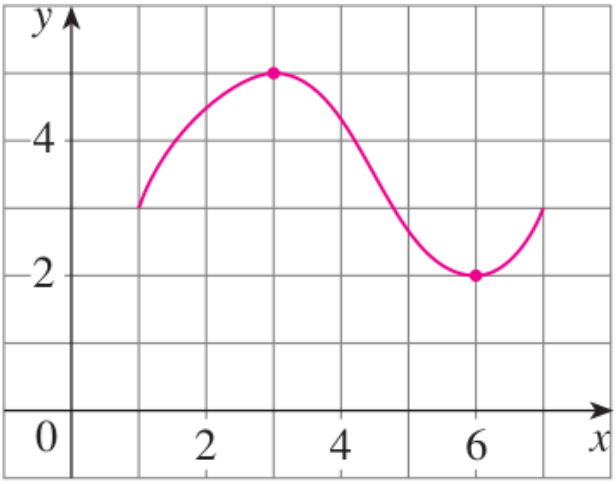
Chapter 3
Applications of Derivatives

3.1 Maximum and Minimum Values

Maximums and minimums.

What would be a maximum value or a minimum value of a function?



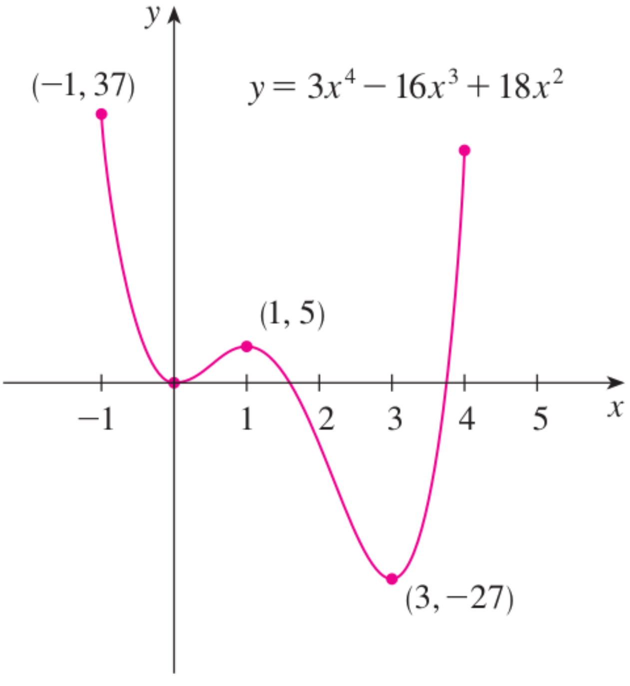
Suggestions/observations:

1)

2)

3)

4)



Suggestions/observations:

1)

2)

3)

4)

Important observations:

a)

b)

