Important: Each problem on this homework worth 5 points (not 10 points).

Section 4.3, Problem 14

Let $g(x) = \int_1^x \frac{z^2}{z^4+1}\, dz$ and $f(x) = \sqrt{x}$. Then, we have

$$h(x) = g(f(x)).$$

Using the Chaine rule, we obtain

$$h'(x) = g'(f(z))f'(z).$$

By FTC part 1, $g'(x) = x^2/(x^4+1)$. Thus,

$$h'(x) = \frac{z}{z^2+1}\Big(\frac{1}{2\sqrt{z}}\Big) = \frac{\sqrt{z}}{2(z^2+1)}.$$

Section 4.3, Problem 20

An antiderivative of x^{100} is $x^{101}/101$. Thus, by FTC part 2, we have

$$\int_{-1}^1 x^{100}\, dx = \frac{x^{101}}{101}\bigg|_{-1}^1 = \frac{2}{101}.$$

Section 4.3, Problem 22

Using linearity, we have

$$\int_0^1 (1 - 8v^3 + 16v^7)\, dv = \int_0^1 dv - 8\int_0^1 v^3\, dv + 16\int_0^1 v^7\, dv.$$

Using the part 2 of the FTC, we have

$$\int_0^1 (1 - 8v^3 + 16v^7)\, dv = v|_0^1 - 8\,\frac{v^4}{4}\bigg|_0^1 + 16\,\frac{v^8}{8}\bigg|_0^1 = 1 - 2 + 2 = 1.$$

Section 4.3, Problem 34

We have $(s^4+1)/s^2 = s^2 + 1/s^2$. Thus,

$$\int_1^2 \frac{s^4+1}{s^2}\, ds = \int_1^2 s^2\, ds + \int_1^2 (1/s^2)\, ds = \frac{s^3}{3}\bigg|_1^2 + \frac{-1}{s}\bigg|_1^2 = \frac{8-1}{3} + 1/2 = 11/6.$$

Section 4.3, Problem 38

We divide the integral in two pars:

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx.$$

According to the definition of the function $f(x)$, we have

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} 2\,dx + \int_{0}^{2} (4-x^2)\,dx = 4 + 8 - 8/3.$$

So the final answer is 28/3.

Section 4.3, Problem 54

We write $g(x)$ as followed

$$g(x) = \int_{1-2x}^{0} t \sin t\,dt + \int_{0}^{1+2x} t \sin t\,dt = -\int_{0}^{1-2x} t \sin t\,dt + \int_{0}^{1+2x} t \sin t\,dt.$$

Let $h(x) = \displaystyle\int_{0}^{x} t \sin t\,dt$, $f_1(x) = 1-2x$ and $f_2(x) = 1+2x$. Thus, we can rewrite g as

$$g(x) = -h(f_1(x)) + h(f_2(x)).$$

Using the Chain Rule and the FTC (part 1), we obtain

$$g'(x) = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x).$$

We can simply this expression using some trig. identities. In $g(x)$, we have the expression

$$2\sin(1-2x) + 2\sin(1+2x) = 4\sin(1)\cos(2x)$$

and the expression

$$-4x\sin(1-2x) + 4x\sin(1+2x) = 8x\sin(2x)\cos(1).$$

We thus obtain

$$g'(x) = 4\sin(1)\cos(2x) + 8x\cos(1)\sin(2x).$$

$\boxed{\textbf{Section 4.3, Problem 60}}$

By the FTC (part 1), we have $F'(x) = f(t)$. So, the function is concave downward when $F'(x)$ varies from being decreasing (corresponding to the second derivative being negative). From the graph of f, we see that f is decreasing on the interval $(-1, 1)$. Thus, F is concave down on $(-1, 1)$.

