

**Section 15.2, Problem 6**

We first have that

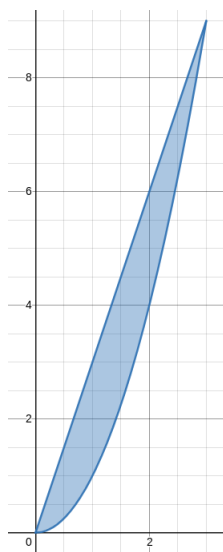
$$\int_0^{e^v} \sqrt{1+e^v} dw = \sqrt{1+e^v} (w)|_0^{e^v} = e^v \sqrt{1+e^v}.$$

So

$$\int_0^1 \int_0^{e^v} \sqrt{1+e^v} dw dv = \int_0^1 e^v \sqrt{1+e^v} dv = \frac{2}{3}((1+e)^{3/2} - 2\sqrt{2}) \approx 2.894.$$

**Section 15.2, Problem 14**

The first thing to do is to draw the region  $D$ .



We see that the curves  $y = x^2$  and  $y = 3x$  intersect at the points  $(0,0)$  and  $(3,9)$ .

**Type I** We have  $0 \leq x \leq 3$  and  $x^2 \leq y \leq 3x$ . So the functions bounding the values of  $y$  are  $x^2$  and  $3x$ . As a type I, the domain is written as

$$D = \{(x, y) : 0 \leq x \leq 3, x^2 \leq y \leq 3x\}.$$

**Type II** We see that  $0 \leq y \leq 9$  and since  $x \geq 0$ , the curves bounding the values of  $x$  are  $x = y/3$  and  $x = \sqrt{y}$ . As a type II, the domain is written as

$$D = \{(x, y) : y/3 \leq x \leq \sqrt{y}, 0 \leq y \leq 9\}.$$

Now the integral is

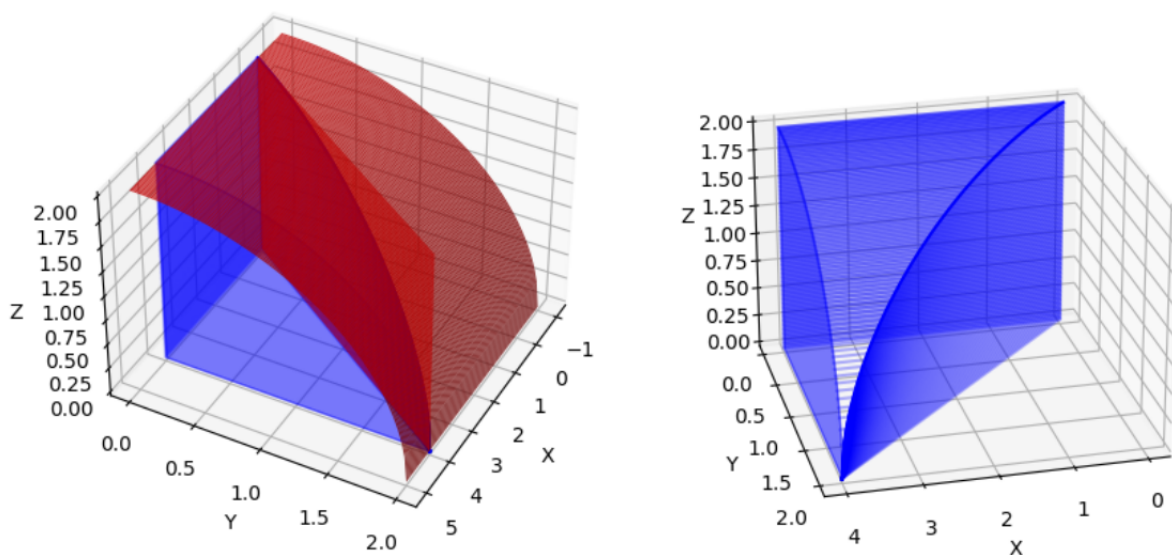
$$\int_0^3 \int_{x^2}^{3x} xy \, dy \, dx = \int_0^3 x \left( \frac{9x^2 - x^4}{2} \right) dx = \int_0^3 \frac{9x^3 - x^5}{2} = \frac{243}{8}.$$

If you chose the other way, then your integral should look like this:

