Example 4

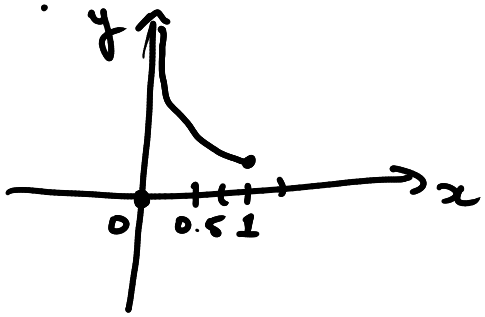
Does the function

$$f(x) = \begin{cases} 1/x & 0 < x \leq 1 \\ 0 & x = 0 \end{cases}$$

have a maximum?

x	$f(x)$
0.5	2
0.25	4
0.10	10
0.01	100

x	$f(x)$
1	1
0.5	2
0.25	4
0.1	10
0.01	100



- One at $x = 1$ → $f(x) = 1 \leq \frac{1}{x}$ $\forall x \leq 1$. $(x \neq 0)$
- Another one at $x = 0$ → $f(0) = 0 \leq 1 \leq \frac{1}{x}$ $\forall x$

Example 7

Let $f(x) = x^3$. What is $f'(0)$? Is $f(0)$ a local maximum or local minimum?

$f'(x) = 3x^2$

The solution to $f'(x) = 0$

$\Leftrightarrow \quad 3x^2 = 0$

$\Leftrightarrow \quad x = 0$

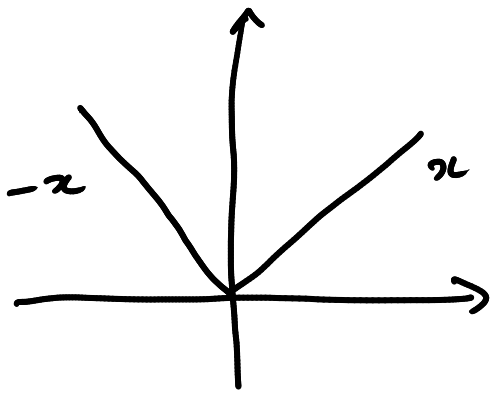
It seems that $x=0$ is a local max or local min.

x	$f(x)$
2	8
1	1
0	0
-1	-1
-2	-8

x is not a maximum and not a minimum!

Example 8

Let $f(x) = |x|$. Is $f(0)$ a local maximum, global maximum, local minimum, or global minimum?



Here $f'(0) \nexists$

But $f(0) = 0$ is an absolute minimum.

x	$f(x)$
-2	2
-1	1
0	0
1	1
2	2

Example 10

Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

1) Find the derivative

$$f'(x) = (x^{3/5})'(4-x) + x^{3/5}\cdot(-1)$$

$$= \frac{3}{5}x^{-2/5}(4-x) - x^{3/5}$$

$$= \frac{3\cdot 4}{5}x^{-2/5} - \frac{3}{5}x^{3/5} - x^{3/5}$$

$$= \frac{12}{5}x^{-2/5} - x^{3/5}\left(\frac{8}{5}\right)$$

$\frac{12}{5x^{2/5}} - \frac{8x^{3/5}}{5}$ ←

$$= \frac{12 - 8x}{5x^{2/5}}$$

$$= \frac{4(3-2x)}{5x^{2/5}}$$

2) Solve $f'(x) = 0$

$$f'(x) = 0 \iff \frac{4(3-2x)}{5x^{2/5}} = 0$$

$$\iff x = \frac{3}{2}$$

3) Find $f'(x)$ where $\nexists$

$f'(x)$ $\nexists$ when $x = 0$

Answer
critical numbers
are 3/2 & 0