**Math 241**
**Spring 2022**
**Midterm 1**
**February 23rd, 2022**

**Name (Print):** _____
**Section Number:** _____

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This exam contains 10 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

**Instructions:**

- You have 75 minutes for the exam.
- You are required to **show your work** and justify your answers for all questions *except where explicitly stated*.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit, if any.
- No notes or books are allowed.
- No electronic devices are allowed.
- No calculators are allowed.

Academic integrity is expected of all University of Hawaii students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.
**Please sign below to indicate that you have read and agree to these instructions.**

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Signature of student

*For official use only:*

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Points: | 9 | 15 | 8 | 10 | 12 | 12 | 10 | 10 | 14 | 100 |
| Score: | | | | | | | | | | |

**Question 1.** (9 points)

The table shows the distance travelled by a bicyclist on a straight line after accelerating from rest.

| Time in seconds | Total distance in feet |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 15 |
| 5 | 30 |
| 6 | 52 |
| 7 | 76 |
| 8 | 101 |

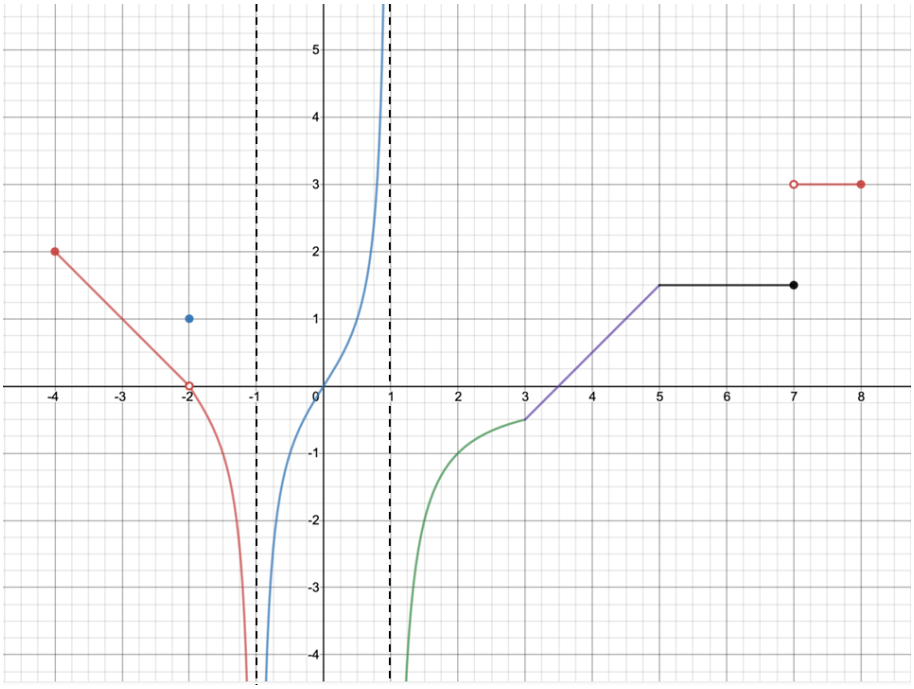
(a) (3 points) Calculate the average speed between 2 and 6 seconds.

(b) (3 points) Compare the average speed of the interval between 0 second and 1 second, and the interval between 1 second and 2 seconds. Between these two intervals, which one has the highest average speed?

(c) (3 points) Estimate the average acceleration of the bicyclist at 7 seconds.
(Hint: The average acceleration can be calculated using two average speeds.)

**Question 2.** (15 points)

The graph of a function $f$ is given below. Assume $f$ has vertical asymptotes at $x = -1$ and $x = 1$. No justifications needed for this problem.




(a) (6 points) Evaluate each of the following limits, or say the limit does not exist. If the limit is either $\infty$ or $-\infty$, specify which (rather than just saying 'does not exist').

1. $\lim_{x \to -2} f(x)$
2. $\lim_{x \to -1^-} f(x)$
3. $\lim_{x \to 1} f(x)$
4. $\lim_{x \to 7^-} f(x)$
5. $\lim_{x \to 7^+} f(x)$
6. $\lim_{x \to 7} f(x)$

(b) (3 points) For which (if any) values in the interval $[-4, 8]$ is the function $f$ not continuous?

