B.I Conditional Probabilities

Example 1. Statement of the example will be written on the board.

Example 2. An urn contains 10 blue balls, 5 red balls, and 10 green balls. A ball is chosen at random from the urn and it is noted that it is not one of the green balls. What is the probability that it is red?

Example 3. Celine is undecided as to whether to take a French or a Chemistry class. The probability of obtaining an A grade if Celine takes French is $1/2$ and if Celine takes Chemistry is $2/3$. If Celine decides to base her course choice on a flip of a fair coin, what is the probability that Celine gets an A in Chemistry?

B.II Bayes' Formula

Example 4. An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that:

- If a person is accident-prone, this person will have an accident at some time within a fixed 1-year period with probability .4.
- If a person is non-accident-prone, this person will have an accident at some time within a fixed 1-year period with probability .2.

We assume that 30% of the population is accident prone.

a) What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

b) If a new policyholder has an accident within a year of purchasing a policy, what is the probability that the person is accident prone?

Example 5. In a show, you are asked to choose between three doors disposed in front of you at random. Behind one of the door is a big money price and behind the other two, nothing... You choose to open the first door, but before the game host opens the door, he opens at random one of the other two doors, say door 3, to show you there is nothing behind it. He then asks you: Do you want to switch door? What should you do, switch door or not?

Partition Theorem

Example 6. Suppose that we have 3 cards identical in form. They each have the following characteristics:

1st card Both sides are red.

2st card Both sides are colored black.

3rd card One side is colored red and the other side black.

The three cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Example 7. If coin X is flipped, it comes up heads with probability 1/4, whereas if coin Y is flipped it comes up heads with probability 3/4. Suppose that one of these coins is randomly chosen and is flipped twice. If both flips land heads, what is the probability that coin Y was the one flipped?

B.III Independent Events

Example 8. A card is selected at random from an ordinary deck of 52 playing cards. If A is the event that the selected card is an ace and B is the event that it is a spade, then show that A and B are independent.

Example 9. Suppose that we toss 2 fair dice. Let A denote the event that the sum of the dice is 6 and B denote the event that the first die equals 3. Are A and B independent?

Example 10. Statement of the example will be written on the board.

Experiments Made Up From Sub-experiments

Example 11. An experiment consists of continually tossing a coin, where each toss are independent from each other. The coin lands on head with probability p and on tail with probability $1 - p$. What is the probability that

a) at least 1 head occurs in the first n tosses;

b) exactly k heads occur in the first n tosses;

c) all tosses result in heads?