Math 644

Chapter 6

Section 6.1: Conformal Maps

Contents

Created by: Pierre-Olivier Parisé

Spring 2023

Definition

Definition 1. A function $f : \Omega \to \mathbb{C}$ is conformal on Ω if

(a) f is analytic in Ω and;

(b) f is one-to-one in Ω.

Example 2. Find an conformal map f from the unit disk $\mathbb{D}$ onto the unit disk $\mathbb{D}$, with $f(0) = \frac{1}{2}$.

THEOREM 3. If there exists a conformal map of a region Ω onto $\mathbb{D}$, then, given any $z_0 \in \Omega$, there exists a unique conformal map f of Ω onto $\mathbb{D}$ such that

$$f(z_0) = 0 \text{ and } f'(z_0) > 0.$$

Proof.

Theorem 4. If φ is a conformal map of a region Ω onto $\mathbb{D}$, then Ω must be simply-connected.

Proof.