

Chapter 2

Functions and Limits

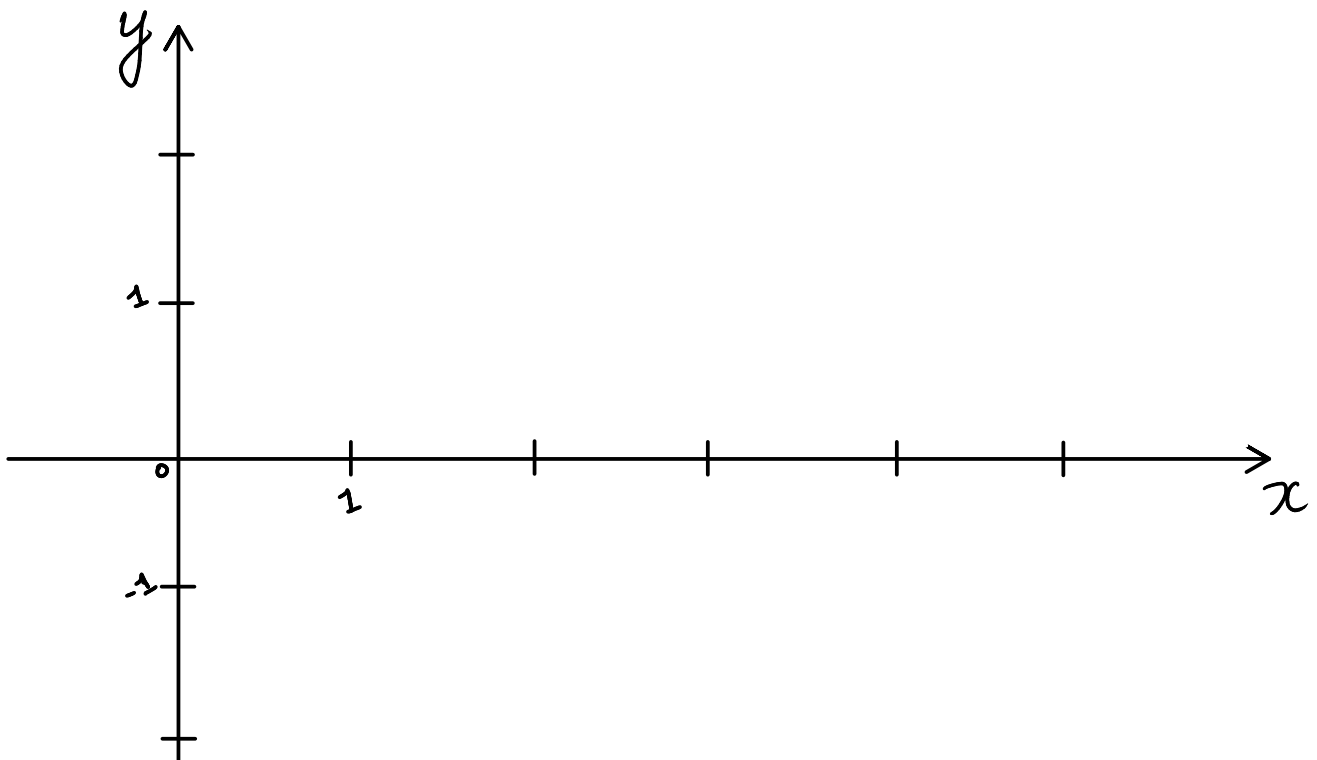
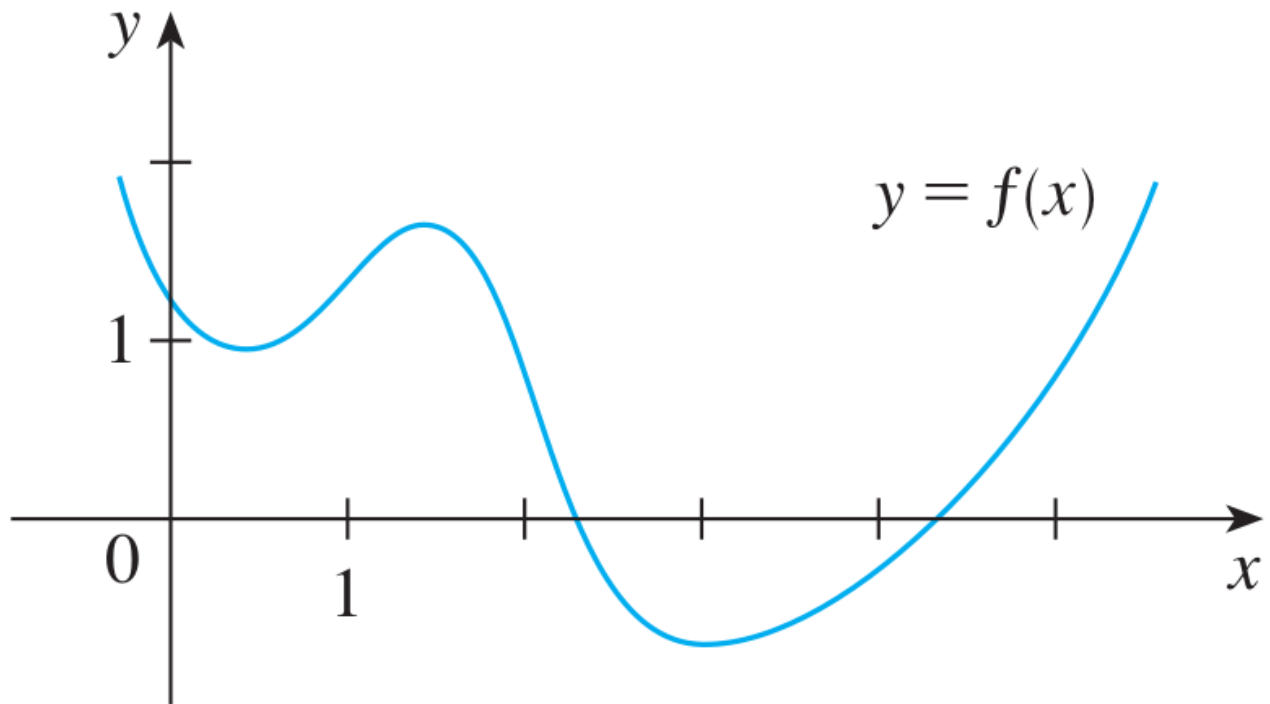
2.2 The Derivatives as a Function

