Chapter 4
Integrals

4.5 The Substitution Rule

Find the indefinite integral of $2x\sqrt{1+x^2}$, that is compute

$$\int 2x\sqrt{1+x^2}\,dx.$$

<u>Goal</u>: Find $F(x)$ s.t. $F'(x) = 2x\sqrt{1+x^2}$.

- $\sqrt{x} \longrightarrow \dfrac{x^{3/2}}{3/2}$

- $\dfrac{(1+x^2)^{3/2}}{3/2} \xrightarrow{d/dx} \dfrac{\frac{3}{2}(1+x^2)^{1/2}}{3/2} \cdot \dfrac{d}{dx}(1+x^2)$

$$\longrightarrow \sqrt{1+x^2} \cdot 2x$$

$$\Rightarrow \int 2x\sqrt{1+x^2}\,dx = \frac{2}{3}(1+x^2)^{3/2} + C$$

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Another example. Compute the indefinite integral

$$\int 2x\sqrt{1+x^2}\,dx.$$

<u>1st</u>: Notice that $\dfrac{d}{dx}(1+x^2) = 2x \Rightarrow d(1+x^2) = 2x\,dx$

<u>2st</u>: Let $u = 1+x^2 \longrightarrow \dfrac{du}{dx} = 2x \longrightarrow du = 2x\,dx$

<u>3rd</u>: $\int 2x\sqrt{1+x^2}\,dx = \int \underbrace{\sqrt{1+x^2}}_{u}\underbrace{2x\,dx}_{du}$

$$= \int \sqrt{u}\,du$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$\Rightarrow I = \frac{2}{3}(1+x^2)^{3/2} + C$$

> **Substitution Rule.** If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then
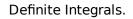> $$\int f'(g(x))g'(x)\,dx = \int f(u)\,du.$$

Relation between du and dx:

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EXAMPLE 1 Find $\displaystyle\int x^3 \cos(x^4 + 2)\,dx$.

EXAMPLE 2 Evaluate $\int \sqrt{2x + 1} \, dx$.

EXAMPLE 3 Find $\int \dfrac{x}{\sqrt{1 - 4x^2}} \, dx$.

EXAMPLE 5 Find $\displaystyle\int \sqrt{1 + x^2}\ x^5\, dx$.

Definite Integrals.

EXAMPLE 7 Evaluate $\int_1^2 \dfrac{dx}{(3 - 5x)^2}$.