2 $\lim_{x\to\infty} f(x) = 0$ $\exists \varepsilon > 0, \exists M > 0$ $x > M$ $|f(x) - 0| < \varepsilon$

$\varepsilon = 1$ $|f(x)| < 1$ $f(x) > 0$ so $f(x) < 1$

Set bounds $[0, M]$. We have continuous bound function, so by EVT, we have c s.t. $f(c) = \max$. Let $C = \max\{f(c), 1\}$

C will be max of $f(x)$ from 0 to ∞.

1 Goal: $|y - x| = \delta \rightarrow |f(y) - f(x)| < \varepsilon$

$|f(y) - f(x)| \leq M|y - x|$; $\frac{|f(y) - f(x)|}{M} \leq |y - x| < \delta$

$\frac{|f(y) - f(x)|}{M} < \delta$ $|f(y) - f(x)| < M\delta$

$\varepsilon = M\delta$ $|y - x| < \delta \rightarrow |f(y) - f(x)| < \varepsilon$

3 $a \leq c \leq b$ so $f(a) \leq f(c) \leq f(b)$ $c = f(c)$

Set $d = \frac{a+b}{2}$. Find what $f(a), f(b), f(d)$ are.

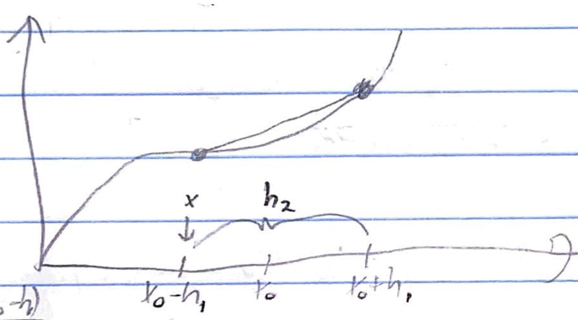
See what to set new bounds as a_1 and b_1.

Check to see if the value c is satisfied by any, if not, set new bounds of either a, b, d where the value of c and $f(c)$ can be found in between them. Set a_1 and b_1 and then $d_1 = \frac{a_1 + b_1}{2}$. Repeat until $c = f(c)$ is found.

HW #5

4 Using Rolle's Thm, if $g(x) = f'(x)$, then $g(c) = g(d)$, then $g'(x) = 0$ for $x \in (c, d)$. If $g(x) = f'(x)$, then $g'(x) = f''(x)$, so $\exists x \in (c,d)$ s.t $f'''(x) = 0$. Since $c < d$ and c and d are in $[a,b]$, $x \in (a,b)$.

5 a.) If f is dif. then $\lim_{x \to x_0} \frac{f(x_0) - f(x)}{x_0 - x}$ exists and $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ exists.



$$\frac{f(x_0 + h_1) - f(x_0 - h_1)}{x_0 + h_1 - (x_0 - h_1)}$$

$$f'(y) = \lim_{h_1 \to 0} \frac{f(x_0 + h_1) - f(x_0 - h_1)}{2h_1}$$ set $x_0 - h_1 = x$ and $h_2 = 2h_1$ w/ change of variables

$$f'(y) = \lim_{h_2 \to 0} \frac{f(x + h_2) - f(x)}{h_2}$$ as $h_1 \to 0$, $x \to x_0$, $h_2 \to 0$

6.) Function $|x|$ at $x_0 = 0$ $\lim_{h \to 0} \frac{|0+h| - |0-h|}{2h} = \lim_{h \to 0} \frac{h - h}{2h}$

$\lim_{h \to 0} \frac{1-1}{2} = \lim_{h \to 0} \frac{0}{2} = 0$ but not differentiable

6 a.) Let $(e^x)' = e^x$ and $(\ln x)' = \frac{1}{x}$

$x^r = e^{\ln(x^r)} = e^{r\ln(x)} = f(x)$

$f'(x)$ using chain rule has $e^{r\ln(x)} \cdot \frac{r}{x} = e^{\ln(x^r)} \cdot \frac{r}{x}$

$x^r \cdot \frac{r}{x} = rx^{r-1}$

b.) $f(x)$ can be written as $g(h(x))$ where g is $x^{\frac{1}{2}}$ and $h = x^2 + \sin x + \cos x$. Addition rules has $h'(x)$ be $(x^2)' + (\sin x)' + (\cos x)' = 2x + \cos x - \sin x$.

