CONFIDENTIAL:
DO NOT RETURN TO STUDENT. SHRED TO DISPOSE.

Dear Professor Pierre-Olivier Parise,

Thank you for working with KOKUA to provide me with appropriate disability-related exam accommodations.

I am acknowledging that I understand the conditions stated below and will take the MATH 311 exam in accordance with these conditions.

NO book allowed, NO notes allowed, Calculator allowed - non-graphing calculator and non-programmable calculator

Lillie J[illegible]
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Jones, Lillie (Lillie)

2/5/24
____
Date

<u>KOKUA Proctor Notes:</u>

The **MATH 311** exam was administered on **2/5/2024** from 10:30 to 11:20.

Return Method: **Via email to parisepo@hawaii.edu**.

UNIVERSITY OF HAWAI'I

Last name: Jones

First name: Lillie

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	6	4	50
Score:							

Instructions:

- Write your complete name on your copy.
- Answer all 6 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam.**
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: Lillie Jones

MAY THE FORCE BE WITH YOU!
PIERRE

$\frac{1}{2} - \frac{6}{2} \quad -\frac{5}{2}$

QUESTION 1 (10 pts)

Find the solution to the following system of linear equations:

$$\begin{cases} x_1 + \ \ x_2 + 3x_3 - x_4 = 3 \\ 2x_1 - 2x_2 + \ \ x_3 + x_4 = 0 \end{cases}$$

Does it have one solution, or infinitely many solutions?

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -1 & 3 \\ 2 & -2 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & -1 & 3 \\ 1 & -1 & 1/2 & 1/2 & 0 \end{array}\right] r_2/2 \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & -1 & 3 \\ 0 & 2 & 5/2 & 3/2 & 3 \end{array}\right] r_1 - r_2 \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 3 & -1 & 3 \\ 0 & 1 & 5/4 & 3/4 & 3/2 \end{array}\right] R_2/2$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 7/4 & -1/4 & 3/2 \\ 0 & 1 & 5/4 & -3/4 & 3/2 \end{array}\right] R_1 - R_2$$

$$x_1 + \frac{7}{4}x_3 - \frac{1}{4}x_4 = 3/2$$
$$x_2 + \frac{5}{4}x_3 - \frac{3}{4}x_4 = 3/2$$

$$\Rightarrow \boxed{\begin{aligned} x_1 &= 3/2 - \tfrac{7}{4}s + \tfrac{1}{4}t \\ x_2 &= 3/2 - \tfrac{5}{4}s + \tfrac{3}{4}t \\ x_3 &= s \\ x_4 &= t \end{aligned}}$$

has infinitely many solutions

QUESTION 2 **(10 pts)**

Consider the following vectors:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \mathbf{z} = \begin{bmatrix} -2 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 9 \\ -8 \\ 0 \\ 1 \end{bmatrix}.$$

We would like to know if $\mathbf{w}$ is a linear combination of $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ and $\mathbf{v}$.

(a) (5 points) Write down the system of linear equations corresponding to this problem. **DO NOT SOLVE THE SYSTEM.**

(b) (5 points) If the RREF of the augmented matrix of the system from part (a) is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right],$$

can you express $\mathbf{w}$ as a linear combination of $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, and $\mathbf{v}$? If so, write $\mathbf{w}$ as a linear combination of the other vectors.

$\vec{w} = a\vec{x} + b\vec{y} + c\vec{z} + d\vec{v}$

$\vec{w} = a\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix} + c\begin{bmatrix} -2 \\ 3 \\ 2 \\ 0 \end{bmatrix} + d\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 9 \\ -8 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b-2c-d \\ a-b+3c+d \\ a+2b+2c+0 \\ a-b+0-d \end{bmatrix} \Leftrightarrow \left[\begin{array}{cccc|c} 1 & 1 & -2 & -1 & 9 \\ 1 & -1 & 3 & 1 & -8 \\ 1 & 2 & 2 & 0 & 0 \\ 1 & -1 & 0 & 1 & 1 \end{array}\right] \cdots \rightarrow \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \end{array}$$

$a = 2$

$b = 2$

$c = -3$

$d = 1$

$\vec{w} = 2\vec{x} + 2\vec{y} - 3\vec{z} + \vec{v}$

QUESTION 3 (10 pts)

Consider the following homogeneous system of linear equations:

$$\begin{cases} x_1 + x_2 + \ x_3 + \ x_4 = 0 \\ 2x_1 - x_2 + \ x_3 - 2x_4 = 0 \\ 3x_1 - x_2 + 2x_3 - \ x_4 = 0 \end{cases}.$$

(a) (2 points) Write the augmented matrix of the system.

