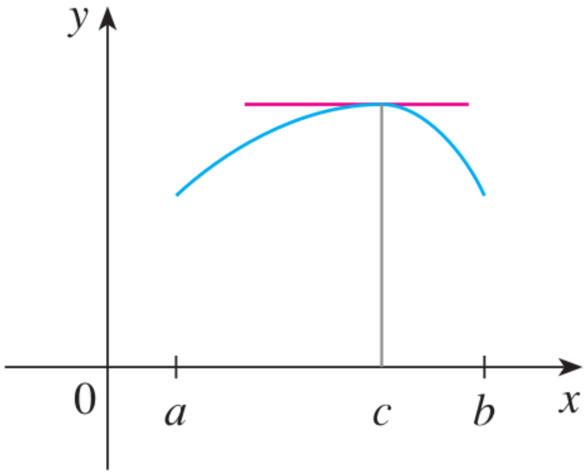
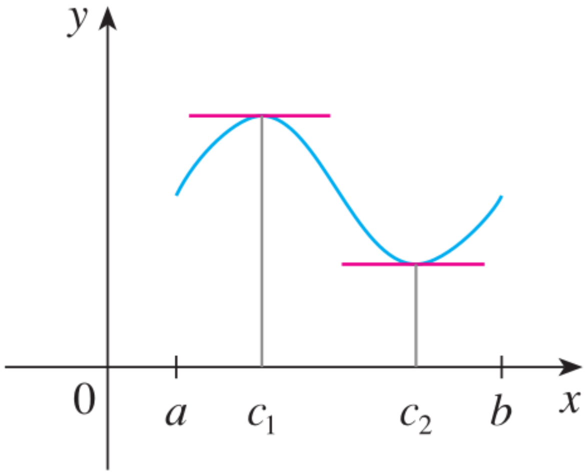
Chapter 3
Applications of Derivatives

3.2 The Mean Value Theorem

The following graphs have a commun geometric property.



(b)



(c)

Is there a condition that garantees the graph of a function has horizontal tangents?

Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

EXAMPLE 2 Prove that the equation $x^3 + x - 1 = 0$ has exactly one real root.

The Mean Value Theorem Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

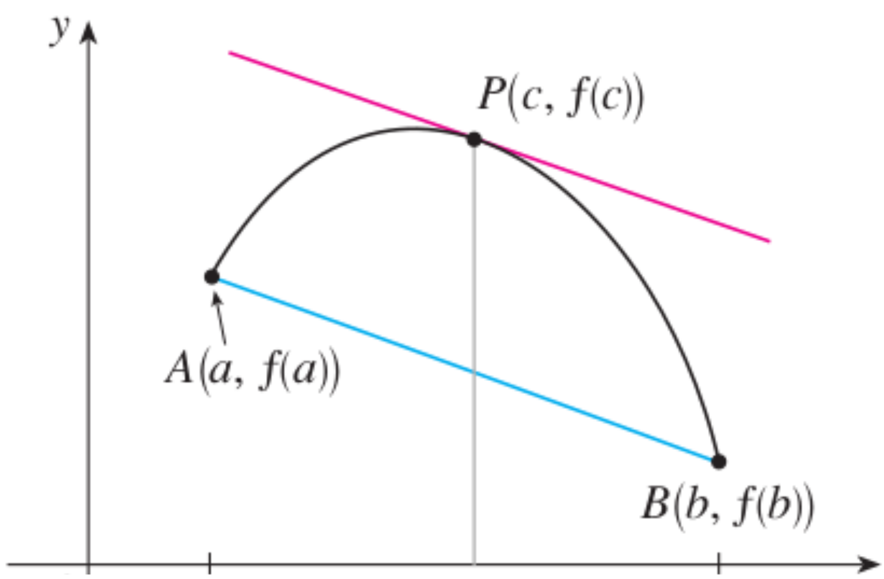
1 $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

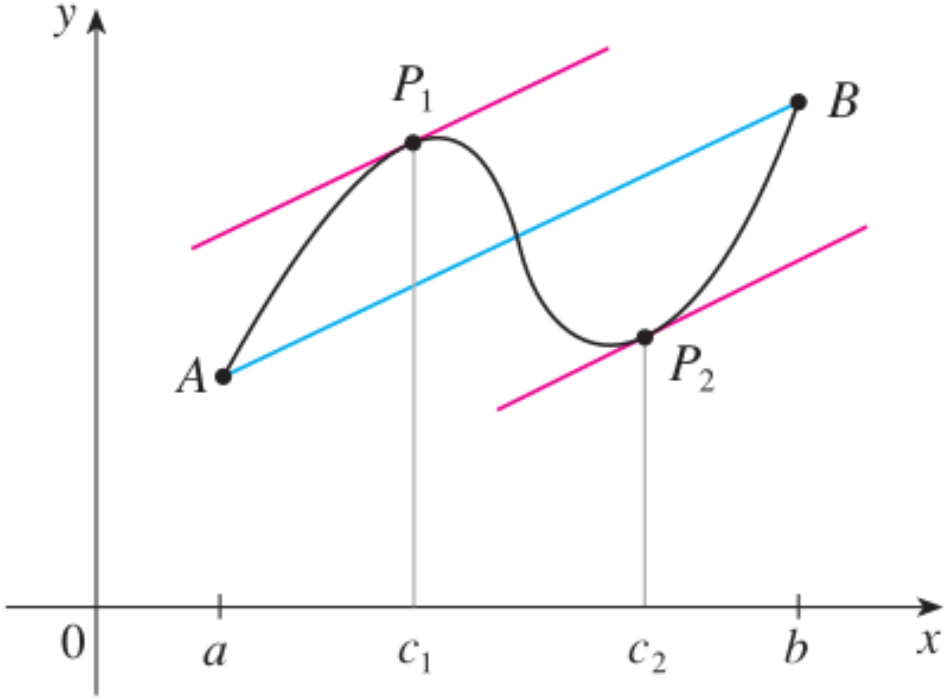
2 $$f(b) - f(a) = f'(c)(b - a)$$

The Meaning: Find an equivalent slope of the secant line with the slope of one of the tangent line.

Only one c.



Multiple c.



Example

Let $f(x) = \sqrt{x}$. Find the number c that satisfies the conclusion of the Mean Value Theorem on the interval [0, 4].

Consequences of the Mean Value Theorem.

$\boxed{5}$ **Theorem** If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

$\boxed{7}$ **Corollary** If $f'(x) = g'(x)$ for all x in an interval (a, b), then $f - g$ is constant on (a, b); that is, $f(x) = g(x) + c$ where c is a constant.

EXAMPLE 5 Suppose that $f(0) = -3$ and $f'(x) \leqslant 5$ for all values of x. How large can $f(2)$ possibly be?