Section 1.1, Problem 2

We have

$$\begin{aligned}
\begin{bmatrix} 2 & 1 & -2 & 0 \\ 2 & -1 & -2 & 0 \\ 1 & 2 & -4 & 0 \end{bmatrix} &\sim \begin{bmatrix} 2 & 1 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & -3 & 6 & 0 \end{bmatrix} \\
&\sim \begin{bmatrix} 4 & 0 & -4 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & 0 \end{bmatrix} \\
&\sim \begin{bmatrix} 12 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 12 & 0 \end{bmatrix} \\
&\sim \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}
\end{aligned}$$

Thus, we get $x = y = z = 0$ (the trivial solution).

$\boxed{\textbf{Section 1.1, Problem 4}}$

We have

$$\begin{bmatrix} 3 & 1 & -2 & 3 \\ 1 & -8 & -14 & -14 \\ 1 & 2 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 3 & 1 & -2 & 3 \\ 0 & 25 & 40 & 45 \\ 0 & -5 & -5 & -3 \end{bmatrix}$$
$$\sim \begin{bmatrix} 75 & 0 & -90 & 30 \\ 0 & 25 & 40 & 45 \\ 0 & 0 & 15 & 30 \end{bmatrix}$$
$$\sim \begin{bmatrix} 225 & 0 & 0 & 630 \\ 0 & 75 & 0 & -105 \\ 0 & 0 & 15 & 30 \end{bmatrix}$$
$$\sim \begin{bmatrix} 1.0 & 0 & 0 & 2.8 \\ 0 & 1.0 & 0 & -1.4 \\ 0 & 0 & 1.0 & 2.0 \end{bmatrix}$$

The solution is then $x = 2.8$, $y = -1.4$, and $z = 2.0$.

Section 1.1, Problem 6

We have

$$\begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & 9 & -4 & 2 \\ 1 & -1 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & -15 & 9 & 0 \\ 0 & 5 & -3 & -2 \end{bmatrix}$$
$$\sim \begin{bmatrix} 10 & 0 & 14 & 20 \\ 0 & -15 & 9 & 0 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

The last line is $0 = -6$, which is impossible. This means there is no solution to the system.

Section 1.1, Problem 18

We have

$$\begin{bmatrix}1 & 2 & -1 & a\\ 1 & 1 & -2 & b\\ 2 & 1 & -3 & c\end{bmatrix} \sim \begin{bmatrix}1 & 2 & -1 & a\\ 0 & 1 & 1 & a-b\\ 0 & 3 & 1 & 2a-c\end{bmatrix}$$
$$\sim \begin{bmatrix}1 & 2 & -1 & a\\ 0 & 1 & 1 & a-b\\ 0 & 0 & -2 & -a+3b-c\end{bmatrix}$$
$$\sim \begin{bmatrix}1 & 2 & -1 & a\\ 0 & 1 & 1 & a-b\\ 0 & 0 & 1.0 & 0.5a-1.5b+0.5c\end{bmatrix}$$

So, there is no condition on a, b, c because the last line is valid (line $[0\,0\,1\,\text{constant}]$. Any number are admissible.

Section 1.1, Problem 24

1st solution: According to the lecture notes, there are 3 variables and 4 equations. There are more equations than variables. This means the system has no solution at all. HOWEVER, THIS IS NOT TRUE AND THERE WAS A MISTAKE IN THE LECTURE NOTES.

2nd solution: We have to reduce to the RREF.

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 3 & -1 & -2 & 0 \\ 2 & -2 & -4 & 0 \\ 1 & 3 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}$$
$$\sim \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\sim \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 2.0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We then conclude that $x = 0$ and $y + 2z = 0$. This is equivalent to

$$x = 0 \quad \text{and} \quad y = -2z.$$

with z a free variable.