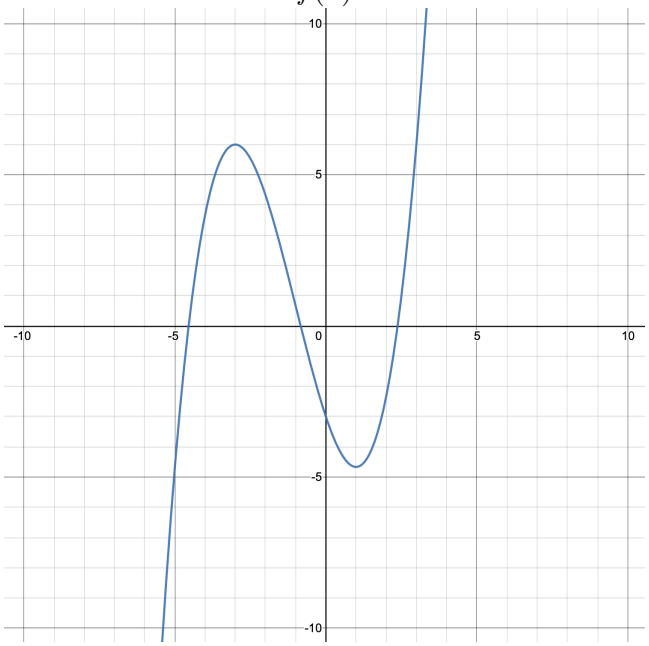
(c) (3 points) For which (if any) values in the interval $[-4, 8]$ is $f$ differentiable but not continuous?

(d) (3 points)For which (if any) values in the interval $[-4, 8]$ is $f$ continuous but not differentiable?

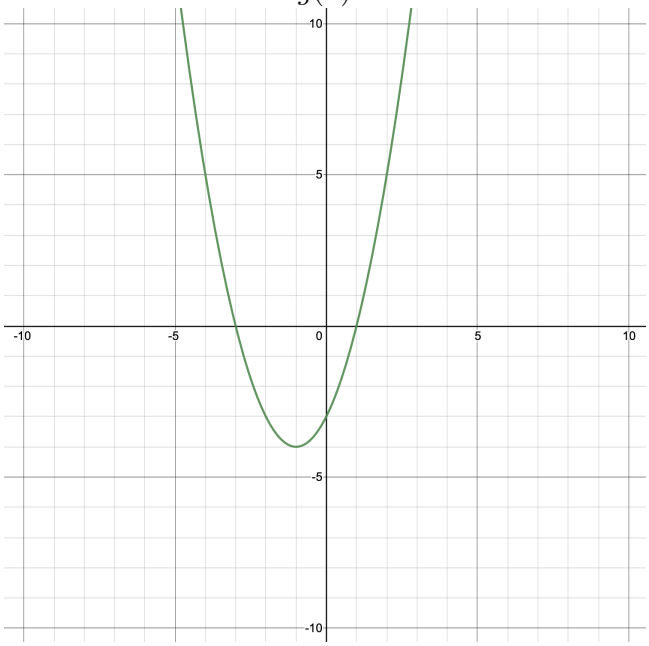
**Question 3.** (8 points)

Given the two graphs below, **roughly** sketch the graphs of their derivative on the blank axes. (4 points for each graph.)

