CONFIDENTIAL:
DO NOT RETURN TO STUDENT. SHRED TO DISPOSE.

Dear Professor Pierre-Olivier Parise,

Thank you for working with KOKUA to provide me with appropriate disability-related exam accommodations.

I am acknowledging that I understand the conditions stated below and will take the MATH 311 exam in accordance with these conditions.

NO book allowed, NO notes allowed, Calculator allowed - Scientific calculator

Lillie Jones

Jones, Lillie (Lillie)

3/11/24

Date

KOKUA Proctor Notes:

The **MATH 311** exam was administered on **3/11/2024** from 10:25 to 11:36.

Return Method: **Via email to parisepo@hawaii.edu**.

MATH-311
Midterm 02 (50min)

Created by Pierre-O. Parisé
March, Spring 2024

UNIVERSITY OF HAWAI'I

Last name: Jones

First name: Lillie

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	6	4	50
Score:							

Instructions:

- Write your complete name on your copy.
- Answer all 6 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam.**
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: Lillie

MAY THE FORCE BE WITH YOU!
PIERRE

1.5, 6a, 6c

$\det(A^{-1}) = \frac{1}{\det(A)}$ $AA^{-1} = I$

$\det(A)$ is invertible if $\neq 0$

QUESTION 1 (10 pts)

Say if the following matrix products are well-defined. If it is well-defined, then compute the matrix products.

$$\underset{3\times 3}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 9 \\ 1 & 1 & -1 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}}.$$

yes well-define b/c both 3x3 matrices so can compute the product

$$\begin{bmatrix} (1)+0+1 & 0+0+1 & -1\ 0+1 \\ 1-1-9 & 0+1-9 & -1+1-9 \\ 1+1+1 & 0\ -1+1 & -1\ -1+1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -9 & -8 & -9 \\ 3 & 0 & -1 \end{bmatrix}$$

$\frac{1}{6}+\frac{12}{6}=\frac{13}{6}$ $\quad \frac{-1}{3}+\frac{6}{3}=\frac{-7}{3}$ $\quad \frac{1}{ad-bc}$ $\frac{1}{6-12}=\frac{-1}{-6}$ $\quad \frac{6}{6}(-2)+\frac{13}{6}$

$\frac{-12}{6}+\frac{13}{6}=\frac{+1}{6}$

QUESTION 2 (10 pts)

Find the values of the entries of the matrix A if

$$\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1} = \begin{bmatrix}1 & 0\\ 2 & 2\end{bmatrix} - 2\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}.$$

$\frac{3}{3}(2)-\frac{7}{3}$

$\frac{6}{3}-\frac{7}{3}=\frac{-1}{3}$

$$\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1} = \begin{bmatrix}1 & 0\\ 2 & 2\end{bmatrix} - \begin{bmatrix}2 & 6\\ 4 & 8\end{bmatrix}$$

$A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$$= \left(\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1}\right)^{-1} = \left(\begin{bmatrix}-1 & -6\\ -2 & -6\end{bmatrix}\right)^{-1}$$

$$= \left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\right) A = \frac{1}{-6}\begin{bmatrix}-6 & 6\\ 2 & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A = \begin{bmatrix}1 & -1\\ -1/3 & 1/6\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ -1/3 & 13/6\end{bmatrix} = \begin{bmatrix}1 & -1\\ 2-7/3 & -2+13/6\end{bmatrix}$$

$$= \begin{bmatrix}1 & -1\\ -1/3 & 1/6\end{bmatrix}$$

$$\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{bmatrix}a & b\\ 2a+c & 2b+d\end{bmatrix}$$

$$\begin{bmatrix}a & b\\ 2a+c & 2b+d\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1/3 & 1/6\end{bmatrix}$$

$a=1, \quad b=-1$

$2a+c=-1/3$ $\qquad 2b+d=1/6$

$\qquad\qquad 2(-1)+d=1/6$

$2+c=-1/3$ $\qquad -2+d=1/6$

$-2 \quad -2$ $\qquad +2 \quad +2$

$c=-1/3-2$ $\qquad d=1/6+2$

$c=\frac{-7}{3}$ $\qquad d=13/6$

$$\boxed{A=\begin{bmatrix}1 & -1\\ -7/3 & 13/6\end{bmatrix}}$$

QUESTION 3 (10 pts)

Find a 2×2 elementary matrix E such that

$$E\begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix} R_1 - R_2$$

$$E = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\underset{2\times 2}{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}} = \begin{bmatrix} 3-2 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

QUESTION 4 **(10 pts)**

Evaluate the determinant of the matrix A.