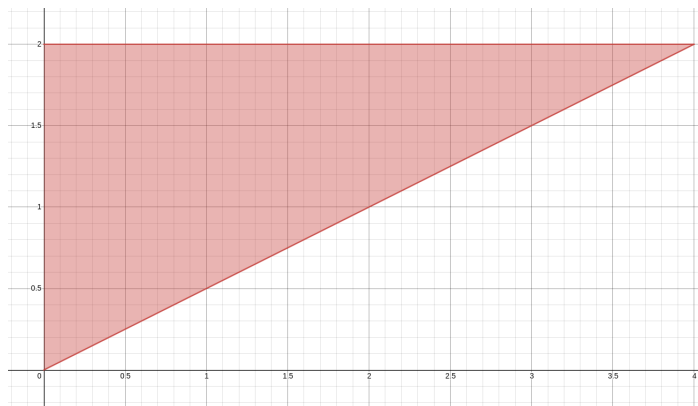
$$\int_0^9 \int_{y/3}^{\sqrt{y}} xy \, dx \, dy.$$

### Section 15.2, Problem 30

The solid we are trying to find the volume is represented in the figure below.



To find the domain of integration  $D$ , we have to project the surfaces  $y^2 + z^2 = 4$  and  $x = 2y$  on the  $XY$ -plane. For the first surface, we obtain  $y = \pm 2$  (two horizontal lines in the  $XY$ -plane) and  $x = 2y$  (a line with slope 1/2). So the domain of integration is the following region: So, the



domain  $D$  is

$$D = \{(x, y) : 0 \leq x \leq 2y, 0 \leq y \leq 2\}.$$

The function to integrate is  $z = \sqrt{4 - y^2}$ . Thus, the volume of the solid  $S$  is given by

$$V(S) = \int_0^2 \int_0^2 y \sqrt{4 - y^2} \, dx dy = \int_0^2 2y \sqrt{4 - y^2} \, dy.$$

The integral with  $2y \sqrt{4 - y^2} \, dy$  is done by a change of variable and we get

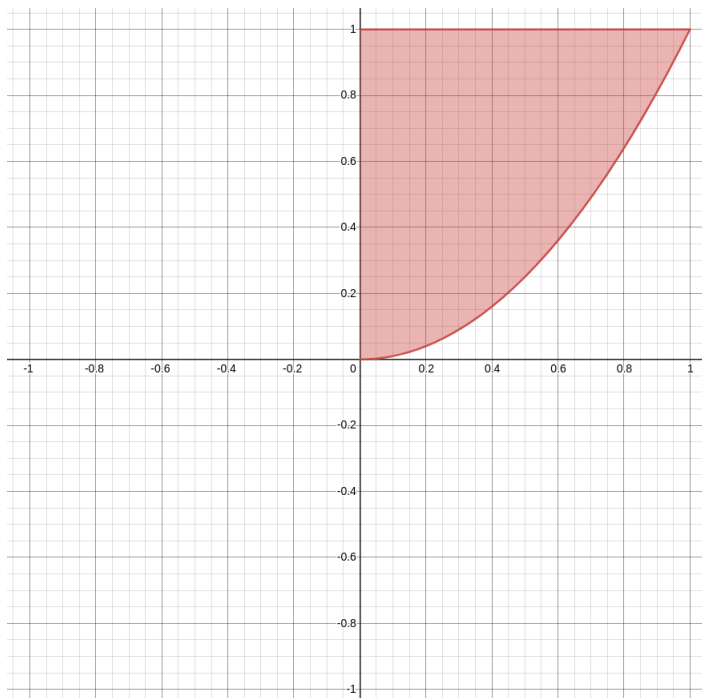
$$\int_0^2 2y \sqrt{4 - y^2} \, dy = \frac{16}{3}.$$

Thus, the volume of the solid is

$$V(S) = \frac{16}{3}.$$

### Section 15.2, Problem 52

From the limits in the integrals, we see that  $0 \leq x \leq 1$  and that  $x^2 \leq y \leq 1$ . So the region of integration looks like this: So the region  $D$  is the region bounded by the curves  $x = 0$ ,  $y = x^2$ ,



and  $y = 1$ . Since  $x \geq 0$ , the region  $D$  is also the region bounded by the curves  $x = 0$ ,  $x = \sqrt{y}$ , and  $y = 1$ . So we can say that

$$D = \{(x, y) : 0 \leq x \leq \sqrt{y}, 0 \leq y \leq 1\}.$$

Thus, the integral now becomes

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y} \sin y \, dx dy = \int_0^1 \sqrt{y} \sin(y) (\sqrt{y} - 0) \, dy = \int_0^1 y \sin y \, dy.$$

