G.1 Mean-Square Law of Large Numbers

Problem 1. Let $X_1, X_2, \ldots$ be a list of random variables which converges to the random variable X in mean square. Show that, for any $a, b \in \mathbb{R}$, $aX_n + b \to aX + b$, as $n \to \infty$.

Problem 2. Let N_m be the number of occurences of 5 or 6 in m throws of a fair die. Show that

$$\frac{1}{m} N_m \to \frac{1}{3} \quad \text{in mean square}$$

as $m \to \infty$.

G.2 Central Limit Theorem

Problem 3. The fracture strength of tempered glass averages 14 (measured in thousands of pounds per square inch) and has standard deviation 2.

a) What is the probability that the average fracture strength of 100 randomly selected pieces of this glass exceeds 14.5?

b) Find an interval that includes, with probability 0.95, the average fracture strength of 100 randomly selected pieces of this glass.