P.-O. Parisé

Section 1.8, Problem 18

As $x \to -2^-$, we have $f(x) \to -\infty$ and as $x \to -2^+$, we have $f(x) \to \infty$. So we have an infinite discontinuity.

Section 1.8, Problem 58 (a)

We have $f(0) = 3$ and $f(-1) = -1 - 1 - 2 + 3 = -1$. So, we have $f(-1) < 0$ and $f(0) > 0$. So, by the intermediate Theorem, with $N = 0$, there is a number $c \in (-1, 0)$ such that $f(c) = 0$.

Section 2.1, Problem 5

The equation of the tangent line at the point $(x_0, y_0) = (2, -4)$ is

$$y + 4 = m(x - 2)$$

where $m = f'(2)$. The derivative is given by the limit of the different quotient:

$$\begin{aligned}\frac{f(2+h) - f(2)}{h} &= \frac{4(2+h) - 3(2+h)^2 + 4}{h} \\ &= \frac{8 + 4h - 3(4 + 4h + h^2) + 4}{h} \\ &= \frac{-4 - 8h - 3h^2 + 4}{h} \\ &= -8 - 3h\end{aligned}$$

and as $h \to 0$, we get $f'(2) = -8$. So, we get

$$y + 2 = -8(x - 2).$$

Section 2.2 (b), (d), Problem 2 (only estimate the derivatives)

(b) It is straigth forward from the graph that $f'(1) \approx 0$.

(d) At $x = 3.5$, $f(3.5) \approx -0.5$ and at $x = 2.5$, $f(2.5) \approx 0.6$. So, we can approximate the derivative, with $h = 0.5$:

$$f'(3) \approx \frac{-0.5 - 0}{0.5} = -1$$

and with $h = -0.5$:

$$f'(3) \approx \frac{0.6 - 0}{-0.5} = -\frac{6}{5}.$$

If we want a better approximation, we can take the average of these values:

$$f'(3) \approx \frac{-1 - 6/5}{2} = -11/10.$$

Section 2.2, Problem 25

The domain of the function is $(-\infty, 9]$. The derivative at x is

$$\begin{aligned} f'(x) = \lim_{h\to 0} \frac{\sqrt{9-x-h} - \sqrt{9-x}}{h} &= \lim_{h\to 0} \frac{9-x-h-9+x}{h(\sqrt{9-x-h} + \sqrt{9-x}} \\ &= \lim_{h\to 0} -\frac{1}{\sqrt{9-x-h} + \sqrt{9-x}} \\ &= -\frac{1}{2\sqrt{9-x}}. \end{aligned}$$

So $f'(x) = -1/2\sqrt{9-x}$ and the domain of f' is $(-\infty, 9)$.