UNIVERSITY OF HAWAI'I

Last name: Jones

First name: Lillie

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	6	4	50
Score:	10	10	10	9	1	2	42

Bien !

Instructions:

- Write your complete name on your copy.
- Answer all 6 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam.**
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: Lillie

MAY THE FORCE BE WITH YOU!
PIERRE

$\det(A^{-1}) = \frac{1}{\det(A)}$ $AA^{-1} = I$

$\det(A)$ is invertible if $\neq 0$

QUESTION 1 (10 pts)

Say if the following matrix products are well-defined. If it is well-defined, then compute the matrix products.

$$\underset{3\times 3}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 9 \\ 1 & 1 & -1 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}}.$$

yes well-defined b/c both 3x3 matrices so can compute the product

$$\begin{bmatrix} (1)+0+1 & 0+0+1 & -1\ 0+1 \\ 1-1-9 & 0+1-9 & -1+1-9 \\ 1+1+1 & 0\ -1+1 & -1\ -1+1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -9 & -8 & -9 \\ 3 & 0 & -1 \end{bmatrix}$$

10/10

$\frac{1}{6} + \frac{12}{6} = \frac{13}{6}$ $\quad \frac{-1}{3} - \frac{6}{3} = -\frac{7}{3}$ $\quad \frac{1}{ad-bc} \cdot \frac{1}{6-12} = \frac{-1}{-6}$ $\quad \frac{6}{6}(-2) + \frac{13}{6}$ $\quad \frac{-12}{6} + \frac{13}{6} = \frac{+1}{6}$

QUESTION 2 (10 pts)

Find the values of the entries of the matrix A if

$$\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1} = \begin{bmatrix}1 & 0\\ 2 & 2\end{bmatrix} - 2\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}.$$

$\frac{3}{3}(2) - \frac{7}{3}$

$\frac{6}{3} - \frac{7}{3} = \frac{-1}{3}$

$\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1} = \begin{bmatrix}1 & 0\\ 2 & 2\end{bmatrix} - \begin{bmatrix}2 & 6\\ 4 & 8\end{bmatrix}$

$A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$= \left(\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1}\right)^{-1} = \left(\begin{bmatrix}-1 & -6\\ -2 & -6\end{bmatrix}\right)^{-1}$

$= \left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\right) A = \frac{1}{-6}\begin{bmatrix}-6 & 6\\ 2 & -1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A = \begin{bmatrix}1 & -1\\ -1/3 & 1/6\end{bmatrix}$

$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} \cdot \begin{bmatrix}1 & -1\\ -1/3 & 13/6\end{bmatrix} = \begin{bmatrix}1 & -1\\ 2 - 7/3 & -2 + 13/6\end{bmatrix}$

$= \begin{bmatrix}1 & -1\\ -1/3 & 1/6\end{bmatrix}$

$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{bmatrix}a & b\\ 2a+c & 2b+d\end{bmatrix}$

$\begin{bmatrix}a & b\\ 2a+c & 2b+d\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1/3 & 1/6\end{bmatrix}$

$a = 1, \quad b = -1$

$2a + c = -1/3$ $\quad 2b + d = 1/6$

$2(-1) + d = 1/6$

$2 + c = -1/3$ $\quad -2 + d = 1/6$

$-2 \quad -2$ $\quad +2 \quad +2$

$c = -1/3 - 2$ $\quad d = 1/6 + 2$

$c = \frac{-7}{3}$ $\quad d = 13/6$

$$\boxed{A = \begin{bmatrix}1 & -1\\ -7/3 & 13/6\end{bmatrix}}$$

10/10

QUESTION 3 (10 pts)

Find a 2×2 elementary matrix E such that

$$E\begin{bmatrix}3 & 0 & 1\\ 2 & -1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 0\end{bmatrix}.$$

$\begin{bmatrix}3 & 0 & 1\\ 2 & -1 & 0\end{bmatrix} \to \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 0\end{bmatrix} R_1 - R_2$

$E = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$

10/10

$\underset{2\times 2}{\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}}\underset{2\times 3}{\begin{bmatrix}3 & 0 & 1\\ 2 & -1 & 0\end{bmatrix}} = \begin{bmatrix}3-2 & 1 & 1\\ 2 & -1 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 2 & -1 & 0\end{bmatrix}$

9/10

Question 4 (10 pts)

Evaluate the determinant of the matrix A.

(a) (2 Pts) $A = \begin{array}{cc} & \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \\ & \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \end{array}$

(b) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$.