1 Definition Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

2 Definition The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.

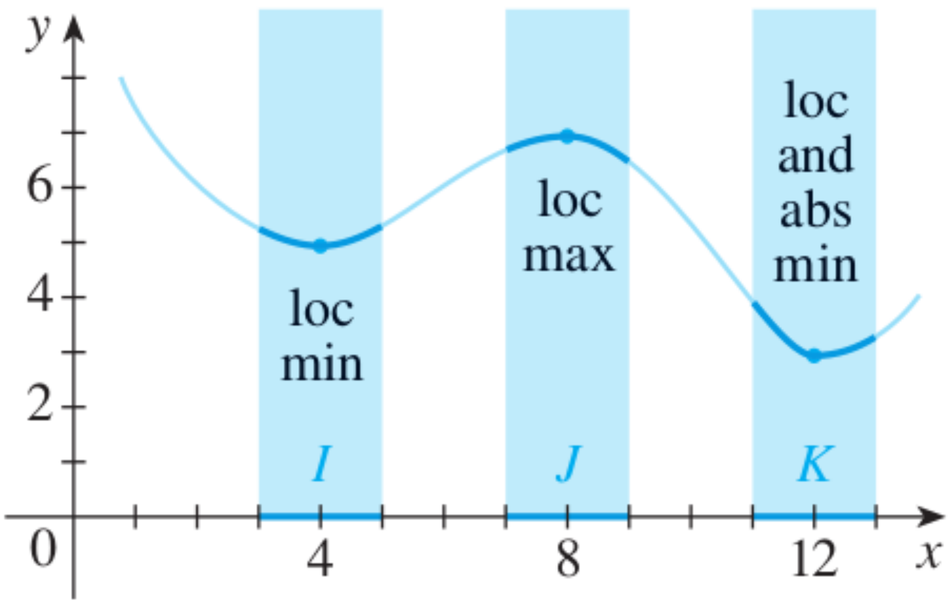


Illustration of the local and absolute max and min.

Terminology.

1) Global maximum or global minimum

2) Extreme values for abs. max. and abs. min.

Example 4. Identify the extremums of the function $f(x) = 3x^4 - 16x^3 + 18x^2$ using the graph of the function.

Extreme Values Theorem.

Which conditions garantee that extreme values exist?

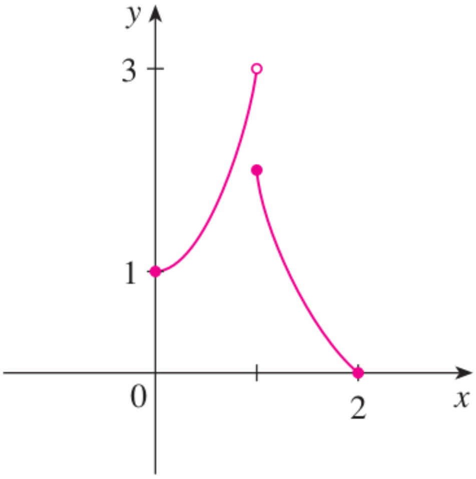


FIGURE 9
This function has minimum value $f(2) = 0$, but no maximum value.

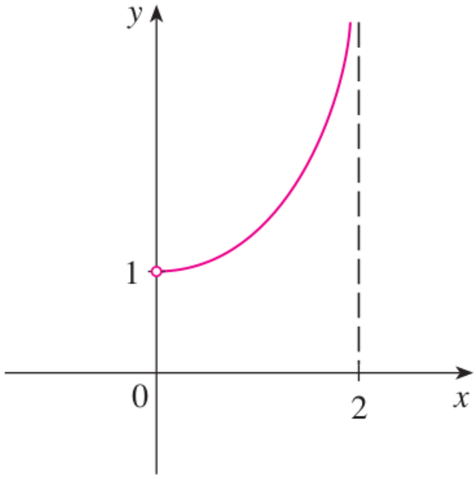
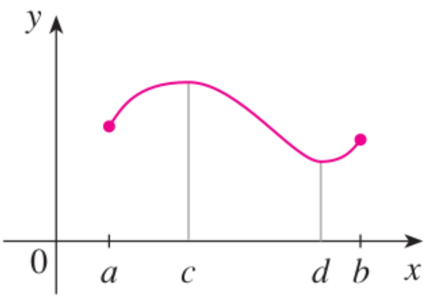
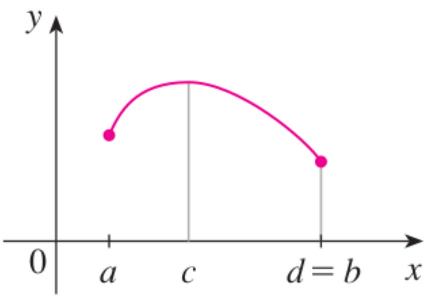


FIGURE 10
This continuous function g has no maximum or minimum.

