3/10

38/70.

a.) i.) Let n has $[a_n, b_n]$. Let m be $[a_m, b_m]$.
$n, m \geq 1$. Let $M = \max\{n, m\}$. If $a_n > b_m$, **± 1**
then the beginning of n would be after
2/5 the end of m, meaning $[a_n, b_n] \supseteq [a_m, b_m]$
is not true, so $a_n \leq b_m$. *disjoint in fact...*

ii.) $[a_1, b_1]$ has $a_1 \leq a_2 \leq a_3 \leq ... \leq a_n$. So a_1 is a
lower bound. So $\sup[a_n] \geq a_1$. But if $a_2 \geq \sup[a_n]$,
it isn't sup, so $\sup[a_n] \geq a_n$. And $a_n \leq b_m$ **X**
so $\sup[a_n] \leq b_m$. a_n is an upper bound. b_m **(-1)**
is also an upper bound. Since $a_n \leq b_m$, b_m is
the highest upper bound. Since a_n is the highest
a can go. So $\sup[a_n] \leq b_m$. *$a_n \leq b_m \ \forall n \geq 1$.*
By AC, sup exists.

iii.) If $c = \sup[a_n]$, $c \geq a_n$ for all $n \in \mathbb{N}$ → *why?* **(-1)**
because c is a supremum. And **X** $c \leq b_m$ for
$m \in \mathbb{N}$. This means it is between each $[a_n, b_n]$
pair because for every b_m, $a_n \leq b_m$ so c lies
in every interval. → *vague. I don't follow.*

B.) For any N, there is a corresponding $[a_n, b_n]$.
If this function starts at $n = 1 \rightarrow [a_1, b_1]$ being the
1/5 largest interval. Let us exclude r_1 from $n = 1$. Then
let us have on $n = 2$ s.t. $[a_2, b_2]$ exclude r_2. We continue
this to a_n excluding r_n numbers. a_n does not
include c which is a real number. The list exclude
all r_n so it not excluding c means it does not
exclude all r_n, thus making r_n uncountable
because $f: \mathbb{N} \rightarrow \mathbb{R}$ is not surjective going through all
$\mathbb{R}$.

See the solution.

RA HW #2

$a_n \to A$ then $|a_n - A| < \varepsilon$ $a_n \leq |a_n|$ $A \leq |A|$

1/5

so $|a_n - A| \leq ||a_n| - |A|| \leq \varepsilon$, $||a_n| - |A|| < \varepsilon$ ✗ you have to

ε is arbitrary here, so $|a_n| \to |A|$ use

$||a_n| - |A|| \leq |a_n - A|$.

3) If $x_n \to X$ $y_n \to Y$ $x_n \leq y_n$ $X \leq Y$

25/5

$a_n \to L$ $c_n \to C$ $a_n \leq c_n$ so $L \leq C$

$c_n \to C$ $b_n \to L$ $c_n \leq b_n$ so $C \leq L$

$C \leq L \leq C$. L cannot be less than L because it needs to be more than L and vise versa so C must be equal to L, so $c_n \to L$

You supposed that (c_n) converges.

4) 1.) $a_n \to 0$ $|a_n| < \varepsilon$ $a_n < \varepsilon$ let $a_n < \varepsilon^2$

4/5

$\sqrt{a_n} < \varepsilon$ $|a_n - 0| < \varepsilon$ epsilon arbitrary. ✓

2.) $|a_n - A| < \varepsilon$ let ε be $|A|/2$

$|a_n - A| \leq |-A|/2$ $|a_n| - |A| = \frac{|-A|}{2}$ $|a_n| < |A| + \frac{|-A|}{2} > \frac{|2A - A|}{2}$

$\frac{|A|}{2} \leq |a_n|$ $\sqrt{\frac{|A|}{2}} \leq \sqrt{|a_n|}$ ①

Wrong inequalities.

