Section 1.4 — Problem 4 — 10 points

(a) Let $y = \cos \pi x$.

(ii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.4\pi)}{0.5 - 0.4} = \frac{-\cos(0.4\pi)}{0.1} = -3.090170.$$

(iii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.49\pi)}{0.5 - 0.49} = -\frac{\cos(0.49\pi)}{0.01} = -3.141076.$$

(iv) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.499\pi)}{0.5 - 0.499} = -\frac{\cos(0.499\pi)}{0.001} = -3.141586.$$

(vi) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.6\pi)}{0.5 - 0.6} = \frac{\cos(0.6\pi)}{0.1} = -3.090170.$$

(vii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.51\pi)}{0.5 - 0.51} = \frac{\cos(0.51\pi)}{0.01} = -3.141076.$$

(viii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.501\pi)}{0.5 - 0.501} = \frac{\cos(0.501\pi)}{0.001} = -3.141586.$$

(b) The slope would be $-\pi$.

(c) The equation of a line with slope $-\pi$ is $y - y_0 = -\pi(x - x_0)$ where the line passes through the point (x_0, y_0). Therefore, since $(0.5, 0)$ is on the line, we get

$$y = \pi(x - 0.5) = \pi x - \pi/2.$$

(d) Plot using Desmos.

Section 1.5 — Problem 4 — 10 points

(a) $\lim_{x \to 2^-} f(x) = 3$.

(b) $\lim_{x \to 2^+} f(x) = 1$.

(c) $\lim_{x \to 2}$ doesn't exist because the limit on the left is different from the limit on the right.

(d) $f(2) = 3$.

(e) $\lim_{x \to 4} f(x) = 4$ because the limit from the left and the limit from the right are 4.

(f) $f(4)$ doesn't exist.

Section 1.5 — Problem 16 — 10 points

Here is a graph that satisfies the conditions.

