

Chapter 5

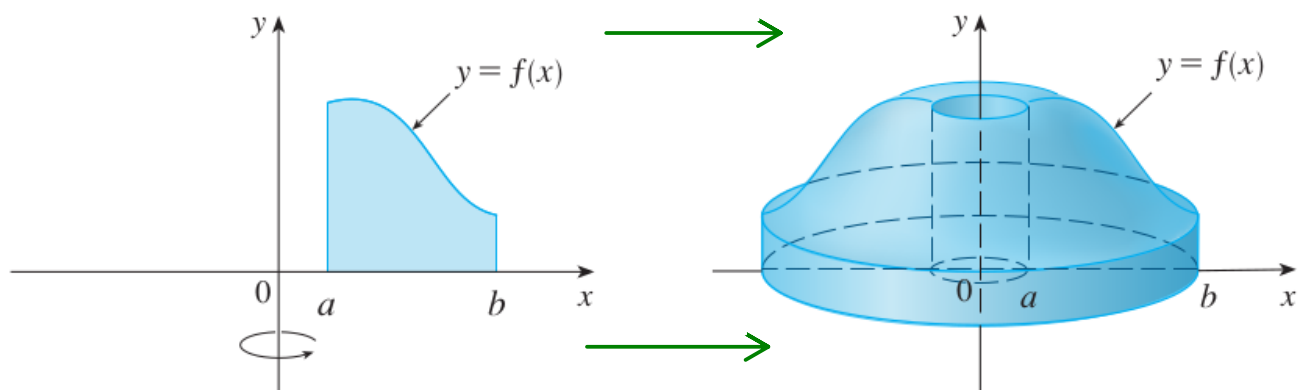
Applications in integration

5.3 Volumes by Cylindrical Shells

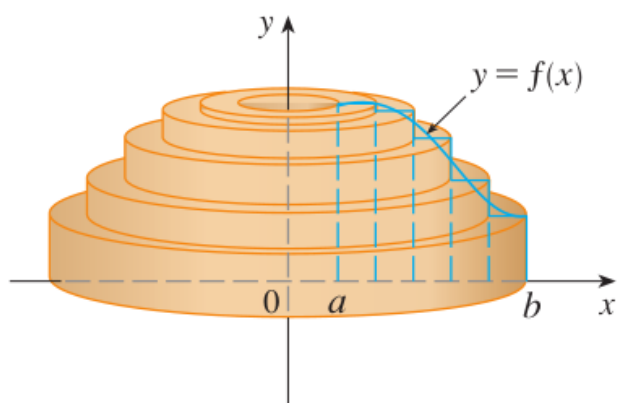
DIY.

EXAMPLE 1 Find the volume of the solid obtained by rotating about the y -axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

Method with Cylindrical Shells. (Rotation about the y-axis)



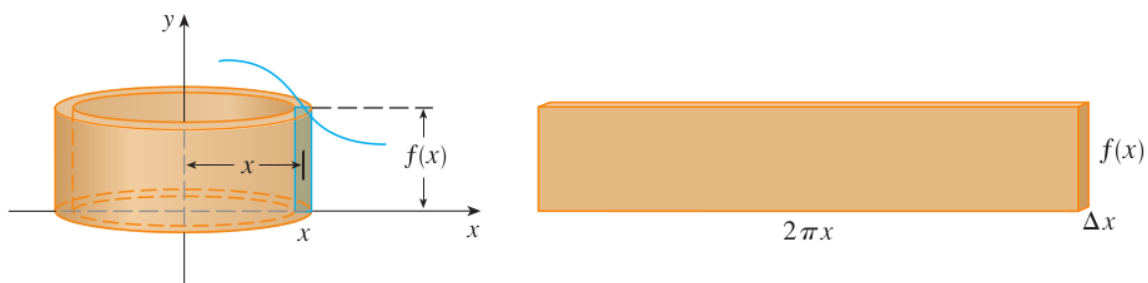
Approximation by spherical Shells.



2 The volume of the solid in Figure 3, obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b , is

$$V = \int_a^b 2\pi x f(x) dx \quad \text{where } 0 \leq a < b$$

$$\int_a^b \underbrace{(2\pi x)}_{\text{circumference}} \underbrace{[f(x)]}_{\text{height}} \underbrace{dx}_{\text{thickness}}$$



Rotation about the x -axis. (Find the radius of the cylindrical shell)

EXAMPLE 3 Use cylindrical shells to find the volume of the solid obtained by rotating about the x -axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

Rotation about another axis.

EXAMPLE 4 Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.