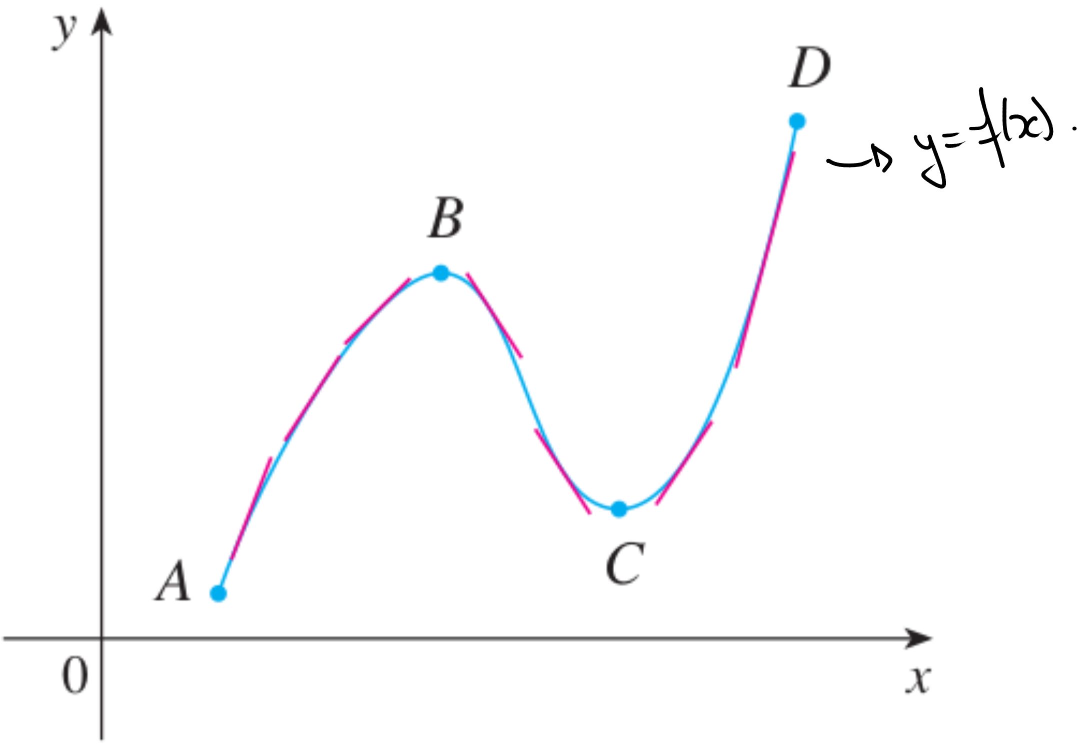
Chapter 3
Applications of Derivatives

3.3 How Derivatives affect the Shape of a Graph

What does f' tells us about f.



	A	between	B	between	C	between	D
$f'(x)$	DNE	+	0	−	0	+	DNE
$f(x)$	Abs. min.	↗	loc. max.	↘	loc. min.	↗	Abs. max.

Conclusion:

Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

EXAMPLE 1 Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.

① <u>Derivative</u> $f'(x) = 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$= 12x(x+1)(x-2)$

