Chapter 16

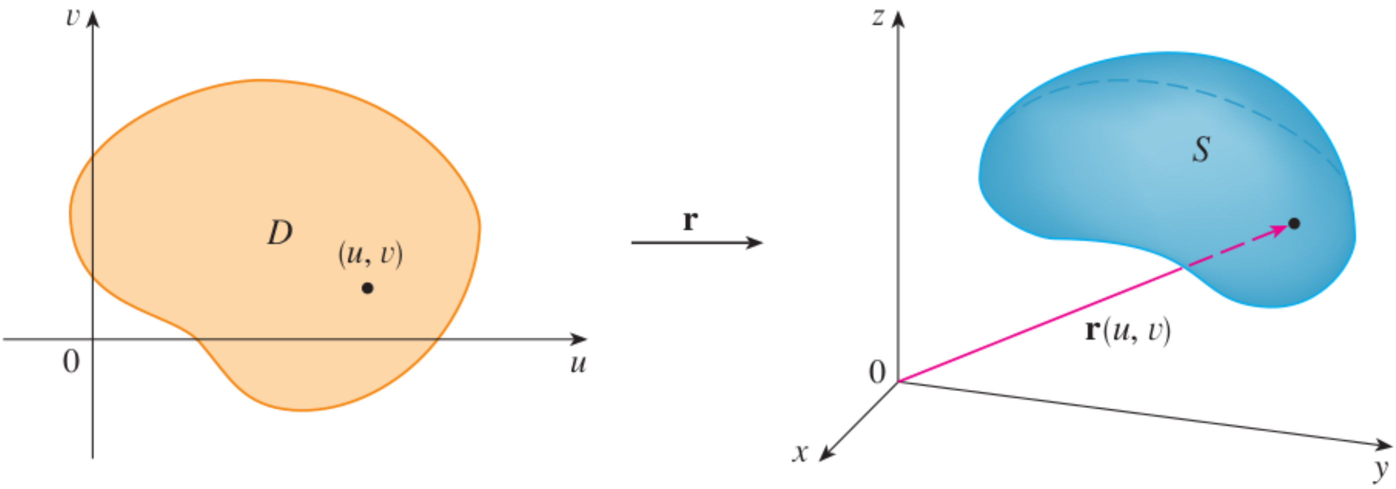
Vector Calculus

16.6 Parametric Surfaces

Generic surfaces in 3D.

EXAMPLE. Using Python, draw the surface described by the equation

$$x^2 + y^2 + z^2 = 1.$$



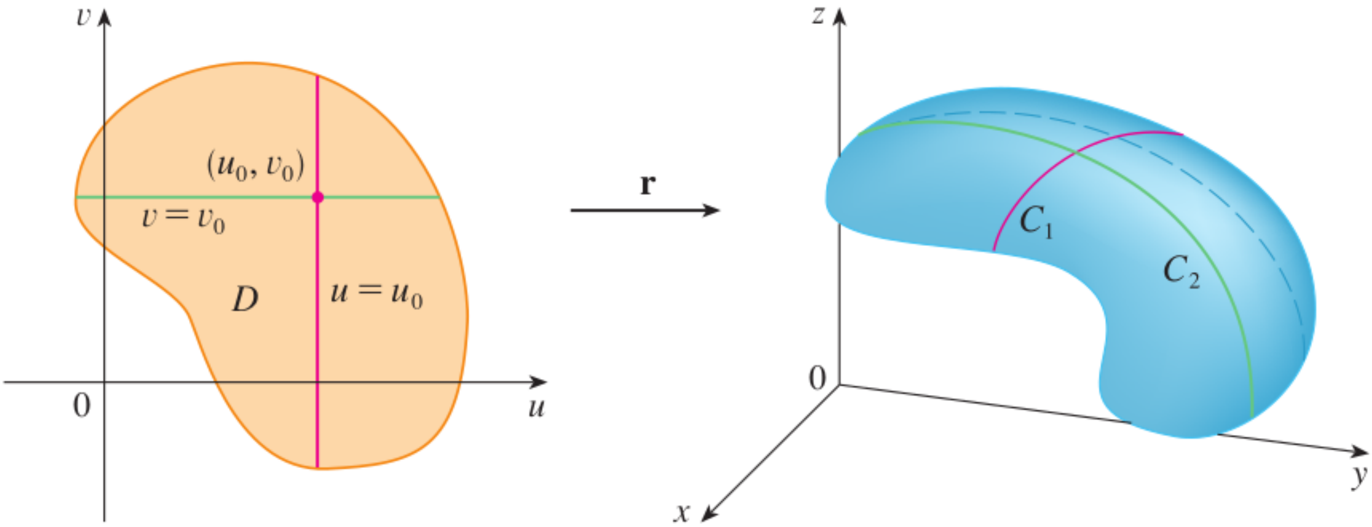
$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle \qquad \Big((u,v) \in D \Big)$$

Vector function or parametrization

3–6 Identify the surface with the given vector equation.

