MATH-302 Created by Pierre-O. Parisé
Midterm 02 2022/11/01, Fall 2022

Last name: _____
First name: _____

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all the questions below and write your answers directly on the questionnaire.
- You have 75 minutes to complete the exam.
- When you are done (or at the end of the 75min period), return your copy.
- No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam.
- **Turn your cellphone off during the exam**.
- You may use a digital calculator (no graphical calculators or symbolic calculators will be allowed).
- You are not allowed to use the lecture notes or the textbook.
- You may bring one 2-sided cheat sheet of handwriting notes.
- You must show ALL your work to have full credit. An answer without justification is worth no point.

Your Signature: _____

May the Force be with you! Pierre-Olivier Parisé

University of Hawai'i

Question 1 **(20 pts)**

For the given ODE, find the general solution.

(a) (10 points) $y'' + 2y' + y = 0$.

(b) (10 points) $y'' + 6y' + 10y = 0$.

Question 2 **(20 pts)**

For the following ODEs, give the form of the particular solution. **Don't solve for the constants**.

(a) (10 points) $y'' + 5y' - 6y = 22 + 18x - 18x^2$.

(b) (10 points) $y'' - 2y' + 5y = e^x\Big((6 + 8x)\cos(2x) + (6 - 8x)\sin(2x)\Big)$.

Question 3 **(20 pts)**

Find the general solution to the following ODE:

$$y'' - 4y' - 5y = -6e^{-x}.$$

Question 4 **(20 pts)**

Find the general solution to the following ODE:

$$x^2y'' + xy' - 4y = -6x - 4$$

knowing that $y_1(x) = x^2$ is a solution to the complementary equation.

Question 5 **(10 pts)**

(a) (5 points) If y_1 and y_2 are two differentiable functions not identically zero, the Wronkians W of $\{y_1, y_2\}$ is

$$W = y_1 y_2' - y_1' y_2.$$

Show that if $\{y_1, y_2\}$ is **not** a set of fundamental solutions for a second order differential equation, then $W = 0$.

(b) (5 points) Solve the following IVP:

$$y'' + y = 0, \quad y(0) = 0, y'(0) = 1.$$

Question 6 **(10 pts)**

Answer the following statements with **True** or **False**. Write your answer on the horizontal line at the end of each statement. Justify your answer in the white space underneath each statement.

(a) (/ 2) $\{x, 1\}$ is a fundamental of solutions to $y'' = 0$.

(a) _____

(b) (/ 2) If $y_1(x) = \cos(2x) + \sin(2x)$ and $y_2(x) = 2\cos(2x) + 2\sin(2x)$ are solutions to $y'' + 4y = 0$, then $y(x) = 3\cos(2x) + 3\sin(2x)$ is a solution to $y'' + 4y = 0$.

(b) _____

(c) (/ 2) In the Spring-mass system model $y'' + (k/m)y = \frac{F_0}{m}\cos(\omega t)$, a resonance occurs when $\sqrt{k/m} = \omega$.

(c) _____

(d) (/ 2) If $y_1 = x$ and $y_2 = e^x$ are solutions to the complementary equation $(x-1)y'' - xy' + y = (x-1)^2$, then the solution should have the form $y(x) = xu_1(x) + e^x u_2(x)$.

(d) _____

(e) (/ 2) The function $y(x) = \sin(x) + \cos(x)$ is a solution to the following IVP: $y'' + y = 0$, $y(0) = 1$, $y'(0) = 1$.

(e) _____

Do not write on this page.

For officials use only:

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	20	10	10	100
Score:							