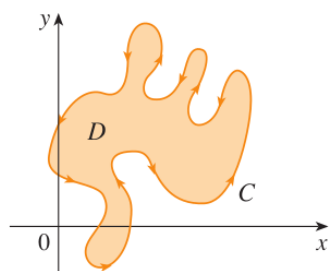
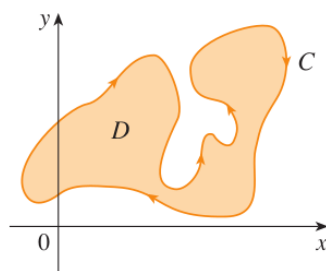


16.4 Green's Theorem.

Some terminology.



(a) Positive orientation



(b) Negative orientation

Green's Theorem Let C be a positively oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C . If P and Q have continuous partial derivatives on an open region that contains D , then

$$\oint_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Remarks on notations.

EXAMPLE 1 Evaluate $\int_C x^4 dx + xy dy$, where C is the triangular curve consisting of the line segments from $(0, 0)$ to $(1, 0)$, from $(1, 0)$ to $(0, 1)$, and from $(0, 1)$ to $(0, 0)$.

EXAMPLE 2 Evaluate $\oint_C (3y - e^{\sin x}) dx + (7x + \sqrt{y^4 + 1}) dy$, where C is the circle $x^2 + y^2 = 9$.

EXAMPLE 3 Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

EXAMPLE 4 Evaluate $\oint_C y^2 dx + 3xy dy$, where C is the boundary of the semiannular region D in the upper half-plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

$$A = \oint_C x \, dy = -\oint_C y \, dx = \frac{1}{2} \oint_C x \, dy - y \, dx$$

EXAMPLE 3 Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Extension to more general regions.

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_{C_1} P dx + Q dy + \int_{C_2} P dx + Q dy = \int_C P dx + Q dy$$