

## L.I Mathematical Statements

**PROBLEM 1.** Are the following sentences statements? If it is, say if it is true or false. If it is not, explain briefly why.

- a)  $|-12| = -12$ .
- b)  $x < 0$ .
- c) Is that an odd integer?
- d) If  $a = 2$  and  $b = 4$ , then  $a + b = 6$ .

## L.II Logic and Mathematical Language

**PROBLEM 2.** Give the converse and the contrapositive of the following conditional statements.

- a) If it is Saturday, then Angela sleeps in.
- b) If it rains outside, then I will use my umbrella.
- c) If I went surfing, then the surf was bigger than 4 feet high.

**PROBLEM 3.** Write useful negations of the following statements in English. You can use symbols to simplify the statement.

- a) It is raining and Charlie is cold.
- b) If it is raining, then Charlie is cold.
- c) For every real number  $x$ , there exists a real number  $y$  such that  $x + y = 0$ .
- d)  $|a| > 0$  if and only if  $a \neq 0$ .

**PROBLEM 4.**

- a) By constructing the truth table of  $P \Rightarrow Q$  and  $Q \Rightarrow P$ , show when a conditional statement and its converse do not have the same truth values.
- b) By constructing the truth table of  $P \Rightarrow Q$  and  $(\neg Q) \Rightarrow (\neg P)$ , show a conditional statement and its contrapositive always have the same truth values.

## L.III Methods of Proof

**PROBLEM 5.** Suppose that  $a$  and  $b$  are integers. Prove each of the following.

- a) If  $a$  and  $b$  are both odd, then  $a + b$  is even.
- b) If  $a$  is even and  $b$  is odd, then  $a + b$  is odd.

**PROBLEM 6.** A rational number is a number  $q$  that can be put in the form of a fraction, that is there exist two integers  $n$  and  $m$  such that  $q = n/m$ . Show that  $\sqrt{2}$  is not rational.

**PROBLEM 7.** Prove that there exist integers  $m$  and  $n$  such that  $2m + 3n = 12$ .

**PROBLEM 8.** For all even integers  $a$  and  $b$ ,  $a/b$  is an integer.