

# MATH 302

## CHAPTER 1

### SECTION 1.3: DIRECTION FIELDS FOR FIRST ORDER EQUATIONS

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**What Is A Direction Field?**

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## WHAT IS A DIRECTION FIELD?

Consider the following first order ODE:

slope of the tangent line  $\rightarrow y'(x) = f(x, y(x))$ .  $\leftarrow$  Value of the slope of the tangent line at  $(x, y)$

If you use this information well, you can get these pictures.

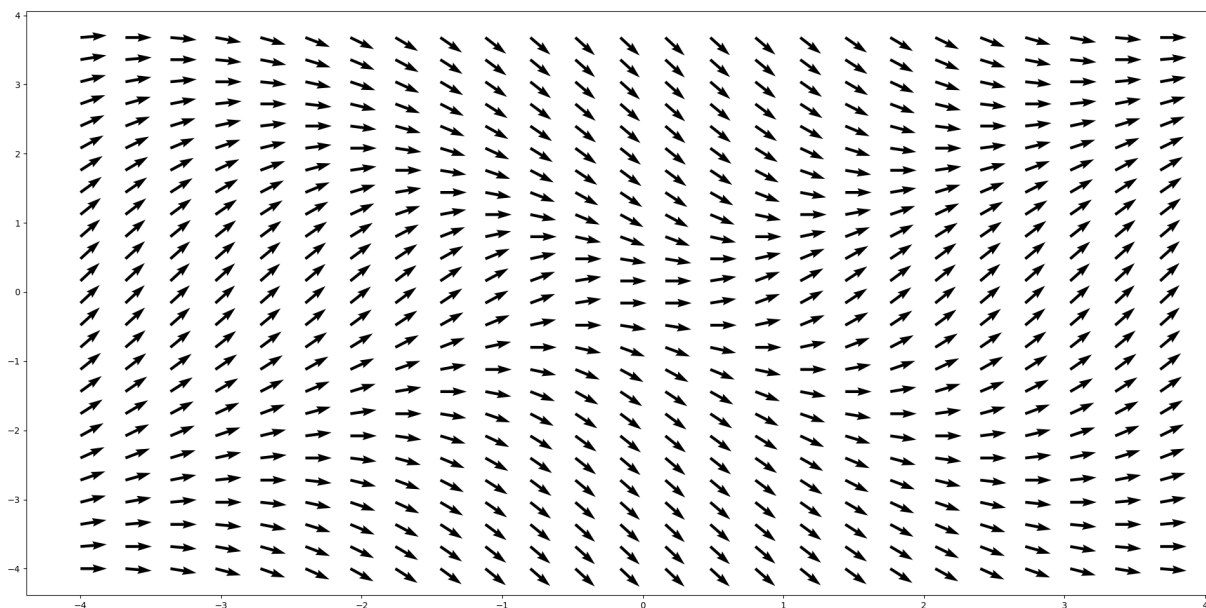


Figure 1: Direction field of  $y' = \frac{x^2 - y^2}{1 + x^2 + y^2}$ .

