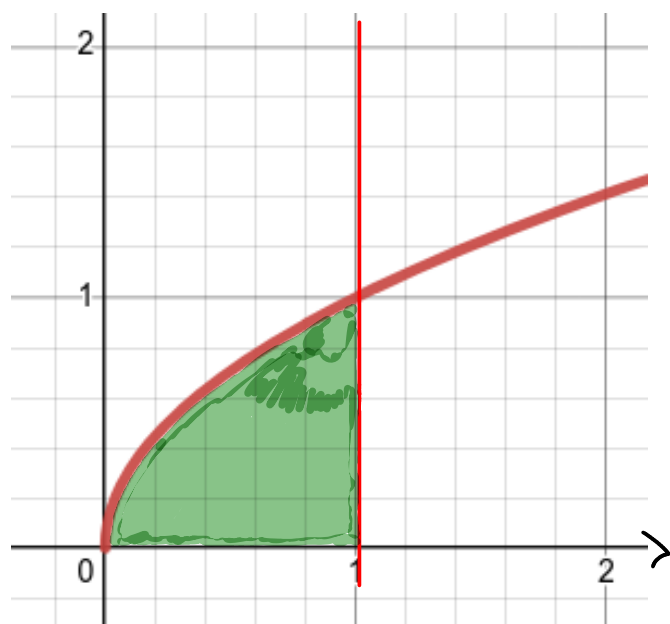


# Chapter 5

## Applications in integration

### 5.2 Volumes

## SOLIDS OF REVOLUTION.



$$f(x) = \sqrt{x}$$

- Consider the region enclosed by

$$x = 0 \quad , \quad x = 1 \quad ,$$

$$y = 0 \quad \text{and} \quad y = \sqrt{x}$$

- Rotate the region about one of the axis.

- About x-axis

- About y-axis

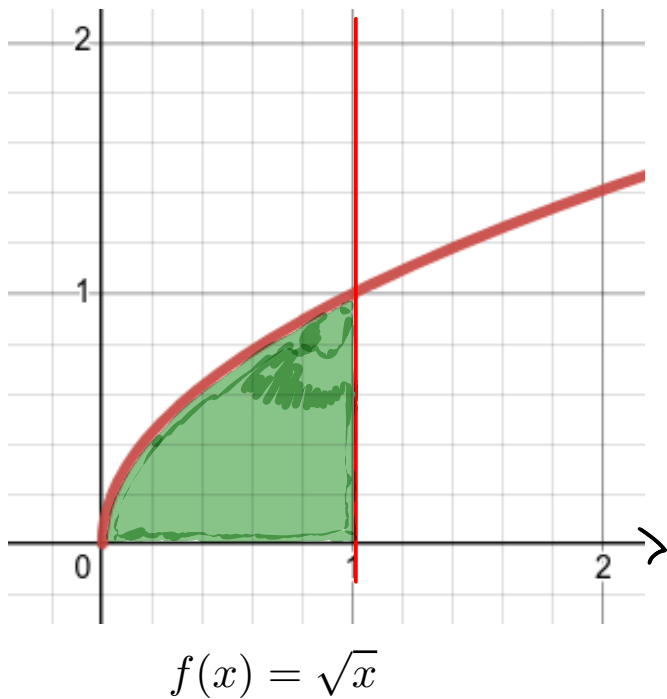
App: <https://c3d.libretexts.org/CalcPlot3D/index.html#Volumes>

### Example.

Rotate the region enclosed by  $y = x$ ,  $y = 1$ ,  $x = 0$  about the  $y$ -axis.

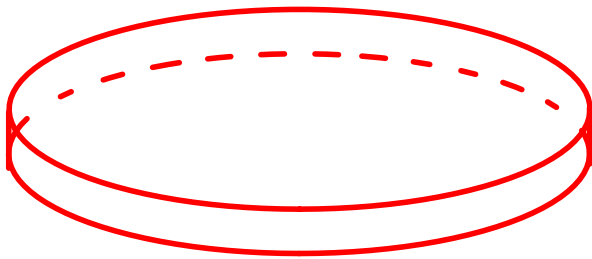
# VOLUME OF SOLIDS OF REVOLUTION.

Rotation about the x-axis.



- Divide into subintervals.
- Create rectangles.
- Rotate those rectangles.
- Generate small cylinders

Typical cylinder:



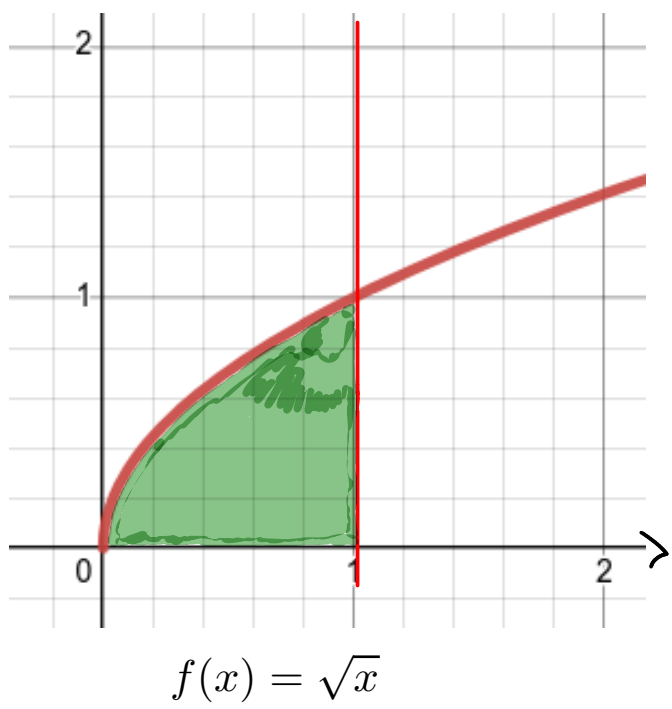
- Radius:
- Height:

Volume of typical cylinder:

$$\text{Vol(Solid)} = \int_a^b \pi(\text{radius})^2 dx$$

**EXAMPLE 2** Find the volume of the solid obtained by rotating about the  $x$ -axis the region under the curve  $y = \sqrt{x}$  from 0 to 1. Illustrate the definition of volume by sketching a typical approximating cylinder.

SKETCH



Rotation around the y-axis.

$$\text{Vol}(\text{Solid}) = \int_a^b \pi(\text{radius})^2 dy$$

**EXAMPLE 3** Find the volume of the solid obtained by rotating the region bounded by  $y = x^3$ ,  $y = 8$ , and  $x = 0$  about the y-axis.

## Cross-section as a washer.

Rotation about  
x-axis

$$\text{Vol}(\text{Solid}) = \int_a^b \pi(r_{\text{out}}^2 - r_{\text{in}}^2) dx$$

Rotation about  
y-axis

$$\text{Vol}(\text{Solid}) = \int_a^b \pi(r_{\text{out}}^2 - r_{\text{in}}^2) dy$$

**EXAMPLE 4** The region  $\mathcal{R}$  enclosed by the curves  $y = x$  and  $y = x^2$  is rotated about the  $x$ -axis. Find the volume of the resulting solid.