② <u>Zeros:</u> $f'(x) = 0 \iff x=0,\ x=-1,\ x=2$

$x<-1$

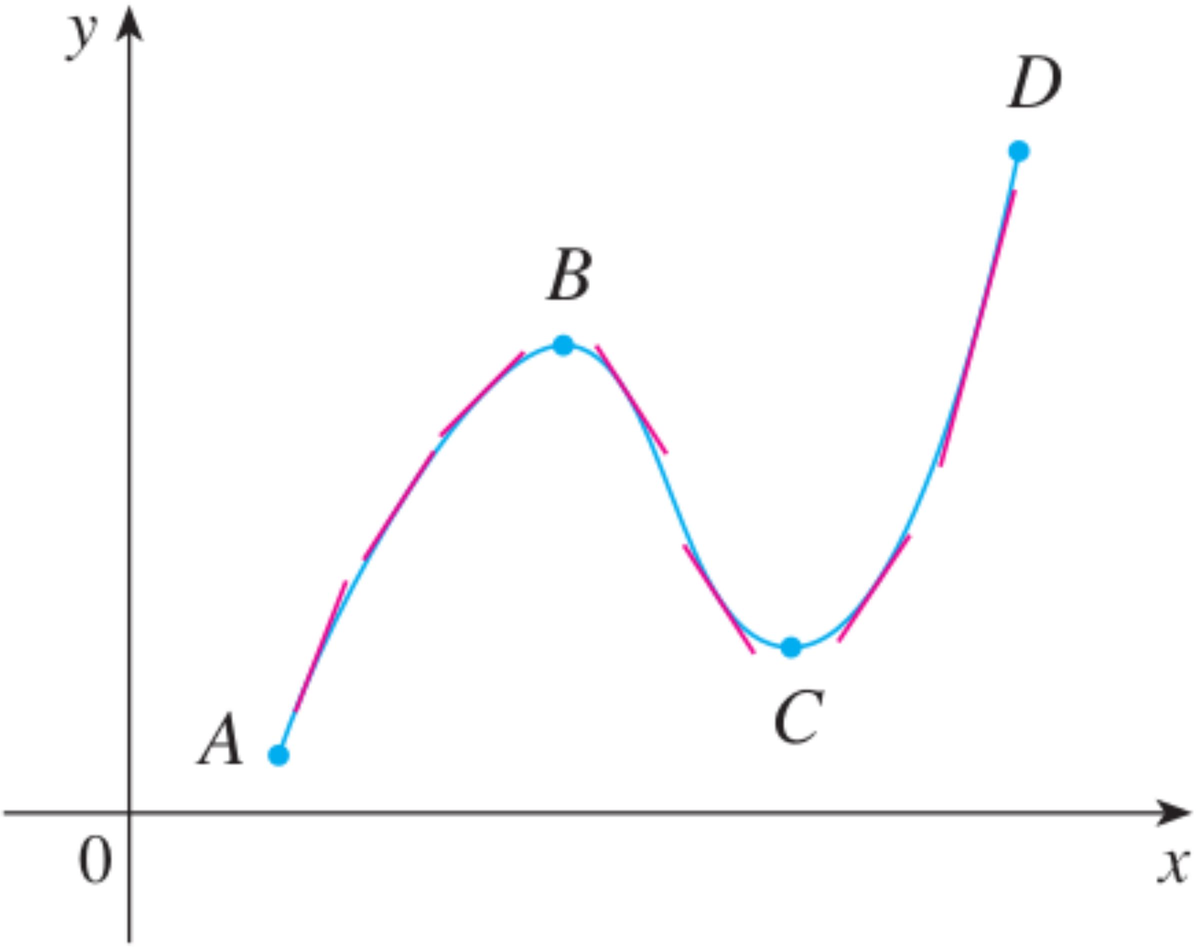
Factors	$x<$	-1	$< x <$	0	$< x <$	2	$< x$
$x+1$	$-$	0	$+$	X	$+$	X	$+$
$x-2$	$-$	X	$-$	X	$-$	0	$+$
x	$-$	X	$-$	0	$+$	X	$+$
$f'(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\searrow$	loc. min	$\nearrow$	loc max	$\searrow$	loc min	$\nearrow$

$x<-1$ $\left[\begin{array}{l} x<-1 \to x+1 < -1+1 \to x+1<0 \\ x<-1 \to x-2 < -1-2 \to x-2<-3 \\ x<-1 \end{array}\right.$

$-1<x<0$ $\left[\begin{array}{l} x>-1 \to x+1>0 \\ -1<x<0 \to -1-2 < x-2 < 0-2 \to -3 < x-2 < -2 \end{array}\right.$

Local Extreme Values.

