Section 2.7 — Problem 6(a) — 5 points

The slope of the tangent line from 0 to 1 is decreasing. Therefore, the object is slowing down between 0 and 1.

At 1, it stoped because the slope is zero.

Between 1 and 0, the object is speeding up because the slope is getting more negative.

Then between 2 and 3, it is slowing down because the slope is getting closer and closer to 0.

At 3, the slope is zero and therefore the object has stoped.

Finally, the object is speeding up when the time is greater than 3.

Section 2.7 — Problem 20(a) — 5 points

Using the power rule, we find that

$$\frac{dF}{dr} = \frac{-2GmM}{r^3}.$$

The derivative means the rate of change of the force each time away of the center of the body. The minus sign means that the force diminishes when we get further and further away from the object.

Section 2.8 — Problem 6 — 10 points

We denote by

- $V(t)$: volume of the sphere (in mm^3).
- $r(t)$: radius of the sphere (in mm).
- t: time in seconds.

We know that

$$\frac{dr}{dt} = 4\mathrm{mm}/s.$$

The goal is to find

$$\left.\frac{dV}{dt}\right|_{r=40}.$$

The connection between V and r is

$$V = \frac{4}{3}\pi r^3.$$

Taking the derivative, we obtain

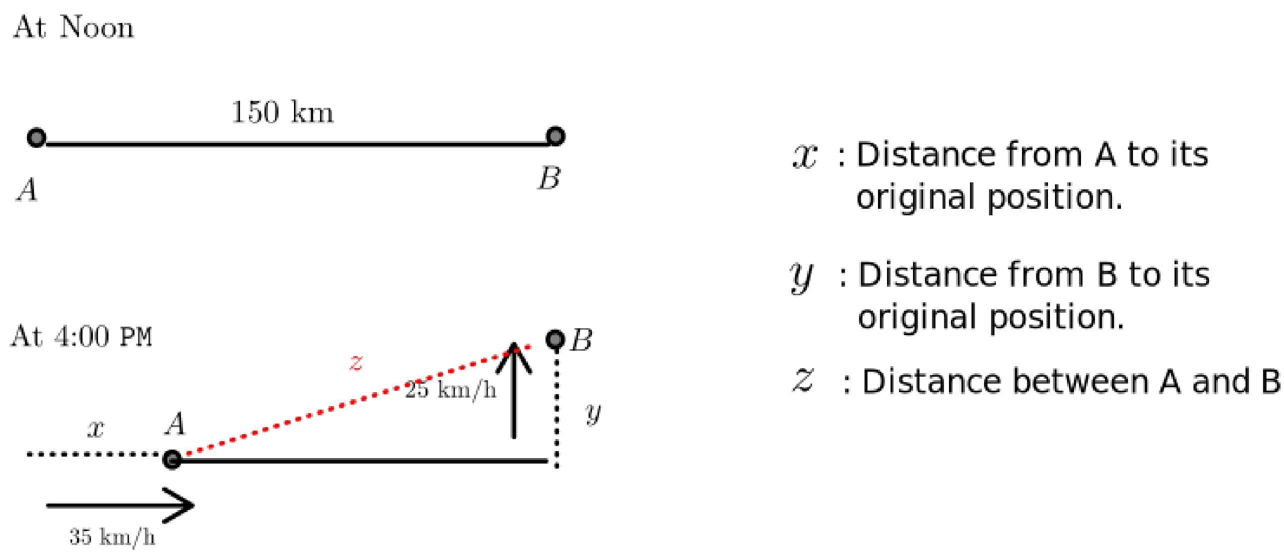
$$\frac{dV}{dt} = 4\pi r^2\Big(\frac{dr}{dt}\Big).$$

Therefore, replacing r by 40, we get

$$\frac{dV}{dt} = 4\pi(40)^2 4 = 25600\pi\,\mathrm{mm}^3.$$

Section 2.8 — Problem 16 — 10 points

First, let's draw a picture and introduce some notations. The known information is $dx/dt = 35$



and $dy/dt = 25$. What we would like to know is dz/dt.

The link between x, y and z is given by the pythagorean Theorem:

$$z^2 = (150 - x)^2 + y^2$$

where $150 - x$ is the distance from the boat A to the original position of the boat B. Taking the derivative with respect to time gives

$$\begin{aligned} & 2z(dz/dt) = 2(150 - x)(-dx/dt) + 2y(dy/dt). \\ \Longleftrightarrow \quad & dz/dt = ((150 - x)/z)(-dx/dt) + (y/z)(dy/dt). \end{aligned}$$

From noon to 4:00PM, the boat A travelled $4 \times 35 = 140$ km and the boat B travelled $4 \times 25 = 100$ km. So $x = 140$, $y = 100$, and $z = \sqrt{10^2 + 100^2} = 10\sqrt{101}$. Replacing everything in the last equations above, we obtain

$$dz/dt = (1/\sqrt{101})(-35) + (10/\sqrt{101})(25) = 215/\sqrt{101} \approx 25 \text{ km/h}.$$

Thus, $dz/dt \approx 25$ km/h.

Section 2.8 — Problem 22 — 10 points

We denote by

- $x(t)$: the distance from the bow of the boat and the bottom of the dock (in meters).
- $z(t)$: the distance from the bow of the boat and the dock.
- t: time in seconds.

We know that

$$\frac{dz}{dt} = 1\text{m/s}.$$

The goal is to find

$$\left.\frac{dx}{dt}\right|_{x=8m}.$$

The connection between z and x is via the pythagorean theorem

$$x^2 + 1^2 = z^2 \quad \Rightarrow \quad x^2 + 1 = z^2.$$

Taking the derivative, we find that

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt} \quad \Rightarrow \quad x\frac{dx}{dt} = z\frac{dz}{dt}.$$

With $x = 8$, we find that $z = \sqrt{1+8^2} = \sqrt{65}$. Therefore, pluging all the information in, we find

$$8\frac{dx}{dt} = \sqrt{65}\cdot 1 \quad \Rightarrow \quad \frac{dx}{dt} = \frac{\sqrt{65}}{8}\text{m/s}.$$

Section 2.9 — Problem 4 — 5 points

The linearization is given by

$$L(x) = f(3) + f'(3)(x - 3).$$

We have

$$f'(x) = \frac{-2x}{(x^2 - 5)^{3/2}}.$$

Therefore, we have $f'(3) = \frac{-6}{8} = -\frac{3}{4}$. We also have $f(3) = 1$. Therefore, the linearization is

$$L(z) = 1 - \frac{3}{4}(x - 3)$$

Section 2.9 — Problem 24 — 5 points

We see that

$$4.002 = 4 + 0.002.$$

Therefore, the value 0.002 will by my x in the linearization and suggest

$$f(x) = \frac{1}{x+4}.$$

We see that $f(0.002) = 1/4.002$. Therefore, a linear approximation of f around $x = 0$ will be useful to approximation $1/4.002$. We have

$$f'(x) = -\frac{1}{(x+4)^2} \quad \Rightarrow \quad f'(0) = -\frac{1}{16}.$$

Since $f(0) = 1/4$, we have

$$L(x) = 1/4 - \frac{x}{16}.$$

Using the linearization of f, we find that

$$\frac{1}{4.002} = f(0.002) \approx L(0.002) = 0.25 - \frac{0.002}{16} = 0.25 - 0.000125 = 0.249875.$$

Therefore, we have

$$\frac{1}{4.002} \approx 0.249875.$$

Total (Points): 50.