Math 307

Chapter 6

Section 6.2: Homogeneous Systems With Constant Coefficients
The Diagonalizable Case

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Real Eigenvalues

Example 1. Determine the general solution to

$$Y' = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} Y.$$

Fact: Suppose A and B are $n \times n$ matrices with $B = P^{-1}AP$ for some invertible $n \times n$ matrix P. Then

- If Z is a solution to $Y' = BY$, then PZ is a solution to $Y' = AY$.
- If $Z_1, Z_2, \ldots, Z_n$ is a fundamental set of solutions of $Y' = BY$, then $PZ_1, PZ_2, \ldots, PZ_n$ is a fundamental set of solutions to $Y' = AY$.

EXAMPLE 2. Solve the initial value problem

$$Y' = \begin{bmatrix} 2 & -3 & -3 \\ 2 & -2 & -2 \\ -2 & 1 & 1 \end{bmatrix} Y, \quad Y(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Complex Exponential Function

For a complex number $z = a + ib$, we define

$$e^z = e^{a+ib} = e^a \cos(b) + ie^a \sin(b).$$

The solution to the differential equation $y' = (a + ib)y$ is

$$y(x) = e^{(a+ib)x}.$$

Finding solutions with complex numbers

EXAMPLE 3. Find the general solution to

$$Y' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} Y.$$

Fact: If $U(x) + iV(x)$ is a solution to $Y' = AY$, then $U(x)$ and $V(x)$ are solutions to $Y' = AY$.

EXAMPLE 4. Find the general solution to

$$Y' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix} Y.$$