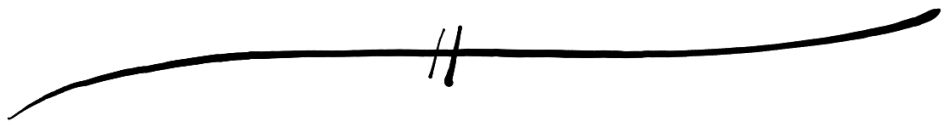


Solution



Worksheet 02



Question 1

(a) By the product rule, we have:

$$\begin{aligned}F'(x) &= (\sin x)' \tan x + \sin x \cdot (\tan x)' \\&= \cos x \tan x + \sin x (\sec^2 x) \\&= \cos x \left(\frac{\sin x}{\cos x} \right) + \frac{\sin x}{\cos^2 x} \\&= \sin x + \frac{\sin x}{\cos^2 x} = \sin x \left(1 + \frac{1}{\cos^2 x} \right)\end{aligned}$$

$$\text{So, } F'(x) = \sin x (1 + \sec^2 x).$$

(d) By the power rule & the product rule:

$$v'(x) = -(\cos x \sin x)^{-2} (\cos x \sin x)'$$

and

$$\begin{aligned}(\cos x \sin x)' &= (\cos x)' \sin x + \cos x (\sin x)' \\&= -\sin^2 x + \cos^2 x \\&= \cos(2x) \quad [\text{trigo. identity}].\end{aligned}$$

So,

$$v'(x) = - \frac{\cos 2x}{(\cos x \sin x)^2} = - \frac{\cos 2x}{(\sin^2 x / 2)^2}$$

$$= -4 \left(\frac{\cos 2x}{\sin 2x} \right) \left(\frac{1}{\sin 2x} \right).$$

$$\text{So, } v'(x) = -4 \cot(2x) \operatorname{cosec}(2x).$$

Question 2.

(a) By the chain rule,

$$(\tan(y))' = \sec^2 y \cdot y'$$

and using implicit differentiation:

$$\sec^2 y \cdot y' = 2x$$

$$\Rightarrow y' = \frac{2x}{\sec^2 y}.$$

(b) By the product rule,

$$(y \sin y)' = y' \sin y + y \cos y \cdot y'$$

$$= y' (\sin y + y \cos y).$$

So, by using implicit differentiation:

$$y' (\sin y + y \cos y) = 1$$

$$\Rightarrow y' = \frac{1}{\sin y + y \cos y}.$$