Pierre-Olivier Parisé

Fall 2021, 20/10/2021

Name of the members of the team: ______________________________

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Team name (if any): ______________________________

Question:	1	2	Total
Points:	10	10	20
Score:			

Instructions: You must answer all the questions in teams of 3 and hand out one copy per team. You are allowed to use the lecture notes only. No other tools such as a cell-phone, a calculator, or a laptop. Only your pen and eraser. The space between the questions are there to write the final versions of your answers.

Question 1 **(10 pts)**

Let $f : [0, 2] \to \mathbb{R}$ be the function defined by $f(x) = x^3 + 2x - 1$ and let $L = 0$. Find the interval $[a_3, b_3]$ constructed in the proof of the Intermediate Value Theorem. (Exceptionnally, you can use the calculator to do some of the calculations.)

Solution: Let $a = 0$ and $b = 1$. Put $d = (0+1)/2$. Then $f(1/2) = 1/8 + 1 - 1 = 1/8$ and so $1/8 > 0$. We put $a_1 = 0$ and $b_1 = 1/8$.

Let $d = (a_1 + b_1)/2 = 1/16$. Then $f(1/16) = -0.874755859375$. Since $f(1/16) < 0$, we put $a_2 = 1/16$ and $b_2 = b_1 = 1/8$.

Let $d = (a_2 + b_2)/2 = 3/32$. Then $f(3/32) = -0.618408203125$. Since $f(3/32) < 0$, then $a_3 = 3/32$ and $b_3 = b_2 = 1/8$.

So, the interval is $[3/32, 1/8]$.

Question 2 **(10 pts)**

Let $S, T \subseteq \mathbb{R}$ be two open sets. Show that $S \cap T$ is an open set. [Hint to start: Try to illustrate the situation and what you want to prove with a picture.]

Solution: Let $x \in S \cap T$. Then $x \in S$ and $x \in T$. Since S is open, there is a $\delta_1 > 0$ such that $(x - \delta_1, x + \delta_1) \subset S$. Similarly, there is a $\delta_2 > 0$ such that $(x - \delta_2, x + \delta_2) \subseteq T$. Let $\delta := \min\{\delta_1, \delta_2\}$. We will now prove that $(x - \delta, x + \delta) \subset S \cap T$. Let $y \in (x - \delta, x + \delta)$. This means that $x - \delta < y < x + \delta$. By the definition of δ, we have $\delta \leq \delta_1, \delta_2$. So, firstly, we get

$$x - \delta_1 \leq x - \delta < y < x + \delta \leq x + \delta_1$$

and since $(x - \delta_1, x + \delta_1) \subseteq S$, then $y \in S$. Secondly, we get

$$x - \delta_2 \leq x - \delta < y < x + \delta \leq x + \delta_2$$

and since $(x - \delta_2, x + \delta_2) \subseteq T$, then $y \in T$. Thus, we conclude that $y \in S$ and $y \in T$, and so $y \in S \cap T$. Since y was arbitrary, we have $(x - \delta, x + \delta) \subseteq S \cap T$.