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① Proof by induction: Let $n=1$ be our base

$$1 = \frac{(n+1)n}{2} = \frac{2(1)}{2} = \frac{2}{2} = 1 \checkmark \checkmark$$

Base case is true. Assume of expression

$X(n) = \frac{(n+1)n}{2}$ and $X(n)$ is true. Must prove $X(n+1)$ is true.

$$X(n+1) = \frac{(n+1)(n+1)(n+1)}{2}$$

be careful $(n+1)+1$ so that we don't confused with multiplication.

$$X(n+1) = \frac{(n+1)n}{2} + \frac{2}{2} = \frac{(n+1)(X(n+1)+1)}{2}$$

$$= \frac{(n+1)(n+2)}{2} = \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$$

$$= n(n+1)$$

not sure of what is going on here.

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There for by induction $n \in \mathbb{N} = \frac{n(n+1)}{2}$.

(2) Proof by induction :

Let $n=1$ be our base case
 $f(1) = 1$ and $2^{1-1} = 2^0 = 1 = 1 \checkmark$
 $1 \leq 1 \checkmark$

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Our base case is true. Let's see if it's true for $n \geq 1$

$$f(n+1) = f(n+1-1) + f(n+1-2) + f(n+1-3) \\ \leq 2^{n+1-1}$$

$$f(n) + f(n-1) + f(n-2) \leq 2^n$$

Let $n=4$ $f(4) + 3 + 2 \leq 2^4$

Since for all $n \geq 4$ the pattern 2^n will be at least 4, $\leq 2^{n-1} \therefore$

$f(n) \leq 2^{n-1}$ for all $n \in \mathbb{N}$ by induction.

not the way.

we have to use PM I 2.

③ Let A, B, C all be in a set of \sim .
 (A) $a \sim a$ because $f(a) = a$ which allows for $a = a$ or $A = A^T$ which is the reflexive property.
 You have to use the definition of \sim .
 $\exists f: A \rightarrow B$ s.t. f is bijection.
 what is A^T ?

(B) $A \sim B$ then $B \sim A$. This is the symmetric property where if $A \sim B$ then $B \sim A$ because $B = (B^+)^+ = (A^+)^+ = A^+$ and it implies $B \sim A$.

(C) This is the transitive property.
 If $A \sim B$ and $B \sim C$ then $A \sim C$.
 since $B = B^+$ then $A = B^+$ and $B = C^+$
 $A \sim C$. Therefore $A = B^+ = B = C^+$. Therefore

(4)

Proof :- Let S be a countable set
 T is a subset of S .

1- If T is finite then obviously it's countable

2- If T is an infinite subset of S

Since S is countable set, its elements can be arranged in any way such as x_1, x_2, x_3, \dots

Since T is an infinite subset of S , T contains infinite numbers and the set of elements of T form an infinite subset P

By well ordering property of \mathbb{N} , P contains a least element
(L.E) then for $a \in P \subseteq T$

Since even when T is an infinite subset of S , you will still have a L.E and a M.E which allows for the set to be countable.

$\therefore T$ is countable.

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See the answers to
the homework problems.

⑤

Let $0 < a < b$ be positive ^{0/5} real numbers
 a) since $a < b$ this would also be ^{example 2} true for $a(a) < b(b)$ or $a^2 < b^2$ because
 lets say $a = n$ and $b = n+1$ to satisfy
 $a < b$ $n^2 = n^2$ and $(n+1)^2 =$
 $n^2 + 2n + 1 > n^2$ resulting in $a^2 < b^2$.

b) additionally the same argument can be made
 for $\sqrt{a} < \sqrt{b}$ since $a < b$
 $a^{\frac{1}{2}} < b^{\frac{1}{2}}$ because say $a = n$ and $b = n+1$
 to satisfy $a < b$ ^{0/5}

$\sqrt{n} < \sqrt{n+1}$ for all n because

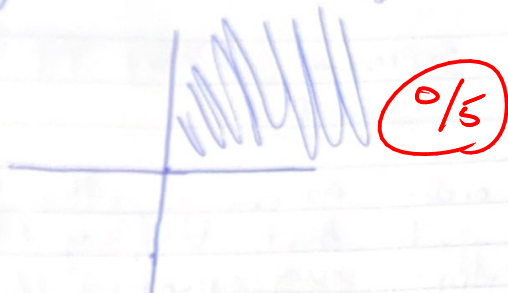
if $n=1$ $\sqrt{1} < \sqrt{2}$ or

$1 < \sqrt{2}$ which is true.

