

# UNIVERSITY OF HAWAI'I



Last name: \_\_\_\_\_

First name: \_\_\_\_\_

Question:	1	2	3	4	5	Total
Points:	20	10	10	5	5	50
Score:						

## Instructions:

- Write your complete name on your copy.
- Answer all 5 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam.**
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: \_\_\_\_\_

MAY THE FORCE BE WITH YOU!  
PIERRE

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QUESTION 1

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(20 pts)

Let  $A = \begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$ .

- (a) (5 Pts) Find the eigenvalues of the matrix  $A$ .
- (b) (10 Pts) Find the eigenvectors associated to each eigenvalue.
- (c) (5 Pts) Is  $A$  diagonalizable? If so, find the matrix  $P$  such that  $P^{-1}AP$  is a diagonal matrix.



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QUESTION 2

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(10 pts)

Let  $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$  be vectors in a vector space  $V$ . Simplify the following expression:

$$2(\mathbf{x} - \mathbf{y}) + 4(\mathbf{z} - \mathbf{y}) + 4(\mathbf{w} - \mathbf{z}) + (\mathbf{x} - 4\mathbf{w}).$$

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QUESTION 3

(10 pts)

Which of the following are subspaces of  $\mathbf{M}_{22}$ , the vector space of all  $2 \times 2$  matrices with usual addition and scalar multiplication of matrices.

- (a) (5 Pts)  $U = \{A : A \in \mathbf{M}_{22} \text{ and } A = A^\top\}$ .
- (b) (5 Pts)  $U = \{A : A \in \mathbf{M}_{22} \text{ and } A^4 = I\}$ .

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QUESTION 4

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(5 pts)

Answer the following questions:

- (a) (3 Pts) Assume that  $A$  is an  $3 \times 3$  matrix and that  $c_A(x)$  is the characteristic polynomial of  $A$ . Show that

$$c_{A^2}(x^2) = (-1)c_A(x)c_A(-x).$$

[Hint: Use the following property of determinants:  $\det(XY) = \det(X)\det(Y)$ .]

- (b) (2 Pts) What does the word “eigen” in “eigen-vectors” and “eigen-values” mean in English?

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QUESTION 5 (5 pts)

Answer the following questions with **True** or **False**. Write down your answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) If  $A$  is a  $2 \times 2$  matrix with two distinct eigenvectors, then  $A$  is diagonalizable. ( / 1)

(a) \_\_\_\_\_  
 (b) If  $A$  is a  $2 \times 2$  matrix with eigenvalues  $\lambda_1 = 3$  and  $\lambda_2 = -1$ , then  $P^{-1}AP = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ . ( / 1)

(b) \_\_\_\_\_  
 (c) If the solution to  $A\mathbf{x} = \lambda\mathbf{x}$  is only  $\mathbf{x} = \mathbf{0}$ , then  $\lambda$  is an eigenvalue. ( / 1)

(c) \_\_\_\_\_  
 (d) If a matrix  $A$  has  $\lambda = 0$  as an eigenvalue, then  $A$  is not invertible. ( / 1)

(d) \_\_\_\_\_  
 (e) The set  $U = \{p : p \in \mathbf{P}_3 \text{ and } p(0) = 1\}$  is a subspace of  $\mathbf{P}_3$ . ( / 1)

(e) \_\_\_\_\_