MATH-331 Intro. to Real Analysis — Pierre-Olivier Parisé
Team test 03 — Fall 2021, 12/03/2021

**Name of the members of the team:** _____

**Team name (if any):** _____

**Instructions:** You must answer all the questions in teams of 3 and hand out one copy per team. There are 2 QUESTIONS on this test. The scores for each questions is on the last page of the test.

You are allowed to use the lecture notes only. No other tools such as a cell-phone, a calculator, or a laptop. Only your pen and eraser. The space between the questions are there to write the final versions of your answers.

Be the better version of yourself!

## Question 1 **(10 pts)**

Let $f : (a, b] \to \mathbb{R}$ be a function where $a < b$.

(a) (5 points) How would you define the Riemann integral of $f$ on $(a, b]$? Explain in details your definition.

(b) (5 points) Find a function $f : (0, 1] \to \mathbb{R}$ that is Riemann integrable on $(0, 1]$ (with respect to your definition) but is unbounded on $(0, 1]$.

## Question 2 **(10 pts)**

Find the limit of the sequence $(a_n)_{n=1}^{\infty}$ if

$$a_n = \sum_{k=1}^{n} \frac{k}{k^2 + n^2}.$$

## Scores table

| Question: | 1 | 2 | Total |
|---|---|---|---|
| Points: | 10 | 10 | 20 |
| Score: | | | |