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University of Hawai'i

Last name: ______________________________

First name: ______________________________

Question:	1	2	3	4	Total
Points:	15	15	15	5	50
Score:					

Instructions:

- Write your complete name on your copy.
- Answer all 4 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam**.
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: ______________________________

May the Force be with you!
Pierre

Question 1 **(15 pts)**

Let $T(x, y, z) = (x, -y, x)$.

(a) (5 Pts) Show that T is a linear transformation.

(b) (5 Pts) Find the kernel and the image of T.

(c) (5 Pts) Find the nullity and the rank of T.

Question 2 (15 pts)

Let $U = \text{span}\{(1,1,1),(-1,0,2)\}$.

(a) (5 Pts) Are $(1,1,1)$ and $(-1,0,2)$ orthogonal?

(b) (5 Pts) Is the vector $(2,-3,1)$ in U?

(c) (5 Pts) Using the Gram-Schmidt process, transform the set $\{(1,1,1),(-1,0,2)\}$ into a set F of orthogonal vectors.

Question 3 (15 pts)

Let $V = \mathbb{R}^3$. Let B be the standard basis for $\mathbb{R}^3$ and let $D = \{(1,1,0),(1,0,1),(0,1,0)\}$ be another basis of $\mathbb{R}^3$. Let $T(a,b,c) = (2a-b, b+c, c-3a)$ be a linear operator.

(a) (5 Pts) Find the change matrix $P_{B\leftarrow D}$.

(b) (5 Pts) Find the matrix representation of T in the basis B.

(c) (5 Pts) Knowing that

$$P_{D\leftarrow B} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix},$$

find the matrix representation of the linear operator T in the basis D.
[Hint: $P^{-1}_{D\leftarrow B} M_D(T) P_{D\leftarrow B} = M_B(T)$ and $P^{-1}_{D\leftarrow B} = P_{B\leftarrow D}$.]

QUESTION 4 **(5 pts)**

Let $T : \mathbf{P_n} \to \mathbf{P_n}$ be the **linear** operator $T(p(x)) = p(x) - p(-x)$.

(a) (3 Pts) Show that $\ker T = \{p : p(x) = p(-x)\}$ and $\operatorname{im} T = \{q : q(-x) = -q(x)\}$.

(b) (2 Pts) If U is the subspace of all **even**[1] polynomials and V is the subspace of all **odd**[2] polynomials, then show that

$$n + 1 = \dim U + \dim V.$$

[1] Recall that a function $f(x)$ is *even* if $f(-x) = f(x)$.

[2] Recall that a function $f(x)$ is *odd* if $f(-x) = -f(x)$.