

PROBLEM 1. Two fair dice are rolled. What is the probability that at least one lands on 6 given that the dice land on different numbers?

PROBLEM 2. Suppose that an urn contains 35 red balls and 20 blue balls. We draw 2 balls from the urn without replacement. If we assume that every ball is equally likely to be drawn, what is the probability that both balls drawn are red? Use the formula in Corollary 1.

PROBLEM 3. A total of 46% of the voters in a certain city classify themselves as Independents, whereas 30% classify themselves as Liberals and 24% as Conservative. In a recent local election, 35% of the Independents, 62% of the Liberals, and 58% of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that the person is a) an Independent? b) a Liberal? c) a Conservative?

PROBLEM 4. When a dice x is tossed it lands on \square with probability $1/2$ and all the other outcomes are equally likely to happen. When a dice y is tossed, it lands on \square with probability $1/2$ and all the other outcomes are equally likely to happen. Suppose that one of these dice is randomly chosen and then tossed. What is the probability that dice x was tossed, if the die landed on \square ?

PROBLEM 5. Let A, B be two events.

- a) If $P(A) = 0.2$, $P(B) = 0.3$, and $P(A \cap B) = 0.3$, are A and B independent?
- b) If $P(A) = 0.2$, $P(B) = 0.3$, and $P(A \cap B) = 0.06$, are A and B independent?

PROBLEM 6. Let (S, \mathcal{A}, P) be a probability space. Suppose that two events A and B are given such that $P(A) > 0$, $P(B) > 0$. Prove that if $P(A) < P(A|B)$, then $P(B) < P(B|A)$.

PROBLEM 7. Suppose that $A \subset B$ and that $P(A) > 0$ and $P(B) > 0$. Show that $P(B|A) = 1$ and $P(A|B) = P(A)/P(B)$.

PROBLEM 8. If A and B are mutually exclusive events and $P(B) > 0$, show that

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}.$$

PROBLEM 9. A system composed of 5 separate components is said to be a parallel system if it functions when at least one of the components functions. For such a system, if component i , independent of other components, functions with probability p_i , $i = 1, 2, \dots, 5$, what is the probability that the system functions?