Math 302

Chapter 5

Section 5.5: The Method of Undetermined Coefficient II

Contents

Created by: Pierre-Olivier Parisé

Fall 2022

We consider the following first basic case:

$$ay'' + by' + cy = F\cos\omega x + G\sin\omega x$$

where F, G and α are fixed real numbers.

Case I

When $\cos\omega x$ and $\sin\omega x$ are not solution to the complementary equation $ay'' + by' + cy = 0$.

Example 1. Find the general solution to

$$y'' - 2y' + y = 5\cos 2x + 10\sin 2x.$$

Case II

When $\cos \omega x$ or $\sin \omega x$ are solutions to the complementary equation.

Example 2. Find the general solution to

$$y'' + 4y = 8\cos 2x + 12\sin 2x.$$

We consider the following second basic case:

$$ay'' + by' + cy = F(x)\cos\omega x + G(x)\sin\omega x$$

where ω is a fixed real number and F, G are two polynomials.

There are still two cases: weither $\cos\omega x$ and $\sin\omega x$ are or are not solutions to the complementary equation.

Example 3. Find the general solution to

$$y'' + 3y' + 2y = (16 + 20x)\cos x + 10\sin x.$$

We now consider the more general case

$$ay'' + by' + c = e^{\alpha x}\left(F(x)\cos\omega x + G(x)\sin\omega x\right)$$

where α, ω are real numbers with $\omega \neq 0$ and F, G are polynomials.

There are also two cases: weither $\cos\omega x$ and $\sin\omega x$ are or are not solutions to the complementary equation.

Example 4. Find the general solution of

$$y'' + 2y' + 5y = e^{-x}\left((6 - 16x)\cos 2x - (8 + 8x)\sin 2x\right).$$

Recap

A particular solution of

$$ay'' + by' + cy = e^{\alpha x}\left(F(x)\cos\omega x + G(x)\sin\omega x\right)$$

where $\omega \neq 0$ has the form

- when $e^{\alpha x} cos \omega x$ and $e^{\alpha x}\sin\omega x$ are not solutions to the complementary equation,

  $$y_{par}(x) = e^{\alpha x}\left(A(x)\cos\omega x + B(x)\sin\omega x\right),$$

  with $A(x)$ and $B(x)$ polynomials of the same degree as the biggest degree between $F(x)$ and $G(x)$

- When $e^{\alpha x}\cos\omega x$ and $e^{\alpha x}\sin\omega x$ are solutions to the complementary equation,

  $$y_{par}(x) = xe^{\alpha x}\left(A(x)\cos\omega x + B(x)\sin\omega x\right),$$

  with $A(x)$ and $B(x)$ are polynomials of the same degree as the highest degree between the polynomials $F(x)$ and $G(x)$.