3.) For $n \geq N_2$, $|a_n - A| < \frac{3}{4} \frac{\varepsilon}{\sqrt{A}}$

4.) $N = \max\{N_1, N_2\}$. $|a_n - A| < \frac{3}{4}\frac{\varepsilon}{\sqrt{A}}$ $\sqrt{a_n} + \sqrt{A} > \sqrt{\frac{A}{2}} + \sqrt{A}$

$\sqrt{a_n} + \sqrt{A} > \sqrt{\frac{3A}{2}}$ $\frac{1}{\sqrt{a_n} + \sqrt{A}} > \sqrt{\frac{2}{3A}}$ $||a_n - A|| \leq \varepsilon$

$\frac{|a_n - A|}{\sqrt{a_n} + \sqrt{A}} < \frac{1}{\sqrt{a_n} + \sqrt{A}} \cdot \varepsilon = \sqrt{\frac{2}{3A}} \cdot \varepsilon$

$|\sqrt{a_n} - \sqrt{A}| < \sqrt{\frac{2}{3A}} \cdot \varepsilon$ ✓   If ε is $\varepsilon \cdot \sqrt{\frac{3A}{2}}$, then we just have $\acute{\varepsilon}$

and $|\sqrt{a_n} - A| \leq \varepsilon$ and ε is arbitrary so

$\sqrt{a_n} \to \sqrt{A}$

#8 and #5

AA

8 $a_n = (-1)^n$ $|a_n - A| < \varepsilon$ diverges $|a_n - A| > \varepsilon$

(4/10) $|(-1)^n - A| \leq \varepsilon$ $\varepsilon \geq |(-1)^n| - |A| \geq |(-1)^n - A|$

$| - |A| \leq \varepsilon$ $| < \varepsilon + |A|$ Take $\varepsilon = -|A|$ ← negative number! ε must be positive **(−2)**

$| < -|A| + |A|$ $| < 0$ #

so $|(-1)^n - A| > -|A|$

b) $\sin\left(\frac{2n+1}{2}\pi\right)_{n=1}^{\infty}$ $-\varepsilon < |\sin\left(\frac{2n+1}{2}\pi\right) - A| \leq |\sin\left(\frac{2n+1}{2}\pi\right)| - |A|$

$\varepsilon + |A| \leq |\sin\left(\frac{2n+1}{2}\pi\right)|$

Let $\varepsilon = -2 - |A|$ → negative $(\varepsilon > 0)$ **(−2)**

$-2 - |A| + |A| < |\sin\left(\frac{2n+1}{2}\pi\right)|$

$-2 < |\sin\left(\frac{2n+1}{2}\pi\right)|$ $\sin\left(\frac{2n+1}{2}\pi\right)$ bounded above by

So an $\varepsilon < |a_n - A|$ exists 1 and below by -1.

for $\forall N \in \mathbb{N}$, so $\sin\left(\frac{2n+1}{2}\pi\right)$ diverges. **(−2)** not correct

5 A divergent sequence where σ_n converges would

(1/5) be $((-1)^n)_{n=1}^{\infty}$ ✓

$|a_n - A| < \varepsilon$ $\varepsilon > 0$ → $\exists N \in \mathbb{N}$. ($n = 1$) $|a_1 - A| < \varepsilon$ all of

$|a_n - A| < \varepsilon$ so $|a_1 - A| < \varepsilon, |a_2 - A| < \varepsilon, \ldots |a_n - A| < \varepsilon$

so $|a_1 - A| + |a_2 - A| + |a_3 - A| \ldots |a_n - A| < \varepsilon + \varepsilon + \varepsilon \ldots \varepsilon = n\varepsilon$

$\left|\frac{a_1 + a_2 + a_3 \ldots + a_n - nA}{n}\right| < \varepsilon \leq |\sigma_n - A| < \varepsilon$

You have separate the sum $\frac{|a_1 - A| + \ldots + |a_n - A|}{n}$ in two parts: from 1 to N−1 (bounded) & N to n ($< \varepsilon$)

ε is arbitrary, so $\sigma_n \to A$

Real Analysis HW #2

6) a.) $a_n = 5 + \frac{1}{n}$, $n \geq 1$, $n \to \infty$, $a_n \to 5$

10/10

Let $\varepsilon > 0$. $|a_n - A| < \varepsilon$, $|5 + \frac{1}{n} - 5| < \varepsilon$, $|\frac{1}{n}| < \varepsilon$

Let $N \in \mathbb{N}$. Let $n \geq N$. $|\frac{1}{n}| < \varepsilon$, $\frac{1}{n} < \varepsilon$, $1 < n\varepsilon$

$\therefore 1 \leq N\varepsilon \leq n\varepsilon$. In A.P. Let $y = 1$ and $x = \varepsilon$.

