0) = 2, \quad y'(0) = -1.$$

Give only the first five coefficients of the power series solution.



---

QUESTION 2

---

(20 pts)

Answer the following questions.

(a) (10 points) Find the Laplace transform of  $f(t) = te^t \cos(2t)$ .

(b) (10 points) Find the inverse Laplace transform of  $F(s) = \frac{1}{(s-2)(s+3)}$ .

---

QUESTION 3

---

(20 pts)

Answer the following questions.

- (a) (10 points) Find the Laplace transform of the function

$$f(t) = \begin{cases} t - 1 & 0 \leq t < 1 \\ t + 1 & 1 \leq t. \end{cases}$$

- (b) (10 points) Find the inverse Laplace transform of the function  $F(s) = \frac{e^{-s}}{(s+1)^2}$ .

---

QUESTION 4

---

(20 pts)

Find the solution to the following IVP using the Laplace transform:

$$y'' - 4y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

---

QUESTION 5

---

(10 pts)

- (a) (5 points) Denote by  $F(s)$  the Laplace transform of  $f(t)$ . Show that if  $h(t) = \int_0^t x f(x) dx$ , then  $L(h(t)) = -\frac{F'(s)}{s}$ .

- (b) (5 points) Find the solution of the following integral equation:

$$y(t) = 1 + \int_0^t y(x) dx.$$

---

QUESTION 6

---

(10 pts)

Answer the following statements with **True** or **False**. Write your answer on the horizontal line at the end of each statement. Justify your answer in the white space underneath each statement.

- (a) (    / 2) The radius of convergence of the power series solution  $\sum_{n=0}^{\infty} a_n(x-3)^n$  of the ODE  $(16+x^2)y'' + xy' + y = 0$  is 5.

(a) \_\_\_\_\_

- (b) (    / 2) If  $f(t) = t$  and  $g(t) = t^2$ , then  $L(f(t)g(t)) = \frac{2}{s^5}$ .

(b) \_\_\_\_\_

- (c) (    / 2) If  $f(t) = 0$  for  $t < 2$ ,  $f(t) = 2$  for  $2 \leq t < 3$  and  $f(t) = t$  for  $t \geq 3$ , then  $f(t) = 2u(t-2) + (t-2)u(t-3)$ .

(c) \_\_\_\_\_

- (d) (    / 2) If  $f(t) = t^2$  and  $g(t) = t^2$ , then  $f(t) * g(t) = \frac{t^5}{30}$ .

(d) \_\_\_\_\_

- (e) (    / 2) The number  $x = 0$  is a singular point of the ODE  $(x^2 + x)y'' + xy' + y = 0$ .

(e) \_\_\_\_\_



DO NOT WRITE ON THIS PAGE.

*For officials use only:*

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							