Section 15.7, Problem 6

The cylindrical coordinates are (r, θ, z) and the transformations to the cartesian coordinates (x, y, z) are $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$. So if $\theta = \pi/6$, then $x = r(\sqrt{2}/2)$ and $y = r(1/2)$. So

$$2x/\sqrt{2} = r = 2y \quad \Rightarrow \quad x/\sqrt{2} = y.$$

So the surface described by the equation $\theta = \pi/6$ is the plane $x/\sqrt{2} - y = 0$.

$\boxed{\textbf{Section 15.7, Problem 8}}$

The transformation from cylindrical coordinates to the cartesian coordinates are $x = r\cos\theta$, $y = r\sin\theta$, and $z = z$. Since $\sin\theta = y/r$, the equation $r = 2\sin\theta$ becomes

$$r = 2y/r \quad \Rightarrow \quad r^2 = 2y.$$

But $r^2 = x^2 + y^2$, and so replacing in the last equation, we obtain

$$x^2 + y^2 = 2y \quad \Longleftrightarrow \quad x^2 + (y-1)^2 = 1.$$

This last equation represents a cylinder of radius 1 with center at $(0, 1)$.

Section 15.7, Problem 12

The inequalities $0 \leq \theta \leq \pi/2$ means that we are in the first octant and the eigth octant. The inequalities $r \leq z \leq 2$ means that the value for z is positive and it lies between the equations $z = r$ and $z = 2$. Since $r = \sqrt{x^2 + y^2}$, then z lies above the cone $z = \sqrt{x^2 + y^2}$. Thus the solid is a quarter of a cone.

$\boxed{\textbf{Section 15.7, Problem 20}}$

In cylindrical coordinate, the solid E has the following description:

$$E = \{(r, \theta, z) \,:\, 1 \leq r \leq 4,\, 0 \leq \theta \leq 2\pi,\, 0 \leq z \leq y + 4\}.$$

So the triple integral can be rewritten as

$$\begin{aligned}
\iiint_E (x - y)\, dV &= \int_0^{2\pi} \int_1^4 \int_0^{r\sin\theta + 4} r\cos\theta - r\sin\theta \, dz r dr d\theta \\
&= \int_0^{2\pi} \int_1^4 (\cos\theta - \sin\theta)(r\sin\theta + 4) r^2 \, dr d\theta \\
&= \int_0^{2\pi} \int_1^4 r^3 \cos\theta \sin\theta + 4r^2 \cos\theta - r^3 \sin^2\theta - 4r^2 \sin\theta \, dr d\theta \\
&= -255\pi/4.
\end{aligned}$$

Section 15.8, Problem 8

Since $\rho = \sqrt{x^2+y^2+z^2}$ and $\cos\phi = z/\rho = z/\sqrt{x^2+y^2+z^2}$. Combining everything together gives the following equation:

$$\sqrt{x^2+y^2+z^2} = \frac{z}{\sqrt{x^2+y^2+z^2}} \quad \Rightarrow \quad x^2+y^2+z^2 = z.$$

So, rewritting the last equation in the following way:

$$x^2+y^2+(z-1/2)^2 = 1/4,$$

we see that the surface is a sphere of radius $1/2$ and centered at $(0,0,1/2)$.

Section 15.8, Problem 26

The equation of the cone is $\phi = \pi/4$ because

$$z = \sqrt{x^2 + y^2} \iff \phi = \pi/4.$$

The equation of the first sphere is $\phi = 1$ and the equation of the second sphere is $\rho = 2$. So the solid can be described in spherical coordinates as followed:

$$E = \{(\rho, \theta, \phi) \,:\, 1 \leq \rho \leq 2, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi/4\}.$$

Using the change of variable formula for spherical coordinates, we obtain

$$\begin{aligned}
\iiint_E \sqrt{x^2 + y^2 + z^2}\, dV &= \int_0^{\pi/4} \int_0^{2\pi} \int_1^2 \rho^3 \sin(\phi)\, d\rho d\theta d\phi \\
&= \Big(\int_0^{\pi/4} \sin\phi\, d\phi \Big) \Big(\int_0^{2\pi} d\theta \Big) \Big(\int_1^2 \rho^3\, d\rho \Big) \\
&= (1 - 1/\sqrt{2})(2\pi)(15/4) = (\sqrt{2} - 1)15\pi/(2\sqrt{2}).
\end{aligned}$$

Section 15.8, Problem 30

The volume of the solid E is given by

$$V(E) = \iiint_E 1\, dV = \iiint_E dV.$$

The solid lies below the cone $z = \sqrt{x^2 + y^2}$ whose equation in spherical coordinates is $\phi = \pi/4$. Since we want the portion below this cone, the angle ϕ is between $\phi/4$ and $\phi/2$ ($\pi/4 \le \phi \le \pi/2$). The equation of the sphere is simply $\rho = 2$. So the description of E in spherical coordinates is

$$E = \{(\rho, \theta, \phi) \,:\, 0 \le \rho \le 2, 0 \le \theta \le 2\pi, \pi/4 \le \phi \le \pi/2\}.$$

Now, using the change of variable formular for spherical coordinates, the volume of E is given by

$$\begin{aligned} V(E) &= \int_{\pi/4}^{\pi/2} \int_0^{2\pi} \int_0^2 \rho^2 \sin(\phi)\, d\rho d\theta d\phi \\ &= \Big(\int_{\pi/4}^{\pi/2} \sin\phi\, d\phi \Big) \Big(\int_0^{2\pi} d\theta \Big) \Big(\int_0^2 \rho^2\, d\rho \Big) \\ &= (1/\sqrt{2})(2\pi)(8/3) = 16\pi/(3\sqrt{2}). \end{aligned}$$