$\exists N_0 \in \mathbb{N}$ s.t. $N_0 \varepsilon > 1$. Take $N = N_0$. If $n \geq N_0$, then

$n\varepsilon \geq N_0 \varepsilon > 1$, so $n\varepsilon > 1$ ($\forall n \geq N_0$) such that

$n \geq N_0 \to \frac{1}{n} < \varepsilon \to |5 + \frac{1}{n} - 5| < \varepsilon$. ✓

Since ε was arbitrary, $5 + \frac{1}{n} \to 5$.

6.) b.) $a_n = \frac{3n}{2n+1}$, $n \to \infty$, $a_n \to \frac{3}{2}$. Let $\varepsilon > 0$

$|\frac{3n}{2n+1} - \frac{3}{2}| < \varepsilon$, $|\frac{6n}{4n+2} - \frac{6n+3}{4n+2}| < \varepsilon$, $|\frac{3}{4n+2}| < \varepsilon$

$3 < \varepsilon(4n+2)$, $3 - 2\varepsilon < 4\varepsilon n$. Let $n \geq N$.

In A.P. Take $y = 3 - 2\varepsilon$ and $x = 4\varepsilon$

$\exists N_0 \in \mathbb{N}$ such that $4\varepsilon N_0 > 3 - 2\varepsilon$

Take $N = N_0$. If $n \geq N_0$, then $4\varepsilon n > 4\varepsilon N_0 > 3 - 2\varepsilon$

so $4\varepsilon n > 3 - 2\varepsilon$ ($\forall n \geq N_0$) such that $n \geq N_0$ then

$4\varepsilon n > 3 - 2\varepsilon \to 3 < \varepsilon(4n+2) \to \frac{3}{4n+2} < \varepsilon \to |\frac{3}{4n+2}| < \varepsilon$ ✓

$|\frac{6n}{4n+2} - \frac{6n+3}{4n+2}| < \varepsilon \to |\frac{3n}{2n+1} - \frac{3}{2}| < \varepsilon$

Since ε was arbitrary, $\frac{3n}{2n+1} \to \frac{3}{2}$

7) $\frac{(2n+1)}{n} = a_n$, $|a_n - a_m| < \varepsilon$, $\varepsilon > 0$, $\varepsilon/2 > 0$

2/5

$\exists N \in \mathbb{N}$ If $n, m \geq N$. Suppose $\frac{2n+1}{n} \to 2$. ← *You have to prove it.* ↗

$|a_n - a_m - 2 + 2| \to |a_n - 2 - a_m + 2| \leq |a_n - 2| + |a_m - 2|$

$n > N \to |a_n - 2| < \varepsilon/2$, $m > N \to |a_m - 2| < \varepsilon/2$

$|a_n - 2| + |a_m - 2| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$, $|a_n - 2| + |a_m - 2| < \varepsilon$

ε is arbitrary, Therefore $\frac{2n+1}{n}$ is Cauchy. ← *You have to use the definit...*

9 $a_n = (-1)^n$ this diverges because it alternates between 1 and 1 $\forall n \in \mathbb{N}$.

$b_n = (-1)^{n+1}$ this diverges because it alternates between 1 and -1 $\forall n \in \mathbb{N}$.

↳ example seen in the class.

$(a_n + b_n)$ converges because $(-1)^n + (-1)^{n+1} = 0$ $\forall n \in \mathbb{N}$ so it converges to 0.

a) $\dfrac{n^2 + 4_n}{n^2 - 5} = \dfrac{n^2(1 + 4/n)}{n^2(1 - 5/n^2)} = \dfrac{1 + 4/n}{1 - 5/n^2}$ $n \to \infty$ $\dfrac{1+0}{1-0} = \dfrac{1}{1} = 1$ ✓

b) $\dfrac{n}{n^2 - 3} = \dfrac{n}{n^2(1 - 3/n^2)} = \dfrac{(1/n)}{(1 - 3/n^2)}$ $n \to \infty = \dfrac{0}{1-0} = 0$

c) $\dfrac{\cos n}{n} = \dfrac{1}{n} \cos n$ $n \to \infty$ $\dfrac{1}{n} \to 0$ $0 \cdot \cos n = 0$.   → justify with a thm.

d) $\left(\sqrt{4 - \tfrac{1}{n}} - 2\right) n$ $n \to \infty$ $\left(\sqrt{4 - 0} - 2\right)\infty = (2-2)\infty = 0 \cdot \infty$

$0 \cdot \infty = 0$ ✗ 0.5