Section 1.1 — Problem 1a (5 Pts)

Replacing in the first equation, we get

$$2(19t-35)+3(25-13t)+t=38t-70+75-39t+t=5$$

and replacing in the second equation

$$5(19t-35)+7(25-13t)-4t=95t-175+175-91t-4t=0.$$

Hence, it is a solution to the system of linear equations.

Section 1.1 — Problem 7 (6 Pts)

a. $\left[\begin{array}{cc|c} 1 & -3 & 5 \\ 2 & 1 & 1 \end{array}\right]$.

b. $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & 1 \end{array}\right]$.

Section 1.1 — Problem 8a (5 Pts)

Here is the system associated to the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 6 & 0 \\ 0 & 1 & 0 & 3 \\ 2 & -1 & 0 & 1 \end{array}\right].$$

Section 1.1 — Problem 14 (12 Pts)

a. False in general. For example

$$\begin{aligned} 2x+3y+2z&=0 \\ x+3y+2z&=1 \end{aligned} \quad \longrightarrow \quad \left[\begin{array}{ccc|c} 2 & 3 & 2 & 0 \\ 1 & 3 & 2 & 1 \end{array}\right]$$

The augmented matrix has 2 rows only, but there are $n=3$ variables.

b. False in general. For an example, see Example 2 in Section 1.1.

c. True, because any system, say S_1 obtained from row operations applied to another system, say S_2, have the same set of solutions. Therefore, S_2 is consistent, then S_1 is consistent.

d. True. If the system S_1 is inconsistent, then there is no solutions to the system S_1, so the set of solutions is the empty set $\varnothing$. Since the new system S_2 obtained from the series of row operations has the same set of solutions, then S_2 must have no solution, that is the set of solution is $\varnothing$.

Section 1.2 — Problem 1 (2 Pts)

REF: c, d and e.
RREF: None.

Section 1.2 — Problem 5 (12 Pts)

a. We have

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 3 & -1 & 1 & 0 \\ -1 & 3 & 4 & -4 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 31 \\ 0 & 0 & 1 & -20 \end{array}\right]$$

Hence, $x = 17$, $y = 31$, $z = -20$.

c. We have

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ -1 & 4 & 5 & -5 \\ 1 & 6 & 3 & 15 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -9/5 & 9 \\ 0 & 1 & 4/5 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The system is consistent and $x = 9 + 9t/5$, $y = 1 - 4t/5$, and $z = t$, for $t \in \mathbb{R}$.

d. We have

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 2 & 5 & -3 & 1 \\ 1 & 4 & -3 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

The system is inconsistent.

Section 1.2 — Problem 11 (8 Pts)

a. We obtain

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $\text{rank}(A) = 3$.

b. We obtain

$$\begin{bmatrix} -2 & 3 & 3 \\ 3 & -4 & 1 \\ -5 & 7 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, $\text{rank}(A) = 2$.