(a) (2 Pts) $A = \begin{array}{cc} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} & \\ \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array}$

(b) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$.

(c) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & -1 & 1 & 4 \\ 1 & -1 & 1 & -1 \end{bmatrix}$.

(d) (2 Pts) $A = \begin{array}{cc} & C_1 \\ R_1 & \begin{bmatrix} 1 & 45 & 3 & 4 & 3 & 4 \\ 0 & 1 & 9 & 100 & 4 & 45 \\ 0 & 0 & 1 & 45 & -3 & -2 \\ 0 & 0 & 0 & 5 & 4 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} \end{array}$. $= 20$

★ (e) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ if $\begin{array}{cc} & C_1 \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} \end{array} = 1.$

a) Expand along R_1:

$= (1)(-1)^{1+1}\det\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} + (-1)(-1)^{1+2}\det\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} + (2)(-1)^{1+3}\det\begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix} = (3+1) + (9-2) - 2(-3-2) = 21$

b) Expand along R_1:

$= (1)(-1)^{1+1}\det\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} + (1)(-1)^{1+2}\det\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} + (1)(-1)^{1+3}\det\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} = (6-6) - (6-6) + (6-6) = 0$

c) Expand along R_2: $\det(A) = 0$

d) Expand along C_1:

$= (1)(-1)^{1+1}\det_{R_1}\begin{bmatrix} \cancel{1} & \cancel{9} & \cancel{100} & \cancel{4} & \cancel{45} \\ 0 & 1 & 45 & -3 & -2 \\ 0 & 0 & 5 & 4 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} = \text{Expand along } C_1: (1)(-1)^{1+1}\det_{R_1}\begin{bmatrix} \cancel{1} & \cancel{45} & \cancel{-3} & \cancel{-2} \\ 0 & 5 & 4 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $= (1)(-1)^{1+1}\det\begin{bmatrix} 5 & 4 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix}$ Expand along C_1:

$= (5)(-1)^{1+1}\det\begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix} = 5(4) = 20$

e) $\det\begin{bmatrix} \cancel{1} & \cancel{1} & \cancel{2} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ Expand along $C_1 = (1)(-1)^{1+1}\det\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} + (1)(-1)^{2+1}\det\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} + (1)(-1)^{3+1}\det\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$= (2-3) - (1-2) + (3-4) = -1$

$A \neq \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$I^2 = I$

$(I+A)(I+A)$ $\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ $AA^{-1} = I$

QUESTION 5 (6 pts)

Let A be an $n \times n$ matrix. Assume that $A^2 = 0$ and $I - A$ is invertible, where I is the $n \times n$ identity matrix. Show that

$$(I - A)^{-1} = I + A.$$

$(I+A)^2 = I^2 + \cancel{A^2}^{\,0} + IA + AI = I + IA + AI = I(1+2A)$

$(1-A) = (AA^{-1}) - \frac{I}{A^{-1}} = \frac{(A^{-1})^2 A - I}{A^{-1}} =$

QUESTION 6 (4 pts)

Answer the following questions with **True** or **False**. Write down you answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) If A is an $n \times n$ matrix and $A^2 = I$, then $A = \pm I$. (/ 1)

(a) True

(b) If A and B are $n \times n$ matrices, then $AB = BA$. (/ 1)

A & B are matrixes and not just #'s matrices dont always have commutative properties.

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\quad AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix}$

$BA = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $\quad BA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 4 & 6 \end{bmatrix}$

(b) False

(c) If A and B are $n \times n$ invertible matrices, then $A + B$ is invertible. (/ 1)

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{-1}{ad-bc} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \neq 0$ $\quad$ Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ + $b = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

A & B both shouldnt = 0 b/c they are invertable

$\frac{1}{ad-bc} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} + \frac{1}{eh-gf} \begin{bmatrix} h & -f \\ -g & e \end{bmatrix}$

(c) True

(d) If A and B are $n \times n$ matrices, then $(AB)^\top = A^\top B^\top$. (/ 1)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 3 \\ 7 & 7 \end{bmatrix}$ $(AB)^T = \begin{bmatrix} 3 & 7 \\ 3 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A^T \cdot B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 6 & 6 \end{bmatrix}$

(d) False