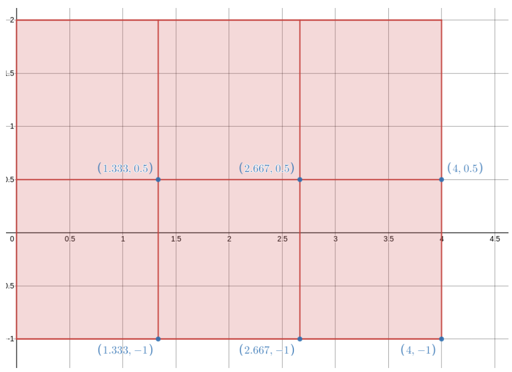
MATH-241 Calculus IV — Pierre-Olivier Parisé

Homework 01 Solutions — Spring 2022

Section 15.1, Problem 2

According to my lecture notes:

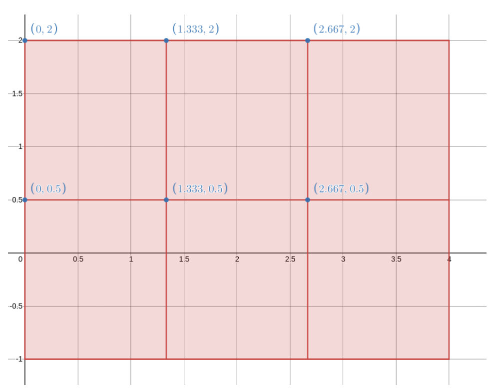
(a) We split the rectangle in 6 smaller rectangles as depicted in the following figure (here $n = 3$ and $m = 2$):



We use $x_1 = 4/3$, $x_2 = 8/3$. $x_3 = 4$, $y_1 = -1$ and $y_2 = 0.5$. We then get

$$\iint_R (1 - xy^2)\, dA \approx \sum_{i=1}^{3} \sum_{j=1}^{2} (1 - x_i y_j^2) A(R_{ij}) = -8.$$

(b) We split the rectangle in 6 smaller rectangles as depicted in the following figure (here $n = 3$ and $m = 2$):



We use $x_1 = 0, x_2 = 4/3$, and $x_3 = 8/3$, $y_1 = 0.5$ and $y_2 = 2$. We then get

$$\iint_R (1 - xy^2)\, dA \approx \sum_{i=1}^{3} \sum_{j=1}^{2} (1 - x_i y_j^2) = -22.$$

Warning: The n and m used in my lecture notes are reversed. The n is the textbook is the number of parts for the y-values, but the n in my lecture notes is the number of parts for the x-values. Same for m: in the textbook, it stands for the number of parts for the x-values, but in my lecture notes, it stands for the number of parts for y-values. Maybe the students will get the following answers for a) -12; b) -8.

Section 15.1, Problem 18

We first compute the inside integral:

$$\int_0^{\pi/2} (\sin x + \sin y)\, dy = (y \sin x - \cos y)\Big|_0^{\pi/2} = (\pi/2)\sin x + 1.$$

Then we can compute the outer integral:

$$\int_0^{\pi/6} (\pi/2)\sin x + 1\, dx = [-(\pi/2)\cos x + x]\Big|_0^{\pi/6} = (8 - 3\sqrt{3})\pi/12 \approx 0.734045.$$

Section 15.1, Problem 32

The integral is over a rectangle, so we use an interated integral. We have

$$\iint_R \frac{x}{1+xy}\, dA = \int_0^1 \int_0^1 \frac{x}{1+xy}\, dydx.$$

We put $u = 1 + xy$, so that $du = xdy$. This implies that

$$\int_0^1 \frac{x}{1+xy}\, dy = \int_1^{1+x} \frac{1}{u}\, du = \ln(1+x).$$

Then, we can evaluate the outer integral:

$$\int_0^1 \ln(1+x)\, dx = 2(\ln(2) - 1).$$

Section 15.1, Problem 36

The function $z = 2 - x^2 - y^2$ is a paraboloide that is going downward and that is 2 units above the XY-plane. We are also integrating on the square $R = [0, 1] \times [0, 1]$. So the solid should look like this: