Problem 1

Using the product rule, we have

$$f'(x) = (x^2)' \sin x + x^2 (\sin x)' = 2x \sin x + x^2 \cos(x).$$

Problem 3

From the sum and difference rules, we have

$$f'(x) = 3(\cot x)' - 2(\cos x)' = -3\csc^2(x) + 2\sin x.$$

Notice that the negative sign became a plus sign because the derivative of $\cos x$ is $-\sin x$.

Problem 5

There are two ways to complete the problem.

1. We can use the product rule:

$$\begin{aligned} \frac{dy}{d\theta} &= \frac{d}{d\theta}(\sec\theta)\tan\theta + \sec\theta \frac{d}{d\theta}(\tan\theta) \\ &= \sec\theta\tan^2\theta + \sec^3\theta. \end{aligned}$$

2. We can rewrite the expression of y as

$$y = \frac{1}{\cos\theta}\frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\cos^2\theta}.$$

   We then use the quotient rule:

$$\begin{aligned} y' &= \frac{\frac{d}{d\theta}(\sin\theta)\cos^2\theta - \sin\theta\frac{d}{d\theta}(\cos^2\theta)}{\cos^4\theta} \\ &= \frac{\cos^3\theta + 2\sin^2\theta\cos\theta}{\cos^4\theta} \\ &= \frac{\cos^2\theta + 2\sin^2\theta}{\cos^3\theta}. \end{aligned}$$

   We can simplify further using $\sin^2\theta + \cos^2\theta = 1$ and then obtain

$$y' = \frac{\cos^2 + \sin^2\theta + \sin^2\theta}{\cos^3\theta} = \frac{1 + \sin^2\theta}{\cos^3\theta}.$$

Problem 9

Using the quotient rule, we have

$$\begin{aligned}\frac{dy}{dx} &= \frac{1(2-\tan x) - x(-\sec^2 x)}{(2-\tan x)^2} \\ &= \frac{2 + x\sec^2 x - \tan x}{(2-\tan x)^2}.\end{aligned}$$

Problem 15

Using the product rule a first time:

$$f'(\theta) = (\theta)'\cos\theta\sin\theta + \theta(\cos\theta\sin\theta)'.$$

Using the product rule a second time:

$$(\cos\theta\sin\theta)' = -\sin^2\theta + \cos^2\theta$$

Therefore, we get

$$\begin{aligned}f'(\theta) &= (1)\cos\theta + \theta(-\sin^2\theta + \cos^2\theta) \\ &= (1 + \theta\cos\theta)\cos\theta - \theta\sin^2\theta.\end{aligned}$$