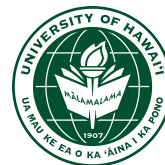


UNIVERSITY OF HAWAI'I



Last name: _____

First name: _____

Question:	1	2	3	4	Total
Points:	15	15	15	5	50
Score:					

Instructions:

- Write your complete name on your copy.
- Answer all 4 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam.**
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: _____

MAY THE FORCE BE WITH YOU!
PIERRE

QUESTION 1

(15 pts)

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $T(x, y, z) = (2y, z, z)$.

- (a) (5 Pts) Show that T is a linear transformation.
- (b) (5 Pts) Find the kernel of T and its dimension.
- (c) (5 Pts) Using the Dimension Theorem, deduce the rank of T .

QUESTION 2

(15 pts)

Let $V = \mathbb{R}^3$. Let B be the standard basis $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and let the set $D = \{(1, 2, 0), (0, -1, 2), (0, 2, 0)\}$ be another basis of \mathbb{R}^3 .

(a) (10 Pts) Knowing that

- $(1, 0, 0) = a(1, 2, 0) + b(0, -1, 2) + c(0, 2, 0)$, where $a = 1, b = 0, c = -1$;
- $(0, 1, 0) = a(1, 2, 0) + b(0, -1, 2) + c(0, 2, 0)$, where $a = 0, b = 0, c = 1/2$;
- $(0, 0, 1) = a(1, 2, 0) + b(0, -1, 2) + c(0, 2, 0)$, where $a = 0, b = 1/2, c = 1/4$;

Find the change matrix $P_{D \leftarrow B}$. Justify carefully your answer.

(b) (5 Pts) Let $T(x, y, z) = (2y, z, z)$. Find the matrix representation of T on the basis B , that is $M_B(T)$.

QUESTION 3

(15 pts)

Let $U = \text{span}\{(-1, 0, 3), (0, -3, 2)\}$.

- (a) (3 Pts) Are $(-1, 0, 3)$ and $(0, -3, 2)$ orthogonal?
- (b) (5 Pts) What is the dimension of U . Justify carefully your answer.
- (c) (5 Pts) Using the Gram-Schmidt process, transform the set of vectors $\{(-1, 0, 3), (0, -3, 2)\}$ in a set of orthogonal vectors F .
- (d) (2 Pts) Illustrate visually the Gram-Schmidt orthogonalization process for two vectors in \mathbb{R}^2 .

QUESTION 4 (5 pts)

Assume that $T : V \rightarrow \mathbb{R}^2$ is a linear transformation where V is a vector space and \mathbb{R} is the vector space of real numbers. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis of V .

- (a) (1 Pt) If $T(\mathbf{v}_1) = (2, 2)$, $T(\mathbf{v}_2) = (1, 2)$ and $T(\mathbf{v}_3) = (2, 1)$, show that $-3\mathbf{v}_1 + 2\mathbf{v}_2 + 2\mathbf{v}_3 \in \ker T$. Explain carefully your answer.
- (b) (4 Pts) If $T(\mathbf{v}_1)$, $T(\mathbf{v}_2)$ and $T(\mathbf{v}_3)$ are defined as in part (a), find the nullity of T . Explain carefully your answer.