(b) (2 points) Are there one solution or infinitely many solutions? Justify your answer.

(c) (6 points) The RREF of the augmented matrix of the system is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{array}\right]$$

Express the solution as a linear combination of basic solution(s).

a) $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & -2 & 0 \\ 3 & -1 & 2 & -1 & 0 \end{array}\right] \cdots \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{array}\right]$

there are many solutions b/c x_4 doesn't have a leading one and can be anything which affects the values of all the other variables.

$x_1 - 3x_4 = 0$

$x_2 = 0$

$x_3 + x_4 = 0$

$\boxed{\begin{array}{l} x_1 = 3s \\ x_2 = 0 \\ x_3 = -s \\ x_4 = s \end{array}}$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s\begin{bmatrix} 3 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

QUESTION 4 **(10 pts)**

Find the entries of the matrix A if A satisfies the equation:

$$\left(2A^{\top} - 5\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\right)^{\top} = 4A - 9\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}.$$

$$\left(2A^{T} - \begin{bmatrix} 5 & 0 \\ -5 & 10 \end{bmatrix}\right)^{T} = 4A - \begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix}$$

$$(2A^{T})^{T} - \begin{bmatrix} 5 & 0 \\ -5 & 10 \end{bmatrix}^{T} = 4A - \begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix}$$

$$2A - \begin{bmatrix} 5 & -5 \\ 0 & 10 \end{bmatrix} = 4A - \begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix}$$

$$-2A \qquad -2A$$

$$-\begin{bmatrix} 5 & -5 \\ 0 & 10 \end{bmatrix} = 2A - \begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix}$$

$$+\begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix} \qquad +\begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 9 \\ -9 & 0 \end{bmatrix} - \begin{bmatrix} 5 & -5 \\ 0 & 10 \end{bmatrix} = 2A$$

$$\cdot\frac{1}{2}\begin{bmatrix} 4 & 14 \\ -9 & -10 \end{bmatrix} = 2A \cdot\frac{1}{2}$$

$$A = \begin{bmatrix} 4 & 14 \\ -9 & -10 \end{bmatrix} \cdot \frac{1}{2}$$

$$A = \begin{bmatrix} 2 & 7 \\ -9/2 & -5 \end{bmatrix}$$

QUESTION 5 **(6 pts)**

A square matrix A is **skew-symmetric** if $A^\top = -A$. Show that if A and B are skew-symmetric, then $A - B$ is skew-symmetric.

$A^T = -A \quad B^T = -B$

$A = -A^T \quad B = -B^T$

has to be true

given A&B are skew-symmetric

$A - B \Rightarrow -A^T - -B^T \Rightarrow (-A^T - -B^T)^T \Rightarrow (-A^T)^T - (-B^T)^T \Rightarrow$

$= (-A) - (-B)$ statement is skew-symmetric if $A^T = -A$ + $B^T = -B$

$A^T - B^T \Rightarrow$

QUESTION 6 (4 pts)

Answer the following questions with **True** or **False**. Write down you answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) A matrix B with RREF $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ has rank $(A) = 2$. (/ 1)

(a) False

(b) A homogeneous system can have no solution. (/ 1)

even if variables = 0

$\begin{bmatrix} 0 & 1 & 0 \\ & & \end{bmatrix}$ - $x_2 = 0$ that is still a solution

(b) False

(c) If $\mathbf{x_1}$ and $\mathbf{x_2}$ are solutions to a system of linear equations denoted by (S), then $2\mathbf{x_1} - \mathbf{x_2}$ is also a solution of the system (S). (/ 1)

$(S) = x_1$

$= x_2$

(c) True

(d) A system of 3 linear equations in 2 variables with a coefficient matrix of rank 2 has a unique solution. (/ 1)

$\begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ 0 & 0 & \end{bmatrix}$ when ref form bottom row should be all zeros which gives you infinite solutions.

(d) false