Chapter 2

Functions and Limits

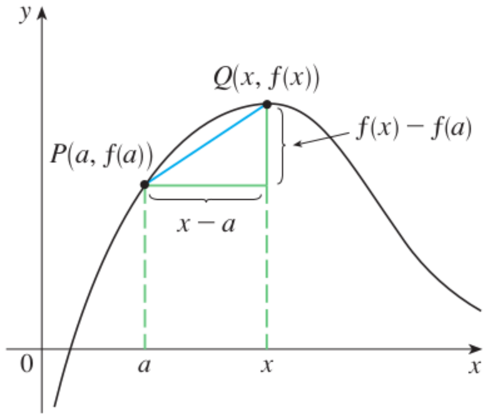
2.1 Derivatives and Rates of Change

Tangents.

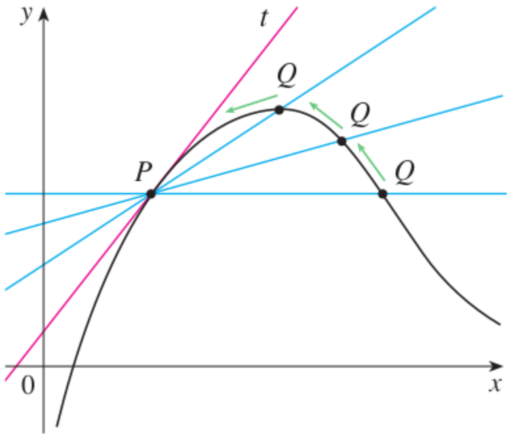
How do we find the tangent at a point P on a curve given by the graph of a function?

Answer:

1) Find the slope of the secant line passing to two points P and Q on the curve:



2) Taking the limit as Q approached P.

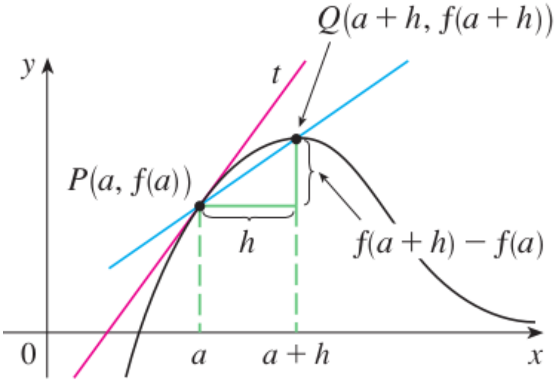


1 Definition The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

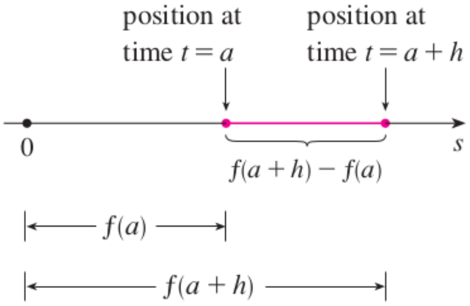
provided that this limit exists.

Another expression for calculating the slope of the tangent line.



$$m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

EXAMPLE 2 Find an equation of the tangent line to the hyperbola $y = 3/x$ at the point (3, 1).



Position function:

Average Velocity:

Instantaneous Velocity.

$$\boxed{v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower, 450 m above the ground.
(a) What is the velocity of the ball after 5 seconds?
(b) How fast is the ball traveling when it hits the ground?

Recall Galileo:

$$s(t) = 4.9t^2$$

The Derivative.

4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

Another notation:

EXAMPLE 4 Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number a.

Tangent line to a curve.

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

$$\boxed{y - f(a) = f'(a)(x - a)}$$

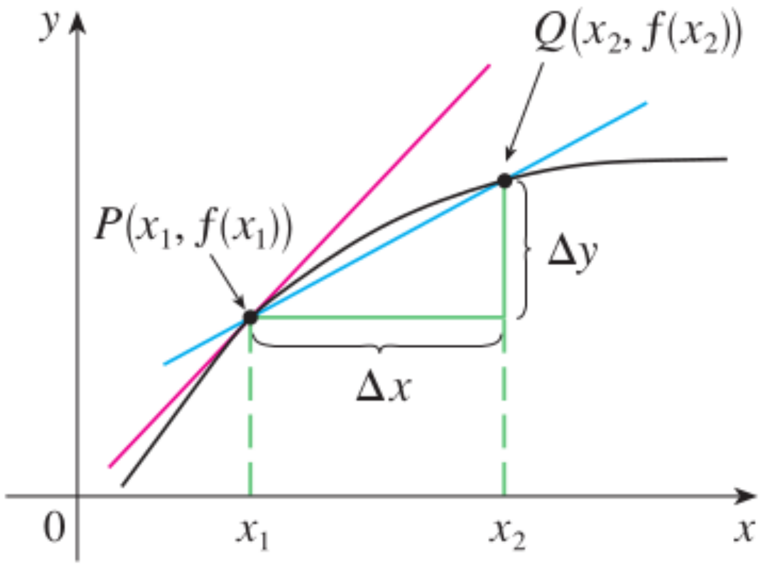
EXAMPLE 5 Find an equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at the point $(3, -6)$.

Rates of Change.

Increment in x.

Increment in y.

Average Change.



$$\boxed{6} \qquad \text{instantaneous rate of change} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

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14. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$.

(a) Find the velocity of the rock after one second.

(b) Find the velocity of the rock when $t = a$.

(c) When will the rock hit the surface?

(d) With what velocity will the rock hit the surface?