## Section 16.1, Problem 30

The function is $f(x) = x^2 + xy$. So its gradient is $\vec{\nabla} f(x, y) = (2x + y)\vec{i} + x\vec{j}$. The $x$-coordinate of the vector field is zero if $y = -2x$. In this case, the vector field looks like

$$\vec{\nabla} f(x, y) = -(y/2)\vec{j}.$$

Also, when $y > -2x$, then the $x$ coordinate of the vector field is positive and all the vectors in the vector field must point to the east (to the right, in the same direction to the positive $x$-axis). When $y < -2x$, then the $x$ coordinate of the vector field is negative and all the vectors in the vector field must point to the west (to the left, in the opposite direction to the positive $x$-axis).

So the corresponding representation is IV.

**Section 16.1, Problem 34**

The vector at $(x, y) = (1, 3)$ is $\vec{F}(1, 3) = \langle 1, -1 \rangle$. So, the new position of the particle would be

$$\langle x_1, y_1 \rangle = (1, 3) + \Delta t \vec{F}(1, 3) = \langle 1, 2 \rangle + 0.05 \langle 1, -1 \rangle = \langle 1.05, 1.95 \rangle .$$

## Section 16.2, Problem 2

We have $x'(t) = 3t^2$ and $y'(t) = 4t^3$. So, the line integral becomes

$$\int_C (x/y)\, ds = \int_1^2 (t^3/t^4)\sqrt{9t^4 + 16t^6}\, dt = 3\int_1^2 t\sqrt{1 + (4t/3)^2}\, dt.$$

By letting $u = 1 + (4t/3)^2$, we get

$$\int_C (x/y)\, ds = \frac{1}{48}(73\sqrt{73} - 125) \approx 10.390.$$

**Section 16.2, Problem 8**

We parametrize the circle $C_1$ described by $x^2 + y^2 = 2$ with $x = 2\cos(t)$ and $y = 2\sin(t)$. Since we only need the part of the circle going from $(2, 0)$ to $(0, 2)$, the parameter lies in $0 \leq t \leq \pi/2$ (a quarter of a circle).

We have $x'(t) = -2\sin(t)$ and $y'(t) = 2\cos(t)$. So, the contour integral is

$$\int_{C_1} x^2 dx + y^2 dy = \int_0^{\pi/2} (-8)\cos^2(t)\sin(t)\, dt + 8\int_0^{\pi/2} \sin^2(t)\cos(t)\, dt = -8\int_0^1 t^2\, dt + 8\int_0^1 t^2\, dt.$$

So we obtain

$$\int_{C_1} x^2 dx + y^2 dy = 0.$$

We parametrized the line segment $C_2$ by $x = 4t$ and $y = 2 + t$ where $0 \leq t \leq 1$. So $x'(t) = 4$ and $y'(t) = 1$. The contour integral is then

$$\int_{C_2} x^2\, dx + y^2\, dy = \int_0^1 64t^2\, dt + \int_0^1 (2+t)^2\, dt = \frac{64}{3} + \frac{8}{3} = 24.$$

If $C = C_1 \cup C_2$, then from the properties of the line integral, we obtain

$$\int_C x^2 dx + y^2 dy = \int_{C_1} x^2 dx + y^2 dy + \int_{C_2} x^2\, dx + y^2\, dy = 24$$

## $\boxed{\textbf{Section 16.2, Problem 22}}$

We have $\vec{r}\,'(t) = (-\sin t)\vec{i} + \cos(t)\vec{j} + \vec{k}$. So,

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,' = -\cos t \sin t + \cos t \sin t + \cos t \sin t = \cos t \sin t.$$

Thus, we obtain

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi} \cos t \sin t \, dt = \int_0^{\pi} (1/2)\sin(2t)\, dt = 0.$$