

F.I Moments

PROBLEM 1. If X is uniformly distributed on (a, b) , show that

$$\text{Exp}(X^k) = \frac{b^{k+1} - a^{k+1}}{(b - a)(k + 1)} \quad \text{for } k = 1, 2, \dots$$

F.II Moment Generating Function

PROBLEM 2. If X has the normal distribution with mean 0 and variance 1, find $E(X^3)$.

PROBLEM 3. Show that, if X has a normal distribution, then so does $aX + b$, for any $a, b \in \mathbb{R}$ with $a \neq 0$.

PROBLEM 4. Suppose that the waiting time for the first customer to enter a retail shop after 9:00AM is a random variable X with an exponential density function given by

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & x > 0, \\ 0 & \text{elsewhere.} \end{cases}$$

- a) Find the moment-generating function of X .
- b) Use the answer from part (a) to find $\text{Exp}(X)$ and $\text{Var}(X)$.