Section 1.2 — Problem 1 — 4 points

a) The highest derivative is d^3y/dx^3, and therefore the order of the ODE is 3.

b) The highest derivative is y'', and therefore the order of the ODE is 2.

c) The highest derivative is y', and therefore the order of the ODE is 1.

d) The highest derivative is y'', and therefore the order of the ODE is 2.

Section 1.2 — Problem 3b — 3 points

Integrate one time to get

$$y(x) = -\int x \sin x \, dx + c = \sin(x) - x\cos(x) + c$$

where c is a constant.

Section 1.2 — Problem 3f — 3 points

Integrate a first time to get

$$y'(x) = x^2 - \cos x + e^x + c_1$$

where c_1 is a constant. Integrate a second time to get

$$y(x) = \frac{x^3}{3} - \sin x + e^x + c_1 x + c_2$$

where c_2 is a constant.

Section 1.2 — Problem 5a — 10 points

We see that

$$y(\pi/4) = \frac{\pi \cos(\pi/4)}{4} = \frac{\pi}{4\sqrt{2}}.$$

The derivative is

$$y'(x) = \cos x - x \sin x.$$

Replacing y in the left-hand side of the differential equation, we see that

$$\cos x - \frac{x \cos x \sin x}{\cos x} = \cos x - x \sin x.$$

Therefore, we see that $y' = \cos x - y \tan x$ and $y(\pi/4) = \pi/4\sqrt{2}$.

Section 1.2 — Problem 5c — 10 points

We see that

$$y(0) = \tan\left(\frac{0^2}{2}\right) = \frac{\sin(0)}{\cos(0)} = \frac{0}{1} = 0.$$

The derivative is

$$y' = x \sec^2\left(\frac{x^2}{2}\right).$$

Replacing y in the left-hand side of the differential equation, we see that

$$x(1+y^2) = x\left(1 + \tan^2\left(\frac{x^2}{2}\right)\right)$$

and, if you remembered some of your trig. identities,

$$1 + \tan^2(A) = \sec^2(A)$$

and therefore, with $A = x^2/2$, we get

$$x(1+y^2) = x \sec^2\left(\frac{x^2}{2}\right).$$

This is exactly the expression of y' and so y satisfies $y' = x(1+y^2)$ and $y(0) = 0$.

Section 1.2 — Problem 9 — 10 points

First, we remark that $e^x - 1 \geq 0$ when $x \geq 0$ because e^x is increasing and $e^0 = 1$ and $1 - e^{-x} < 0$ when $x < 0$ because e^{-x} is decreasing and $e^0 = 1$. Therefore, the left-hand side of the differential equation is

$$|y| + 1 = e^x - 1 + 1 = e^x$$

if $x \geq 0$ and

$$|y| + 1 = -(1 - e^{-x}) + 1 = e^{-x}$$

if $x < 0$.

For the left-hand side, when $x \geq 0$, then $y(x) = e^x - 1$ and therefore

$$y'(x) = e^x = |y| + 1.$$

When $x < 0$, then $y(x) = 1 - e^{-x}$ and therefore

$$y'(x) = e^{-x} = |y| + 1.$$

We just verified that for any x, $y' = |y| + 1$ and therefore y satisfies the differential equation.

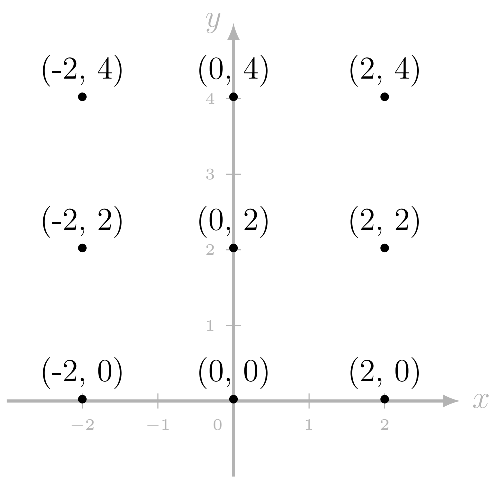
Section 1.3 — Problem 15 — 10 points

- Create a rectangular grid.

  The rectangular grid is given: from -2 to 2 in x and from 0 to 4 in y. We place the nods at

  - $x_0 = -2$.
  - $x_1 = 0$.
  - $x_2 = 2$.
  - $y_0 = 0$.
  - $y_1 = 2$.
  - $y_2 = 4$.