3 **The Extreme Value Theorem** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



attained inside



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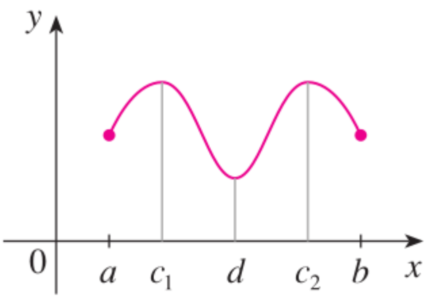
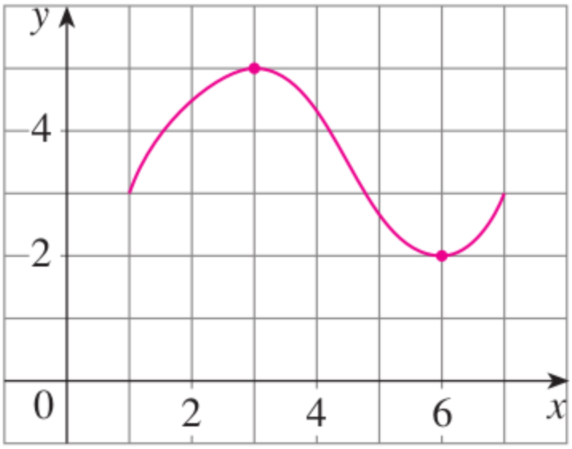


Attained multiple times

Fermat's Theorem.

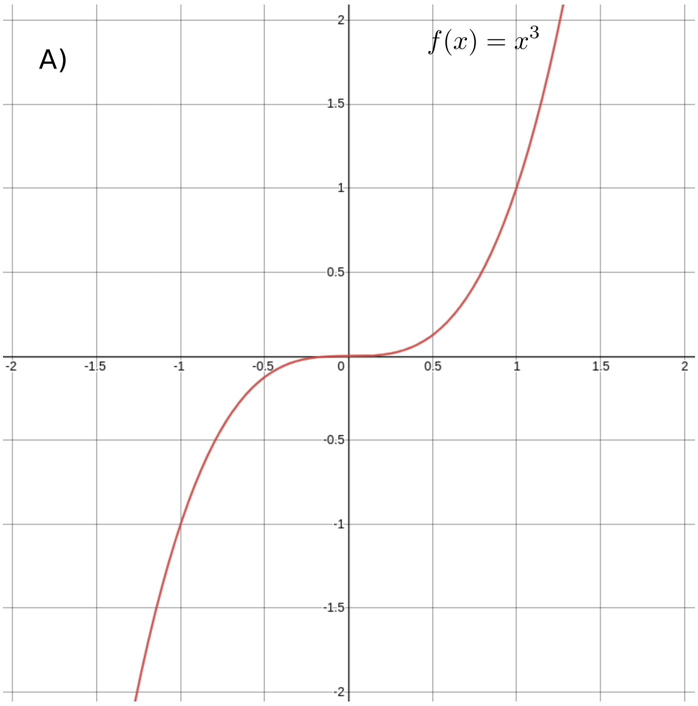
An observation:

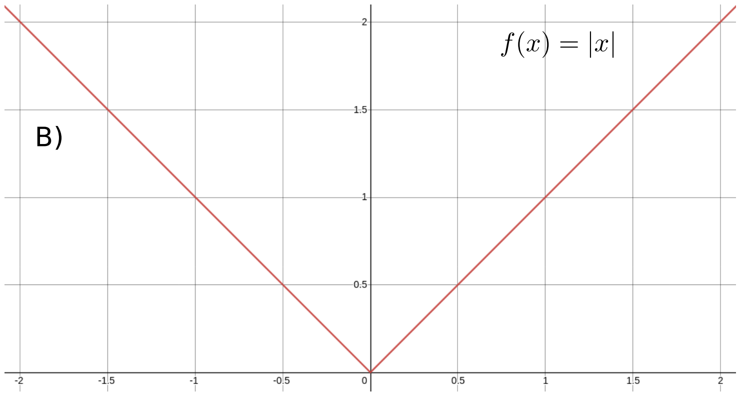


4 **Fermat's Theorem** If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Interested in the proof: see page 207 in the textbook.

BE CAREFUL!!





A)

B)

6 **Definition** A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

EXAMPLE 7 Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).

2. Find the values of f at the endpoints of the interval.

3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

EXAMPLE 8 Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1 \qquad -\tfrac{1}{2} \leqslant x \leqslant 4$$