MATH-241 Calculus 1 — Created by P.-O. Parisé
Homework 05 — Fall 2021

Assigned date: 10/25/2021 9am
Due date: 11/01/2021 5pm

Last name: _____
First name: _____
Section: _____

Question:	1	2	3	Total
Points:	20	15	15	50
Score:				

Instructions: You must answer all the questions below and upload your solutions (in a PDF format) to Gradescope (go to `www.gradescope.com` with the Entry code GEK6Y4). Be sure that after you scan your copy, it is clear and readable. You must name your file like this: `LASTNAME_FIRSTNAME.pdf`. A homework may not be corrected if it's not readable and if it's not given the good name. No other type of files will be accepted (no PNG, no JPG, only PDF) and no late homework will be accepted.

Make sure to show all your work!

Good luck!

QUESTION 1 **(20 points)**

Approximate $\sqrt{3}$ with Newton's method.

(a) (5 points) Explain why we can't use $x_0 = 0$ as an initial guess.

Solution: From Newton's method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

But, in this situation, we have $f(x) = x^2 - 3$ and so $f'(x) = 2x$. We can't start with $x_0 = 0$ because we will divide by 0.

(b) (10 points) Use the starting point $x_0 = 20$ and do 5 iterations.

Solution: With the function $f(x) = x^2 - 3$, we have

$$x_{n+1} = x_n - \frac{x^2 - 3}{2x}.$$

So, we get

- $x_1 = 20 - \frac{20^2-3}{40} = 10.075.$
- $x_2 = 10.075 - \frac{(10.075)^2-3}{2 \cdot 10.075} \approx 2.186383375.$
- $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 2.882410569.$
- $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \approx 1.961603043.$
- $x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} \approx 1.745482228.$

(c) (5 points) How many iterations are required to obtain an approximation of $\sqrt{3}$ right for 10 decimal places? (You can use a spreadsheet to do that. Make sure to join your spreadsheet to your homwork as a justification of your work.)

Solution: It takes at least 8 iterations to obtain an approximation of $\sqrt{3}$ right for 10 decimal places.

n	x_n	20
0	20	
1	10.075	
2	5.18638337469	
3	2.8824105691	
4	1.96160304332	
5	1.74548222763	
6	1.7321024847	
7	1.73205080834	
8	1.73205080757	
9	1.73205080757	
10	1.73205080757	

QUESTION 2 **(15 points)**

(a) (5 points) Express each sum with the summation $1+1/3+1/5+1/7+1/9+1/11+1/13$ in compact form with the symbol $\sum$.

Solution: The answer is

$$\sum_{k=1}^{7} \frac{1}{2k-1}.$$

(b) (5 points) Express each sum with the summation $1+4+7+10+13+16+19+22$ in compact form with the symbol $\sum$.

Solution: The answer is

$$\sum_{k=0}^{7}(3k+1).$$

(c) (5 points) Expand the expression $\sum_{k=0}^{n}(\frac{k+1}{2})^2$.

Solution: The solution is

$$\frac{1}{4}+1+\frac{9}{4}+\cdots+\frac{(n+1)^2}{4}.$$

QUESTION 3 **(15 points)**

Using the definition of the integral and the upper sums, compute the following definite integral

$$\int_1^2 x\,dx.$$

Solution: Take $\Delta x = \frac{2-1}{n} = \frac{1}{n}$ and $x_i = 1 + i\Delta x = 1 + \frac{i}{n}$. Here, we have $f(x) = x$. So,

$$\begin{aligned}
\int_1^2 x\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)\frac{1}{n} \\
&= \lim_{n\to\infty} \frac{1}{n^2} \sum_{i=1}(i+n) \\
&= \lim_{n\to\infty} \frac{1}{n^2}\left(\sum_{i=1}^{n} i + \sum_{i=1}^{n} n\right) \\
&= \lim_{n\to\infty} \frac{1}{n^2}\left(\frac{n(n+1)}{2} + n^2\right) \\
&= \lim_{n\to\infty} \frac{n+1}{2n} + \frac{n}{n+1} \\
&= \frac{1}{2} + 1 \\
&= \frac{3}{2}.
\end{aligned}$$

So, we get

$$\int_0^2 x\,dx = \frac{3}{2}.$$