Appendix L, Problem 1 (4 Pts)

a) Yes it is a statement. The statement is false since $|-12| = 12$ (absolute value turns negative numbers into positive numbers).

b) No, this is not a statement. The value of x is not specify, so there is no truth value that can be associated to this sentence.

c) No, this is not a statement. A question is not a statement.

d) Yes, this is a statement. It is true, because assuming that $a = 2$ and $b = 4$, we have $a + b = 2 + 4 = 6$.

Appendix L, Problem 2 (6 Pts)

a) **Converse:** If Angela sleeps in, then it is a Saturday.
Contrapositive: If Angela does not sleep in, then it is not Saturday.

b) **Converse:** If I use my umbrella, then it rains outside.
Contrapositive: If I don't use my umbrella, then it does not rain outside.

c) **Converse:** If the surf was bigger than 4 feet high, then I went surfing.
Contrapositive: If the surf was smaller than 4 feet high, then I did not go surfing.

Appendix L, Problem 3 (8 Pts)

a) The negation is "It is not the case that it is raining and Charlie is cold.". The negation of a statement $P \wedge Q$, is $(\neg P) \vee (\neg Q)$. So, letting P: "It is raining" and Q: "Charlie is cold", a useful reformulation of the negation is "it is not raining or Charlie is not cold".

b) The negation is "It is not the case that if is raining, then Charlie is cold". The negation of a statement $P \Rightarrow Q$ is $P \wedge (\neg Q)$. So, a useful reformulation of the negation is "It is raining and Charlie is not cold".

c) Let's simplify the statement using mathematical symbols. We can equivalently and compactly rewrite the statement as " $\forall x$ real, $\exists y$ real such that $x + y = 0$". The negation is then "It is not the case that $\forall x$ real, $\exists y$ real such that $x+y = 0$". The negation of a universal statement "$\forall x$, $P(x)$" is "$\exists x$, $\neg P(x)$". Let $P(x)$: "$\exists y$ real such that $x + y = 0$". Then we can rewrite the negation of the statement as "$\exists x$ real such that $\neg P(x)$" or

$$\exists x \text{ real such that it is not the case that there exists } y \text{ real such that } x + y = 0.$$

The negation of an existential "$\exists y,\ Q(y)$" is "$\forall y,\ \neg Q(y)$. For a fixed x, let $Q(y)$: "$x+y=0$". Then we can rewrite the negation of "$\exists y$ real such that $x+y=0$" as "$\forall x$ real, $\neg Q(y)$", or "$\forall x$ real, $x+y\neq 0$". Therefore, the negation of the whole statement is

$$\exists x \text{ real such that } \forall y \text{ real, } x+y\neq 0\ .$$

d) Let P : "$|a|>0$" and Q : "$a\neq 0$". The statement $P \iff Q$ can be written as

$$(P\Rightarrow Q)\wedge(Q\Rightarrow P).$$

Therefore, the negation of the $P \iff Q$ is

$$(P\wedge\neg Q)\vee(Q\wedge\neg P).$$

Replacing what is P and Q, the statement $P\wedge\neg Q$ becomes " $|a|>0$ and $a=0$ " and the statement $Q\wedge\neg P$ becomes "$a\neq 0$ and $|a|\leq 0$". Hence, the negation of the full statement is

$$(|a|>0 \text{ and } a=0) \text{ or } (a\neq 0 \text{ and } |a|\leq 0).$$

Appendix B, Problem 1 (12 Pts)

a) **Proof of the implication.** Assume that n is an even integer. Then $n=2k$, for some integer k. Hence, $n^2=(2k)^2=4k^2$ and n^2 is a multiple of 4.
Proof of the converse. Assume that n^2 is a multiple of 4. Then $n^2=4k$, for some integer k. Rearranging the equation for n and k, we get $(n/2)^2=k$. If n was an odd integer, then $(n/2)^2$ would be a rational number (a fraction). However, from the equation $(n/2)^2=k$, $(n/2)^2$ should be a whole number. Therefore, n should be even.

d) **Proof of the implication.** Assume that $x^2-5x+6=0$. Factoring the polynomial, we find that $(x-2)(x-3)=0$. Therefore, $x-2=0$ or $x-3=0$. Hence, $x=2$ or $x=3$.
Proof of the converse. There are two cases to verify. Assume $x=2$. Then $2^2-5(2)+6=4-10+6=0$. So $x=2$ satisfies the conclusion. Now assume that $x=3$. Then $3^2-5(3)+6=9-15+6=0$. So $x=3$ satisfies the conclusion.

Appendix B, Problem 2b (5 Pts)

Assume that n is an odd integer. Then $n=2m+1$, for some integer m. Squaring n, we get

$$n^2=(2m+1)^2=4m^2+4m+1=4(m^2+m)+1.$$

Since m is an integer, it can be ether odd or even. We can split the proof into two cases:

1. **Assume that m is even.** Then $m=2j$, for some integer j. Replacing it in the expression for n^2, we get

$$n^2=4(4j^2+2j)+1=4(2(2j^2+j))+1=8(2j^2+j)+1.$$

Let $k=2j^2+2$, then $n^2=8k+1$.

2. **Assume that m is odd.** Then $m = 2j+1$, for some integer j. Replacing it in the expression for n^2, we get

$$n^2 = 4(4j^2 + 4j + 1 + 2j + 1) + 1 = 4(4j^2 + 6j + 2) + 1 = 8(2j^2 + 3j + 1) + 1.$$

   Let $k = 2j^2 + 3j + 1$, then $n^2 = 8k + 1$.

Thus, in both cases, we can write $n^2 = 8k + 1$, for some integer k.

Appendix B, Problem 3a **(10 Pts)**

Assume that $n > 2$ and n is a prime. To give a proof by contradiction, assume further that n is not an odd integer. In other words, assume that n is an even integer. Since n is even, then n is divisible by 2. But a prime number is only divisible by itself and 1. Since $n > 2$, n can not be divisible by 2. This is a contradiction. Hence, n should be an odd integer.
The converse is false though. For example, take $n = 9$. Then n is an odd integer, but it is divisible by 3, so it is not a prime.

Appendix B, Problem 4a **(5 Pts)**

Assume that x and y are positive numbers. Assume further that the conclusion is not true, so that $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$. Then squaring the last equation:

$$x + y = (\sqrt{x} + \sqrt{y})^2 = x + 2\sqrt{x}\sqrt{y} = y$$

which, after simplifications, turns out to be

$$0 = xy.$$

However, two numbers are zero only when one of them is 0. But, we assumed that both x and y are not zero. A contradiction. Hence,

$$\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}.$$