Math 311

Last Chapter

Section 8.1: Orthogonal Complements and Projections

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Created by: Pierre-Olivier Parisé
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Gram-Schmidt Orthogonalization

Example 1. Let $V = \mathbb{R}^2$ and $B = \{(1, -1), (2, 1)\}$. Notice that B is not an orthogonal basis. Using the vectors from B, construct an orthogonal basis F.

Solution. Geometric intuition: https://www.desmos.com/geometry/e9mrgozxmb.

The Gram-Schmidt Orthogonalization Algorithm

Let $B = \{\mathbf{b_1}, \mathbf{b_2}, \ldots, \mathbf{b_m}\}$ be a basis of a subspace U of $V = \mathbb{R}^n$.

To transform B into an orthogonal basis $F = \{\mathbf{f_1}, \mathbf{f_2}, \ldots, \mathbf{f_m}\}$, we define

- $\mathbf{f_1} = \mathbf{b_1}$.
- $\mathbf{f_2} = \mathbf{b_2} - \dfrac{\mathbf{b_2} \cdot \mathbf{f_1}}{\|\mathbf{f_1}\|^2}\mathbf{f_1}$.
- $\mathbf{f_3} = \mathbf{b_3} - \dfrac{\mathbf{b_3} \cdot \mathbf{f_1}}{\|\mathbf{f_1}\|^2}\mathbf{f_1} - \dfrac{\mathbf{b_3} \cdot \mathbf{f_2}}{\|\mathbf{f_2}\|^2}\mathbf{f_2}$.
- $\ldots$
- $\mathbf{f_k} = \mathbf{b_k} - \dfrac{\mathbf{b_k} \cdot \mathbf{f_1}}{\|\mathbf{f_1}\|^2}\mathbf{f_1} - \dfrac{\mathbf{b_k} \cdot \mathbf{f_2}}{\|\mathbf{f_2}\|^2}\mathbf{f_2} - \cdots - \dfrac{\mathbf{b_k} \cdot \mathbf{f_{k-1}}}{\|\mathbf{f_{k-1}}\|^2}\mathbf{f_{k-1}}$.
- $\ldots$
- $\mathbf{f_m} = \mathbf{b_m} - \dfrac{\mathbf{b_m} \cdot \mathbf{f_1}}{\|\mathbf{f_1}\|^2}\mathbf{f_1} - \dfrac{\mathbf{b_m} \cdot \mathbf{f_2}}{\|\mathbf{f_2}\|^2}\mathbf{f_2} - \cdots - \dfrac{\mathbf{b_m} \cdot \mathbf{f_{m-1}}}{\|\mathbf{f_{m-1}}\|^2}\mathbf{f_{m-1}}$.