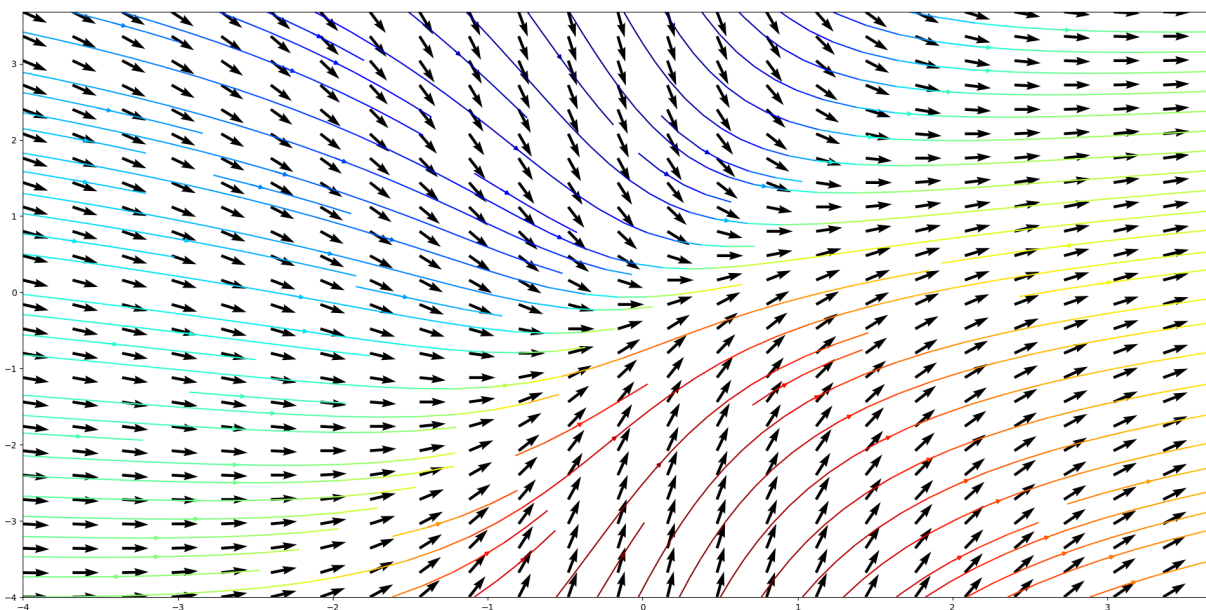
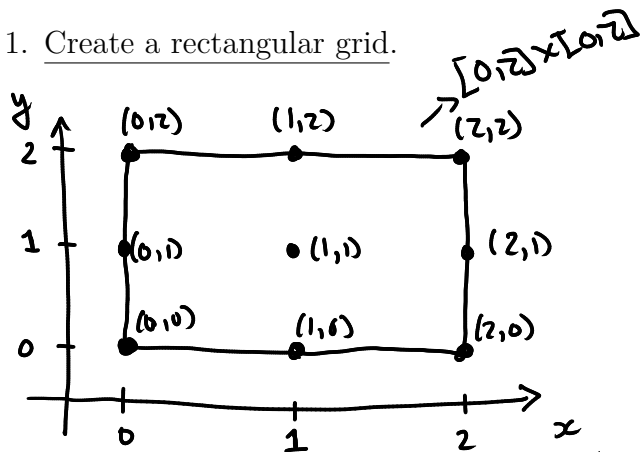


Figure 2: Direction field of  $y' = \frac{x - y}{1 + x^2}$ .

**EXAMPLE 1.** Draw the direction field of the following ODE:

$$y' = 1 + xy^2 \rightarrow f(x, y)$$

1. Create a rectangular grid.



interval  $[0, 2]$  for  $x$

3 pts:  $x_0 = 0$   
 $x_1 = 1$   
 $x_2 = 2$

interval  $[0, 2]$  for  $y$

3 pts:  $y_0 = 0$   
 $y_1 = 1$   
 $y_2 = 2$

2. Find the slopes  $y'$  in each points of the grid (nods).

$$(0,0): y' = 1 + 0 \cdot 0^2 = 1$$

$$(1,0): y' = 1$$

$$(0,1): y' = 1 + 0 \cdot 1^2 = 1$$

$$(1,1): y' = 1 + 1 \cdot 1^2 = 2$$

$$(0,2): y' = 1 + 0 \cdot 2^2 = 1$$

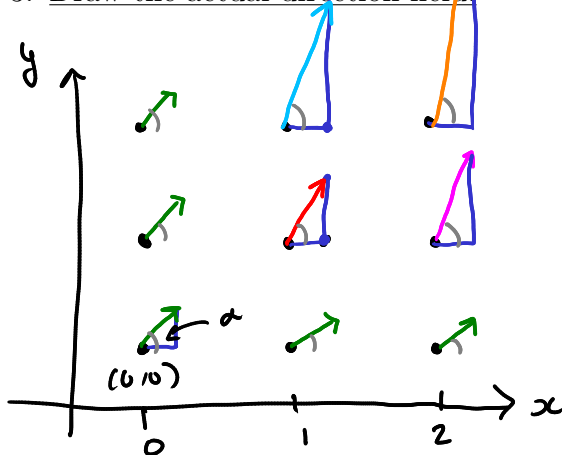
$$(1,2): y' = 5$$

$$(2,0): y' = 1$$

$$(2,1): y' = 3$$

$$(2,2): y' = 9$$

3. Draw the actual direction field.



$$\alpha = \arctan(\text{slope}) = \arctan(1) = 45^\circ$$

$$(0,0): \text{slope} = 1 \quad (x=0.25 \rightarrow y=1 \cdot 0.25 = 0.25)$$

$$(0,1): \text{same}$$

$$(0,2): \text{same}$$

$$(1,0): \text{same (slope} = 1)$$

$$(1,1): \text{slope} = 2 \quad (x=0.25 \rightarrow y=2 \cdot 0.25 = 0.5)$$

$$(1,2): \text{slope} = 5 \quad (x=0.25 \rightarrow y=5 \cdot 0.25 = 1.25)$$

$$(2,0): \text{slope} = 1$$

$$(2,1): \text{slope} = 3$$

$$(2,2): \text{slope} = 9$$