After an integration by parts, we get the value of the integral:

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y} \sin y \, dx dy = \sin(1) - \cos(1) \approx 0.301168$$

### Section 15.3, Problem 12

Let  $x = r \cos \theta$  and  $y = r \sin \theta$  (we change from cartesian to polar coordinates). The equation of the circle of radius 2 centered at the origin is simply  $r = 2$ . Also, in polar coordinates, we have  $dA = r dr d\theta$  and so

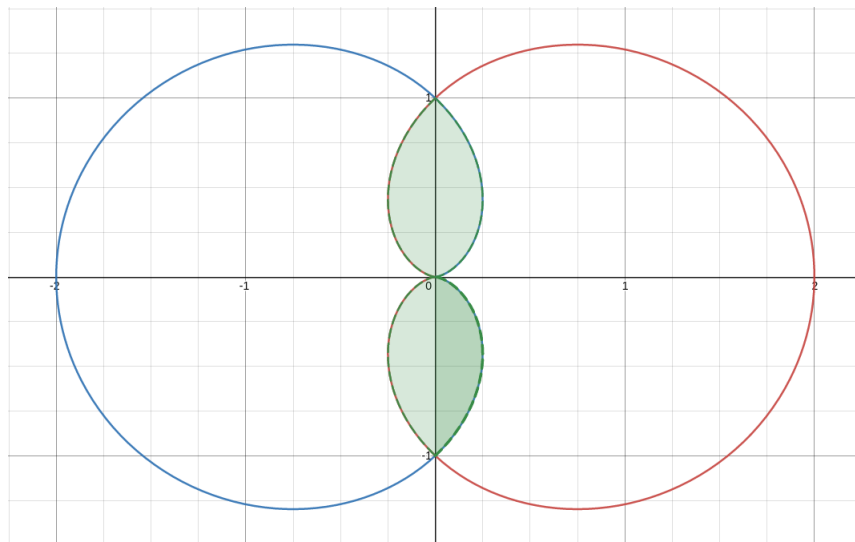
$$\iint_D \cos \sqrt{x^2 + y^2} \, dA = \int_0^{2\pi} \int_0^2 (\cos r) r \, dr d\theta = \left( \int_0^{2\pi} d\theta \right) \left( \int_0^2 r \cos r \, dr \right).$$

After an integration by parts, the value of the integral is

$$\iint_D \cos \sqrt{x^2 + y^2} \, dA = 2\pi(-1 + 2 \sin(2) + \cos(2)).$$

### Section 15.3, Problem 16

We draw the region between the two cardioids. Here are the two regions (in green) enclosed within the two cardioids: The two cardioids meet when  $1 + \cos \theta = 1 - \cos \theta$ . After rearranging, we



have to solve the equation  $2 \cos \theta = 0$ . This occurs only when  $\theta$  is  $\pi/2 + k\pi$ . We choose the values  $\theta = \pi/2$  and  $\theta = -\pi/2$ . So, the polar coordinates of the two points of intersection are

$$(1, \pi/2) \quad \text{and} \quad (1, -\pi/2)$$

which corresponds to the following points in the cartesian plane:

$$(0, 1) \quad \text{and} \quad (0, -1).$$

We have now to setup the integral. Let  $D$  denote the region enclosed by the two cardioids. The area is given by

$$A(D) = \iint_D dA.$$

In polar coordinates, we have  $dA = r dr d\theta$ . Due to the symmetry of the domain, we can only compute the area of the petal with a positive (or zero)  $y$  coordinate (above the  $x$ -axis) and then multiply our result by 2. Call this region  $D_1$ .

The argument  $\theta$  will vary from 0 to  $\pi$ . However, we have to split the interval  $[0, \pi]$  into the intervals  $[0, \pi/2]$  and  $[\pi/2, \pi]$  because the cardioids intersect at  $\theta = \pi/2$ . We can apply again the symmetry argument because the region  $D_1$  is symmetric with respect to the  $y$  axis and multiply by 2 to get the area of  $D_1$ . Denote half of the petal by  $D_2$ . So we have

$$A(D_2) = \int_0^{\pi/2} \int_0^{1-\cos(\theta)} r dr d\theta = \int_0^{\pi/2} \frac{(1 - \cos \theta)^2}{2} d\theta = \frac{3\pi}{8} - 1.$$

Thus,

$$A(D) = 2A(D_1) = 4A(D_2) = \frac{3\pi}{2} - 4.$$