The derivative as a function.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Dom of f' :

EXAMPLE 1 The graph of a function f is given in Figure 1. Use it to sketch the graph of the derivative f' .



EXAMPLE 3 If $f(x) = \sqrt{x}$, find the derivative of f . State the domain of f' .
Illustrate this formula by comparing the graphs of f and f' .

EXAMPLE 4 Find f' if $f(x) = \frac{1-x}{2+x}$.

Other notations for the derivative.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$$

Evaluating in the Leibniz notation:

Example. What is the value of $\left. \frac{dy}{dx} \right|_{x=2}$ if $y = f(x) = x^2$.

3 Definition A function f is **differentiable at a** if $f'(a)$ exists. It is **differentiable on an open interval (a, b)** [or (a, ∞) or $(-\infty, a)$ or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

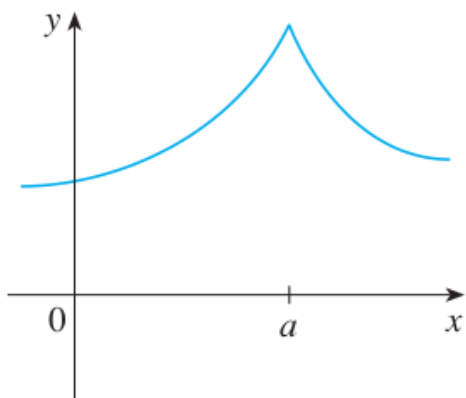
EXAMPLE 5 Where is the function $f(x) = |x|$ differentiable?

