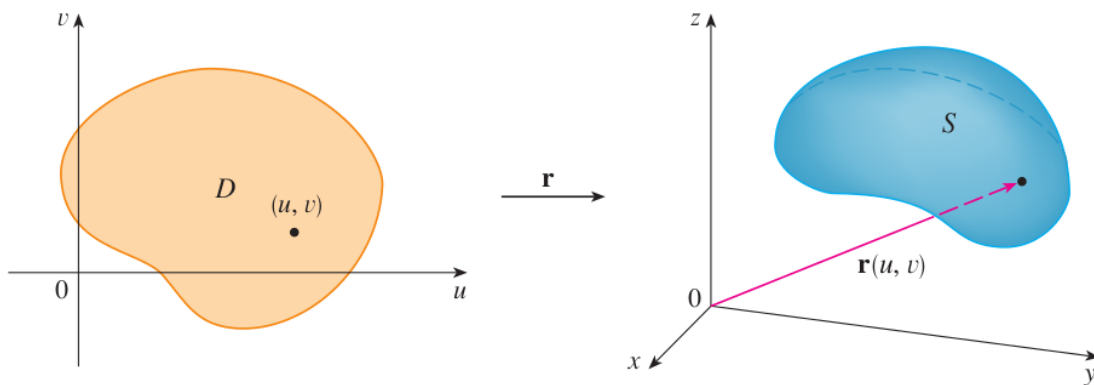


16.6 Parametric surfaces and Their Areas.



Vector expression.

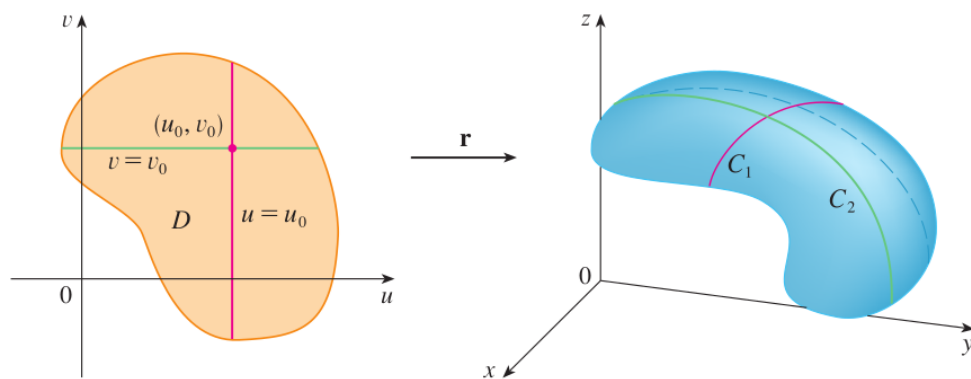
Parametric equations.

EXAMPLE 1 Identify and sketch the surface with vector equation

$$\mathbf{r}(u, v) = 2 \cos u \mathbf{i} + v \mathbf{j} + 2 \sin u \mathbf{k}$$

Question: What happen to the surface if we restric one of the parameter?

Grid curves.



EXAMPLE 2 Use a computer algebra system to graph the surface

$$\mathbf{r}(u, v) = \langle (2 + \sin v) \cos u, (2 + \sin v) \sin u, u + \cos v \rangle$$

Which grid curves have u constant? Which have v constant?

EXAMPLE 3 Find a vector function that represents the plane that passes through the point P_0 with position vector \mathbf{r}_0 and that contains two nonparallel vectors \mathbf{a} and \mathbf{b} .

