University of Hawai'i

Last name: ______________________________

First name: ______________________________

Question:	1	2	3	4	5	Total
Points:	20	10	10	5	5	50
Score:						

Instructions:

- Write your complete name on your copy.
- Answer all 5 questions below.
- Write your answers directly on the questionnaire.
- Show ALL your work to have full credit.
- Draw a square around your final answer.
- Return your copy when you're done or at the end of the 50min period.
- No electronic devices allowed during the exam.
- Scientific calculator allowed only (no graphical calculators).
- **Turn off your cellphone(s) during the exam**.
- Lecture notes and the textbook are not allowed during the exam.

Your Signature: ______________________________

May the Force be with you!
Pierre

Question 1 **(20 pts)**

Let $A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$.

(a) (5 Pts) Find the eigenvalues of the matrix A.

(b) (10 Pts) Find the eigenvectors associated to each eigenvalue.

(c) (5 Pts) Is A diagonalizable? If so, find the matrix P such that $P^{-1}AP$ is a diagonal matrix.

Question 2 **(10 pts)**

Let $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{w}$ be vectors in a vector space V. Simplify the following expression:

$$2(\mathbf{x} - \mathbf{y}) + 4(\mathbf{z} - \mathbf{y}) + 4(\mathbf{w} - \mathbf{z}) + (\mathbf{x} - 4\mathbf{w}).$$

Question 3 **(10 pts)**

Which of the following are subspaces of $\mathbf{M_{22}}$, the vector space of all 2×2 matrices with usual addition and scalar multiplication of matrices.

(a) (5 Pts) $U = \{A \,:\, A \in \mathbf{M_{22}} \text{ and } A = -A^{\top}\}$.

(b) (5 Pts) $U = \{A \,:\, A \in \mathbf{M_{22}} \text{ and } A^2 = I\}$.

QUESTION 4 **(5 pts)**

Answer the following questions:

(a) (3 Pts) Assume that A is an 3×3 matrix and that $c_A(x)$ is the characteristic polynomial of A. Show that

$$c_{A^2}(x^2) = (-1)c_A(x)c_A(-x).$$

[Hint: Use the following property of determinants: $\det(XY) = \det(X)\det(Y)$*.]*

(b) (2 Pts) What does the word "eigen" in "eigen-vectors" and "eigen-values" mean in English?

Question 5 (5 pts)

Answer the following questions with **True** or **False**. Write down you answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) The set $U = \{p : p \in \mathbf{P_3} \text{ and } p(0) = 1\}$ is a subspace of $\mathbf{P_3}$. (/ 1)

(a) ____________

(b) If the solution to $A\mathbf{x} = \lambda\mathbf{x}$ is only $\mathbf{x} = \mathbf{0}$, then λ is an eigenvalue. (/ 1)

(b) ____________

(c) If a matrix A has $\lambda = 0$ as an eigenvalue, then A is not invertible. (/ 1)

(c) ____________

(d) If A is a 2×2 matrix with two distinct eigenvectors, then A is diagonalizable. (/ 1)

(d) ____________

(e) If A is a 2×2 matrix with eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -1$, then $P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. (/ 1)