CONFIDENTIAL:
DO NOT RETURN TO STUDENT.
SHRED TO DISPOSE.

Dear Professor **Pierre-Olivier Parise:**

I acknowledge that I understand the stated conditions below and will take the MATH 302 exam in accordance with these conditions:

CLOSED BOOK, NOTES ALLOWED - ((1) DOUBLE SIDED NOTE SHEET), USE OF CALCULATOR (NO GRAPHING ALLOWED)

I certify that I will not use any unauthorized materials, written or electronic, and that I have not communicated nor will I communicate with anyone regarding this exam which was administered to me today at the KOKUA Program office. Thank you for working with KOKUA to provide me with appropriate testing accommodations.

Jasmine Carpena

12/15/22
Date

12/15/2022

Dear Professor Pierre-Olivier Parise:

Please find enclosed your **MATH 302** exam taken by **Jasmine Carpena** at the KOKUA Program. In the interest of exam security and validity, we are informing you of service provided to Jasmine.

The student requested:

USE OF KOKUA FACILITIES, TIME EXTENSION

The examination was administered by KOKUA on 12/15/2022 from 12:15 to 1:55.

This envelope and all of its enclosures were returned to **parisepo@hawaii.edu** at **Professor Pierre-Olivier Parise** on **12/15/2022** by KOKUA staff. Thank you very much for your invaluable cooperation!

KOKUA Program
Office of Student Equity, Excellence and Diversity
(V/TTY) 956-7612 or (V/TTY) 956-7511

Created by Pierre-O. Parisé
2022/15/12, Fall 2022

Last name: Carpena

First name: Jasmine

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all the questions below and write your answers directly on the questionnaire.
- You have 120 minutes (2 hours) to complete the exam.
- When you are done (or at the end of the 120min period), return your copy.
- No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam.
- **Turn off your cellphone during the exam**.
- You may use a digital calculator (no graphical calculators or symbolic calculators will be allowed).
- You are not allowed to use the lecture notes or the textbook.
- You may bring one 2-sided cheat sheet of handwriting notes.
- You must show ALL your work to have full credit. An answer without justification is worth no point.

Your Signature:

May the Force be with you!

Pierre-Olivier Parisé

UNIVERSITY OF HAWAI'I

QUESTION 1 **(20 pts)**

Find the solution of the following ODE using the power series method.

$$(1+x^2)y'' + xy' + y = 0, \quad y(0) = 2,\ y'(0) = -1.$$

Give only the first five coefficients of the power series solution.

$$(1+x^2)\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + x\sum_{n=1}^{\infty} na_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} n(n-1)a_n x^n + \sum_{n=0}^{\infty} na_n x^n + \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}x^n + \sum_{n=0}^{\infty}[n(n-1)+n+1]a_n x^n$$

$$(n+2)(n+1)a_{n+2} = (n^2+1)a_n \rightarrow a_{n+2} = \frac{n^2+1}{(n+2)(n+1)}a_n$$

$$a_2 = \frac{1}{2\cdot 1}a_0 = \frac{1}{2}a_0 \qquad a_3 = \frac{2}{3\cdot 2} = \frac{2}{6}a_1 = \frac{1}{3}a_1$$

$$a_4 = \frac{5}{4\cdot 3}a_2 = \frac{5}{12}\cdot\frac{1}{2}a_0 \qquad a_5 = \frac{10}{5\cdot 4} = \frac{10}{20}a_3 = \frac{1}{2}a_3 = \frac{1}{2}\cdot\frac{1}{3}a_1$$

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \dots$$

$$y(0) = a_0 + \dots = 2 \Rightarrow a_0 = 2$$

$$y'(0) = a_1 + \dots = -1 \Rightarrow a_1 = -1$$

$$y = 2 - x + x^2 - \frac{1}{3}x^3 + \frac{5}{12}x^4 - \frac{1}{6}x^5 + \dots$$

Question 2 (20 pts)

Answer the following questions.

(a) (10 points) Find the Laplace transform of $f(t) = te^t \cos(2t)$.

$$L\left[te^t \cos(2t)\right] \Rightarrow \frac{s}{(s-1)^4 + 4}$$

$$L[t] = \frac{1}{s^2}$$

$$L\left[te^t\right] = \frac{1}{(s-1)^2}$$

$$L\left[te^t \cos(2t)\right] = \frac{s}{((s-1)^2)^2 + (2)^2}$$

(b) (10 points) Find the inverse Laplace transform of $F(s) = \dfrac{1}{(s-2)(s+3)}$.

$$\frac{1}{(s-2)(s+3)} = \frac{A}{s-2} + \frac{B}{s+3} \rightarrow 1 = A(s+3) + B(s-2)$$

$$1 = (A+B)s + (3A-2B)$$

$$\left.\begin{array}{l} A+B=0 \\ 3A-2B=1 \end{array}\right\} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 3 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -5 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & -1/5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 1/5 \\ 0 & 1 & -1/5 \end{array}\right] \rightarrow \begin{array}{l} A = 1/5 \\ B = -1/5 \end{array}$$

$$L^{-1}\left[\frac{1}{5(s-2)} - \frac{1}{5(s+3)}\right] = \frac{1}{5}e^{2t} - \frac{1}{5}e^{-3t}$$

$$L^{-1}\left[\frac{1}{(s-2)(s+3)}\right] = \frac{1}{5}e^{2t} - \frac{1}{5}e^{-3t}$$

QUESTION 3 (20 pts)

Answer the following questions.

