Questions	Scores
1	7
2	10
3	5
4	2
5	1
6	10
7	1
8	3
9	2
10	4

TOTAL.

45/65

Real Analysis HW #6

1 a.) $b-a$ is length of segment. set $\ell := \min\{N, b-a\}$
Where $\frac{b-a}{N} < \delta$ (by A.P.) Then a partition with lengths $\frac{b-a}{\ell}$,
the $\|P\|$ will be less than δ Be more precise...
Describe P. 2/5

b.) Suppose $\int_a^b f_1 \neq \int_a^b f_2$ are the two RI values
of f. f is RI, so $\|P\| < \delta_1$ $|S(P,f) - \int_a^b f_1| < \varepsilon/2$
and $\|P\| < \delta_2$ $|S(P,f) - \int_a^b f_2| < \varepsilon/2$

7/10

Set $\delta := \min\{\delta_1, \delta_2\}$ And $|\int_a^b f_1 - \int_a^b f_2| \neq 0$

$\|P\| < \delta$, then $|\int_a^b f_1 - \int_a^b f_2| = |\int_a^b f_1 - S(f,P) + S(f,P) - \int_a^b f_2|$
$\leq |S(f,P) - \int_a^b f_1| + |S(f,P) - \int_a^b f_2| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} < \varepsilon$
Set $\varepsilon = \frac{|\int_a^b f_1 - \int_a^b f_2|}{2}$
$|\int_a^b f_1 - \int_a^b f_2| < \frac{|\int_a^b f_1 - \int_a^b f_2|}{2}$ $1 < \frac{1}{2}$ $2 < 1$ #

✓ 5/5

$\int_a^b f$ is unique

2 a.) $\|P\| < \delta_1 \to |S(f,P) - \int_a^b f| < \varepsilon/2$ $\|P\| < \delta_2 \to |S(g,P) - \int_a^b g| < \varepsilon/2$

10/10

set $\delta = \min\{\delta_1, \delta_2\}$ then $\|P\| < \delta$ $|S(f+g,P) - \int_a^b f - \int_a^b g| < \varepsilon$
$S(f+g,P) = \sum_{i=1}^{N} (f+g)(c_i)(x_i - x_{i-1}) = \sum_{i=1}^{N} f(c_i)(x_i - x_{i-1}) + \sum_{i=1}^{N} g(c_i)(x_i - x_{i-1})$
5/5
number so distribute
$S(f+g,P) = S(f,P) + S(g,P) \to |S(f,P) - \int_a^b f + S(g,P) - \int_a^b g| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} < \varepsilon$

want

2 b.) $f(x) \le g(x)$ $\int_a^b f \le \int_a^b g$

$S(f,P) \le S(g,P) \to \sum_{i=1}^{n} f(c_i)(x_i - x_{i-1}) \le \sum_{i=1}^{n} g(c_i)(x_i - x_{i-1})$ because $f \le g$

Let $\varepsilon = \frac{\int_a^b f - \int_a^b g}{2}$ $\|P\| < \delta_1 \to |S(f,P) - \int_a^b f| < \frac{\int_a^b f + \int_a^b g}{2}$

$\|P\| < \delta_2 \to |S(g,P) - \int_a^b g| < \frac{\int_a^b f + \int_a^b g}{2}$

Take $\delta := \min\{\delta_1, \delta_2\}$

then $-\frac{\int_a^b f + \int_a^b g}{2} < S(f,P) - \int_a^b f < \frac{\int_a^b f + \int_a^b g}{2}$

then $-\frac{\int_a^b f + \int_a^b g}{2} < S(g,P) - \int_a^b g < \frac{\int_a^b f + \int_a^b g}{2}$

$\frac{\int_a^b f + \int_a^b g}{2} < S(f,P)$ $S(g,P) < \frac{\int_a^b f + \int_a^b g}{2}$

so $S(g,P) < \frac{\int_a^b f + \int_a^b g}{2} < S(f,P)$

Which is a contradiction of $S(f,P) \le S(g,P)$ #

Okay you argue by contrad. Make it clear from the beginning. ✓

5/5

3 We know that $\int_a^b k = k(b-a)$ for some constant k. If we set $M = k$ then $\int_a^b M = M(b-a)$. We know that $|f(x)| \le M$ so from #2b, we know that $\int_a^b f \le \int_a^b M = M(b-a)$ so $\int_a^b f \le M(b-a)$ ✓

5/5

Use the def. of R.I. and make $\|P_n\| < \delta$.

4 $\lim_{n\to\infty} \|P_n\| \to 0$ $|\|P_n\| - 0| < \varepsilon$ $\forall \varepsilon > 0, \exists N$ s.t. $n \ge N$ then $\|P_n\| < \varepsilon$

$n \ge N$ then $|S(f,P_N) - \int_a^b f| < \varepsilon$ $S(f,P_N) = \sum_{i=1}^{N} f(c_i)(x_i - x_{i-1})$

$(x_i - x_{i-1}) \to 0$ so 0 $|0 - \int_a^b f| < \varepsilon$ $\int_a^b f < \varepsilon$ ✗

2/5

10/10

6 a) Let P be tagged partition with $\|P\|<\delta$.

