Math 302

Chapter 5

Section 5.3: Nonhomogeneous Linear Equations

Contents

Particular Solutions **2**

The Principle of Superposition **5**

Created by: Pierre-Olivier Parisé

Fall 2022

Particular Solutions

Our goal is to find the solutions to

$$y'' + p(x)y' + q(x)y = f(x). \tag{1}$$

<u>Nomenclature:</u>

- the equation $y'' + p(x)y' + q(x)y = 0$ is the **complementary equation** for (1).
- a **particular solution** is a solution y_{par} of (1).

Example 1. Find a particular solution to the following ODE:

$$y'' - 2y' + y = 4x.$$

Assumptions:

1) Suppose $\{y_1, y_2\}$ is a fundamental set of solutions to

$$y'' + p(x)y' + q(x)y = 0.$$

2) Suppose y_{par} is a particular solution to

$$y'' + p(x)y' + q(x)y = f(x).$$

Conclusion:

- Then the $y = y_{par} + c_1 y_1 + c_2 y_2$ is the general solution of

$$y'' + p(x)y' + q(x)y = f(x).$$

EXAMPLE 2.

a) Find the general solution of

$$y'' - 2y' + y = -3 - x + x^2.$$

b) Solve the following IVP:

$$y'' - 2y' + y = -3 - x + x^2, \quad y(0) = -2,\ y'(0) = 1.$$

EXAMPLE 3. Suppose that we know that $y_1(x) = x^4/15$ is a particular solution to

$$x^2y'' + 4xy' + 2y = 2x^4$$

and that $y_2(x) = x^2/3$ is a particular solution to

$$x^2y'' + 4xy' + 2y = 4x^2.$$

Find a particular solution to

$$x^2y'' + 4xy' + 2y = 2x^4 + 4x^2.$$

General Fact: If y_1 is a particular solution to

$$y'' + p(x)y' + q(x)y = f_1(x)$$

and y_2 is a particular solution to

$$y'' + p(x)y' + q(x)y = f_2(x)$$

then $y_{par} = y_1 + y_2$ is a particular solution to

$$y'' + p(x)y' + q(x)y = f_1(x) + f_2(x).$$