Section 2.2 — Problem 20 — 5 points

Using the definition, we have

$$\begin{aligned} f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{m(x+h) + b - (mx+b)}{h} \\ &= \lim_{h\to 0} \frac{mx + mh + b - mx - b}{h} \\ &= \lim_{h\to 0} \frac{mh}{h} = m. \end{aligned}$$

Therefore, $f'(x)$ exists for any x and $f'(x) = m$.

Section 2.2 — Problem 28 — 5 points

The domain of the function is $[0, \infty)$. Using the definition, we have

$$\begin{aligned}
f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0} \frac{(x+h)^{3/2}-x^{3/2}}{h} \\
&= \lim_{h\to 0} \left(\frac{(x+h)^{3/2}-x^{3/2}}{h}\right)\left(\frac{(x+h)^{3/2}+x^{3/2}}{(x+h)^{3/2}+x^{3/2}}\right) \\
&= \lim_{h\to 0} \frac{(x+h)^3-x^3}{h((x+h)^{3/2}+x^{3/2})} \\
&= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h((x+h)^{3/2}+x^{3/2})} \\
&= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3}{h((x+h)^{3/2}+x^{3/2})} \\
&= \lim_{h\to 0} \frac{3x^2+3xh+h^2}{(x+h)^{3/2}+x^{3/2}} \\
&= \frac{3x^2}{2x^{3/2}} \\
&= \frac{3}{2}x^{1/2}.
\end{aligned}$$

Therefore, we get $f'(x) = (3/2)x^{1/2}$. The domain of the derivative is $[0, \infty)$.

Section 2.2 — Problem 40 — 5 points

The function is not differentiable at $x = -1$ because f is not continuous.

The function is not differentiable at $x = 2$ because there is a corner in the graph of f (the limit slope from the left and from the right are not the same).

Section 2.3 — Problem 4 — 5 points

Using the fact that the derivative of a sum is the sum of the derivatives and the power rule, we have

$$\begin{aligned} g'(x) = \frac{dg}{dx} &= \frac{d}{dx}\left(\frac{7}{4}x^2\right) - 3\frac{d}{dx}(x) + \frac{d}{dx}(12) \\ &= (7/4)2x - 3(1) + 0 \\ &= (7/2)x - 3. \end{aligned}$$

Therefore, $g'(x) = (7/2)x - 3$.

Section 2.3 — Problem 18 — 5 points

We first simplify the expression:

$$\frac{\sqrt{x}+x}{x^2} = \frac{1}{x^{3/2}} + \frac{1}{x} = x^{-3/2} + x^{-1}.$$

Therefore, we have

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^{-3/2}) + \frac{d}{dx}(x^{-1}) \\ &= (3/2)x^{-5/2} - x^{-2}.\end{aligned}$$

So, we obtain $y' = \frac{(3/2)}{x^{5/2}} - \frac{1}{x^2}$.

Section 2.3 — Problem 26 — 5 points

Using the product rule, we have

$$B'(u) = (2u^2 - 4u - 2)\frac{d}{du}(u^3 + 1) + (u^3 + 1)\frac{d}{du}(2u^2 - 4u - 2).$$

Then, using the sum rule and the power rule for derivatives, we get

$$\frac{d}{du}(u^3 + 1) = 3u^2$$

and

$$\frac{d}{du}(2u^2 - 4u - 2) = 4u - 4.$$

Plugging in back in $B'(u)$, we get

$$\begin{aligned} B'(u) = (2u^2 - 4u - 2)3u^2 + (u^3 + 1)(4u - 4) &= 6u^4 - 12u^3 - 6u^2 + 4u^4 - 4u^3 + 4u - 4 \\ &= 10u^4 - 16u^3 - 6u^2 + 4u - 4. \end{aligned}$$

Therefore, $B'(u) = 10u^4 - 16u^3 - 6u^2 + 4u - 4$.

Section 2.3 — Problem 30 — 5 points

Using the quotient rule, we get

$$\begin{aligned} h'(t) &= \frac{(6t-1)\frac{d}{dt}(6t+1) - (6t+1)\frac{d}{dt}(6t-1)}{(6t-1)^2} \\ &= \frac{(6t-1)6 - (6t+1)6}{(6t-1)^2} \\ &= -\frac{36}{(6t-1)^2}. \end{aligned}$$

Section 2.3 — Problem 66 — 5 points

Using the power rule for derivatives, we see that

$$S'(A) = (0.882)(0.842)A^{-0.158} = (0.742644)A^{-0.158}.$$

Using the formula for $S'(A)$, we find that

$$S'(100) = (0.882)(0.842)(100)^{-0.158} \approx 0.35874 \text{ trees/m}^2$$

where m^2 means square meters.

Section 2.3 — Problem 58 — 10 points

The equation of the tangent line at $(4, 0.4)$ is

$$y - 0.4 = f'(0.4)(x - 4).$$

We have $f(x) = \sqrt{x}/(x+1)$. Using the quotient rule, we get

$$f'(x) = \frac{\frac{x+1}{2\sqrt{x}} - \sqrt{x}}{(x+1)^2} = \frac{1-x}{2\sqrt{x}(x+1)^2}.$$

Therefore, we have $f'(4) = -0.03$. Plugging this into the equation of the tangent line and after simplifying, we get

$$y = 0.52 - 0.03x.$$

Total (Points): 50.