Example 1

Find the general indefinite integral of

$$\int (10x^4 - 2\sec^2 x)\, dx.$$

$$\int 10x^4 - 2\sec^2 x \, dx = \int 10x^4 dx + \int (-2\sec^2 x)\, dx$$

$$= 10\int x^4 dx - 2\int \sec^2 x \, dx$$

$$= 10\frac{x^5}{5} + C_1 - 2\tan x + C_2$$

$$= 10\frac{x^5}{5} - 2\tan x + C$$

where $C = C_1 + C_2$.

$$\int 10x^4 - 2\sec^2 x \, dx = 2x^5 - 2\tan x + C$$

Example 2

Evaluate the indefinite integral $\int \frac{\cos\theta}{\sin^2\theta}\, d\theta.$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$

$$\int \frac{\cos\theta}{\sin^2\theta}\, d\theta = \int \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}\, d\theta = \int \cot\theta \cdot \csc\theta\, d\theta$$

$$= \csc\theta + C$$

Example 3

Evaluate $\displaystyle\int \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\,dt.$

$t^2 \cdot t^{1/2} = t^{2+1/2} = t^{5/2}$ $\qquad \frac{t^{5/2}}{t^2} = t^{5/2-2}$

$$\int \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\,dt = \int \frac{2t^2}{t^2} + \frac{t^2\sqrt{t}}{t^2} - \frac{1}{t^2}\,dt$$

$$= \int 2 + \sqrt{t} - \frac{1}{t^2}\,dt$$

$$= 2\int 1\,dt + \int \underbrace{\sqrt{t}}_{t^{1/2}}\,dt - \int \frac{1}{t^2}\,dt \qquad \frac{1}{t^2} = t^{-2}$$

$$= 2t + C_1 + \frac{t^{3/2}}{3/2} + C_2 - \frac{t^{-1}}{-1} + C_3$$

$$= 2t + \frac{2}{3}t^{3/2} + \frac{1}{t} + \underbrace{C_1 + C_2 + C_3}_{=C}$$

$$= 2t + \frac{2}{3}t^{3/2} + \frac{1}{t} + C$$

Example 4

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

a) Find the displacement of the particle during the time period $1 \le t \le 4$.

b) Find the distance traveled during this time period.

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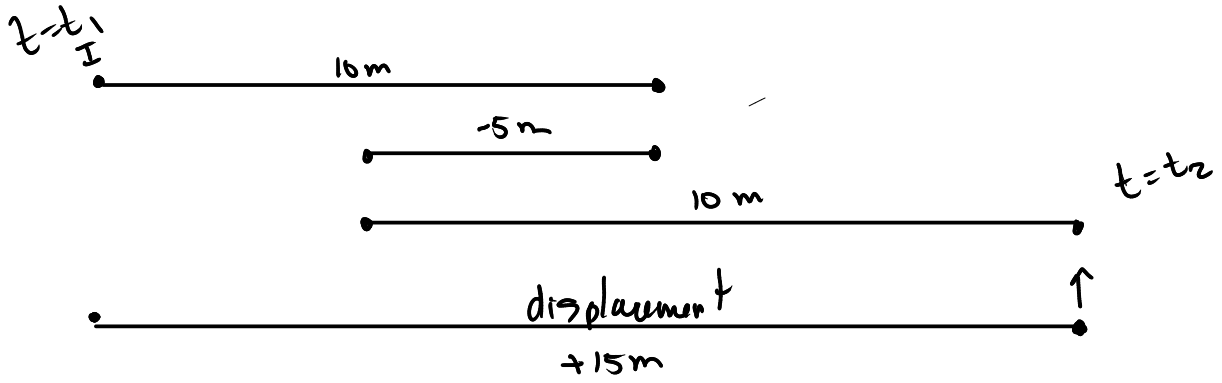
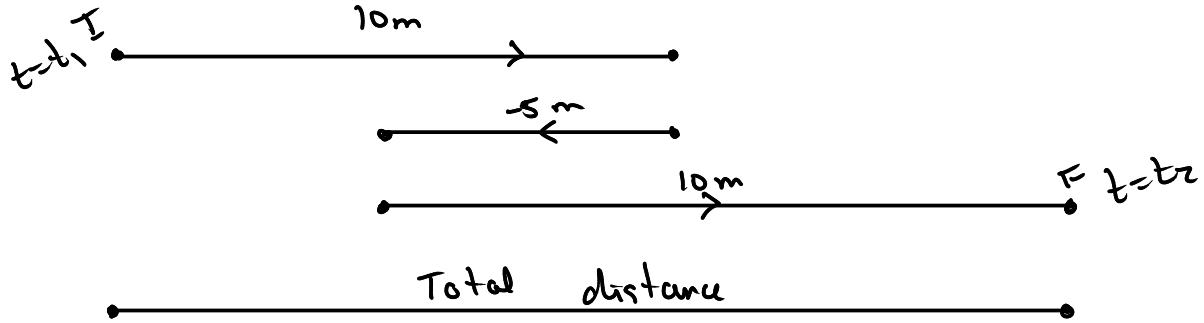
two concepts.

