Math 241

Chapter 3

Section 3.9: Antiderivatives

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Definition

A function F is an **antiderivative** of a function f if $F'(x) = f(x)$.

Example 1. Find an antiderivative of the following functions.

(a) $f(x) = x^2$.

(b) $g(x) = 3x^3 + \cos(x)$.

(c) $h(x) = x^{2/3} + 4\sec^2(x)$.

Remarks:

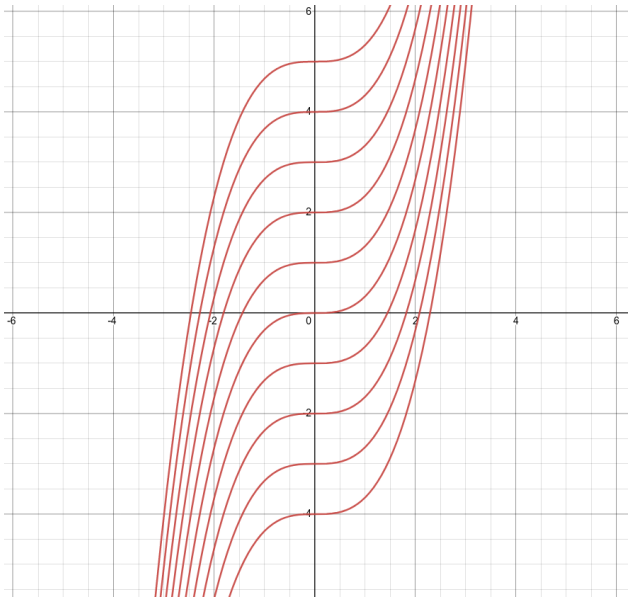
- Recall that $f'(x) = g'(x)$ if and only if $f(x) = g(x) + C$ for some constant C.
- There are more than just one antiderivative!

General Antiderivatives

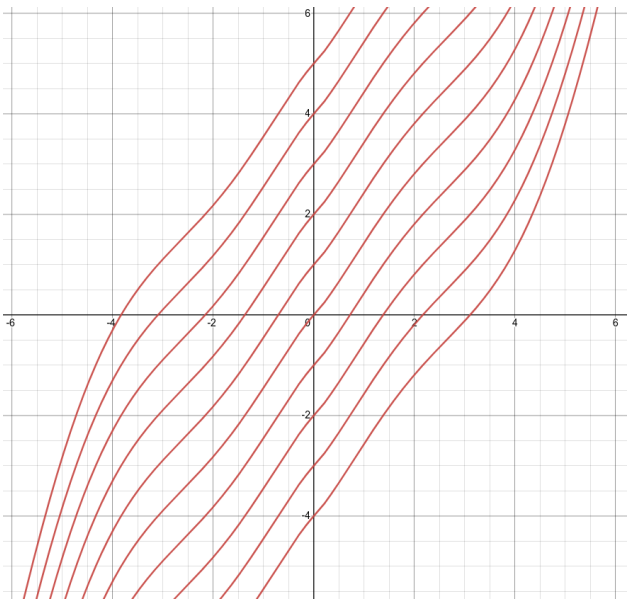
The **most general antiderivative** of a function f is

$$F(x) + C,$$

where C is a constant.



(a) Several Antiderivatives of $f(x) = x^2$, that is $\frac{x^3}{3} + C$



(b) Several antiderivatives of $f(x) = x^{2/3} + \cos(x)$, that is $\frac{3}{5}x^{5/3} + \sin(x) + C$.

Example 2. Find the most general antiderivative of each of the following functions.

(a) $f(x) = \sin x$.

(b) $f(x) = x^n$, $n \geq 0$.

Table of Antiderivatives

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\cos x$	$\sin x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sin x$	$-\cos x$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec^2 x$	$\tan x$
		$\sec x \tan x$	$\sec x$

Figure 2: Properties and some Antiderivatives

Example 3. Find all functions g such that

$$g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}.$$

Initial Condition

Example 4. Find F if $F'(x) = x\sqrt{x}$ and $F(1) = 2$.

Example 5. Find F if $F'(x) = \frac{1}{x^2}$ and $F(1) = 2$.