0

$$y = x + |x| - |y|$$

(b)



~~State $x + |x|$ will never be ≤ 0~~

$$x + |x| \geq 0 \text{ because if } x = -1$$

$-1 + 1 = 0 \geq 0$ ✓, the absolute function cancels out any negative answers and puts them to zero

same applies for $y + |y| \geq 0$ allowing for the function to be in Q2

① If $x \geq 0$ and $y \geq 0$ then $\sqrt{xy} \leq \frac{x+y}{\sqrt{2}}$
~~that is, $\sqrt{xy} \leq \frac{x+y}{\sqrt{2}}$~~

Let $x=3$ and $y=3$ be our base.

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$$\sqrt{9} \leq \frac{6}{\sqrt{2}}$$

this is an example.

$$3 \leq \frac{6}{\sqrt{2}} \quad \text{because } \sqrt{2} < 2 \quad \text{which means } 3 \leq \frac{6}{\sqrt{2}} \checkmark$$

~~Let's see if it's true for $x=a, y=b$~~

Let $x=a$ and $y=b$ where $a, b \geq 0$

$$\sqrt{ab} \leq \frac{a+b}{\sqrt{2}}$$

$$\sqrt{2} \leq \frac{a+b}{\sqrt{ab}}$$

since $\sqrt{2}$ is $1 < \sqrt{2} < 2$

and if a and b are 1 then

$$\frac{2}{\sqrt{1}} = \frac{2}{1} = 2 \quad \text{which is } \sqrt{2} \leq 2 \checkmark$$

1 is our least element in $x, y \geq 0$

if the least element is true then it's true for all $x, y \geq 0 \checkmark$

Exercises 8 & 9??
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⑩ a) Does supremum of set rE for $r > 0$
 $= \sup E$ times r ?

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Due to the order axioms 11, there exists a nonzero number r such that $\forall x, y \in \mathbb{R}, x \leq y \iff rx \leq ry$. If we multiply every number of the set E by r with x as our supren; rx is the greatest of every element in E . However, if x is the supren of set E and we multiply it by r with $0 < r < 1$, x will be \leq the lowest term in the set. Therefore $r \sup(E) \neq \sup(rE)$??

b) ?

Feedback.

Exercise 9. I suggest that you follow these lines:

- Suppose toward a contradiction that there is a bijection $f: A \rightarrow P(A)$.
- Use the hint: define $C := \{x : x \in A \text{ \& } x \notin f(x)\}$.
- Now use the fact that f is a bijection:
 $\exists x \in A$ s.t. $f(x) = C$.
- What can you say about $x \in f(x)$?

Exercise 10

I suggest that you use the definition!

Let $s := \sup E$.

a) Then try to prove that

1) $r+s$ is an upper bound for $r+E$, that is
 $\forall x \in E, \quad r+x \leq r+s \quad (r+E = \{r+x : x \in E\})$.

2) For any upper bound b of $r+E$, you have
 $r+s \leq b$.

(try to argue by contradiction).

b) try to follow the same line of reasoning as in

a). Try to prove

1) $r+x \leq r+s \quad \forall x \in E \quad (r+s \text{ u.b. for } r+E)$.

2) $r+s \leq b$ for any u.b. b of $r+E$.

(argue by contradiction).

General comments

- To help you prove something, try to look at your definition and explain what you want to prove in precise terms.
- Write down what you know related to the problem (Axioms, Theorems, Properties that can be used).