5. $\mathbf{r}(s,t) = \langle s\cos t, s\sin t, s\rangle$ where $0 \le s \le 2$, and $0 \le t \le 2\pi$.

Grid curves.



- $C_1 : \vec{r}(v) = \vec{r}(u_0, v)$
- $C_2 : \vec{r}(u) = \vec{r}(u, v_0)$

7–12 Use a computer to graph the parametric surface. Get a printout and indicate on it which grid curves have u constant and which have v constant.

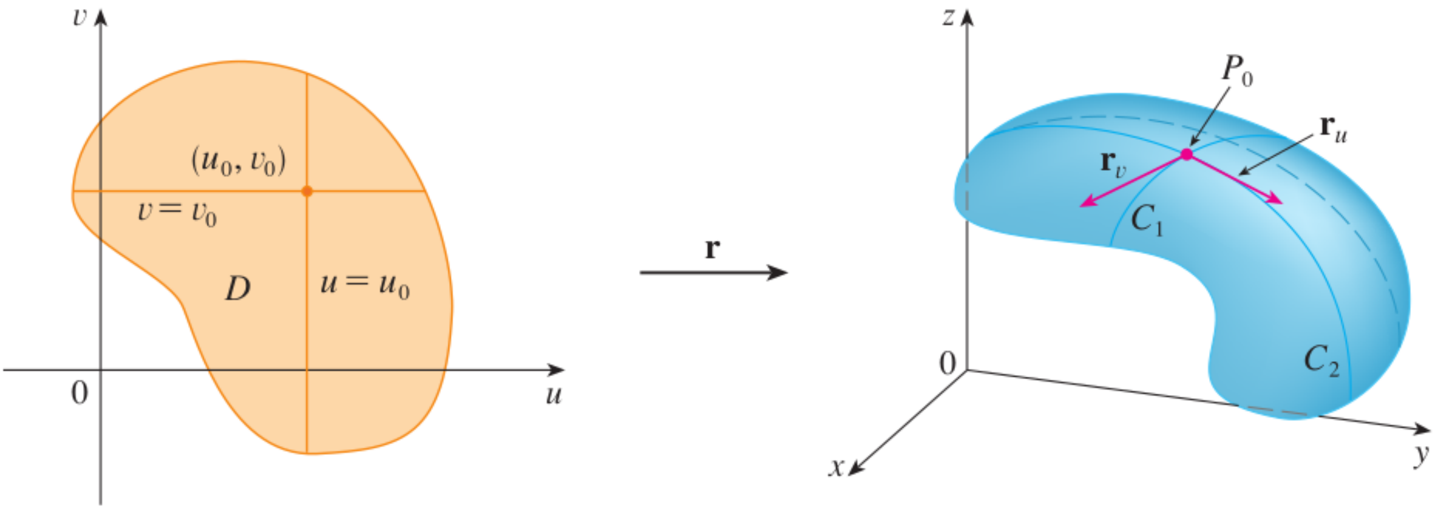
7. $\mathbf{r}(u, v) = \langle u^2, v^2, u + v \rangle$,
$-1 \leqslant u \leqslant 1, \ -1 \leqslant v \leqslant 1$

EXAMPLE 3 Find a vector function that represents the plane that passes through the point P_0 with position vector $\mathbf{r}_0$ and that contains two nonparallel vectors $\mathbf{a}$ and $\mathbf{b}$.

19–26 Find a parametric representation for the surface.

23. The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$

Tangent Planes.



- $u = u_0$ (constant). $\vec{r}(v) = \vec{r}(u_0, v)$ represents the curve C_1. Hence, $\vec{r}_v$ is the tangent vector to C_1 at P_0.
- $v = v_0$ (constant). $\vec{r}(u) = \vec{r}(u, v_0)$ represents the curve C_2. Hence, $\vec{r}_u$ is the tangent vector to C_2 at P_0.

Equation of the tangent plane at P_0 :

$$\vec{r}(u, v) = \langle x_0, y_0, z_0 \rangle + u\vec{r}_u(u_0, v_0) + v\vec{r}_v(u_0, v_0)$$

where $-\infty < u < \infty, -\infty < v < \infty$.

37–38 Find an equation of the tangent plane to the given parametric surface at the specified point. Graph the surface and the tangent plane.

37. $\mathbf{r}(u, v) = u^2\,\mathbf{i} + 2u\sin v\,\mathbf{j} + u\cos v\,\mathbf{k}$; $\quad u = 1,\ v = 0$