Math 307

Chapter 5

Section 5.2: The Algebra of Linear Transformations

Contents

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Summer 2022

Addition

If $T : V \to W$ and $S : V \to W$ are two linear transformations, then their sum $T + S$ is the new linear transformation defined by

$$(T + S)(v) = T(v) + S(v) \quad v \text{ in } V.$$

EXAMPLE 1. Let T and S be the following linear transformations:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + 2y \end{bmatrix} \quad \text{and} \quad S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 3y \\ x - y \end{bmatrix}.$$

Find $T + S$.

Scalar Multiplication

If $T : V \to W$ is a linear transformation and c is a real number, then the function cT is the linear transformation defined by

$$(cT)(v) = cT(v) \quad v \text{ in } V.$$

EXAMPLE 2. With T and S as in the previous example, find $S + 4T$.

Let $B(V, W)$ be the set of all linear transformations $T : V \to W$.

THEOREM 3. The set $B(V, W)$ equipped with the addition and scalar multiplication is a vector space.

Composition or Multiplication of Operators

If $T : V \to W$ and $S : W \to U$ are two linear transformations, then the composite $ST : V \to U$ is the linear transformation defined by

$$ST(v) = S(T(v)) \quad v \text{ in } V.$$

Example 4. Find ST with S and T as in example 1.