

Name of the members of the team: _____

Team name: _____

Question:	1	2	Total
Points:	10	10	20
Score:			

Instructions: You must answer all the questions in teams of 3 and hand out one copy per team. You are allowed to use the lecture notes only. No other tools such as a cell-phone, a calculator, or a laptop. Only your pen and eraser. The space between the questions are there to write the final versions of your answers.

QUESTION 1

(10 pts)

Which of the following statements are true? Give a proof or a counter-example.

- (a) (2 points) $|x_1 - x_2 + x_3| \leq |x_1| - |x_2| - |x_3|$ for any $x_1, x_2, x_3 \in \mathbb{R}$.
- (b) (2 points) $|x_1 - x_2 + x_3| \geq |x_1| - |x_2| + |x_3|$ for any $x_1, x_2, x_3 \in \mathbb{R}$.
- (c) (2 points) $|x_1 - x_2 + x_3| \geq |x_1| - |x_2| - |x_3|$ for any $x_1, x_2, x_3 \in \mathbb{R}$.
- (d) (2 points) $((-1)^n/n)_{n=1}^{\infty}$ converges to 0.
- (e) (2 points) $((-1)^n + (-1)^{n+1})_{n=1}^{\infty}$ converges.

QUESTION 2

(10 pts)

Suppose $(a_n)_{n=1}^{\infty}$ converges to A , and define the new sequence $(b_n)_{n=1}^{\infty}$ by $b_n = \frac{a_n + a_{n-1}}{2}$ for all $n \geq 1$. Prove that the sequence $(b_n)_{n \geq 1}$ converges to A .