Math 471 — **Homework Problems Set** — Chapter A — Fall 2023

Problem 1. Let $(S, \mathcal{A}, P)$ be a probability space. If A and B are two events, then show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Problem 2. Let S be a non-empty set and let A be a non-empty subset of S such that A is not all of S. If $\mathcal{A} = \{\varnothing, A, \overline{A}, S\}$, then show that all probability measures on $\mathcal{A}$ have the form

$$P(\varnothing) = 0 \qquad P(A) = p$$
$$P(\overline{A}) = 1 - p \qquad P(S) = 1$$

for some number p satisfying $0 \leq p \leq 1$.

Problem 3. Let $S = \{s_1, s_2, \ldots, s_N\}$ be a sample space with exactly N outcomes, and let $\mathcal{A}$ be the family of all subsets of S. Show that the function $P : \mathcal{A} \to \mathbb{R}$ defined by

$$P(A) = \frac{|A|}{N} \quad (A \text{ is an event})$$

is a probability measure.

Problem 4. A boxcar contains six complex electronic systems. Two of the six are to be randomly selected for thorough testing and then classified as defective or not defective. Two of the six systems are defective. Find the probability that one of the two systems selected will be defective.

Problem 5. A poker hand consists of 5 cards. If the cards have distinct consecutive numerical values and are not all of the same suit, we say that the hand is straight. For instance, the hands 5♠ 6♠ 7♠ 8♠ 9♥ or 6♥ 7♠ 8♦ 9♣ 10♥ are straight hands. What is the probability that one is dealt a straight hand? Assume that all possible pocker hands are equally likely.

Problem 6. Three teams of three people have to be selected from a group of 5 mathematicians, 2 engineers, 1 astrophysicist and 1 atmospheric scientist. If the selection is made randomly, what is the probability that at least one person in each team is a mathematician?

Problem 7. [Extra] A poker hand consists of 5 cards. What is the probability that a poker hand consists of all cards of the same suite and not consecutive values[1]. A valid example would be 2♥ 4♥ 5♥ 6♥ 7♥. Examples of non-valid poker hands are 2♥ 3♥ 4♥ 5♥ 6♥ and 2♥ 4♠ 5♠ 6♠ 7♠. Also, it is prohibited to create sequences of consecutive numbers by connecting the Ace with the 2. For example, the poker hand K♥ A♥ 2♥ 3♥ 4♥ is not valid. Assume all poker hands are equally likely.

Problem 8. Let B_1, B_2, … be the list of events defined in the proof of Theorem 1. Show that

$$\bigcup_{i=1}^{\infty} A_i = \bigcup_{i=1}^{\infty} B_i.$$

[1] Here, by the "value" of a card, we mean the numerical values from 2 to 10 together with the Ace, King, Queen, Jack.