

MATH 302

CHAPTER 1

SECTION 1.3: DIRECTION FIELDS FOR FIRST ORDER EQUATIONS

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WHAT IS A DIRECTION FIELD?

Consider the following first order ODE:

$$y'(x) = f(x, y(x)).$$

slope of the
tangent line.

value of the
slope of tangent
line at (x, y) .

If you use this information well, you can get these pictures.

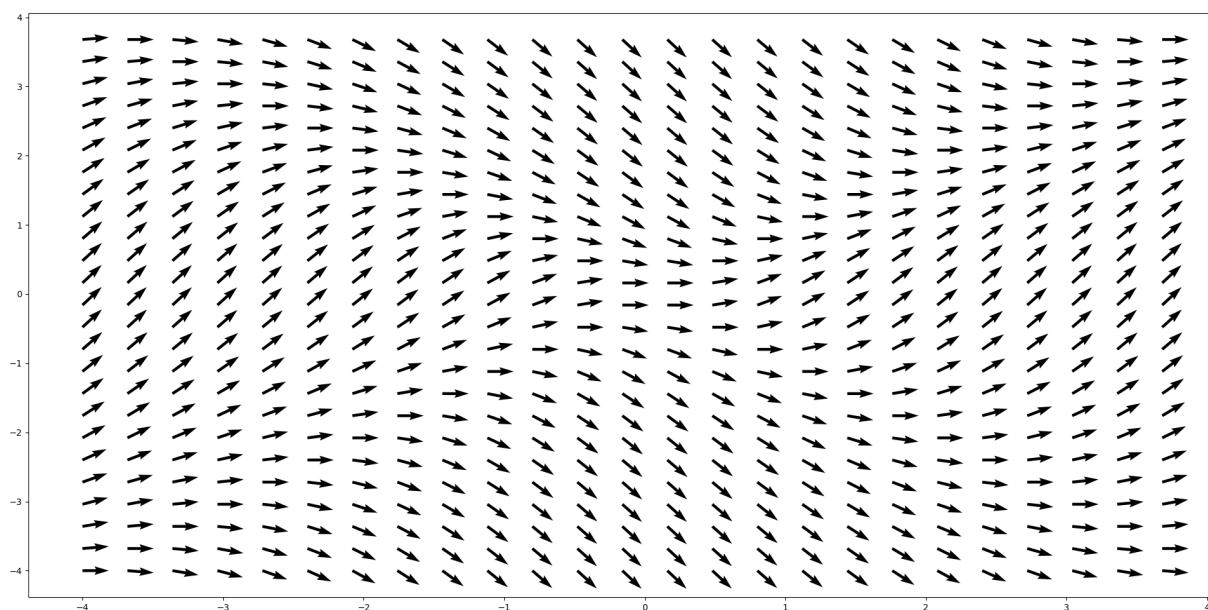


Figure 1: Direction field of $y' = \frac{x^2 - y^2}{1 + x^2 + y^2}$.

