Section 3.9 — Problem 2 — 5 points

We have $(x^3/3)' = x^2$, $(-3x^2/2)' = -3x$, and $(2x)' = 2$. Therefore, the most general antiderivative is

$$\frac{x^3}{3} - \frac{3}{2}x^2 + 2 + C.$$

Section 3.9 — Problem 6 — 5 points

Write the expression of $f(x)$ as

$$u^5,$$

where $u = x - 5$. Therefore, we see that $u^6/6$ is an antiderivative of u^5. However, from the chain rule, we will get

$$\left(\frac{u^6}{6}\right)' = u^5 \frac{du}{dx}.$$

Since $u = x - 5$, we have $u' = 1$. This means that

$$\left(\frac{(x-5)^6}{6}\right)' = (x-5)^5.$$

Therefore, the most general antiderivative is

$$\frac{1}{6}(x-5)^6 + C.$$

Section 3.9 — Problem 12 — 5 points

We simplify the expression $f(x)$ to

$$f(x) = x^{2/3} + x^{3/2}.$$

From the power rule, we have

$$\Big(\frac{x^{5/3}}{5/3}\Big)' = x^{2/3} \quad \text{and} \quad \Big(\frac{x^{5/2}}{5/2}\Big)' = x^{3/2}.$$

Therefore, the most general antiderivative is

$$\frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + C.$$

Section 3.9 — Problem 14 — 5 points

We simplify the expression of g to

$$g(x) = 5x^{-6} - 4x^{-3} + 2.$$

From the power rule, we have

$$\left(\frac{x^{-5}}{-5}\right)' = x^{-6}, \quad \left(\frac{x^{-2}}{-2}\right)' = x^{-3} \quad \text{and} \quad (x)' = 1.$$

Therefore, using the algebraic rules of differentiation, the most general antiderivative is

$$-x^{-5} + 2x^{-2} + 2x + C.$$

$\boxed{\textbf{Section 3.9 — Problem 16 — 5 points}}$

We have

$$\frac{d}{dt}(\sin t) = \cos t \quad \text{and} \quad \frac{d}{dt}(\cos t) = -\sin t.$$

Therefore, the most general antiderivative is

$$3\sin(t) + 4\cos(t) + C.$$

Section 3.9 — Problem 18 — 5 points

We have

$$\frac{d}{dv}(v) = 1 \quad \text{and} \quad \frac{d}{dv}(\tan v) = \sec^2(v).$$

Therefore, the most general antiderivative is

$$5v + 3\tan(v) + C.$$

Section 3.9 — Problem 22 — 5 points

The general antiderivative of $f(x)$ is

$$F(x) = \frac{x^2}{2} - 2\cos(x) + C.$$

Therefore, if $F(0) = -6$, we have to solve for C the following equation

$$-6 = F(0) = \frac{0^2}{2} - 2\cos(0) + C$$

which simplifies to

$$-6 = -2 + C.$$

After isolating C, we get $C = -4$ and therefore

$$F(x) = \frac{x^2}{2} - 2\cos(x) - 4.$$

Section 3.9 — Problem 30 — 5 points

The most general antiderivative is

$$f(x) = x^5 - x^3 + 4x + C.$$

Assuming that $f(-1) = 2$, and plugging $x = -1$ in the expression of $f(x)$, we get

$$2 = (-1)^5 - (-1)^3 - 4 + C = -4 + C$$

and therefore $C = 6$. The function we were looking for is

$$f(x) = x^5 - x^3 + 4x + 6.$$

Section 4.1 — Problem A — 5 points

We have $f(x) = \frac{1}{1+x^2}$.