EXAMPLE 4 Find a parametric representation of the sphere

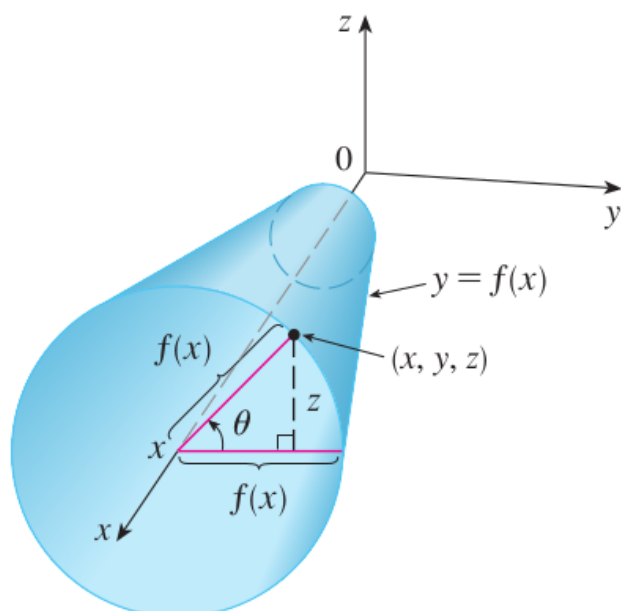
$$x^2 + y^2 + z^2 = a^2$$

EXAMPLE 6 Find a vector function that represents the elliptic paraboloid $z = x^2 + 2y^2$.

EXAMPLE 7 Find a parametric representation for the surface $z = 2\sqrt{x^2 + y^2}$, that is, the top half of the cone $z^2 = 4x^2 + 4y^2$.

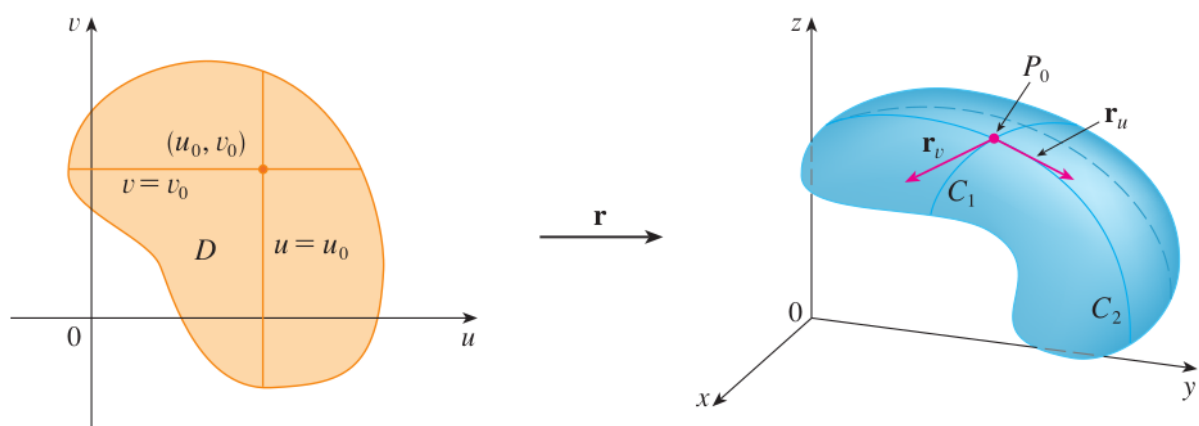
Surfaces of revolution.

Equations.

