Section 2.2 — Problem 2a (5 Pts)

We let $A = \begin{bmatrix} 1 & -1 & 3 \\ -3 & 1 & 1 \\ 5 & -8 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ -6 \\ 9 \end{bmatrix}$. Then the system can be rewritten as

$$A\mathbf{x} = \mathbf{b} \iff \begin{bmatrix} 1 & -1 & 3 \\ -3 & 1 & 1 \\ 5 & -8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \\ 9 \end{bmatrix}.$$

Section 2.3 — Problem 1 (20 Pts)

b. The answer should be

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -6 & -2 \\ 0 & 6 & 10 \end{bmatrix}.$$

d. The answer should be

$$\begin{bmatrix} 1 & 3 & -3 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & -15 \end{bmatrix}.$$

f. The answer should be

$$\begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -8 \end{bmatrix} = \begin{bmatrix} -23 \end{bmatrix}$$

g. The answer should be

$$\begin{bmatrix} 2 \\ 1 \\ -7 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 6 \\ 1 & -1 & 3 \\ -7 & 7 & -21 \end{bmatrix}.$$

Section 2.3 — Problem 2a (10 Pts)

We first consider the squares A^2, B^2, C^2.

- The matrix A is a 2×3 and hence $A^2 = AA$ is not defined.
- Similarly, the matrix C is a 3×2 matrix and hence $C^2 = CC$ is not defined.
- The matrix B is a 2×2 and the product B^2 is defined. The result is

$$\begin{bmatrix} 1 & -2 \\ 1/2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1/2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 2 & 8 \end{bmatrix}$$

Now we consider the possible products between A and B.

- The matrix A is 2×3 and B is 2×2. So AB is undefined.
- The matrix B is 2×2 and A is 2×3. Hence, BA is defined. Then

$$BA = \begin{bmatrix} 1 & -2 \\ 1/2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 3 \\ -5/2 & 1 & 3/2 \end{bmatrix}.$$

Now we consider the possible products between A and C.

- The matrix A is 2×3 and C is 3×2. So AC is defined. The result is

$$AC = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 5 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 19 \\ 1 & 0 \end{bmatrix}.$$

- The matrix C is 3×2 and A is 2×3, so CA is also defined. The result is

$$CA = \begin{bmatrix} -1 & 0 \\ 2 & 5 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -3 \\ -3 & 4 & 6 \\ -3 & 0 & 0 \end{bmatrix}.$$

Now we consider the possible products between B and C.

- The matrix B is 2×2 and C is 3×2. Hence BC is undefined.
- The matrix C is 3×2 and B is 2×2. Hence CB is defined. The result is

$$CB = \begin{bmatrix} -1 & 0 \\ 2 & 5 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0.5 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4.5 & 11 \\ 1.5 & 9 \end{bmatrix}$$

Section 2.3 — Problem 16d (5 Pts)

We have

$$(A - B)(C - A) = AC - A^2 - BC + (-B)(-A) = AC - A^2 - BC + BA$$

and

$$(C - B)(A - C) = CA + C(-C) - BA + BC = CA - C^2 - BA + BC$$

and

$$(C - A)^2 = (C - A)(C - A) = C^2 - CA - AC + A^2.$$

Plugging that into the original equation, denote it by E, we get

$$\begin{aligned} E &= AC - A^2 - BC + BA + CA - C^2 - BA + BC + C^2 - CA - AC + A^2 \\ &= (AC - AC) + (A^2 - A^2) + (BC - BC) + (BA - BA) + (CA - CA) - C^2 + C^2 \\ &= 0. \end{aligned}$$

Section 2.3 — Problem 34a (10 Pts)

Assume that $AB = BA$. Then

$$(A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2.$$

Since $AB = BA$, then $AB + BA = AB + AB = 2AB$ and hence

$$(A+B)^2 = A^2 + 2AB + B^2.$$

Now, assume that $(A+B)^2 = A^2 + 2AB + B^2$. Then we get

$$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2.$$

Substracting A^2, B^2 and AB on each side, we get

$$BA = 2AB - AB = AB.$$

Hence $BA = AB$.