C.I Discrete Random Variables

Example 1. Example will be written on the board.

Example 2. Let $(S, \mathcal{A}, P)$ be a probability space in which

$$S = \{1,2,3,4,5,6\}, \quad \mathcal{A} = \{\varnothing, \{2,4,6\}, \{1,3,5\}, S\}.$$

Let U and V be defined by

$$U(s) = s, \quad V(s) = \begin{cases} 1 & \text{if } s \text{ is even,} \\ 0 & \text{if } s \text{ is odd} \end{cases}$$

for $s \in S$.

a) Is U a discrete random variable?

b) Is V a discrete random variable?

Example 3. Statement will be written on the board.

Example 4. Let $S = \{0, 1, 2, \ldots\}$ and let $X : S \to \mathbb{R}$ be a discrete random variable with $\operatorname{Im} X = \{0, 1, 2, 3, 4, \ldots\}$. Define the function $p : S \to [0, 1]$ by $p(x) = c2^x/x!$, for $x = 0, 1, 2, \ldots$,

a) For what value of c is the function p a pmf?

b) Find $P(X = 0)$.

c) Find $P(X > 2)$.

C.II Functions of Discrete Random Variables

Example 5. The statement will be written on the board.

C.III Expectation and Variance

Example 6. Consider a fair 6-faced die and the following game. After tossing the die, if the face lands on an even number, then you win 2 US dollars. But if the face lands on even, then you loose 1 US dollar. Would you like to play this game?

Example 7. The manager of an industrial plant is planning to buy a new machine of either type a or type b. If t denotes the number of hours of daily operations, the number of daily repairs Y_1 required to maintain a machine of type a is a random variable with mean and variance both equal to $t/10$. The number of daily repairs Y_2 for a machine of type b is a random variable with mean and variance both equal to $3t/25$. The daily cost of operating a is $C_a(t) = 10t + 30Y_1^2$; for b it is $C_b(t) = 8t + 30Y_2^2$. Assume that the repairs take negligible time and that each night the machines are tuned so that they operate essentially like new machines at the start of the next day. Which machine minimizes the expected daily cost if a workday consists of (a) 10 hours (b) 20 hours?

C.IV Examples of Discrete Random Variables

Example 8. There are $n = 6$ machines to test if they are working properly or not. According to a recent survey, a machine is working properly in 75% of the time. What is the probability that 4 machines are working properly.

Example 9. Consider an experiment that consists of counting the number of α-particles given off in a 1-second interval by 1 gram of radioactive material. If we know from past experience that, on the average, 3.2 such α-particles are given off, what is a good approximation to the probability that no more than 2 α-particles will appear?

Example 10. A geological study indicates that an exploratory oil well drilled in a particular region should strike oil with probability 0.2. Find the probability that the third oil strike comes on the fifth well drilled.

Example 11. An urn contains 10 red balls and 20 blue balls. Ball are randomly selected, one at a time, until a red one is obtained. If we assume that each selected ball is replaced before the next one is drawn, what is the probability that

a) exactly 3 draws are needed?

b) at least 6 draws are needed.