# University of Hawai'i

**Last name:** ______________________________

**First name:** ______________________________

| Question: | 1 | 2 | 3 | 4 | Total |
|---|---|---|---|---|---|
| Points: | 15 | 15 | 15 | 5 | 50 |
| Score: | | | | | |

**Instructions:**

- Write your complete name on your copy.
- Answer all 4 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam**.
- Lecture notes and the textbook are not allowed during the exam.

**Your Signature:** ______________________________

May the Force be with you!
Pierre

## Question 1 **(15 pts)**

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2y, z, z)$.

(a) (5 Pts) Show that $T$ is a linear transformation.

(b) (5 Pts) Find the kernel of $T$ and its dimension.

(c) (5 Pts) Using the Dimension Theorem, deduce the rank of $T$.

## QUESTION 2 **(15 pts)**

Let $V = \mathbb{R}^3$. Let $B$ be the standard basis $\{(1,0,0),(0,1,0),(0,0,1)\}$ and let the set $D = \{(1,2,0),(0,-1,2),(0,2,0)\}$ be another basis of $\mathbb{R}^3$.

(a) (10 Pts) Knowing that

- $(1,0,0) = a(1,2,0) + b(0,-1,2) + c(0,2,0)$, where $a = 1$, $b = 0$, $c = -1$;
- $(0,1,0) = a(1,2,0) + b(0,-1,2) + c(0,2,0)$, where $a = 0$, $b = 0$, $c = 1/2$;
- $(0,0,1) = a(1,2,0) + b(0,-1,2) + c(0,2,0)$, where $a = 0$, $b = 1/2$, $c = 1/4$;

Find the change matrix $P_{D \leftarrow B}$. Justify carefully your answer.

(b) (5 Pts) Let $T(x,y,z) = (2y,z,z)$. Find the matrix representation of $T$ on the basis $B$, that is $M_B(T)$.

## Question 3 **(15 pts)**

Let $U = \text{span}\{(-1,0,3),(0,-3,2)\}$.

(a) (3 Pts) Are $(-1,0,3)$ and $(0,-3,2)$ orthogonal?

(b) (5 Pts) What is the dimension of $U$. Justify carefully your answer.

(c) (5 Pts) Using the Gram-Schmidt process, transform the set of vectors $\{(-1,0,3),(0,-3,2)\}$ in a set of orthogonal vectors $F$.

(d) (2 Pts) Illustrate visually the Gram-Schmidt orthogonalization process for two vectors in $\mathbb{R}^2$.

## Question 4 (5 pts)

Assume that $T : V \to \mathbb{R}^2$ is a linear transformation where $V$ is a vector space and $\mathbb{R}$ is the vector space of real numbers. Let $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be a basis of $V$.

(a) (1 Pt) If $T(\mathbf{v_1}) = (2, 2)$, $T(\mathbf{v_2}) = (1, 2)$ and $T(\mathbf{v_3}) = (2, 1)$, show that $-3\mathbf{v_1} + 2\mathbf{v_2} + 2\mathbf{v_3} \in \ker T$. Explain carefully your answer.

(b) (4 Pts) If $T(\mathbf{v_1})$, $T(\mathbf{v_2})$ and $T(\mathbf{v_3})$ are defined as in part (a), find the nullity of $T$. Explain carefully your answer.