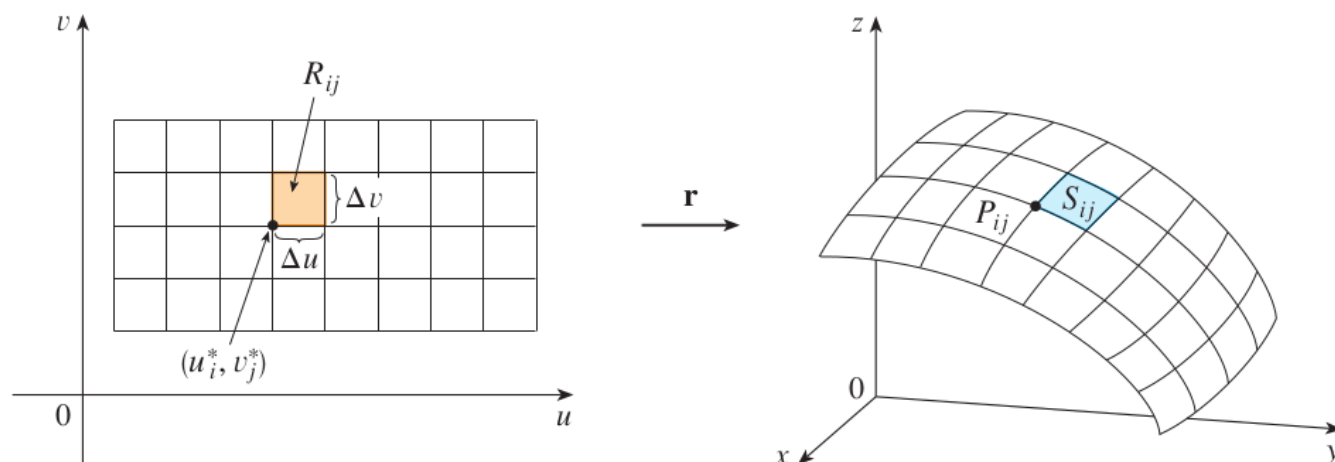


EXAMPLE 8 Find parametric equations for the surface generated by rotating the curve $y = \sin x$, $0 \leq x \leq 2\pi$, about the x -axis. Use these equations to graph the surface of revolution.

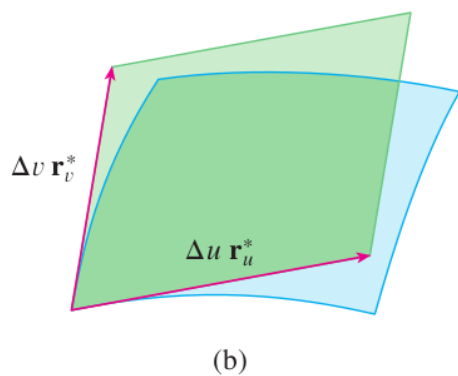
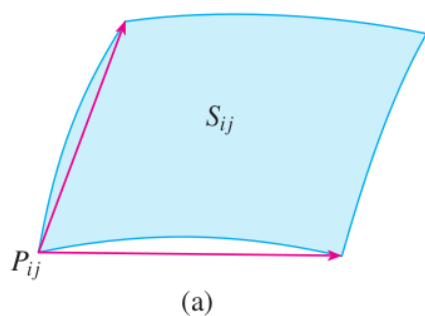
Question: What are the equations of a surface obtained by rotating a function about another axis?



EXAMPLE 9 Find the tangent plane to the surface with parametric equations $x = u^2$, $y = v^2$, $z = u + 2v$ at the point $(1, 1, 3)$.



Closer look.



6 Definition If a smooth parametric surface S is given by the equation

$$\mathbf{r}(u, v) = x(u, v) \mathbf{i} + y(u, v) \mathbf{j} + z(u, v) \mathbf{k} \quad (u, v) \in D$$

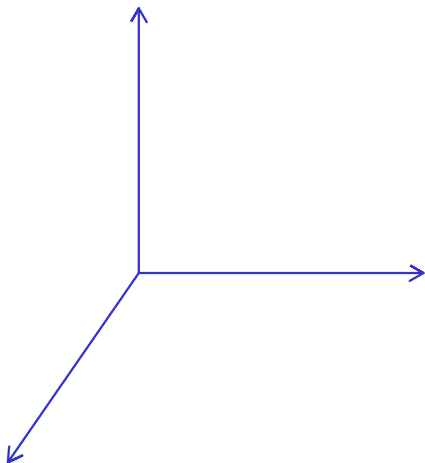
and S is covered just once as (u, v) ranges throughout the parameter domain D , then the **surface area** of S is

$$A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA$$

where $\mathbf{r}_u = \frac{\partial x}{\partial u} \mathbf{i} + \frac{\partial y}{\partial u} \mathbf{j} + \frac{\partial z}{\partial u} \mathbf{k}$ $\mathbf{r}_v = \frac{\partial x}{\partial v} \mathbf{i} + \frac{\partial y}{\partial v} \mathbf{j} + \frac{\partial z}{\partial v} \mathbf{k}$

- 41.** The part of the plane $x + 2y + 3z = 1$ that lies inside the cylinder $x^2 + y^2 = 3$ (Find the area)

Surface Area of a Graph of a Function.



$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} dA$$

EXAMPLE 11 Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.