CRITICAL POINTS $\not\longrightarrow$ EXTREME VALUES (MAX OR MIN)



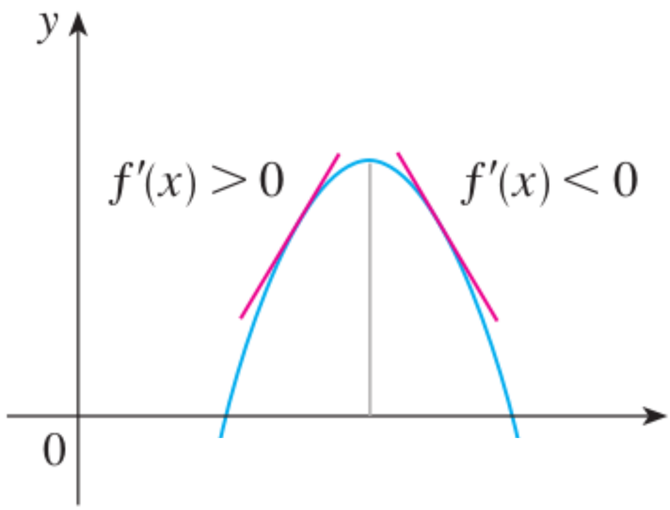
	A		B		C		D
$f'(x)$	$\nexists$	+	0	-	0	+	$\nexists$
$f(x)$	abs. min	$\nearrow$	max	$\searrow$	min	$\nearrow$	abs. max

The First Derivative Test Suppose that c is a critical number of a continuous function f.

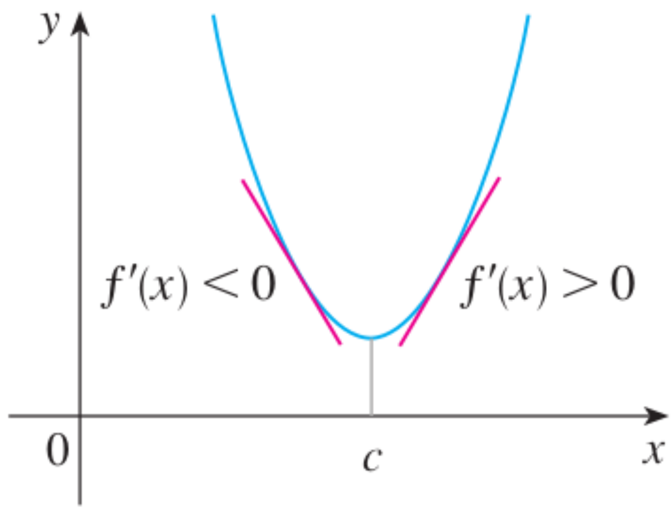
(a) If f' changes from positive to negative at c, then f has a local maximum at c.

(b) If f' changes from negative to positive at c, then f has a local minimum at c.

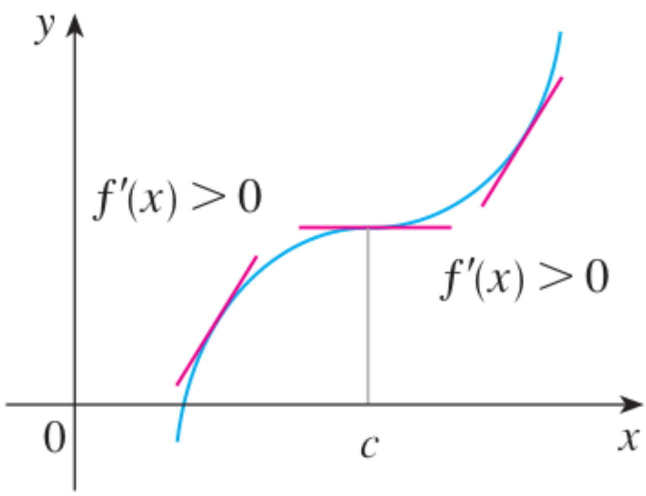
(c) If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.



