Section 3.8, Problem 6

Let $f(x) = 2x^3 - 3x^2 + 2$ and $x_1 = -1$. The derivative is given by $f'(x) = 6x^2 - 6x$.

To find the next approximation x_n, we use Newton's method:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} = x_{n-1} - \frac{2x_{n-1}^3 - 3x_{n-1}^2 + 2}{6x_{n-1}^2 - 6x_{n-1}}$$

For x_2, we obtain

$$x_2 = -1 - \frac{2(-1)^3 - 3(-1)^2 + 2}{6(-1)^2 - 6(-1)} = -\frac{3}{4}$$

For x_3, we obtain

$$x_3 = -0.75 - \frac{2(-0.75)^3 - 3(-0.75)^2 + 2}{6(-0.75)^2 - 6(-0.75)} \approx -0.46825.$$

$\boxed{\textbf{Section 3.8, Problem 34}}$

To find the maximum value, we will use the interval method. We have to find the critical numbers inside $(0, \pi)$. The derivative of f is

$$f'(x) = \cos x - x \sin x.$$

The derivative exists everywhere, so the critical numbers are the zero of f'. We will use Newton's method to find the zero of f'. If $x = 0$, then $f'(x) = 0$. But x is not inside the interval $(0, \pi)$. We will search for another zero inside $(0, \pi)$. The Newton's method tells us that the critical number c will be approximated by

$$x_{n-1} - f'(x_{n-1})/f''(x_{n-1})$$

where x_1 is an initial guess within $(0, \pi)$.

Let $x_1 = \pi/2$. We have $f''(x) = -2\sin x - x\cos x$. So

$$c \approx x_{n-1} - \frac{\cos x_{n-1} - x_{n-1}\sin(x_{n-1})}{-2\sin x_{n-1} - x_{n-1}\cos x_{n-1}}.$$

Apprying Newton's method several times, we get the following approximations of c:

Iteration	x_n
2	0.7853981633974483
3	0.8624434632122491
4	0.8603349794247831
5	0.8603335890199867
6	0.8603335890193797

We see that, after the fifth iteration, the first six digits are stable. So $c \approx 0.860333$.

Now, we have

$$\max f(x) = \max\{f(0), f(0.860333), f(\pi)\} = \max\{0, 0.561096, -3.141592\} = 0.561096.$$

Section 4.1, Problem 4

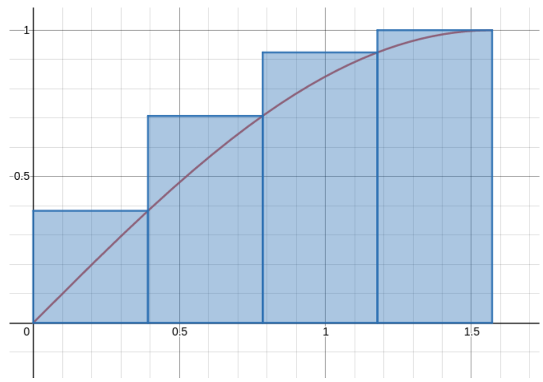
a) We have $n = 4$ and so $\Delta x = (\pi/2 - 0)/4 = \pi/8$. The sample points are

$$x_1 = \pi/8,\ x_2 = \pi/4,\ x_3 = 3\pi/8,\ x_4 = \pi/2.$$

So, we obtain

$$\begin{aligned} A &\approx \sin(\pi/8)\Delta x + \sin(\pi/4)\Delta x + \sin(3\pi/8)\Delta x + \sin(\pi/2)\Delta x \\ &= (\pi/8)(\sin(\pi/8) + \sin(\pi/4) + \sin(3\pi/8) + \sin(\pi/2)) \\ &\approx 1.18346. \end{aligned}$$

Here is the graph of the function and the approximate squares. We see that we overestimated the area under the curve.



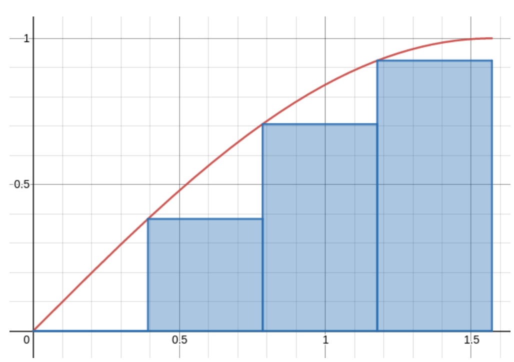
b) We have $n = 4$ and so $\Delta x = (\pi/2 - 0)/4 = \pi/8$. The sample points are

$$x_1 = 0,\ x_2 = \pi/8,\ x_3 = \pi/4,\ x_4 = 3\pi/8.$$

We then obtain

$$\begin{aligned} A &\approx \sin(0)\Delta x + \sin(\pi/8)\Delta x + \sin(\pi/4)\Delta x + \sin(3\pi/8)\Delta x \\ &= (\pi/8)(\sin(0) + \sin(\pi/8) + \sin(\pi/4) + \sin(3\pi/8)) \approx 0.790766. \end{aligned}$$

Here is the graph of the function and the approximate squares. We see that we overestimated the area under the curve.



Section 4.1, Problem 14 (except c))

a) We have $t_1 = 0$, $t_2 = 10$, $t_3 = 20$, $t_4 = 30$, $t_5 = 40$, and $t_6 = 50$ as our sample points and $\Delta x = 10$. So, the distance is estimated by

$$10(182.9) + 10(168.0) + 10(106.6) + 10(99.8) + 10(124.5) + 10(176.1) \approx 8579.0 \text{ miles.}$$

b) We have this time $t_1 = 10$, $t_2 = 20$, $t_3 = 30$, $t_4 = 40$, $t_5 = 50$, and $t_6 = 60$ as our sample points and $\Delta x = 10$. So, the distance is estimated by

$$10(168.0) + 10(106.6) + 10(99.8) + 10(124.5) + 10(176.1) + 10(175.6) \approx 8506.0 \text{ miles.}$$