$\boxed{\textbf{Section 4.3, Problem 75}}$

By the FTC (part 1), we have

$$\frac{f(x)}{x^2} = \frac{1}{\sqrt{x}}.$$

Thus, $f(x) = x^{3/2}$. Now, using the FTC (part 2), we have

$$6 + \int_a^x t^{-1/2}\,dt = 2\sqrt{x} \quad \Rightarrow \quad 6 + 2\sqrt{x} - 2\sqrt{a} = 2\sqrt{x}.$$

We then find $2\sqrt{a} = 6$ and so $a = 9$.

The desire function and number a are $f(x) = x^{3/2}$ and $a = 9$.

Section 4.4, Problem 18

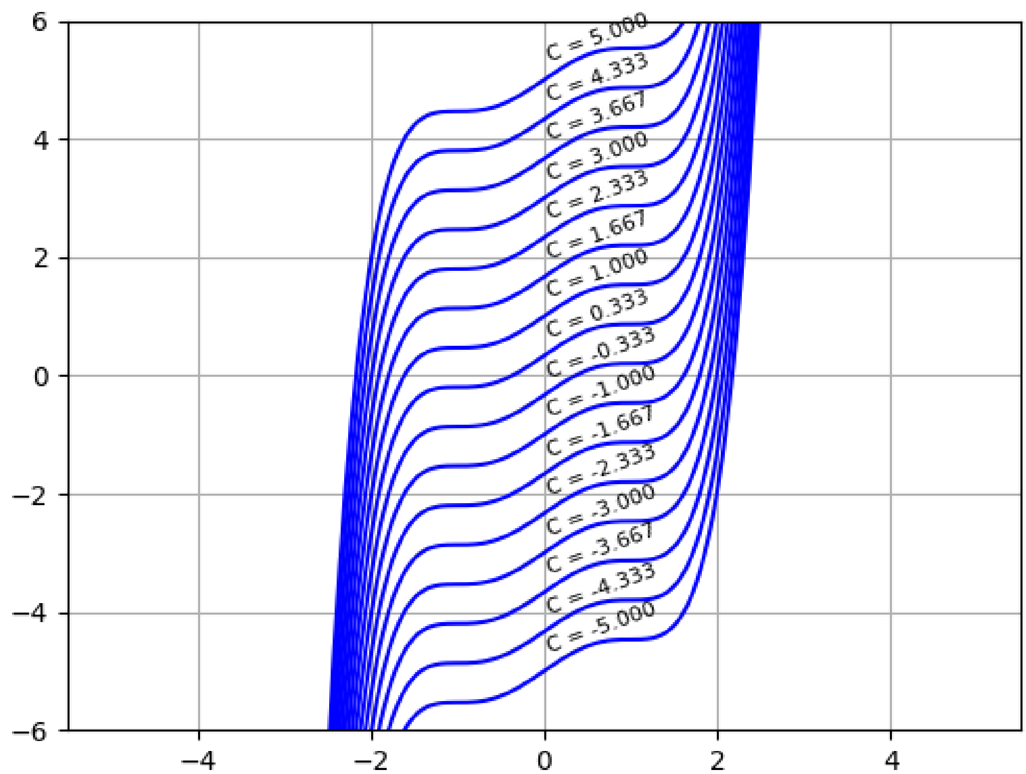
We have

$$(1 - x^2)^2 = 1 - 2x^2 + x^4$$

and so

$$\int (1 - x^2)^2 \, dx = \int dx - 2 \int x^2 \, dx + \int x^4 \, dx = x - \frac{2}{3}x^3 + \frac{x^5}{5} + C.$$

Here is the graph of several antiderivatives with different constants C.



Section 4.4, Problem 58

(a) The velocity is given by $\int 2t + 3\,dt = t^2 + 3t + C$. Now, $v(0) = -4$, so $C = -4$. We then get

$$v(t) = t^2 + 3t - 4 = (t+4)(t-1).$$

(b) The total distance traveled is given by

$$\int_0^3 |v(t)|\,dt = \int_0^1 -(t^2 + 3t - 4)\,dt + \int_1^3 t^2 + 3t - 4\,dt = 14\tfrac{5}{6}.$$