

**Last name:** \_\_\_\_\_

**First name:** \_\_\_\_\_

Question:	1	2	3	4	Total
Points:	10	15	15	10	50
Score:					

**Instructions:** Make sure to write your complete name on your copy. You must answer all the questions below and write your answers directly on the questionnaire. At the end of the 50 minutes, hand out your copy.

No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam. You are not allowed to use the lecture notes and the textbook also.

Make sure to show all your work. State clearly any theorem or definition you are using in your proofs or your calculations.

Good luck!

Pierre-Olivier Parisé

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QUESTION 1

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(10 pts)

Define the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) := \begin{cases} x + 1 & \text{if } x \in \mathbb{Q} \\ -x + 1 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Show that the limit of  $f$  at  $x = -1$  doesn't exist.

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**QUESTION 2****(15 pts)**

From the Intermediate Value Theorem, we know that there is at least one real number  $x$  such that  $\cos x = 2x$ . Why is there no more than one solution? Justify.<sup>1</sup>

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<sup>1</sup>State the Theorem you use and verify all the hypothesis of the Theorem before using it.

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QUESTION 3

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(15 pts)

Let  $f(x) = x^3 + 3x + 1$ .

- (a) (5 points) Show that  $f$  is strictly increasing on  $\mathbb{R}$ .
- (b) (10 points) Let  $g$  be the inverse of  $f$ . Find  $g'(5)$ .

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QUESTION 4

(10 pts)

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be two differentiable functions on  $\mathbb{R}$ . Suppose that  $f'(x) = g(x)$  for any  $x \in \mathbb{R}$  and  $g'(x) = -f(x)$  for any  $x \in \mathbb{R}$ . Prove that  $f^2 + g^2$  is constant on  $\mathbb{R}$ .