Chapter 2

Derivatives

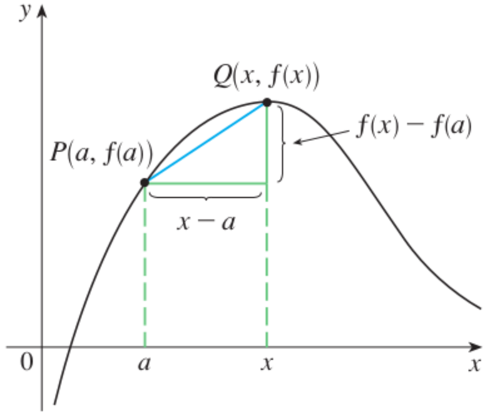
2.1 Derivatives and Rates of Change

Tangents.

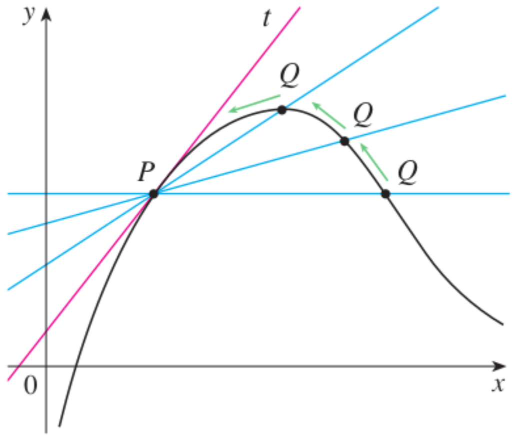
How do we find the tangent at a point P on a curve given by the graph of a function?

Answer:

1) Find the slope of the secant line passing to two points P and Q on the curve:



2) Taking the limit as Q approaches P.

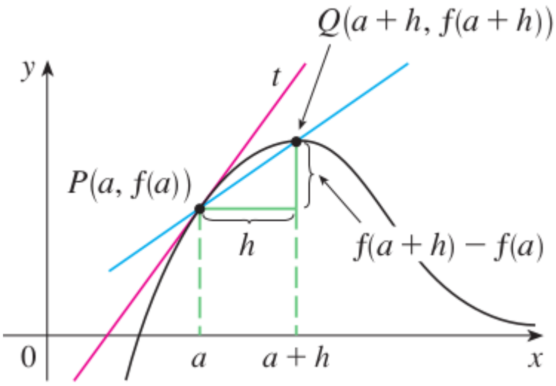


1 **Definition** The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that this limit exists.

Another expression for calculating the slope of the tangent line.



$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

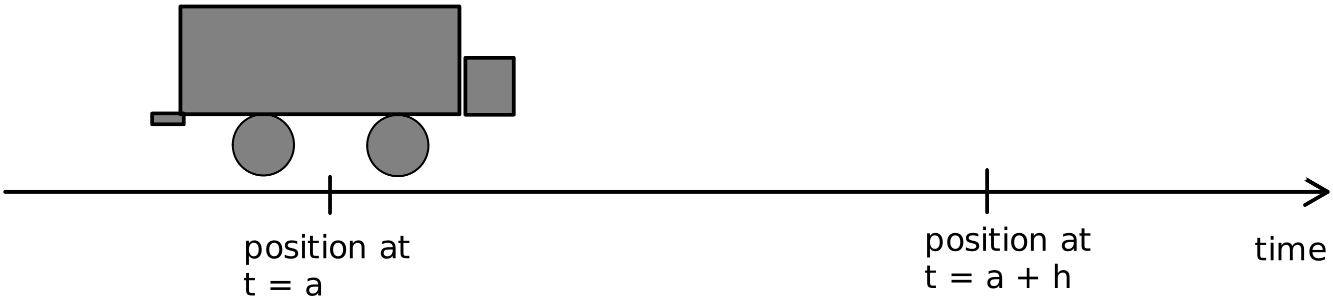
EXAMPLE 2 Find an equation of the tangent line to the hyperbola $y = 3/x$ at the point $(3, 1)$. https://www.desmos.com/calculator/24yre04iat

Equation of a line passing through a point (a, b) with slope m is : $y - b = m(x - a)$

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

$$\boxed{y - f(a) = f'(a)(x - a)}$$

Velocities



Position at t = a:

Position at t = a + h:

Total distance:

Average velocity:

Take the limit as h goes to 0:

$$v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

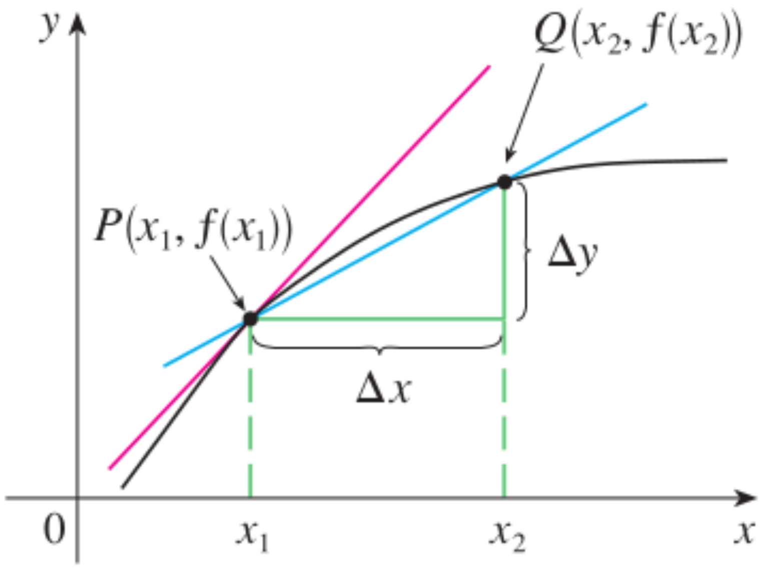
Instantaneous Velocity

Rates of Change.

Increment in x.

Increment in y.

Average Change.



6 instantaneous rate of change $= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

The Derivative.

4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

if this limit exists.

Another notation:

EXAMPLE 4 Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a.