16.5 Curl and Divergence.

Curl.

Definition.

Cross product formula.

$$\boxed{\text{curl}\,\mathbf{F} = \nabla \times \mathbf{F}}$$

EXAMPLE 1 If $\mathbf{F}(x, y, z) = xz\,\mathbf{i} + xyz\,\mathbf{j} - y^2\,\mathbf{k}$, find curl $\mathbf{F}$.

3 Theorem If f is a function of three variables that has continuous second-order partial derivatives, then

$$\text{curl}(\nabla f) = \mathbf{0}$$

EXAMPLE 2 Show that the vector field $\mathbf{F}(x, y, z) = xz\,\mathbf{i} + xyz\,\mathbf{j} - y^2\,\mathbf{k}$ is not conservative.

4 Theorem If $\mathbf{F}$ is a vector field defined on all of $\mathbb{R}^3$ whose component functions have continuous partial derivatives and curl $\mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is a conservative vector field.

EXAMPLE 3

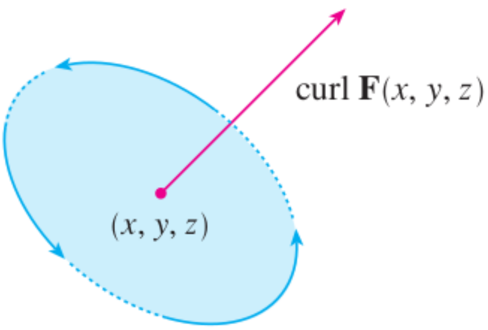
(a) Show that

$$\mathbf{F}(x, y, z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$$

is a conservative vector field.

(b) Find a function f such that $\mathbf{F} = \nabla f$.

Physical interpretation.



DIvergence.

Definition.

Dot product formula.

$$\boxed{\operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F}}$$

EXAMPLE 4 If $\mathbf{F}(x, y, z) = xz\,\mathbf{i} + xyz\,\mathbf{j} - y^2\,\mathbf{k}$, find div $\mathbf{F}$.

11 Theorem If $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$ is a vector field on $\mathbb{R}^3$ and P, Q, and R have continuous second-order partial derivatives, then

$$\operatorname{div} \operatorname{curl} \mathbf{F} = 0$$

EXAMPLE 5 Show that the vector field $\mathbf{F}(x, y, z) = xz\,\mathbf{i} + xyz\,\mathbf{j} - y^2\,\mathbf{k}$ can't be written as the curl of another vector field, that is, $\mathbf{F} \neq \operatorname{curl} \mathbf{G}$.

Incompressible Flow.

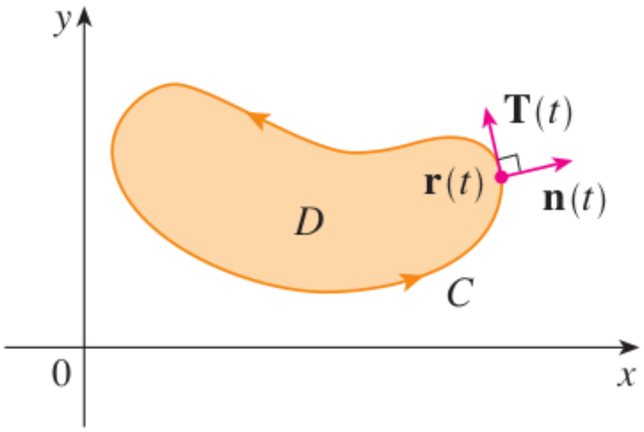
Laplace's Equation.

Vector Forms of Green's Theorem.

I. First Formula with curl.

$$\boxed{\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_D (\operatorname{curl} \mathbf{F}) \cdot \mathbf{k} \, dA}$$

II. Second formula with divergence.



$$\boxed{\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \iint_D \operatorname{div} \mathbf{F}(x, y) \, dA}$$