(c) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & -1 & 1 & 4 \\ 1 & -1 & 1 & -1 \end{bmatrix}$.

(d) (2 Pts) $A = \begin{array}{cc} & c_1 \\ R_1 & \begin{bmatrix} 1 & 45 & 3 & 4 & 3 & 4 \\ 0 & 1 & 9 & 100 & 4 & 45 \\ 0 & 0 & 1 & 45 & -3 & -2 \\ 0 & 0 & 0 & 5 & 4 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} \end{array}$. $= 20$

★(e) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ if $\begin{array}{cc} & c_1 \\ R_1 & \\ R_2 & \\ R_3 & \end{array}\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1$.

a) Expand along R_1: 1/2

$= (1)(-1)^{1+1}\det\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} + (-1)(-1)^{1+2}\det\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} + (2)(-1)^{1+3}\det\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} = (3+1)+(9-2) - 2(-3-2) = \cancel{21}$ +2 1

b) Expand along R_1:

$= (1)(-1)^{1+1}\det\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} + (1)(-1)^{1+2}\det\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} + (1)(-1)^{1+3}\det\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} = (6-6) - (6-6) + (6-6) = 0$ 2/

c) Expand along R_2: $\det(A) = 0$ 2/

d) Expand along C_1:

$= (1)(-1)^{1+1}\det_{R_1}\begin{bmatrix} \cancel{1\ 9\ 100\ 4\ 45} \\ 0\ 1\ 45\ -3\ -2 \\ 0\ 0\ 5\ 4\ -1 \\ 0\ 0\ 0\ 1\ -1 \\ 0\ 0\ 0\ 0\ 4 \end{bmatrix}$ = Expand along C_1: $(1)(-1)^{1+1}\det_{R_1}\begin{bmatrix} \cancel{1\ 45\ -3\ -2} \\ 0 & 5 & 4 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $= (1)(-1)^{1+1}\det\begin{bmatrix} 5 & 4 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix}$ (Expand along C_1:)

$= (5)(-1)^{1+1}\det\begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix} = 5(4) = 20$ 2/

e) $\det\begin{bmatrix} \cancel{1\ 1\ 2} \\ 1\ 2\ 3 \\ 1\ 1\ 1 \end{bmatrix}$ Expand along $C_1 = (1)(-1)^{1+1}\det\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} + (1)(-1)^{2+1}\det\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} + (1)(-1)^{3+1}\det\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$= (2-3) - (1-2) + (3-4) = -1$ 2/

$A \neq \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$I^2 = I$

$(I+A)(I+A)$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AA^{-1} = I$

QUESTION 5 (6 pts)

Let A be an $n \times n$ matrix. Assume that $A^2 = 0$ and $I - A$ is invertible, where I is the $n \times n$ identity matrix. Show that

$$(I - A)^{-1} = I + A.$$

$(I+A)^2 = I^2 + \overset{0}{A^2} + IA + AI = I + IA + AI = I(1+2A)$

$(I - A) = \frac{(AA^{-1}) - I}{A^{-1}} = \frac{(A^{-1})^2 A - I}{A^{-1}} =$

1/6

Here's what to do:

1. Show that (I - A) (I + A) = I.
2. Show that (I + A) (I - A) = I.

2/4

QUESTION 6 (4 pts)

Answer the following questions with **True** or **False**. Write down you answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) If A is an $n \times n$ matrix and $A^2 = I$, then $A = \pm I$. (0/ 1)

(a) ~~True~~

(b) If A and B are $n \times n$ matrices, then $AB = BA$. (1/ 1)

A & B are matrices and not just #'s Matrices dont always have commutative properties.

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\quad AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix}$

$BA = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 4 & 6 \end{bmatrix}$

(b) False

(c) If A and B are $n \times n$ invertible matrices, then $A + B$ is invertible. (0/ 1)

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{-1}{ad-bc} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \neq 0$ $\quad$ Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

A & B both shouldnt = 0 b/c they are invertable

$\frac{1}{ad-bc} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} + \frac{1}{eh-gf} \begin{bmatrix} h & -f \\ -g & e \end{bmatrix}$

(c) ~~True~~

(d) If A and B are $n \times n$ matrices, then $(AB)^\top = A^\top B^\top$. (1/ 1)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix}$ $\quad (AB)^T = \begin{bmatrix} 3 & 7 \\ 3 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $\quad B^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^T \cdot B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 6 & 6 \end{bmatrix}$

(d) False