(a) (10 points) Find the Laplace transform of the function

$$f(t) = \begin{cases} t-1 & 0 \le t < 1 \\ t+1 & 1 \le t. \end{cases}$$

$$f(t) = (t-1)u(t) - (t-1)u(t-1) + (t+1)u(t-1)$$

$$= (t-1)u(t) + 2u(t-1)$$

$$L\left[(t-1)u(t) + 2u(t-1)\right] = \frac{1}{s}L\left[t-1\right] + 2\left[\frac{e^{-s}}{s}\right]$$

$$= \frac{1}{s}\left[\frac{1}{s^2} - \frac{1}{s}\right] + \frac{2e^{-s}}{s}$$

$$= \frac{1}{s^3} - \frac{1}{s^2} + \frac{2e^{-s}}{s}$$

(b) (10 points) Find the inverse Laplace transform of the function $F(s) = \frac{e^{-s}}{(s+1)^2}$.

$$L^{-1}\left[\frac{e^{-s}}{(s+1)^2}\right] = u(t-1) \cdot L^{-1}\left[\frac{1}{(s+1)^2}\right]$$

$$= te^{-t}u(t-1)$$

QUESTION 4 (20 pts)

Find the solution to the following IVP using the Laplace transform:

$$y'' - 4y' - 5y = 0, \quad y(0) = 1,\ y'(0) = 0.$$

$$L[y'' - 4y' - 5y] = s^2Y - sy(0) - y'(0) - 4[sY - y(0)] - 5Y$$

$$0 = s^2Y - s - 4sY + 4 - 5Y$$

$$0 = [s^2 - 4s - 5]Y - s + 4$$

$$s - 4 = (s-5)(s+1)Y$$

$$Y = \frac{s-4}{(s-5)(s+1)} \rightarrow \frac{A}{(s-5)} + \frac{B}{(s+1)} \rightarrow s - 4 = A(s+1) + B(s-5)$$

$$s - 4 = (A+B)s + (A - 5B)$$

$$\begin{cases} A + B = 1 \\ A - 5B = -4 \end{cases} \rightarrow 6B = 5 \Rightarrow B = \frac{5}{6} \therefore A = \frac{1}{6}$$

$$Y = \frac{1}{6(s-5)} + \frac{5}{6(s+1)}$$

$$L^{-1}\left[\frac{1}{6(s-5)} + \frac{5}{6(s+1)}\right] = \frac{1}{6}e^{5t} + \frac{5}{6}e^{-t}$$

$$y = \frac{1}{6}e^{5t} + \frac{5}{6}e^{-t}$$

QUESTION 5 (10 pts)

(a) (5 points) Denote by $F(s)$ the Laplace transform of $f(t)$. Show that if $h(t) = \int_0^t x f(x)\, dx$, then $L(h(t)) = -\frac{F'(s)}{s}$.

$L[f(t)] = F(s)$

$\int_0^t x f(x)\,dx = x \int f(x)\,dx - \iint f(x)\,dx\,dx$

$u = x \qquad dv = f(x)\,dx$

$du = dx \qquad v = \int f(x)\,dx$

(b) (5 points) Find the solution of the following integral equation:

$$y(t) = 1 + \int_0^t y(x)\, dx.$$

$L[1 + y] = \frac{1}{s} + Y$

$Y = -\frac{1}{s}$

$L^{-1}\left[-\frac{1}{s}\right] = -1$

$y(t) = -1$

QUESTION 6 (10 pts)

Answer the following statements with **True** or **False**. Write your answer on the horizontal line at the end of each statement. Justify your answer in the white space underneath each statement.

(a) (/ 2) The radius of convergence of the power series solution $\sum_{n=0}^{\infty} a_n(x-3)^n$ of the ODE $(16+x^2)y''+xy'+y=0$ is 5.

(a) False

(b) (/ 2) If $f(t)=t$ and $g(t)=t^2$, then $L(f(t)g(t)) = \frac{2}{s^5}$.

$f(t)\cdot g(t) = t\cdot t^2 = t^5$

$L[t^5] = \frac{5!}{s^6}$

(b) False

(c) (/ 2) If $f(t)=0$ for $t<2$, $f(t)=2$ for $2\le t<3$ and $f(t)=t$ for $t\ge 3$, then $f(t) = 2u(t-2)+(t-2)u(t-3)$.

$$f(t) = \begin{cases} 0 & \text{for } t<2 \\ 2 & \text{for } 2\le t<3 \\ t & \text{for } t\ge 3 \end{cases}$$

$f(t) = 2u(t-2) - 2u(t-3) + tu(t-3)$

(c) True

(d) (/ 2) If $f(t)=t^2$ and $g(t)=t^2$, then $f(t)*g(t) = \frac{t^5}{30}$.

$(f*g)(t) = \int_0^t f(x)g(t-x)\,dx$

$= f(t)\,g($

(d) False

(e) (/ 2) The number $x=0$ is a singular point of the ODE $(x^2+x)y''+xy'+y=0$.

$(0^2+0)y'' + (0)y' + y = 0$

$y=0$

(e) True

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For officials use only:

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							