Chapter 4 Integrals

4. Indefinite Integrals and the Net Change Theorem

Previously on Calc I:

We introduce a notation for the antiderivatives:

$$\int f(x) dx = F(x) \qquad \text{means} \qquad F'(x) = f(x)$$

Example.

a)
$$\int x^2 dx =$$

b)
$$\int \cos x \, dx =$$

c)
$$\int \sec^2 x \, dx =$$

Table of Indefinite integrals

$$\int cf(x) dx = c \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Remark: We adopt the convention that the general indefinite integral is valid on a given interval. Thus we write t = t

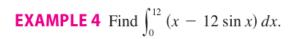
 $\int \frac{1}{x^2} \, dx = -\frac{1}{x} + C$

with the understanding that it is valid on the interval $(0,\infty)$ or on the interval $(-\infty,0)$.

EXAMPLE 1 Find the general indefinite integral

$$\int (10x^4 - 2\sec^2 x) \, dx$$

EXAMPLE 2 Evaluate $\int \frac{\cos \theta}{\sin^2 \theta} d\theta$.



EXAMPLE 5 Evaluate $\int_{1}^{9} \frac{2t^{2} + t^{2}\sqrt{t} - 1}{t^{2}} dt.$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x) \, dx = F(b) - F(a)$$

a) Displacement:

b) Total distance traveled:

c) Acceleration:

EXAMPLE 6 A particle moves along a line so that its velocity at time t is

 $v(t) = t^2 - t - 6$ (measured in meters per second).

- (a) Find the displacement of the particle during the time period $1 \le t \le 4$.
- (b) Find the distance traveled during this time period.