S.1 Terminology

PROBLEM 1. Let $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2\}$, and $B = \{1, 2, 3\}$. Show that $A \subset B$.

PROBLEM 2. Let $U = \{1, 2, 3, 4\}$. Show that U has 2^4 subsets. Bonus: In general, show that if U has a finite number of elements, then U has $2^{(\#U)}$ subsets.

PROBLEM 3. Show that if A is a set, then $\varnothing \subset A$.

PROBLEM 4. Let A, B, and C be subsets of a universal set U. Show that if $A \subset B$ and $B \subset C$, then $A \subset C$.

S.2 Operations With Sets

PROBLEM 5. Let A and B be two subsets of a universal set U.

a) Show that $A \cap B \subset A$.

b) Show that $A \subset A \cup B$.

c) Show that if $A \subset B$, then $A \cup B = B$.

d) Show that if $A \subset B$, then $A \cap B = A$.

PROBLEM 6. Let A and B be two subsets of a universal set U.

a) Show that $A \cup \overline{A} = U$.

b) Show that if $A \subset B$, then $\overline{B} \subset \overline{A}$.

PROBLEM 7. Let $A = \{n : n \text{ is an odd integer}\}$ and let $B = \{n : n \text{ is an even integer}\}$. Show that $A \cap B = \varnothing$. *[Hint: To prove $A \cap B \subset \varnothing$, use the method of proof by contradiction.]*

S.3 Important Laws For Set Algebra

PROBLEM 8. Let A, B, and C be subsets of a universal set U.

a) Prove that $A \cap B = B \cap A$.

b) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

c) $\overline{A \cup B} = \overline{A} \cap \overline{B}$.