Math 302

Chapter 5

Section 5.3: Nonhomogeneous Linear Equations

Contents

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Fall 2022

Particular Solutions

Our goal is to find the solutions to

$$y'' + p(x)y' + q(x)y = f(x). \tag{1}$$

<u>Nomenclature:</u>

- the equation $y'' + p(x)y' + q(x)y = 0$ is the **complementary equation** for (1).
- a **particular solution** is a solution y_{par} of (1).

Example 1. Find a particular solution to the following ODE:

$$y'' - 2y' + y = 4x.$$

Assumptions:

- Suppose y_{par} is a particular solution to

$$y'' + p(x)y' + q(x)y = f(x).$$

- Suppose $\{y_1, y_2\}$ is a fundamental set of solutions to

$$y'' + p(x)y' + q(x)y = 0.$$

Conclusion:

- Then the $y = y_{par} + c_1y_1 + c_2y_2$ is the general solution of

$$y'' + p(x)y' + q(x)y = f(x).$$

Example 2.

a) Find the general solution of

$$y'' - 2y' + y = -3 - x + x^2.$$

b) Solve the following IVP:

$$y'' - 2y' + y = -3 - x + x^2, \quad y(0) = -2, \, y'(0) = 1.$$

If y_1 is a particular solution to

$$y'' + p(x)y' + q(x)y = f_1(x)$$

and y_2 is a particular solution to

$$y'' + p(x)y' + q(x)y = f_2(x)$$

then $y_{par} = y_1 + y_2$ is a particular solution to

$$y'' + p(x)y' + q(x)y = f_1(x) + f_2(x).$$

EXAMPLE 3. Find a particular solution to

$$y'' + y = 1 + x^2 + x^4.$$