Math 302

Chapter 1

Section 1.1: Applications Leading to DEs

Contents

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EXAMPLE 1. Poor some hot water in a teapod and take its temperature with a thermometer. Take the temperature every 5 minutes. Record your data in a table and plot them in a Times VS Temperature graph.

TABLES

DATA FROM CLASS

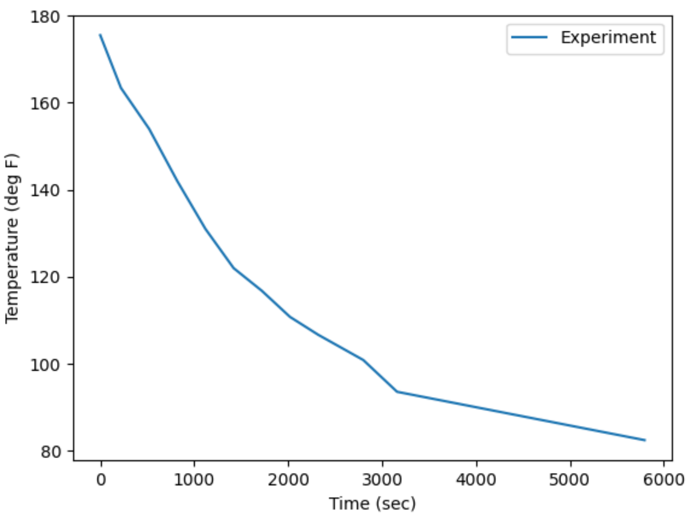
	Time	Temperature
0 sec.	11:56:20	175.5 °F.
220 sec.	12:00:00	163.4°F
520 sec.	12:05:00	154.0°F
820 sec.	12:10:00	142.0°F
1120 sec	12:15:00	131.0°F
1420 sec.	12:20:00	122.0°F
1720 sec.	12:25:00	116.8°F
2020 sec.	12:30:00	110.8°F
2320 sec.	12:35:00	106.7°F
2800 sec	12:43:00	100.9°F
3160 sec.	12:57:00	93.6°F
5795 sec.	13:40:55	82.5°F

MY DATA (From 2022-08-22)

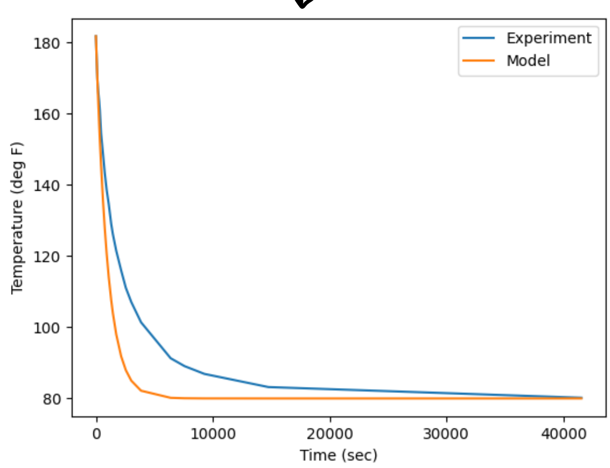
	Time	Temperature
0 sec.	19:00:00	181.8 °F
126 sec.	19:02:06	169.8°F
200 sec.	19:03:20	166.3°F
280 sec.	19:04:40	163.6°F
340 sec.	19:05:40	160.9°F
443 sec.	19:07:23	154.5°F
600 sec.	19:10:00	149.0°F
760 sec.	19:12:40	143.3°F
918 sec.	19:15:18	138.7°F
1100 sec.	19:18:20	134.6°F
1312 sec.	19:21:52	129.0 °F
1450 sec.	19:24:10	126.2°F
1740 sec.	19:29:00	121.5 °F
2160 sec.	19:36:00	116.0 °F
2560 sec.	19:42:40	111.1°F
3026 sec.	19:50:26	107.1 °F
3863 sec.	20:04:23	101.4°F
6396 sec.	20:46:36	91.3°F
7574 sec.	21:06:14	89.1°F
9280 sec.	21:34:40	86.9°F
4,755 sec.	23:05:55	83.2°F
11,526 sec.	6:32:06	80.2°F

PLOTS

DATA FROM CLASS.



My data



EXAMPLE 2. Let $T = T(t)$ be the temperature of a body at time t and let T_m be the temperature of its surrounding. Assuming that

- the rate of cooling of the body is directly proportial to the temperature difference of the surface area exposed
- the temperature of the surrounding does not change

deduce a model describing the evolution of the temperature $T(t)$ of the body.

Information:

rate of change of T $\swarrow$ $\frac{dT}{dt} \propto T - T_m$ $\to$ Temperature difference.

$\uparrow$ proportional.

$\frac{dT}{dt} < 0$ if $T > T_m$ (decrease)

$\frac{dT}{dt} > 0$ if $T < T_m$ (increase).

Therefore

$$\boxed{\frac{dT}{dt} = -k(T - T_m)}$$

($k > 0$ positive)

Later on, if T_m remains constant

$$\Rightarrow T(t) = T_m + (T_0 - T_m)e^{-kt}$$

T_0: temperature at $t = 0$.

Assuming that the medium (surrounding) remains at constant temperature seems reasonable if we're considering a cup of tea/coffee cooling in a room.

What if the body warms or cools its surrounding, resulting in changing drastically the surrounding temperature?

EXAMPLE 3. Let $T = T(t)$ be the temperature of the body at time t and let $T_m = T_m(t)$ be the temperature of its surrounding. Assuming that

- the rate of cooling of the body is directly proportial to the temperature difference of the surface area exposed
- the energy is preserved

deduce a model describing the evolution of the temperature $T(t)$ of the body.

$T_0 := T(0)$

$T_{m0} := T_m(0)$

From Example 1:

$$\frac{dT}{dt} = -k\,(T - \underset{\substack{\uparrow \\ \text{depends on } T.}}{T_m}) \qquad (*) .$$

We will assume further that

- Change in heat of the object as $T(t)$ is increasing from T_0 to $T(t)$ is $a(T - T_0)$ $(a > 0)$.
- Change in heat of the object as $T_m(t)$ is increasing from T_{m0} to $T_m(t)$ is $a_m(T_m - T_{m0})$ $(a_m > 0)$.

By conservation:

$$a(T - T_0) + a_m(T_m - T_{m0}) = 0$$

$$\Rightarrow \quad T_m = \frac{-a}{a_m}(T - T_0) + T_{m0}$$

Replace in $(*)$ $\swarrow$ DE

$$\Rightarrow \quad T' = -k\left(1 + \frac{a}{a_m}\right)T + k\left(T_{m0} + \frac{a}{a_m}T_0\right) \quad (**)$$

Later on, we will able to find:

$$T(t) = \frac{aT_0 + a_m T_{m0}}{a + a_m} + \frac{a_m(T_0 - T_{m0})}{a + a_m} e^{-k\left(1 + \frac{a}{a_m}\right)t}.$$