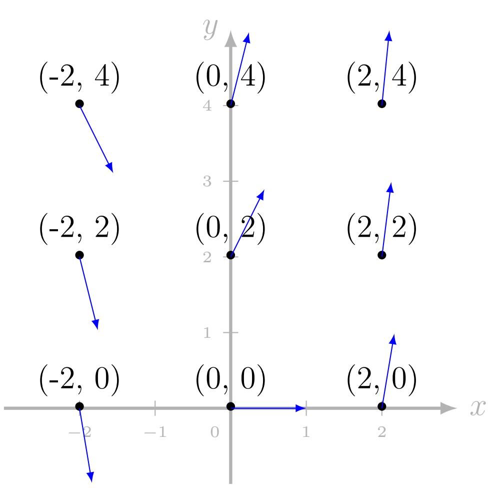
  This creates the following dotted grid:



- Find the slopes at each point of the grid.

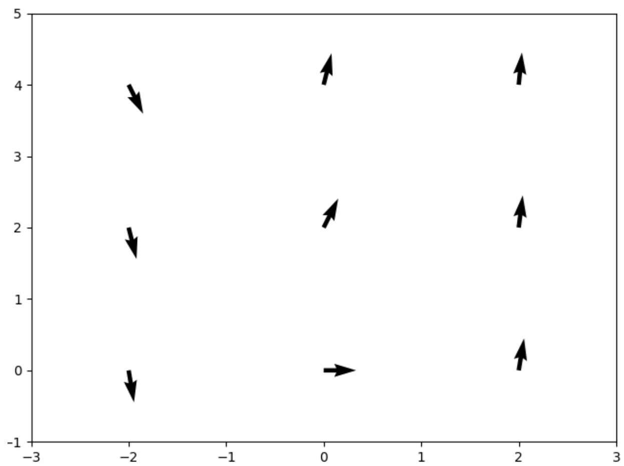
  - $(-2, 0)$: $y' = -6$
  - $(-2, 2)$: $y' = -4$
  - $(-2, 4)$: $y' = -2$
  - $(0, 0)$: $y' = 0$
  - $(0, 2)$: $y' = 2$
  - $(0, 4)$: $y' = 4$
  - $(2, 0)$: $y' = 6$
  - $(2, 2)$: $y' = 8$
  - $(2, 4)$: $y' = 10$

- Draw the actual direction field. (All vectors are normalized)

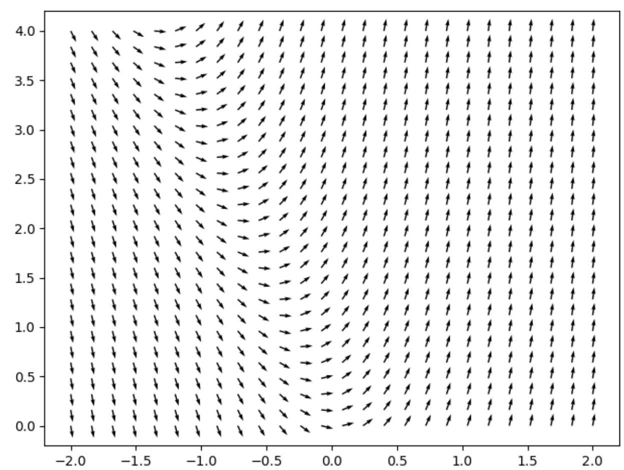


  - $(-2, 0)$: $y' = -6$, $\measuredangle = -80.54°$.
  - $(-2, 2)$: $y' = -4$, $\measuredangle = -75.96°$.
  - $(-2, 4)$: $y' = -2$, $\measuredangle = -63.43$.
  - $(0, 0)$: $y' = 0$, $\measuredangle = 0°$.
  - $(0, 2)$: $y' = 2$, $\measuredangle = 63.43°$.
  - $(0, 4)$: $y' = 4$, $\measuredangle = 75.96°$.
  - $(2, 0)$: $y' = 6$, $\measuredangle = 80.54°$.
  - $(2, 2)$: $y' = 8$, $\measuredangle = 82.87°$.
  - $(2, 4)$: $y' = 10$, $\measuredangle = 84.29°$.

Using Python, you would get (all vectors are normalized)



With a 3×3 grid



With a 26×26 grid

Total (Points): 50.