MIDTERM 02 SPRING 2024

SOLUTIONS M311

Question 1

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 9 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -9 & -8 & -9 \\ 3 & 0 & -1 \end{bmatrix}$$

Question 2

$$A^{-1}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1-2 & 0-6 \\ 2-4 & 2-8 \end{bmatrix}$$

$$\iff A^{-1} = \begin{bmatrix} -1 & -6 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\iff A^{-1} = \begin{bmatrix} -13 & -6 \\ -14 & -6 \end{bmatrix}$$

$$\iff A = \begin{bmatrix} -13 & -6 \\ -14 & -6 \end{bmatrix}^{-1}$$

$$\iff \boxed{A = -\frac{1}{6}\begin{bmatrix} -6 & 6 \\ 14 & -13 \end{bmatrix}}$$

Question 3

Let $E = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. Then

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}.$$

Question 4

(a) $\det A = (1)\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} - (-1)\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + (2)\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}$

$= (1)(4) + (1)(7) + (2)(-5)$

$= 4 + 7 - 10 = \boxed{1}$.

(b) $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 3 & 3 \end{vmatrix} = 0$ (two identical lines).

(c) $|A| = \boxed{0}$ (line of zeros).

(d) $|A| = (1)(1)(1)(5)(1)(4)$ (triangle).

$= \boxed{20}$

(e) $A \to B$ by performing $R_1 + R_2$ of A.

$\Rightarrow$ $|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1$.

Question 5

Assume $A^2 = 0$ and $I - A$ is invertible.

We have

$$(I-A)(I+A) = II + IA - AI - A^2$$

$$= I + A - A - 0$$

$$= I.$$

and

$$(I+A)(I-A) = II - IA + AI - A^2$$

$$= I - A + A - 0$$

$$= I.$$

Hence,

$$(I-A)^{-1} = I + A.$$

$\square$

Question 6

(a) False.

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{but} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \neq \pm I.$$

(b) False.

$$\overset{A}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}\overset{B}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$\neq$

(c) False.

$A = I$ and $B = -I$ are invertible

but $A + B = 0$ is not invertible.

(d) False.

$$\overset{A}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}\overset{B}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \to (AB)^T = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\overset{A^T}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}\overset{B^T}{\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$\neq$