$g'(x) = \frac{1}{2\sqrt{x}}$. Using chain rule, $f'(x) = g'(h(x)) \cdot h'(x)$ which is $\frac{1}{2\sqrt{x^2 + \sin x + \cos x}} \cdot (2x + \cos x - \sin x)$

7 $\mathbb{R} \setminus S$ is open $x \in \mathbb{R} \setminus S$, $\exists \delta > 0$ s.t. $(x-\delta, x+\delta) \subseteq \mathbb{R}/S$

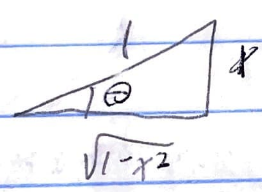
Say S is also open, then $x \in S$ $\exists \delta > 0$ s.t. $(x-\delta, x+\delta) \subseteq S$

If S and $\mathbb{R} \setminus S$ are both open, then the end points of S are not in S or $\mathbb{R} \setminus S$. This means there is a pt outside of both meaning $\mathbb{R} \setminus S$ isn't $\mathbb{R}$, which is a contradiction.

8 x^2 and x^3 are dif on $\mathbb{R}$ and so is $f(x)$.
$f(x^3)$ is a chain rule of $h(x)=x^3$ so $f \circ h$ is dif with $f'(x^3) = f'(x^3) \cdot 3x^2$
By product rules, x^2 and $f(x^3)$ are dif so
$g'(x) = (x^2)' f(x^3) + x^2 f'(x^3) = 2x f(x^3) + x^2 f'(x^3) \cdot 3x^2$
$g'(x) = 2x f(x^3) + 3x^4 f'(x^3)$

9 Let $y = \sin^{-1} x \to \sin y = x$ If we dif. both sides we get $\frac{dy}{dx} \cos y = 1$ plug y back in, we get

$\frac{dy}{dx} \cos(\arcsin(x)) = 1$



$\sin\theta = \frac{o}{h} = x$ $\theta = \sin^{-1}(x)$ $\cos\theta = \frac{a}{h} = \frac{\sqrt{1-x^2}}{1}$

$\frac{dy}{dx} \cos(\arcsin(x)) = \frac{dy}{dx} \sqrt{1-x^2} = 1$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$

10 a.) $ny^{n-1}(x-y) \le x^n - y^n \le nx^{n-1}(x-y)$ $f(t) = t^n$ $y \in [0,x]$
MVP → $ny^{n-1} = \frac{x^n}{\cancel{x}} \to ny^{n-1} = x^n$ $f(y) = y^n$ $f(x) = x^n$
ny^n is positive so $ny^{n-1}x - ny^n \le ny^{n-1}x$ $f'(y) = ny^{n-1}$ $f'(x) = nx^{n-1}$
So $ny^{n-1}(x-y) \le x^n - y^n$ MVP w/ x, $nx^{n-1} = \frac{x^n}{\cancel{x}}$ $x^n \ge y^n$ so $x^n - y^n \le nx^{n-1}(x-y)$

b.) $f(t) = \sqrt{1+t}$ $f'(t) = \frac{1}{2\sqrt{1+t}}$ $y \in (0,x)$ $f'(y) = \frac{f(x) - 0}{x - 0}$
$\frac{1}{2\sqrt{1+x}} = \frac{\sqrt{1+x} - 1}{x}$ $\frac{1}{\sqrt{1+x}} = 2\left(\frac{\sqrt{1+x} - 1}{x}\right)$ $1 > \frac{1}{\sqrt{1+x}}$ $x > 0$
$2\left(\frac{\sqrt{1+x} - 1}{x}\right) = \frac{1}{\sqrt{1+x}} < 1$ $2\left(\frac{\sqrt{1+x} - 1}{x}\right) < 1$ $\sqrt{1+x} - 1 < \frac{1}{2}x$ $\sqrt{1+x} < \frac{1}{2}x + 1$