Section 2.5, Problem 8

By the Chain Rule, we have

$$F'(x) = 99(1 + x + x^2)^{98}(1 + 2x) = 99(1 + 2x)(1 + x + x^2)^{98}.$$

$\boxed{\textbf{Section 2.5, Problem 10}}$

We have, by the chain rule,

$$g'(x) = \frac{3}{2}(2 - \sin x)^{1/2}(-\cos x) = -\frac{3\cos x\sqrt{2 - \sin x}}{2}.$$

Section 2.5, Problem 30

We use the chaine rule and we get

$$ds/dt = \frac{1}{2}\Big(\frac{1+\sin t}{1+\cos t}\Big)^{-1/2}\frac{d}{dx}\Big(\frac{1+\sin t}{1+\cos t}\Big).$$

The derivative of $(1+\sin t)/(1+\cos t)$ is

$$\frac{\cos t(1+\cos t)-(1+\sin t)(-\sin t)}{(1+\cos t)^2} = \frac{\cos t+\cos^2 t+\sin t+\sin^2 t}{(1+\cos t)^2} = \frac{1+\sin t+\cos t}{(1+\cos t)^2}.$$

Thus, the final answer looks like

$$ds/dt = \frac{1+\sin t+\cos t}{2(1+\cos t)^{3/2}\sqrt{1+\sin t}}.$$

Section 2.6, Problem 12

We suppose that $y = f(x)$. Set $y' = dy/dx$. We differentiate with respect to x on each side of the equation:

$$-\sin(xy)(y + xy') = \cos(y)y'$$

and so

$$-y\sin(xy) - xy'\sin(xy) = y'\cos(y)$$

and then

$$y' = -\frac{y\sin(xy)}{x\sin(xy) + \cos(y)}.$$

$\boxed{\textbf{Section 2.6, Problem 32}}$

We suppose that $y = f(x)$ and differentiate each side of the equation. We obtain

$$2yy'(y^2 - 4) + 2y^3y' = 2x(x^2 - 5) + 2x^3.$$

So, now we have to isolate y'. After distributing y and x, we obtain

$$y'(2y^3 - 4y + 2y^3) = 2x^3 - 10x + 2x^3.$$

We then find

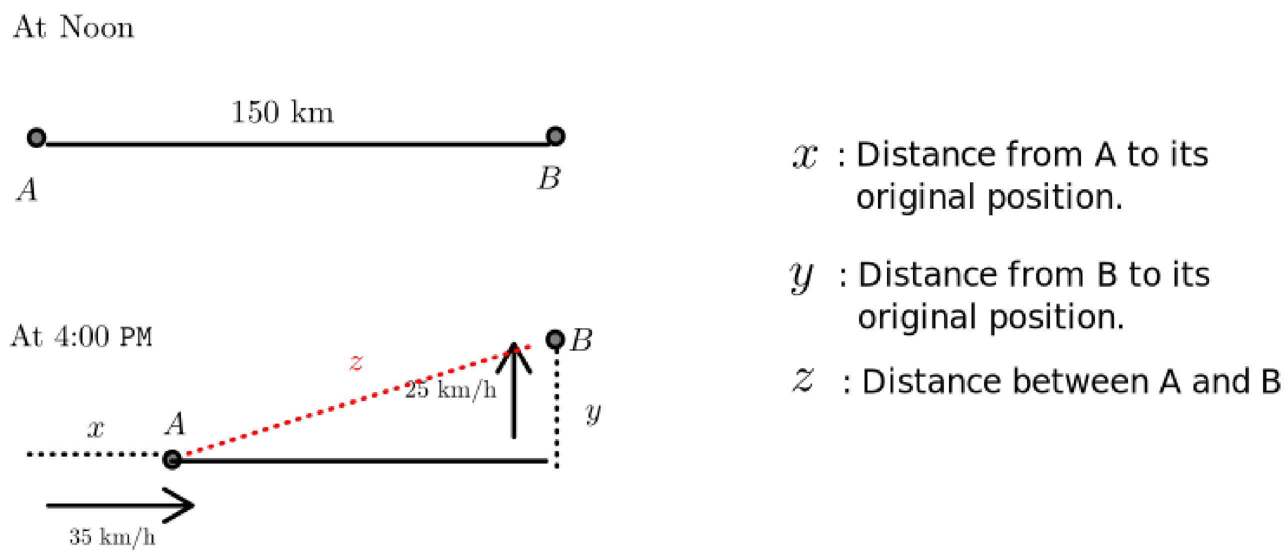
$$y' = \frac{x(2x^2 - 5)}{2y(y^2 - 1)}$$

The equation of the tangent line is $y + 2 = m(x - 0)$ where $m = y'(0)$. So, replacing $x = 0$ and $y = -2$ in the above equation for y', we get $m = 0$. Thus, we obtain

$$y = -2.$$

Section 2.8, Problem 16

First, let's draw a picture and introduce some notations. The known information is $dx/dt = 35$



and $dy/dt = 25$. What we would like to know is dz/dt.

The link between x, y and z is given by the pythagorean Theorem:

$$z^2 = (150 - x)^2 + y^2$$

where $150 - x$ is the distance from the boat A to the original position of the boat B. Taking the derivative with respect to time gives

$$\begin{aligned} & 2z(dz/dt) = 2(150 - x)(-dx/dt) + 2y(dy/dt). \\ \iff \quad & dz/dt = ((150 - x)/z)(-dx/dt) + (y/z)(dy/dt). \end{aligned}$$

From noon to 4:00PM, the boat A travelled $4 \times 35 = 140$ km and the boat B travelled $4 \times 25 = 100$ km. So $x = 140$, $y = 100$, and $z = \sqrt{10^2 + 100^2} = 10\sqrt{101}$. Replacing everything in the last equations above, we obtain

$$dz/dt = (1/\sqrt{101})(-35) + (10/\sqrt{101})(25) = 215/\sqrt{101} \approx 25 \text{ km/h}.$$

Thus, $dz/dt \approx 25$ km/h.