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Real Analysis Hw #1

1.) $n \in \mathbb{N}$, $1+2+\dots+n = \frac{n(n+1)}{2}$

Base case: -2

$1+\dots+n+n+1 = \frac{n(n+1)}{2} + n+1$

$\frac{n(n+1)}{2} + \frac{2n+2}{2}$

$\frac{n^2+n+2n+2}{2}$ okay.

$\frac{n^2+3n+2}{2}$

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$\frac{(n+1)(n+2)}{2}$ which is the $n+1$ equation

By induction, $1+2+\dots+n = \frac{n(n+1)}{2}$ for all $n \in \mathbb{N}$

2.) $n=4 = 3+2+1 = 7$ $\quad 2^{4-1} = 8$

$f(n) \leq 2^{n-1}$

$f(n-1)+f(n-2)+f(n-3) \leq 2^{n-1}$

Lets say the expression holds for all $\mathbb{N}$ n, then $n+1$ would be equal to

$2(f(n-1)+f(n-2)+f(n-3)) \leq 2^n$ ← How did you get to this? You should explain more carefully.

$f(n)+f(n-1)+f(n-2)+f(n-3) \leq 2^n$

$f(n+1)+f(n-3) \leq 2^n$

$f(n-3) > 0 \quad f(n-3)+f(n+1) > f(n+1)$

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$f(n+1) \leq 2^n$

By strong induction, $f(n) \leq 2^{n-1}$ is true for all $n \in \mathbb{N}$

3.) A) $A \sim A$, there exists a bijection $x = x$ from $A \to A$.

Thus, $A \sim A$ okay.

B) $A \sim B$, so there exists a bijection $f(x): A \to B$.

$f^{-1}(x): B \to A$ thus exists because a bijection has an inverse, and an inverse is a bijection.

So $f^{-1}(x)$ is a bijection from $B \to A$, so $B \sim A$. okay.

this doesn't make sens in the context.

Here, a function is written $f: A \to B$, not $f(x): A \to B$.

3c) If $A \sim B$, there exists a bijection $f(x): A \to B$. If $B \sim C$, there exists a bijection $g(x): B \to C$. Let $h(x)$ be $g \circ f$ s.t. $h(x) = g(f(x))$ so that $h(x): A \to C$. Thus a bijection exists between A and C so $A \sim C$.

You have to mention why h is a bijection.

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4.) Let E be a countable set. There exists a bijection $f(x): E \to \mathbb{N}$. If E is finite, its subset will be finite, and thus countable. If E is infinitely countable, then a subset can be finite, or infinite. A finite subset is countable since it is finite. And an infinite subset, can be thought of as a subset A of E. Let B be the subset of everything in E, but not in A. $A + B = E$. Since E is countable, and the sum of two countable sets is countable while the sum of an uncountable set is uncountable, A and B must be countable. Great

$A \cup B$

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You need to justify a little more. Which Axiom did you use?

5A) $0 < a < b$ $\quad a < b$ $\quad a \cdot a < a \cdot b$ $\quad a^2 < ab$ $\quad a < b$ $\quad a \cdot b < b \cdot b$ $\quad ab < b^2$

$a^2 < ab < b^2$ $\quad a^2 < b^2$ 4/5

Which Axiom?

B) $0 < a < b$ so $\sqrt{a} < \sqrt{b}$, $\sqrt{a} = \sqrt{b}$, $\sqrt{a} > \sqrt{b}$. If $\sqrt{a} > \sqrt{b}$, then $\sqrt{a} \cdot \sqrt{a} > \sqrt{a} \cdot \sqrt{b}$ $\quad a > \sqrt{ab}$ $\quad \sqrt{a} > \sqrt{b}$ $\quad \sqrt{ab} > b$ $\quad a > \sqrt{ab} > b$ $\quad a > b$ #

If $\sqrt{a} = \sqrt{b}$ then $\sqrt{a} \cdot \sqrt{a} = \sqrt{a} \cdot \sqrt{b}$ $\quad a = \sqrt{ab}$ $\quad \sqrt{a} \cdot \sqrt{b} = \sqrt{b} \cdot \sqrt{b}$ $\quad \sqrt{ab} = b$

$a = \sqrt{ab} = b$ $\quad a = b$ #

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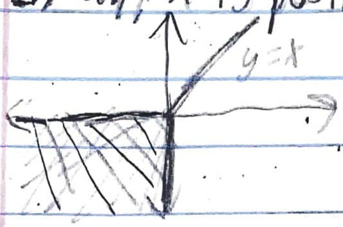
So $\sqrt{a} < \sqrt{b}$.