With 3 rectangles. The data are $a = -1$, $b = 1$ and $n = 3$. Therefore, we get

$$\Delta x = (1+1)/3 = \frac{2}{3}$$

and

$$\begin{aligned} x_1 &= -1 + \Delta x = -\frac{1}{3} \\ x_2 &= -1 + 2\Delta x = \frac{1}{3} \\ x_3 &= b = 1. \end{aligned}$$

The right endpoints rule gives, with 3 rectangles,

$$\begin{aligned} R_3 &= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x \\ &= f(-1/3)\frac{2}{3} + f(1/3)\frac{2}{3} + f(1)\frac{2}{3} \\ &= \frac{23}{15} \approx 1.5333. \end{aligned}$$

With 4 rectangles. The data are $a = -1$, $b = 1$ and $n = 4$. Therefore, we get

$$\Delta x(1+1)/4 = \frac{1}{2}$$

and

$$\begin{aligned} x_1 &= -1 + \Delta x = \frac{-1}{2} \\ x_2 &= -1 + 2\Delta x = 0 \\ x_3 &= -1 + 3\Delta x = \frac{1}{2} \\ x_4 &= b = 1. \end{aligned}$$

The right endpoints rule gives, with 4 rectangles,

$$\begin{aligned} R_4 &= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x \\ &= f(-1/2)\frac{1}{2} + f(0)\frac{1}{2} + f(1/2)\frac{1}{2} + f(1)\frac{1}{2} \\ &= \frac{31}{20} = 1.55. \end{aligned}$$

Section 4.1 — Problem B — 5 points

Let $f(x) = x^2 + 1$, $a = 0$, $b = 1$, and $\Delta x = (b - a)/n = 1/n$. We have

$$x_1 = a + \Delta x = \frac{1}{n}, \quad x_2 = a + 2\Delta x = \frac{2}{n}$$
$$\vdots$$
$$x_i = a + i\Delta x = \frac{i}{n}$$
$$\vdots$$
$$x_{n-1} = a + (n-1)\Delta x = \frac{n-1}{n}, \quad x_n = b = \frac{n}{n}.$$

Therefore, the right Riemann sum is

$$\begin{aligned}
R_n &= f(x_1)\Delta x + f(x_2)\Delta x + \ldots + f(x_i)\Delta x + \ldots + f(x_n)\Delta x \\
&= f(1/n)\frac{1}{n} + f(2/n)\frac{1}{n} + \ldots + f(i/n)\frac{1}{n} + \ldots + f(n/n)\frac{1}{n} \\
&= \frac{1}{n}\Big(\frac{1}{n^2} + 1 + \frac{2^2}{n^2} + 1 + \ldots + \frac{i^2}{n^2} + 1 + \ldots + \frac{n^2}{n^2} + 1\Big) \\
&= \frac{1}{n}\sum_{i=1}^{n}\Big(\frac{i^2}{n^2} + 1\Big) \\
&= \frac{1}{n}\Big(\sum_{i=1}^{n}\frac{i^2}{n^2} + \sum_{i=1}^{n} 1\Big) \\
&= \frac{1}{n}\sum_{i=1}^{n}\frac{i^2}{n^2} + \frac{1}{n}\sum_{i=1}^{n} 1 \\
&= \frac{1}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n}\sum_{i=1}^{n} 1.
\end{aligned}$$

From the sum formulas, we have

$$\sum_{i=1}^{n} i^2 = \frac{n(n-1)(2n-1)}{6} \quad \text{and} \quad \sum_{i=1}^{n} 1 = n.$$

Therefore, replacing in the last equality, we obtain

$$R_n = \frac{n(n-1)(2n-1)}{6n^3} + \frac{n}{n} = \frac{n(2n^2 - 3n + 1)}{6n^3} + 1 = \frac{2n^3 - 3n^2 + n}{6n^3} + 1.$$

Treating n as a variable x and letting x goes to $+\infty$, we obtain

$$\lim_{n\to\infty} R_n = \lim_{x\to\infty}\Big(\frac{2x^3 - 3x^2 + x}{6x^3} + 1\Big) = \frac{2}{6} + 1 = \frac{4}{3}.$$

Therefore, the area under the graph of $f(x) = x^2 + 1$ is $\frac{4}{3}$.

TOTAL (POINTS): 50.