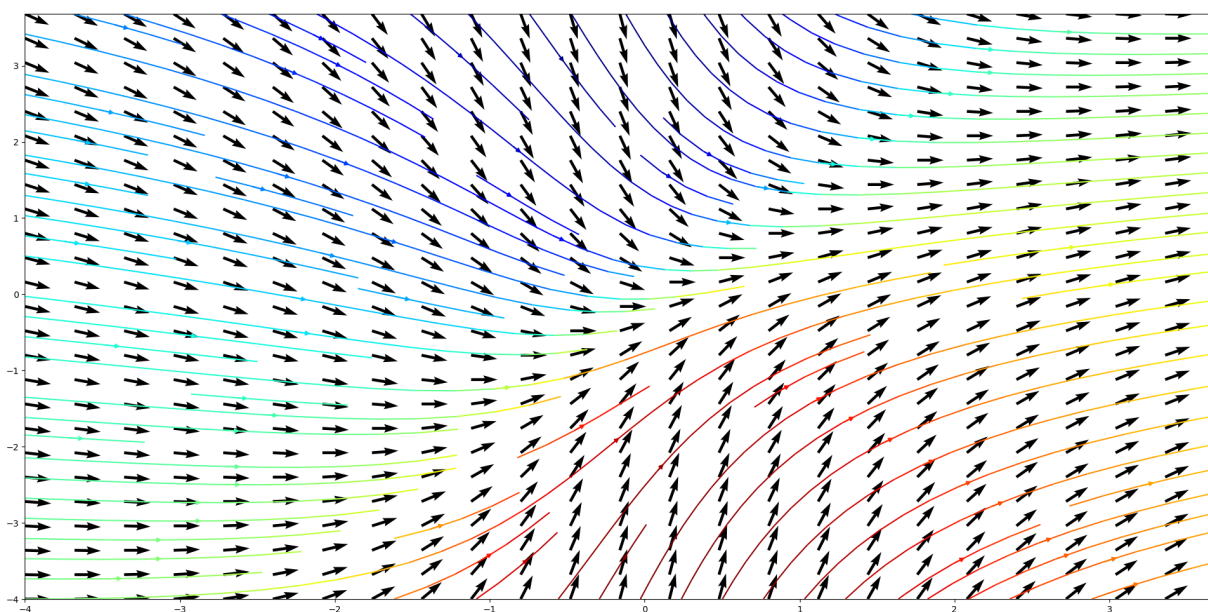
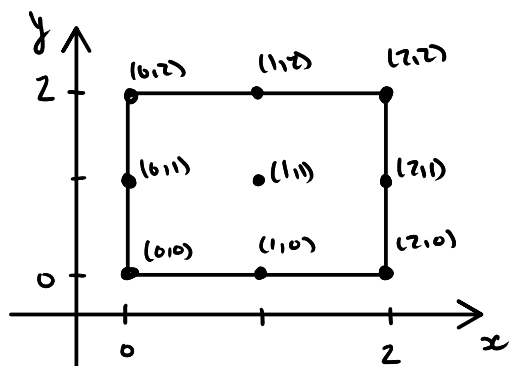


Figure 2: Direction field of $y' = \frac{x - y}{1 + x^2}$.

EXAMPLE 1. Draw the direction field of the following ODE:

$$y' = 1 + xy^2.$$

1. Create a rectangular grid.



interval x : $[0, 2]$ nb. div: 3

nods \rightarrow $x_0 = 0$ $x_2 = 2$
 $x_1 = 1$

interval y : $[0, 2]$ nb. div: 3

nods \rightarrow $y_0 = 0$ $y_2 = 2$
 $y_1 = 1$

2. Find the slopes in each points of the grid (nods).

$$(0,0): y' = 1$$

$$(0,1): y' = 1$$

$$(0,2): y' = 1$$

$$(1,0): y' = 1$$

$$(1,1): y' = 2$$

$$(1,2): y' = 5$$

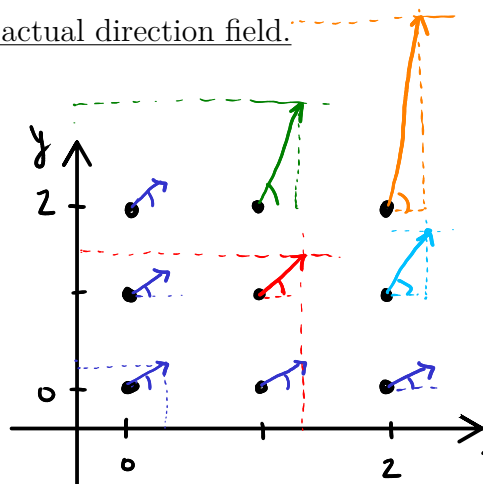
$$(2,0): y' = 1$$

$$(2,1): y' = 3$$

$$(2,2): y' = 9$$

3. Draw the actual direction field.

$\angle: 45^\circ$
 $\angle: 63.4^\circ$
 $\angle: 78.7^\circ$
 $\angle: 71.5^\circ$
 $\angle: 83.6^\circ$



$$(0,0): \text{slope} = 1 = \frac{1}{1} \quad (x=0.25 \rightarrow y=0.25)$$

$$(0,1): \text{slope} = 1 = \frac{1}{1} \quad (x=0.25 \rightarrow y=0.25)$$

$$(0,2): \text{same.}$$

$$(1,0): \text{same}$$

$$(1,1): \text{slope} = 2 = \frac{2}{1} \quad (x=0.25 \rightarrow y=0.5)$$

$$(1,2): \text{slope} = 5 = \frac{5}{1} \quad (x=0.25 \rightarrow y=1.25)$$

$$(2,0): \text{same}$$

$$(2,1): \text{slope} = \frac{3}{1} \quad (x=0.25 \rightarrow y=0.75)$$

$$(2,2): \text{slope} = \frac{9}{1} \quad (x=0.25 \rightarrow y=2.25)$$