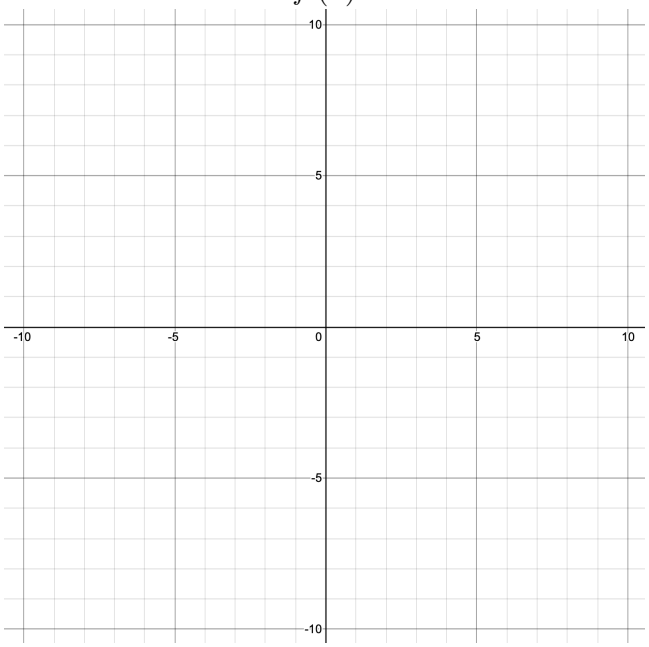
$f(x)$



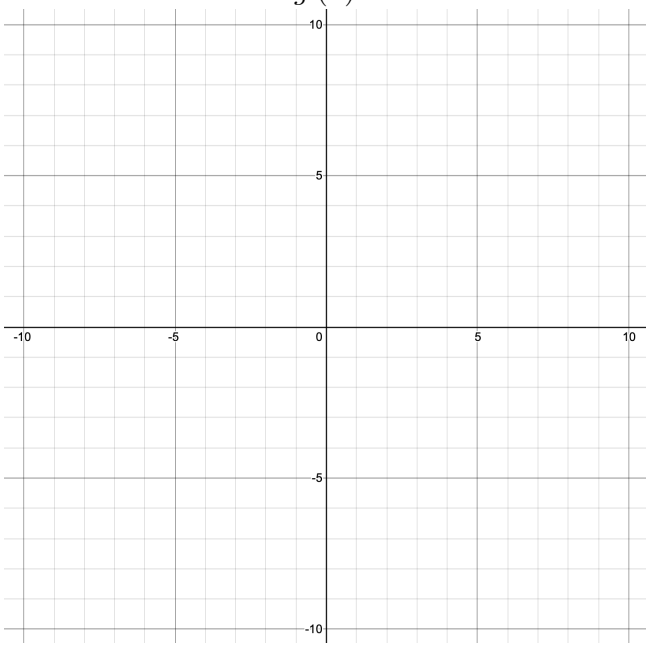

$g(x)$




$f'(x)$




$g'(x)$



**Question 4.** (10 points)

Suppose $f$ is a continuous function that satisfies the following limits:

$$\lim_{x\to -1} f(x) = -2, \qquad \lim_{x\to 0} f(x) = 3$$

Evaluate the following limits. (5 points each.) You may not use L'Hospital's rule, i.e., if you use L'Hospital's rule, you will not get points.

(a) $\lim_{x\to -1} \dfrac{(x^2-3x-4)}{x+1} f(x)$

(b) $\lim_{x\to 0} \dfrac{\sqrt{3x^2+16}-4}{x^2 f(x)}$

**Question 5.** (12 points)

(a) (8 points) Using *the definition of derivative* (also called the limit process), find the derivative of the function $f(x) = \frac{1}{x+4}$.
You will NOT get any credit unless you use the definition of the derivative!

(b) (4 points) Using the function in (a), find the equation of the tangent line to $y = f(x)$ at $(0, \frac{1}{4})$.

**Question 6.** (12 points)

Let $f(x)$ be defined by

$$f(x) = \begin{cases} (x-a)^2 + 2 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ a + x & \text{if } x > 2 \end{cases}$$

(a) (8 points) Find all values of $a$ so that $\lim_{x \to 2} f(x)$ exists.

(b) (4 points) Find all possible values of $a$ so that $f(x)$ is continuous at $x = 2$, or show that none exist. Justify your answer.

**Question 7.** (10 points)

Suppose $f(x)$ is a function where $f(1) = 1$ and $f'(1) = -1$.

(a) (5 points) Let $g(x) = x^3 f(x) + 2$. Find $g'(1)$.

(b) (5 points) Let $h(x) = \sqrt{4\sin(\pi x) + 3f(x)}$. Find $h'(1)$.

**Question 8.** (10 points)

Use implicit differentiation to find an equation of the tangent line to the following cardioid

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2 \text{ at the point } \left(0, -\frac{1}{2}\right)$$

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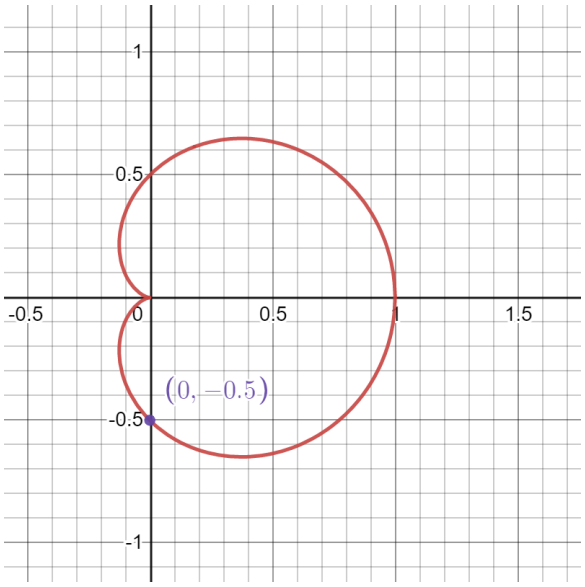

**Question 9.** (14 points)

Suppose that an object moves along a line over time. Its position is given by

$$x(t) = -0.02t^2 + 50t + 100.$$

(a) (4 points) What is the average speed of the object between the time $t = 0$ and $t = 1000$?

(b) (5 points) What is the velocity of the object when $t = 500$?

(c) (5 points) What is the acceleration of the object when $t = 10$?