

B.I Conditional Probabilities

PROBLEM 1. Let (S, \mathcal{A}, P) be a probability space. Suppose two events A and B are given such that $P(A) = 0.5$, $P(B) = 0.3$, and $P(A \cap B) = 0.1$. Find

- a) $P(A|B)$. b) $P(A|A \cup B)$. c) $P(A \cap B|A \cup B)$.

PROBLEM 2. Suppose that a balanced die is tossed once. What is the probability the die lands on a 1, given that an odd number was obtained?

B.II Bayes' Formula

PROBLEM 3. Let (S, \mathcal{A}, P) be a probability space. If A, B are events, then show that

$$\frac{P(A|B)}{P(\bar{A}|B)} = \frac{P(A) P(B|A)}{P(\bar{A}) P(B|\bar{A})}.$$

PROBLEM 4. A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a “false positive” result for 1% of the healthy persons tested¹. If 0.5% of the population actually has the disease, what is the probability a person has the disease given that the test result is positive?

B.III Independent Events

PROBLEM 5. Three brands of coffee, x , y , and z , are to be ranked according to taste by a judge. Define the following events: A for “Brand x is preferred to y ”, B for “Brand x is ranked best”, C for “Brand x is ranked second best” and D for “Brand x is ranked third best”. If the judge actually has no taste preference and randomly assigns ranks to the brands, is event A independent of (a) event B ? (b) event C ? (c) event D ?

PROBLEM 6. Cards are dealt, one at a time, from a standard 52-card deck. If A_i denotes the event “the i -th card dealt is a spade”. Are A_1 and A_2 independent?

PROBLEM 7. Let (S, \mathcal{A}, P) be a probability space. Prove that if A and B are independent events with $0 < P(A), P(B) < 1$, then so are A and \bar{B} . Are \bar{A} and \bar{B} independent?

¹That is, if a healthy person is tested, then, with probability 0.01, the test result will imply the person has the disease.