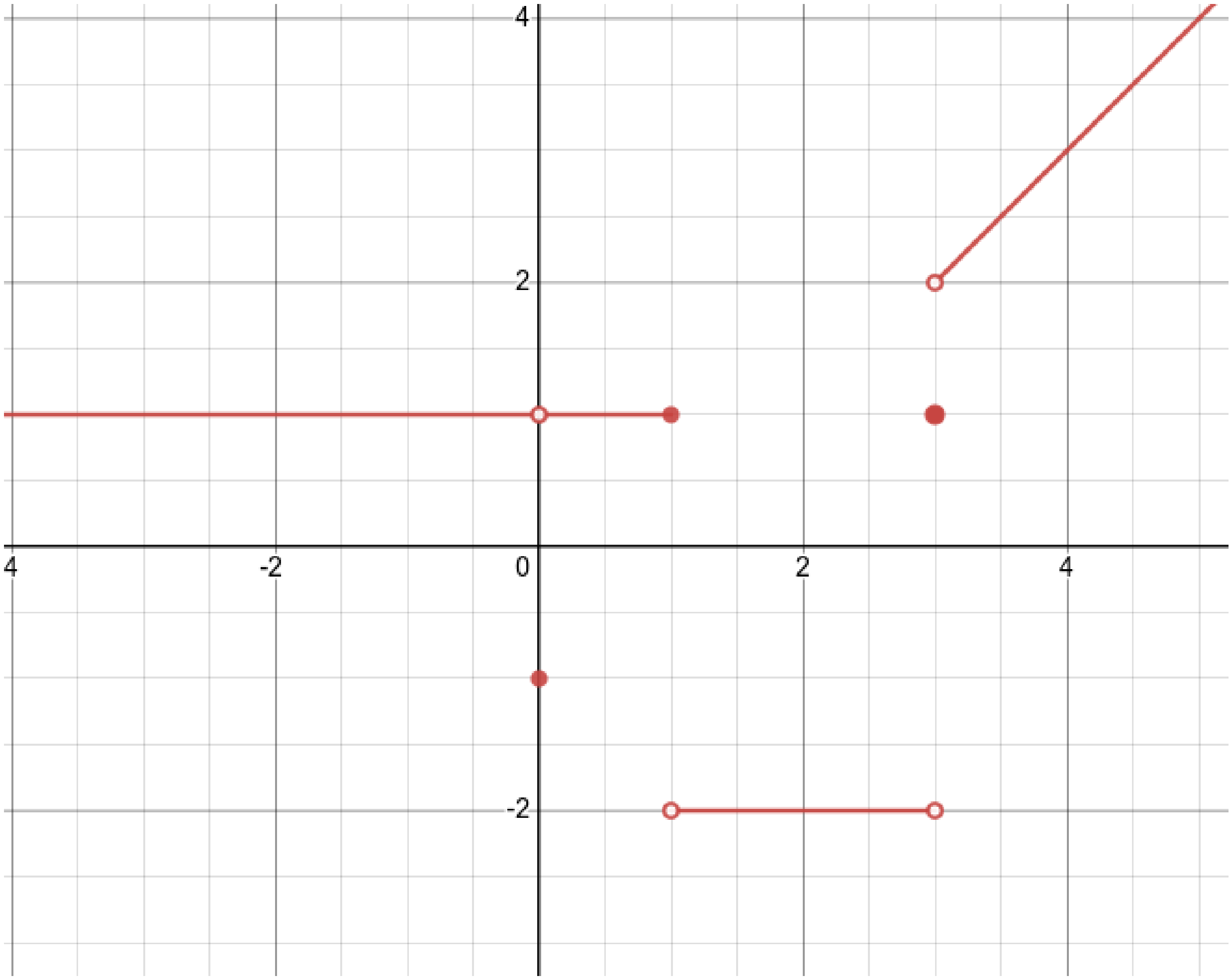

Figure 1: Graph of the function f

We see from the graph that

- $f(0) = -1$ and $f(3) = 1$;
- $\lim_{x\to 0} f(x) = 1$;
- $\lim_{x\to 3^-} f(x) = -2$;
- $\lim_{x\to 3^+} f(x) = 2$.

Section 1.5 — Problem 22 — 5 points

We build a table

x	$f(x)$
0.500000	0.142857
0.100000	0.032258
0.010000	0.003322
0.001000	0.000333
0.000100	0.000033
-0.500000	-0.200000
-0.100000	-0.034483
-0.010000	-0.003344
-0.001000	-0.000333
-0.000100	-0.000033

We can guess from the values from the table that

$$\lim_{h\to 0} \frac{(2+h)^5 - 32}{h} = 0.$$

Section 1.5 — Problem 34 — 10 points

As x approaches 0^- (so x approaches 0 from the left), then $x-1$ approaches -1^- (so a number closed to -1 from the left) and $x+2$ approaches 2^- (so a number closed to 2 from the left). Therefore, the quotient $(x-1)/(x^2(x+2))$ approaches $-1^-/(0^-)^2(2^-) = -\infty$ because we divided by the square of a small negative number (really really close to zero, but negative) which turns out to be a small positive number.

As x approaches 0^+ (so x approaches 0 from the right), then $x-1$ approaches -1^+ (so a number closed to -1 from the right) and $x+2$ approaches 2^+ (so a number closed to 2 from the left). Therefore, the quotient $(x-1)/(x^2(x+2))$ approaches $-1^+/(0^+2^+) = -\infty$ because we divided by the square of a small positive number (really really close to zero, but positive) which turns out to be a small positive number.

Therefore, from the above observations, we conclude that

$$\lim_{x\to 0^-} \frac{x-1}{x^2(x+2)} = \lim_{x\to 0^+} \frac{x-1}{x^2(x+2)} = -\infty.$$

Section 1.5 — Problem 38 — 5 points

First, we notice that

$$\frac{x^2-2x}{x^2-4x+4} = \frac{x(x-2)}{(x-2)^2} = \frac{x}{(x-2)}.$$

Therefore, we see that, as x approaches 2 from the left, $x-2$ will be a small negative number (approaching 0 from the left) and x will approach 2. Therefore,

$$\lim_{x\to 2^-} \frac{x}{x-2} = \frac{2}{0^-} = -\infty.$$

Total (Points): 50.