Math 302

Chapter 2

Section 2.6: Integrating Factors

Contents

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Fall 2022

What's An Integrating Factor

Example 1. Verify if

$$(3x + 2y^3)dx + 2xydy = 0$$

is exact.

A function $\mu = \mu(x, y)$ is an **integrating factor** for

$$M(x, y)dx + N(x, y)dy = 0$$

if the equation

$$\mu(x, y)M(x, y)dx + \mu(x, y)N(x, y)dy = 0$$

is exact.

Finding Integrating Factors

Let's start with the equation

$$\mu(x,y)M(x,y)dx + \mu(x,y)N(x,y)dy = 0. \tag{1}$$

Trick:

General Facts: Let M, N, M_y, N_x be continuous on an open rectangle R.

- if $(M_y - N_x)/N$ is independent of y, then

  $$\mu(x,y) = \pm e^{\int p(x)\,dx}$$

  is an integrating factor for (1) where $p(x) = (M_y - N_x)/N$.
- if $(N_x - M_y)/M$ is independent of x, then

  $$\mu(x,y) = \pm e^{\int q(y)\,dy}$$

  is an integrating factor for (1) where $q(y) = (N_x - M_y)/M$.

EXAMPLE 2. Find an integrating factor for the equation

$$(2xy^3 - 2x^3y^3 - 4xy^2 + 2x)dx + (3x^2y^2 + 4y)dy = 0$$

and solve the equation.

EXAMPLE 3. Find an integrating factor for the equation

$$2xy^3dx + (3x^2y^2 + x^2y^3 + 1)dy = 0$$

and solve the equation.