Chapter 3
Applications of Derivatives

3.5 Summary of Curve Sketching

EXAMPLE 1 Use the guidelines to sketch the curve $y = \dfrac{2x^2}{x^2 - 1}$. $= f(x)$

(A) Domain. $x^2-1 = (x-1)(x+1) = 0 \iff x = -1,\ x = 1$

is $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$.

(B) y-interc. & x-interc.

$f(0) = \frac{0}{?}$ & $f(x) = 0 \iff 2x^2 = 0$

$\iff x = 0$?

(C) Symmetries.

- Odd or even: $f(-x) = \dfrac{2(-x)^2}{(-x)^2-1} = \dfrac{2x^2}{x^2-1} = f(x)$

↳ even. ?

(D) HA & VA.

HA: $\lim\limits_{x\to\infty} \dfrac{2x^2}{x^2-1} = \dfrac{2}{1} = 2 \to y = 2$ HA. $x \to \infty$

$\lim\limits_{x\to-\infty} \dfrac{2x^2}{x^2-1} = \dfrac{2}{1} = 2 \to y = 2$ HA $x \to -\infty$

VA: $x = 1$ $\lim\limits_{x\to1^-} \dfrac{2x^2}{x^2-1} = \dfrac{2}{0^-} = -\infty$

$\lim\limits_{x\to1^+} \dfrac{2x^2}{x^2-1} = \dfrac{2}{0^+} = +\infty$ VA ?

$x = -1$ $\lim\limits_{x\to-1^-} \dfrac{2x^2}{x^2-1} = \dfrac{2}{0^+} = +\infty$

$\lim\limits_{x\to-1^+} \dfrac{2x^2}{x^2-1} = \dfrac{2}{0^-} = -\infty$ VA ?

(E) Interv. increase/decr.

$$f'(x) = -\frac{4x}{(x^2-1)^2} = \frac{-4x}{((x-1)(x+1))^2}$$ → C.N.: $0, -1, 1$

$(x^2-1)^2 \geq 0$ → $f'(x) > 0$ when $-4x > 0$

when $x < 0$

→ f is ↑ on $(-\infty, 0)$

$(x^2-1)^2 \geq 0$ → $f'(x) < 0$ when $-4x < 0$

when $x > 0$

→ f is ↘ on $(0, \infty)$.

(F) Max & Min.

$f \nearrow$ on $(-\infty, 0)$ & $f \searrow$ on $(0, \infty)$

$\Rightarrow$ $x = 0$ is a loc. Max.

& $f(0) = 0$.

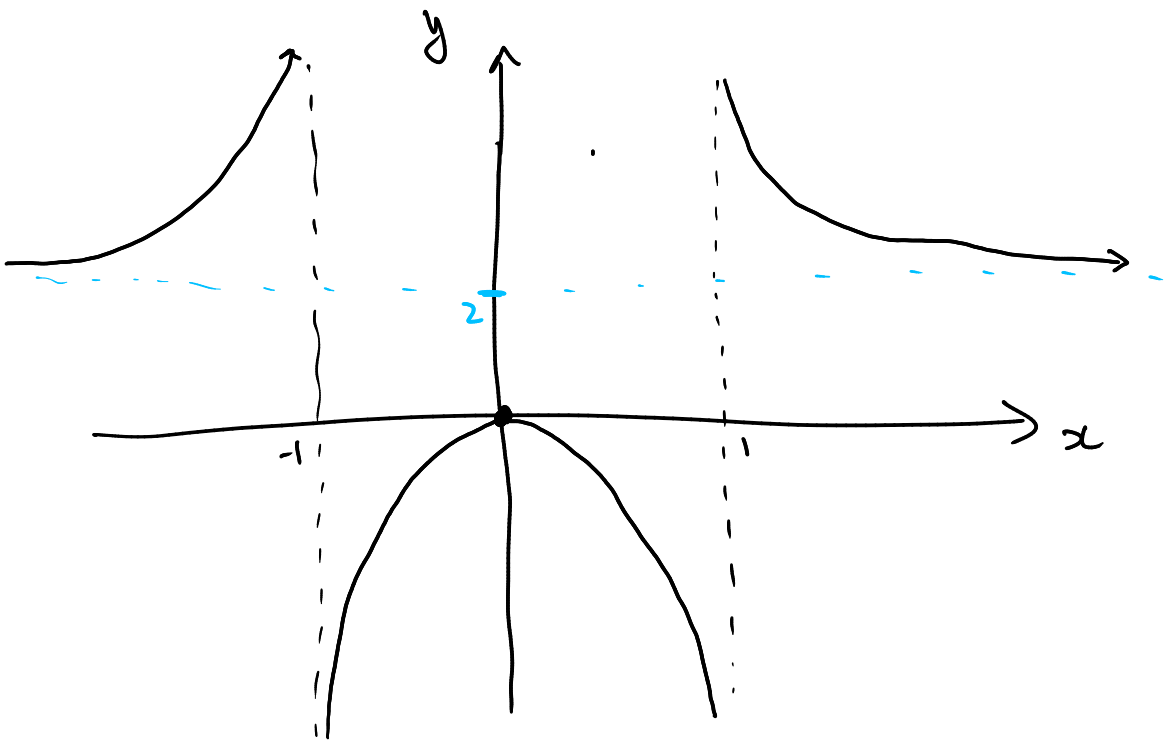
(G) Concavity.

$$f''(x) = \frac{12x^2+4}{(x^2-1)^3} = \frac{12x^2+4}{((x-1)(x+1))^3}$$ ($12x^2+4 \geq 0$) → f'' ∄ when $x = 1$ & $x = -1$

factors	$x <$	-1	$< x <$	1	$< x$
$(x-1)^3$	−	∄	−		+
$(x+1)^3$	−		+		+
$f''(x)$	+		−		+
	concave up on $(-\infty, -1)$		concave down on $(-1, 1)$		concave up on $(1, \infty)$.

(H.) Sketch.

x	$x <$	-1	$< x <$	0	$< x <$	1	$< x$
$f'(x)$	$+$ ↗	$\nexists$	$+$ ↗	0	$-$ ↘	$\nexists$	$-$ ↘
$f''(x)$	$+$ ∪	$\nexists$	$-$ ∩	$*$	$-$ ∩	$\nexists$	$+$ ∪
$f(x)$	↗	$\nexists$ V.A.	↗	loc max $f(0)=0$	↘	$\nexists$ V.A.	↘



Guidelines for Sketching a Curve.

A. Find the domain of the function.

B. Find the y-intercept and x-intercept, that is $f(0)$ and when $y = 0$.

C. Search for symmetries in the function (facultative)

- If $f(x) = f(-x)$, then the function is even.
- If $-f(x) = f(-x)$, then the function is odd.
- If $f(x + p) = f(x)$, then the function repeats itself after a period p (it is periodic).

D. Find the asymptotes of the function:

- The Horizontal asymptotes.
- The Vertical asymptotes.

E. Find the intervals of increase and decrease.

F. Find the local maximum and minimum values.

G. Find the concavity and the points of inflections.

H. Sketch.