① Proof by induction: Let $n=1$ be our base

$$1 = \frac{(n+1)n}{2} = \frac{2(1)}{2} = \frac{2}{2} = 1 \checkmark$$

Base case is true. Assume of expression

$X(n) = \frac{(n+1)n}{2}$ and $X(n)$ is true. Must prove $X(n+1)$ is true.

$$X(n+1) = \frac{(n+1)((n+1)+1)}{2}$$

$$X(n+1) = \frac{(n+1)n}{2} + (n+1) = \frac{(n+1)((n+1)+1)}{2}$$

$$= \frac{(n+1)(n+2)}{2} = \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$$

$$= n(n+1)$$

There for by induction $n \in \mathbb{N} = \frac{n(n+1)}{2}$.

(2) Proof by induction

Let $n=1$ be our base case

$f(1) = 1$ and $2^{1-1} = 2^0 = 1 = 1$ ✓

$1 \leq 1$ ✓

Our base case is true. Lets see if its true for ~~all~~ $n+1$

$f(n+1) = f(n+1-1) + f(n+1-2) + f(n+1-3)$

$\leq 2^{n+1-1}$

$f(n) + f(n-1) + f(n-2) \leq 2^n$

Let $n=4$ $f(4) + 3 + 2 \leq 2^{4}$

Since for all $n \geq 4$ the power 2^n will be at least 4, $\leq 2^{n-1}$ ∴

$f(n) \leq 2^{n-1}$ for all $n \in \mathbb{N}$ by induction.

③ Let A, B, C all be in a set

(A) $a \sim a$ because $f(a) = a$ which allows for $a = a$ or $A = A^T$ The reflexive property.

(B) $A \sim B$ then $B \sim A$. This is the symetric property where if $A \sim B$ then $B \sim A$ because $B = (B^T)^T = (A^T) = A^T$ and it implies $B \sim A$.

(C) This is the transitive property.
If $A \sim B$ and $B \sim C$ then $A = B^T$ and $B = C^T$
since $B = B^T$ Then $A = B^T = B = C^T$. Therefore
$A \sim C$.

(4)

Proof :- Let S be a countable set
T is a subset of S.

1- If T is finite then obviously its countable
2- If T is an infinite subset of S
Since S is countable set, its elements can be arrange in any way such as $x_1, x_2, x_3 \dots$
Since T is an infinite subset of S, T contains infinite numbers and the setting of element of T form and infinite subset P

By well ordering property of $\mathbb{N}$, P contains a least element (L.E) therefore $a_{L.E} \in T$

Since even when T is an infinite subset of S, you will still have a L.E and a M.E which allows for the set to be countable.

$\therefore$ T is countable.

(5) Let $0 < a < b$ be positive real numbers

a) since $a < b$ this would also be true for $a(a) < b(b)$ or $a^2 < b^2$ because lets say $a = n$ and $b = n+1$ to satisfy $a < b$. $n^2 = n^2$ and $(n+1)^2 = n^2 + 2n + 1 > n^2$ resulting in $a^2 < b^2$.

b) additionally the same argument can be made for $\sqrt{a} < \sqrt{b}$ since $a < b$ $a^{1/2} < b^{1/2}$ because say $a = n$ and $b = n+1$ to satisfy $a < b$

$\sqrt{n} < \sqrt{n+1}$ for all n because

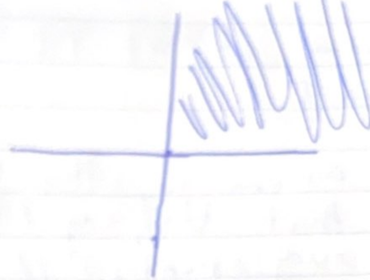
if $n = 1$ $\sqrt{1} < \sqrt{2}$ or

$1 < \sqrt{2}$ which is true.

0

$y = x + |x| - |y|$

(b)



~~State $x + |x|$ will never be~~ ≥ 0

$x + |x| \geq 0$ because if $x = -1$

$-1 + 1 = 0 \geq 0$ ✓, the absolute function cancels out any negative answers and puts them to zero

same applies for $y + |y| \geq 0$ allowing for the function to be in Q2

(1) If $x \geq 0$ and $y \geq 0$ then $\sqrt{xy} \leq \frac{x+y}{\sqrt{2}}$

Let $x = 3$ and $y = 3$ be our base.

$\sqrt{9} \leq \frac{6}{\sqrt{2}}$

$3 \leq \frac{6}{\sqrt{2}}$ because $\sqrt{2} < 2$ which

means $3 \leq \frac{6}{\sqrt{2}}$ ✓.

~~Lets see if this holds for $x+1$, $y+1$~~

Let $x = a$ and $y = b$ wher a & $b \geq 0$

$\sqrt{ab} \leq \frac{a+b}{\sqrt{2}}$

$\sqrt{2} \leq \frac{a+b}{\sqrt{ab}}$ since $\sqrt{2}$ is $1 < \sqrt{2} < 2$

and if a & b are 1 then

$\frac{2}{\sqrt{1}} = \frac{2}{1} = 2$ which is $\sqrt{2} \leq 2$ ✓

1 is our least element in x & $y \geq 0$

if the least element is true then its true for

all x & $y \geq 0$ ✓.

(10) a) Does supremum of set rE for $r > 0$
= sup E times r ?

Due to the order axiom 11, there exists a nonzero number z such that $\forall x, < y \in R$, $xz < yz$. If we multiply every number of the set E, by r with x as our suprem; xr is the greatest of every element in E. However, If x is the supremum of set E and we multiply it by r with $0 < r < 1$, x will be $\leq$ the lowest term in the set. Therefore $r\sup(E) \neq \sup(rE)$

b) ?