Section 3.4, Problem 12

We see that

$$\frac{4x^3+6x^2-2}{2x^3-4x+5} = \frac{4+6/x-2/x^3}{2+4/x^2+5/x^3}.$$

So, by the quotient rule for limits, we have

$$\lim_{x\to-\infty} \frac{4+6/x-2/x^3}{2+4/x^2+5/x^3} = \frac{4}{2} = 2.$$

Section 3.4, Problem 40

We first compute the limit at ∞. Since $x \to \infty$, the variable x will be eventually positive and so $\sqrt{x^2} = x$. Then, we can write

$$\frac{x-9}{\sqrt{4x^2+3x+2}} = \frac{1-9/x}{\sqrt{4+3/x+2/x^2}}.$$

We then find, by the quotient rule and the root rule,

$$\lim_{x\to\infty} \frac{1-9/x}{\sqrt{4+3/x+2/x^2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}.$$

So $y = 1/2$ is an HA at ∞.

Finally, we compute the limit at $-\infty$. Since $x \to -\infty$, the variable x will eventually be negative and so $\sqrt{x^2} = -x$. We then can write

$$\frac{x-9}{\sqrt{4x^2+3x+2}} = -\frac{1-9/x}{\sqrt{4+3/x+2/x^2}}.$$

We then find, by the quotient rule and the root rule,

$$\lim_{x\to\infty} -\frac{1-9/x}{\sqrt{4+3/x+2/x^2}} = \frac{1}{\sqrt{4}} = -\frac{1}{2}.$$

So $y = -1/2$ is an HA at $-\infty$.

Section 3.7, Problem 8

Let x be the width and y be the heigth of the rectangle. The total area is fixed and is 1000 m^2. This means that

$$xy = 1000 \quad \Rightarrow \quad y = 1000/x.$$

The function for the perimeter is $P = 2x + 2y$. This formulea becomes

$$P(x) = 2x + 2000/x.$$

The domain of P is $x > 0$ since a negative or zero width is not possible.

We take the derivative, and see that $P'(x) = 2 - 2000/x^2$. The function P' is defined everywhere on $(0, \infty)$, so the critical numbers correspond to its zeros. We see that

$$P'(x) = 0 \iff x^2 = 1000 \iff x = \pm\sqrt{1000}.$$

Since x must be positive, we reject $-\sqrt{1000}$ and keep $\sqrt{1000}$. If we take the second derivative, we see that

$$P''(x) = \frac{4000}{x^3}$$

and, since $x > 0$, we have $P''(x) > 0$ for every $x > 0$. Thus, the number $x = \sqrt{1000}$ correspond to an absolute minimum by the second derivative test.

Finally, we have $x = \sqrt{1000}$, $y = 1000/x = \sqrt{1000}$, and the perimeter is $P = 4\sqrt{1000}$ m.

Section 3.7, Problem 36

Let x (width) and y (height) be the dimensions of the poster. The total area is $xy = 180$. The total of printed area is

$$A = (x - 2)(y - 3).$$

Since $xy = 180$, we have $y = 180/x$ and so

$$A = (x - 2)(180/x - 3) = 3(x - 2)(60/x - 1).$$

The derivative of A is

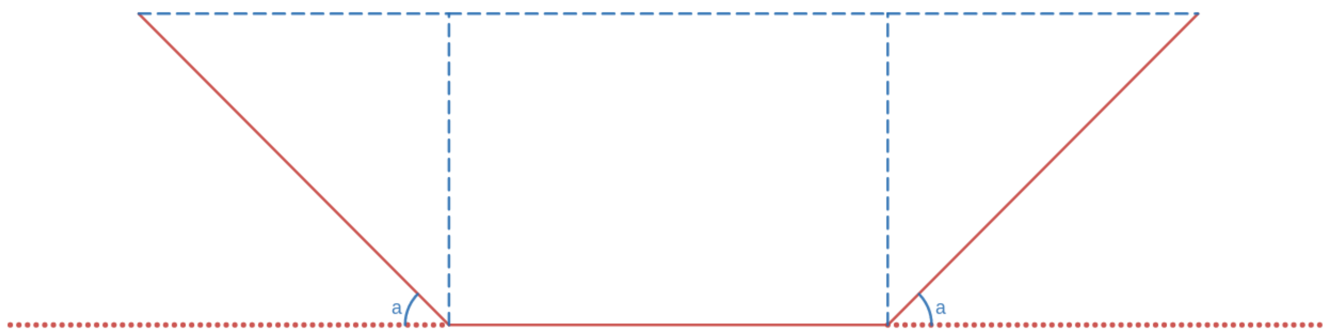
$$A' = 3(60/x - 1 - 60(x - 2)/x^2)) = 3(120/x^2 - 1) = 3(120 - x^2)/x^2.$$

So $A' = 0$ if and only if $x = \pm\sqrt{120}$. We discard $-\sqrt{120}$ and keep $x = \sqrt{120}$. We see that $A' > 0$ when $0 < x < \sqrt{120}$ and $A' < 0$ when $x > \sqrt{120}$. So x corresponds to a global maximum of A on $(0, \infty)$.

So the dimensions are $x = \sqrt{120} \approx 10.95$ in and $y = 180/\sqrt{120} \approx 16.43$ in.

Section 3.7, Problem 76

Let a be the angle as illustrated in the picture below. To determine the maximum amount of water, we will determine the maximum area of the rain gutter. As the figure below shows, we can split the rain gutter in three simple shapes: two identical triangles and one rectangle.



The area of the triangles is

$$A_\Delta = \frac{100\cos a \sin a}{2} = 50\cos a \sin a$$

The area of the rectangle is

$$A_\Box = 10 \cdot 10 \sin a = 100 \sin a.$$

So the total area is

$$A = 2A_\Delta + A_\Box = 100 \sin a(\cos a + 1).$$

The domain of the function a is between 0 and $\pi/2$.

We take the derivative with respect to a. We obtain

$$A' = 100\cos a(\cos a + 1) - 100\sin^2 a = 100(\cos^2 a - \sin^2 a) + 100\cos a.$$

We can replace $\cos^2 a - \sin^2 a$ by $\cos(2a)$ and we find

$$A' = 100\cos(2a) + 100\cos a.$$

We see that

$$A' = 0 \iff \cos 2a = -\cos a \iff \cos 2a = \cos(-a - \pi) \iff 2a = -a + (2k-1)\pi.$$

Taking $k = 0$, we see that $A' = 0 \iff a = -\pi/3$. Taking $k = 1$, we see that $A' = 0 \iff a = \pi/3$. This last angle is in the domain of our function A.

Now, $A' > 0$ when $0 < a < \pi/3$ and $A' < 0$ when $\pi/3 < a < \pi/2$. So, by the first derivative test, we see that $a = \pi/3$ corresponds to an absolute maximum on the interval $(0, \pi/2)$. The area corresponding to this angle is

$$A = 100\sin(\pi/3)(\cos(\pi/3) + 1) \approx 129.90 \text{ m}^2 \text{ of water.}$$