University of Hawai'i

Last name: _____

First name: _____

Question:	1	2	3	4	5	6	Total
Points:	10	10	10	10	6	4	50
Score:							

Instructions:

- Write your complete name on your copy.

- Answer all 6 questions below.

- Write your answers directly on the questionnaire.

- Show ALL your work to have full credit.

- Draw a square around your final answer.

- Return your copy when you're done or at the end of the 50min period.

- No electronic devices allowed during the exam.

- Scientific calculator allowed only (no graphical calculators).

- **Turn off your cellphone(s) during the exam**.

- Lecture notes and the textbook are not allowed during the exam.

Your Signature: _____

May the Force be with you!
Pierre

Find the solution to the following system of linear equations:

$$\begin{cases} x_1 + x_2 + 3x_3 = 3 \\ 2x_1 - 2x_2 + x_3 = 0 \\ x_1 - x_2 + x_3 = 2 \end{cases}$$

Does it have one solution, infinitely many solutions or no solution?

Consider the following vectors:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \; \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \; \mathbf{z} = \begin{bmatrix} -2 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \; \mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \; \mathbf{w} = \begin{bmatrix} 9 \\ -8 \\ 0 \\ 1 \end{bmatrix}.$$

We would like to know if $\mathbf{w}$ is a linear combination of $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ and $\mathbf{v}$.

(a) (5 points) Write down the system of linear equations corresponding to this problem. **DO NOT SOLVE THE SYSTEM**.

(b) (5 points) If the RREF of the augmented matrix of the system from part (a) is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix},$$

can you express $\mathbf{w}$ as a linear combination of $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, and $\mathbf{v}$? If so, write $\mathbf{w}$ as a linear combination of the other vectors.

Consider the following homogeneous system of linear equations:

$$\begin{cases} x_1 - x_2 - x_3 + 2x_5 = 0 \\ x_1 + 2x_2 + x_3 + 2x_4 + 2x_5 = 0 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 = 0 \\ x_1 + 2x_2 + 2x_3 + 3x_4 + 2x_5 = 0 \end{cases}.$$

(a) (2 points) Write the augmented matrix of the system.

(b) (8 points) The RREF of the augmented matrix of the system is

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Express the solution as a linear combination of basic solution(s).

Find the entries of the matrix A if A satisfies the equation:

$$2A^\top - 5\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \left(4A - 9\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}\right)^\top.$$

Show that if A is a square matrix and $A = kA^\top$ for some scalar $k \neq \pm 1$, then $A = 0$.

Answer the following questions with **True** or **False**. Write down you answers on the line at the end of each question. Justify briefly your answer in the space after the statement of the problem.

(a) A matrix B with RREF $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ has rank $(B) = 2$. (/ 1)

(a) ——————

(b) A system of 3 linear equations in 2 variables with a augmented matrix of rank 2 has a unique solution. (/ 1)

(b) ——————

(c) If $\mathbf{x_1}$ and $\mathbf{x_2}$ are solutions to a system of homogeneous linear equations denoted by (S), then $2\mathbf{x_1} - \mathbf{x_2}$ is also a solution of the system (S). (/ 1)

(c) ——————

(d) A system of linear equations can have no solution. (/ 1)

(d) ——————