4 Theorem If f is differentiable at a , then f is continuous at a .

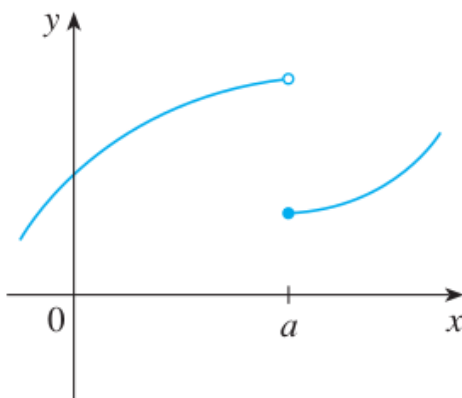
Proof.

Remark:

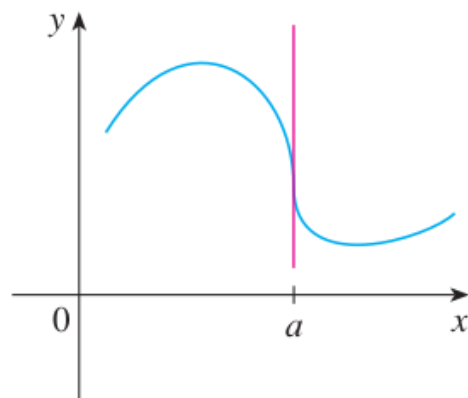
How can a Function Fail to be differentiable?



(a) A corner



(b) A discontinuity



(c) A vertical tangent

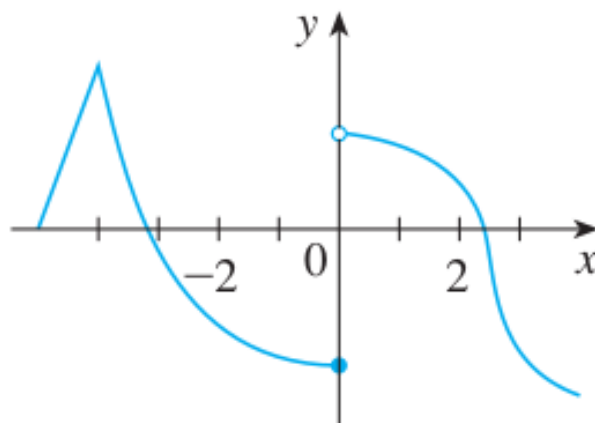
(a)

(b)

(c)

39–42 The graph of f is given. State, with reasons, the numbers at which f is *not* differentiable.

39.



Higher Derivatives.

Second derivative:

$$\underbrace{\frac{d}{dx}}_{\substack{\text{derivative} \\ \text{of}}} \underbrace{\left(\frac{dy}{dx}\right)}_{\substack{\text{first} \\ \text{derivative}}} = \underbrace{\frac{d^2y}{dx^2}}_{\substack{\text{second} \\ \text{derivative}}}$$

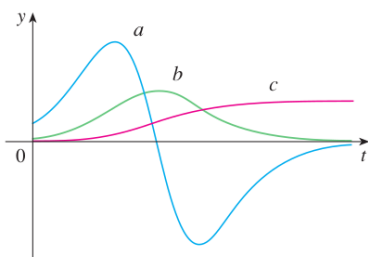
Other notations:

EXAMPLE 6 If $f(x) = x^3 - x$, find and interpret $f''(x)$.

Acceleration:

Example

49. The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



Third Derivative.

$$y''' = f'''(x) = \frac{d}{dx} \left(\frac{d^2 y}{dx^2} \right) = \frac{d^3 y}{dx^3}$$

Jerk: $j = \frac{da}{dt} = \frac{d^3 s}{dt^3}$

n-th Derivative.

$$y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n}$$

EXAMPLE 7 If $f(x) = x^3 - x$, find $f'''(x)$ and $f^{(4)}(x)$.