tangent

$y = mx + b$

$$\boxed{m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

Chapter 2

Derivatives

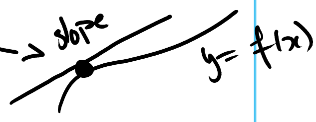
2.1 Derivatives and Rates of Change

The Derivative.

4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

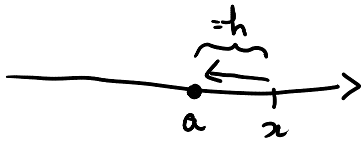
if this limit exists.



Another notation:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

let $h = x - a$



EXAMPLE 4 Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number $a = 1$

By def.:

$$f'(1) = \lim_{h \to 0} \frac{f(\overset{1}{a}+h) - f(\overset{1}{a})}{h}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \to \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 8(1+h) + 9 - 2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} + 2h + h^2 \cancel{-8} - 8h + \cancel{7}}{h}$$

$$= \lim_{h \to 0} \frac{-6h + h^2}{h}$$

$$= \lim_{h \to 0} (-6 + h)$$

$$= -6 + 0 = \boxed{-6}$$

Example. Find the derivative at a = 3 of the function

$$f(x) = \frac{3}{x}.$$

From the def.:

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - \frac{3}{3}}{h} \rightarrow \frac{0}{0}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - \frac{3+h}{3+h}}{h}$$

$$= \lim_{h \to 0} \left(\frac{3-(3+h)}{3+h}\right) \Big/ h$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{(3+h)\cancel{h}}$$

$$= \lim_{h \to 0} -\frac{1}{3+h}$$

$$= \frac{-1}{3+0} = \boxed{\frac{-1}{3}}$$

Tangents.

How do we find the tangent at a point P on a curve given by the graph of a function?

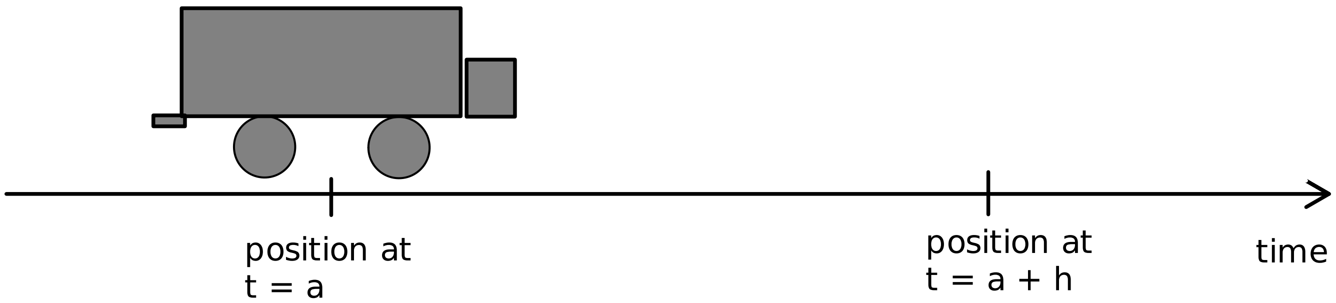
Answer:

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

$$\boxed{y - f(a) = f'(a)(x - a)}$$

value at a slope point compute derivative.

Velocities



- Position at t: $s(t) = t^2$

- Average velocity from t = a to t = a + h: $\dfrac{s(a+h) - s(a)}{h}$

- Instantaneous Velocity at t = a:

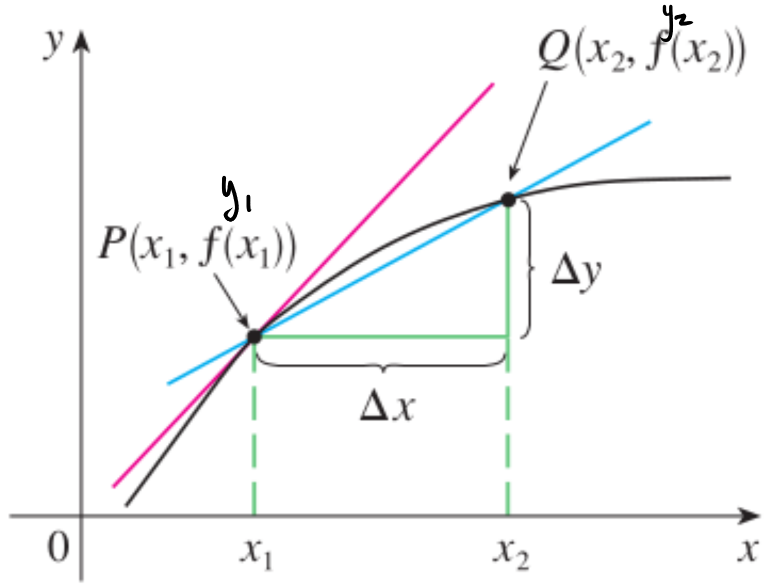
$$\boxed{v(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}} \rightarrow s'(a)$$

Rates of Change.

- Average rate of change when y varies by Δy and x varies by Δx :

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x}$$

- Take limit as $\Delta x \to 0$



$$\boxed{6} \quad \text{instantaneous rate of change} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(\overset{x}{x_2}) - f(\overset{a}{x_1})}{\underset{x}{x_2} - \underset{a}{x_1}} = f'(x_1)$$