MATH-311
Midterm 02 (50min)

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Last name: ____________________

First name: ____________________

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	6	4	50
Score:							

Instructions:

- Write your complete name on your copy.
- Answer all 6 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam**.
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: ____________________

May the Force be with you!
Pierre

Question 1 **(10 pts)**

Say if the following matrix products are well-defined. If it is well-defined, then compute the matrix products.

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 9 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}.$$

Question 2 **(10 pts)**

Find the values of the entries of the matrix A if

$$\left(\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix} A\right)^{-1} = \begin{bmatrix}1 & 0\\ 2 & 2\end{bmatrix} - 2\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}.$$

Question 3 **(10 pts)**

Find a 2×2 elementary matrix E such that

$$E \begin{bmatrix} 3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}.$$

Question 4 **(10 pts)**

Evaluate the determinant of the matrix A.

(a) (2 Pts) $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -1 & 3 \end{bmatrix}$.

(b) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$.

(c) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & -1 & 1 & 4 \\ 1 & -1 & 1 & -1 \end{bmatrix}$.

(d) (2 Pts) $A = \begin{bmatrix} 1 & 45 & 3 & 4 & 3 & 4 \\ 0 & 1 & 9 & 100 & 4 & 45 \\ 0 & 0 & 1 & 45 & -3 & -2 \\ 0 & 0 & 0 & 5 & 4 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$.

(e) (2 Pts) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ if $\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1$.

Question 5 **(6 pts)**

Let A be an $n \times n$ matrix. Assume that $A^2 = 0$ and $I - A$ is invertible, where I is the $n \times n$ identity matrix. Show that

$$(I - A)^{-1} = I + A.$$

QUESTION 6 (4 pts)

Answer the following questions with **True** or **False**. Write down you answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) If A is an $n \times n$ matrix and $A^2 = I$, then $A = \pm I$. (/ 1)

(a) ____________

(b) If A and B are $n \times n$ matrices, then $AB = BA$. (/ 1)

(b) ____________

(c) If A and B are $n \times n$ invertible matrices, then $A + B$ is invertible. (/ 1)

(c) ____________

(d) If A and B are $n \times n$ matrices, then $(AB)^\top = A^\top B^\top$. (/ 1)

(d) ____________