6.) In Q_1 (x,y) are positive so $x+x=y+y$ $2x=2y$ $x=y$
In Q_2 x is negative, y is positive so $-x+x=y+y$ $0=2y$ $y=0$
In Q_3, (x,y) are negative so $-x+x=-y+y$ $0=0$ so all (x,y)
In Q_4, x is positive, y is negative $x+x=-y+y$ $2x=0$ $x=0$



Great 5/5

How did you do that? Mention the properties.

7.) $x\geq 0$ $y\geq 0$ $x\geq 0$ $\sqrt{xy}\leq\frac{x+y}{\sqrt{2}}$ $\sqrt{2xy}\leq x+y$

why is it equal to 2xy?

$(\sqrt{2xy})^2\leq(x+y)^2$ $2xy\leq x^2+2xy+y^2$ $0\leq x^2+y^2$

$0\leq x$ $0\leq x^2$ $0\leq y$ $0\leq y^2$ $0+0\leq x^2+y^2$ $0\leq x^2+y^2$ ✓

$2xy\leq x^2+2xy+y^2$ $\sqrt{2xy}\leq x+y$ $\sqrt{xy}\leq\frac{x+y}{\sqrt{2}}$ 4/5

State all the Axioms and propurties that you use.

justify.

8.) a.) $E:=\{x\in\mathbb{R}: x\geq 0 \text{ and } x^2\leq 9\}$ $x\geq 0$ $x^2\leq 9$ $-3\leq x\leq 3$ $x\leq 3$
The set goes from $x=0$ to $x=\sqrt{3}$. ← no inf $E=0$
because x can't be <0 because $x\geq 0$ and sup $E=3$ 3/5
because the case of $x>3$, then $x^2>9$ $x\notin E$

this is the right justification. sup $E=3$ because $\forall x\in E$ $x\leq 3$ & $x=3$ for some $x\in E$.

b.) $E:=\{\frac{4n+5}{n+1}: n\in\mathbb{N}\}$ $n=1$ $\frac{4(1)+5}{1+1}=\frac{9}{2}$ $n\to\infty, E\to 4$.
sup $E=\frac{9}{2}$ because 1 is lowest value that can be input to get 9/2. To get a higher value, you would need to get a smaller number than 1, which is impossible because $n\in\mathbb{N}$

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8 6.) Say $\inf E = x$. $\boxed{4 \le \frac{4n+5}{n+1}} \le 9/2$ You didn't prove that yet.

We can have $x = 4$, $x < 4$, $x > 4$ minimum is 4

Suppose $x > 4$ $a > 4 - x$

$\frac{4n+5}{n+1} < x$ $4n+5 < xn + x$ $4n - xn < x - 5$ $n(4-x) < x - 5$

In AP, take $\underbrace{x = 4 - x}$ and $y = x - 5$, then $n(4-x) < x - 5$. #

No, x should be positive ...

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9.) A.) A power set can be represented as a binary number with a length of the elements in A. A set with 2 elements Power set can be represented with a 2 digit binary number. With A having n elements, the coorresponding n-digit binary number will have 2^n different values.

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If a the xth digit is 1 in the binary number, it means n_x is in the sub-set. If it is 0, n_x is not in the sub-set. Because the size of the power set is 2^n while A is n, they are not equivalent.

5/10 That's for A finite.

B.) Let all $\subseteq \mathbb{N}$ coorespond to an infinite binary number.

Are taking an element or a subset?

If 1 is in the subset, the first digit is a one, if two is not in the subset, let the second digit be zero. Let this occur for every $n \in \mathbb{N}$ coorresponding the the x_n digit. If we say it is countable and have a list of every binary representation, we can take the first digit of the first number on the list, the second digit of the second number, and so on, and switch ever 0 to 1 and every 1 to 0. This we every number on the list,

X It is not clear to me what you are doing.

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9.) and changes a digit. This means that it is a number not on the list, meaning the list is incomplete. This is a contradiction that we could have a list of all of these, and so, $P(\mathbb{N})$ is uncountable.

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10.) a.) suppose that $\sup(E)=X$. $rE = rx$ $x \in E$
$\sup(E) = X$ so $\sup(rE) = rX$ with $X = \sup(E)$, so rX.
$r\sup(E) \to r \cdot (X) = rX$ there is much more to prove.

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There is much more to prove here also.

b.) $\sup(E) = X$. Adding $r+E$ shifts the entire function r because $r+E := \{r+x : x \in E\}$. So the $\sup(r+E)$ is shifted r, and so if $X = \sup(E)$, then the function $r+E$ has $\sup(r+E)$ at $r+X$. And $r + \sup E = r + X$.

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