① Displacement $= \int_{t_1}^{t_2} v(t)\,dt$



② Distance $= \int_{t_1}^{t_2} |v(t)|\,dt$



$10 + 5 + 10 = 25$ m.

a) Displacement $= \int_1^4 t^2 - t - 6\,dt = \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big|_1^4$

$= \frac{4^3}{3} - \frac{4^2}{2} - 6\cdot 4 - \left(\frac{1}{3} - \frac{1}{2} - 6\right)$

$= -4.5$ m.

b) Total distance $= \int_1^4 |t^2 - t - 6t|\,dt$

We have to find an explicit expression for $|t^2 - t - 6|$:

$$|t^2 - t - 6| = \begin{cases} t^2 - t - 6 & \text{if } t^2 - t - 6 \ge 0 \\ -(t^2 - t - 6) & \text{if } t^2 - t - 6 < 0. \end{cases}$$

When is $v(t) \geq 0$

$t^2 - t - 6 \geq 0$ if $(t-6)(t+1) \geq 0$

if $t-6 \geq 0$ and $t+1 \geq 0$

or $t-6 \leq 0$ and $t+1 \leq 0$

if $t \geq 6$ and $t \geq -1$

or

$t \leq 6$ and $t \leq -1$

if $\boxed{t \geq 6 \text{ or } t \leq -1}$

Since $1 \leq t \leq 4$, then $v(t) \leq 0$

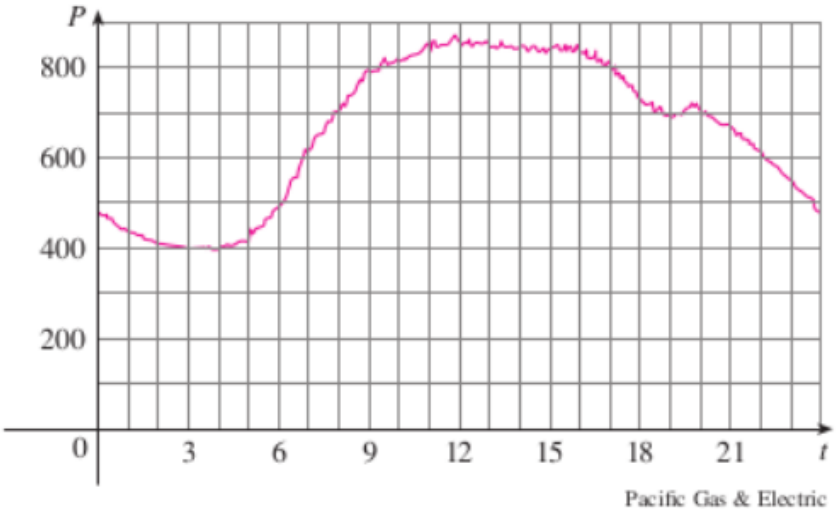
So, $\int_1^4 |v(t)|\,dt = \int_1^4 |t^2 - t - 6|\,dt = \int_1^4 -(t^2 - t - 6)\,dt$

$= -\int_1^4 t^2 - t - 6\,dt$

$= -(-4.5)$ $\boxed{= 4.5 \text{ m.}}$

Example 5

The figure shows the power consumption in the city of San Francisco for a day in September (P is measured in megawatts; t is measured in hours starting at midnight). Estimate the energy used on that day.



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Example 6

Find the indefinite integral $\int 2x\sqrt{1+x^2}\,dx$.

Example 7

Find the indefinite integrals:

a) $\int x^3 \cos(x^4 + 2)\, dx.$

b) $\int \sqrt{2x + 1}\, dx.$

c) $\int \sqrt{1 + x^2} x^5\, dx.$

Example 9

Compute the value of $\int_0^4 \sqrt{2x+1}\, dx$.

Example 10

Compute the value of the definite integrals.

a) $\int_0^4 \sqrt{2x+1}\, dx.$

b) $\int_1^2 \frac{dx}{(3-5x)^2}.$

Example 11

Compute the value of

- $\int_{-1}^{1} x^2\,dx$.
- $\int_{-1}^{1} x^3\,dx$.