MATH 307

Chapter 6

Section 6.3: Homogeneous Systems With Constant Coefficients The Nondiagonalizable Case

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THE UPPER TRIANGULAR CASE

EXAMPLE 1. Find the general solution to the system

$$Y' = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} Y.$$

Solving One Nonhomogeneous ODE

Given an nonhomogeneous ODE

$$y' = ay + g \tag{*}$$

the general solution is given by

$$y = y_H + y_P$$

where

- y_H is the general solution to the homogeneous ODE y' = ay.
- y_P is a particular solution to the ODE (\star) and it has the following form:

$$y_P(x) = e^{ax} \int e^{-ax} g(x) dx.$$

EXAMPLE 2. Complete the previous example.

THE GENERAL CASE

When A in Y' = AY is not diagonalizable, we can use the Jordan Canonical Form B of A.

EXAMPLE 3. Find the general solution of Y' = AY for

$$A = \begin{bmatrix} 1 & -2 & -6 \\ -2 & 2 & -5 \\ 2 & 1 & 8 \end{bmatrix}.$$