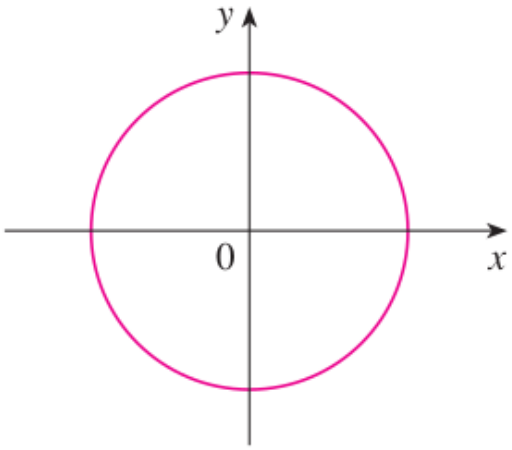
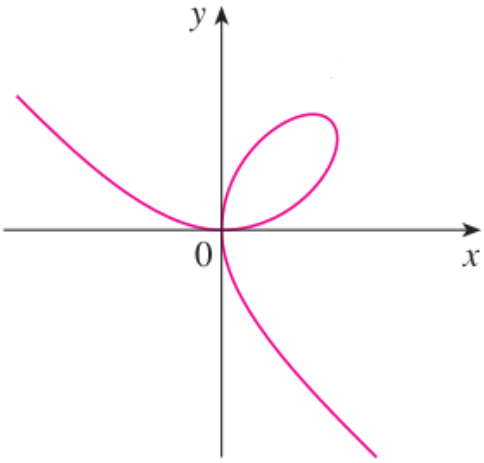
Chapter 2
Derivatives
2.6 Implicit Differentiation

Functions defined implicitly.

Geometry of curves.





In Natural Science (Gas' Law).

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

- P: Pressure
- V: Volume
- T: Temperature
- R, a, b are constants depending on the gas.

HOW DO WE FIND THE SLOPE/DERIVATIVE OF A FUNCTION $y = f(x)$ IF THE RULE IS GIVEN BY AN IMPLICIT EQUATION?

EXAMPLE 1

(a) If $x^2 + y^2 = 25$, find $\dfrac{dy}{dx}$.

(b) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

Main steps for implicit differentiation:

1) Take the derivative on each side of the relation.

2) Use the chain rule and other rules to make the computations.

3) Isolate the derivative dy/dx.

EXAMPLE 2

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

(c) At what point in the first quadrant is the tangent line horizontal?

Desmos: https://www.desmos.com/calculator/efjuccxlrz

EXAMPLE 3 Find y' if $\sin(x + y) = y^2 \cos x$.