$S(f,P)=\sum_{i=1}^{N} f(c_i)(x_i-x_{i-1})$ $f(c_i)$ $\forall c_i \in c_i,\dots,c_N = k$

$S(f,P)=\sum_{i=1}^{N} k(x_i-x_{i-1})$ since P is tagged partition,

$\sum_{i=1}^{N}(x_i-x_{i-1})=(b-a)$ $\sum_{i=1}^{N} k(x_i-x_{i-1}) = k\sum_{i=1}^{N}(x_i-x_{i-1}) = k(b-a)$

So $|S(f,P)-\int_a^b f|<\varepsilon$ for $\varepsilon>0$ $|S(f,P)-k(b-a)|=|k(b-a)-k(b-a)|$

$=0<\varepsilon$ so $|S(f,P)-\int_a^b f|<\varepsilon$ so f is RI on $[a,b]$

and $\int_a^b f = k(b-a)$ 5/5

b.) $\int_a^b \sin^2(x)$ $\sin^2(x)=\frac{1-\cos(2x)}{2}=\frac{1}{2}-\frac{1}{2}\cos(2x)$

5/5 $\int_a^b \frac{1}{2}-\frac{1}{2}\cos(2x) \to \int_a^b \frac{1}{2} + \int_a^b -\frac{1}{2}\cos(2x)$ ✓ $2\in\mathbb{R}$ so RI

$\frac{1}{2}(a-b)-\frac{1}{2}\int_a^b \cos(2x)$ so $\int_a^b \sin^2(2x)$ is RI on $[a,b]$

1/5

7 Let P be t.p. of $[0,1]$ s.t. $\|P\|<\delta$.

Say $\int_0^1 f=\frac{1}{2}$ then $|S(f,P)-\frac{1}{2}|<\varepsilon$

$S(f,P)=\sum_{i=1} f(c_i)(x_i-x_{i-1})$ If $c_i\geq 1/2$, then $f(c_i)=0$ and

if $c_i<\frac{1}{2}$, then $f(c_i)=1$ so $S(f,P)=\sum_{i=1}^{N}(x_{i-1}-x_i)=\frac{1}{2}-0$

$=\frac{1}{2}$ so $|\frac{1}{2}-\frac{1}{2}|<\varepsilon$ $0<\varepsilon$ ×   You didn't use the def..

ε was arbitrary, f is RI on $[0,1]$.

3/5

8 Say $\int_0^1 f=0$ b.c. more $\mathbb{R}$ than $\mathbb{N}$. P be t.p. $[0,1]$ s.t.

$\|P\|<\delta$ then $|S(f,P)|<\varepsilon$ $S(f,P)=\sum_{i=1}^{N} f(c_i)(x_{i-1}-x_i)$

$E_\varepsilon=\{x=\frac{1}{N}:\frac{1}{N}\geq\frac{\varepsilon}{2}\}$ For $x\in E_\varepsilon$ then $N\leq\frac{2}{\varepsilon}$ there is a N_0 s.t. $N_0=\lfloor\frac{2}{\varepsilon}\rfloor$

[0,1) is infinite!

Since set $[0,1]$ is finite E_ε is finite $P_0\subseteq P$ s.t. $P_0=\{(c_i,[x_i-x_{i-1}]): f(c_i)\geq\frac{\varepsilon}{2}\}$?!

$P=P\setminus P_0\cup P_0$, P_0 is finite b.c. E_ε is. $S(f,P)=S(f,P\setminus P_0)+S(f,P_0)$

$S(f,P_0)=\sum_{k=1}^{N_0} f(c_{n_k})(x_{n_k}-x_{n_{k-1}})\leq\frac{\delta}{N}$ $S(f,P\setminus P_0)=\sum^{\text{card}(P)} f(c_i)(x_i-x_{i-1})<\frac{\varepsilon}{2}$

$\delta=\frac{N\varepsilon}{2}$ $|S(f,P)|=S(f,P_0)+S(f,P\setminus P_0)<\frac{\varepsilon}{2}+\frac{\delta}{N}=\frac{\varepsilon}{2}+\frac{N\varepsilon}{2N}=\varepsilon$ So $\int_0^1 f=0$

I don't understand... It's not clear...

4 Let P be a t.p. of $[0,1]$ s.t $\|P\| < \delta$. then say $\int_0^1 f = 0$. Then $|S(f,P) - 0| < \varepsilon \rightarrow |S(f,P)| < \varepsilon$

$S(f,P) = \sum_{i=1}^{N} f(c_i)(x_{i-1} - x_i)$ if $c_i > 0$, then $f(c_i) = 0$

so $\sum_{i=1}^{N} 4\,(0-0) = 0$ so $S(f,P) = 0$

$0 < \varepsilon$ ε wts arbitrary so f is RI on $[0,1]$

2/5

not true! It's only when $c_i = 0$ so, $4(x_1 - x_0)$.

10 $\|P\| = 0.2$ So $\|P_0\| \le \frac{0.2}{3}$ Have 18 partitions from -1 to 2 that are $\frac{0.2}{3}$ units long so that $\|P_0\| = \frac{0.2}{3}$.

4/5

Write it down explicitly... You have to use Cauchy Crit. this may not exist...

5 $c = b - \varepsilon$ $\|P_1\| < \delta$ $|S(f,P_1) - \int_a^c f| < \varepsilon$ P_1 t.p. of $[a,c]$

P_2 t.p. of $[a,b] = [a, c+\varepsilon]$ $S(f,P_2) = S(f,P_1) + f(c_b)(c+\varepsilon-c)$

$S(f,P_2) = S(f,P_1) + f(c_b)\,\varepsilon$

$|S(f,P_2) - \int_a^b f| < \varepsilon$ $|S(f,P_1) + \varepsilon f(c_b) - \int_a^b f| \le |S(f,P_1) - \int_a^b f| - \varepsilon f(c_b)$

$|S(f,P_1) - \int_a^b f| < \varepsilon(1 + f(c_b))$ X

Set ε in beginning to $\frac{\varepsilon}{1+f(c_b)}$

ε arbitrary so f is RI on $[a,b]$.

1/5