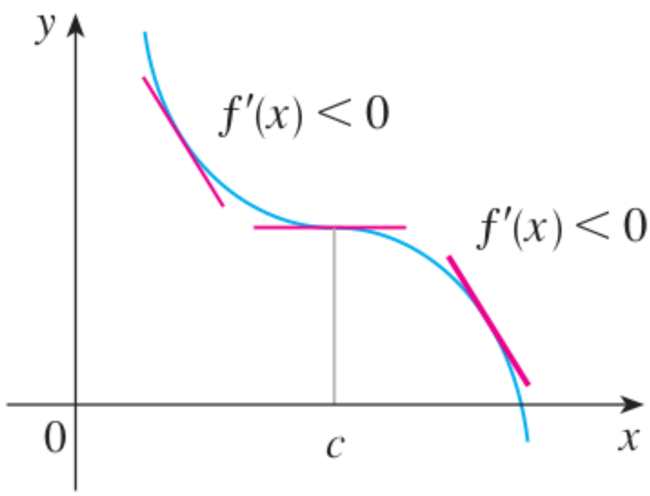
(a) Local maximum



(b) Local minimum



(c) No maximum or minimum



(d) No maximum or minimum

EXAMPLE 3 Find the local maximum and minimum values of the function

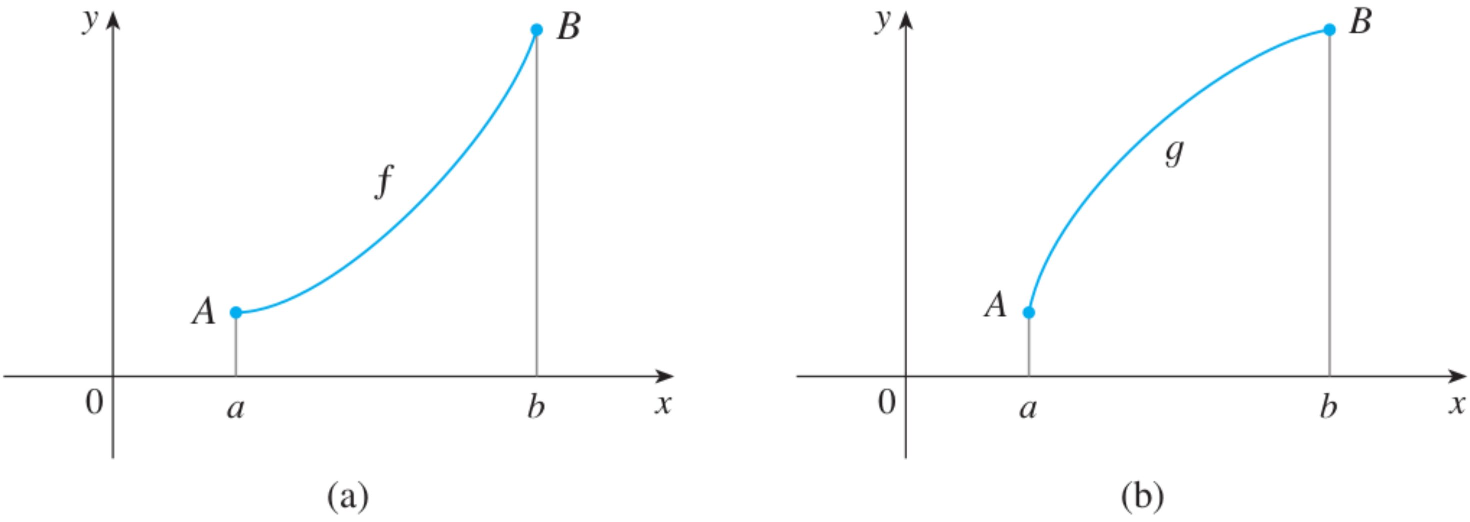
$$g(x) = x + 2 \sin x \qquad 0 \leqslant x \leqslant 2\pi$$

What does f'' tell us about f?

Two important definitions:

1) **Definition** If the graph of f lies above all of its tangents on an interval I, then it is called **concave upward** on I. If the graph of f lies below all of its tangents on I, it is called **concave downward** on I.

2) **Definition** A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P.



Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Note: There is an inflection point when the second derivative is zero.

Example. Find the interval(s) of concavity of the fur ion .

$$f(x) = x^3 - 3x^2 - 9x + 4$$

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

REMARK!

EXAMPLE. Find the extreme values of the function $f(x) = x^3 + 3x^2$.