Chapter 2

Derivatives

2.3 Differentiation Formulas

Constant Function.

$$f(x) = c \quad \longrightarrow \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = 0$$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

Power Functions.

n = 1. $y = x$ $\boxed{\frac{dy}{dx} = 1}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{x+h-x}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1$$

$$= 1$$

n = 2. $y = x^2$ $\boxed{\frac{dy}{dx} = 2x}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2x\cancel{h} + h^{\cancel{2}1}}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h = 2x$$

n = 3. $y = x^3$ $\boxed{dy/dx = 3x^2}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{3x^2\cancel{h} + 3xh^{\cancel{2}1} + h^{\cancel{3}2}}{\cancel{h}}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2$$

$$= 3x^2$$

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

EXAMPLE 4 Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal.

Goal: Find where y' is zero.

$$y' = \lim_{h \to 0} \frac{(x+h)^4 - 6(x+h)^2 + 4 - x^4 + 6x^2 - 4}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^4 - x^4 - 6(x+h)^2 + 6x^2 + 4 - 4}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h} + \lim_{h \to 0} -6\left(\frac{(x+h)^2 - x^2}{h}\right) + \lim_{h \to 0} \frac{4-4}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h} - 6 \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} + \lim_{h \to 0} \frac{4-4}{h}$$

$$= \frac{d}{dx}(x^4) - 6 \frac{d}{dx}(x^2) + \frac{d}{dx}(4)$$

$$= 4x^3 - 6 \cdot 2x + 0$$

$$\Rightarrow \quad y' = 4x^3 - 12x$$

Solution:

$y' = 0 \Leftrightarrow 4x^3 - 12x = 0$

$\Leftrightarrow (4x^2 - 12)x = 0$

$\Leftrightarrow \boxed{x = 0 \text{ or } x = \pm\sqrt{3}}$

Multiplication by a constant.

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Sum.

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Difference.

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Product.

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Caution!!!

$$\boxed{\frac{d}{dx}(fg) \neq \frac{d}{dx}(f)\frac{d}{dx}(g).}$$

Example.

Example. Find the derivative of the function $f(x) = (5x^2 - 2)(x^3 + 3x)$.

Quotient.

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Caution !!

$$\boxed{\frac{d}{dx}\left(\frac{f}{g}\right) \neq \frac{\frac{d}{dx}(f)}{\frac{d}{dx}(g)}}$$

Example.

EXAMPLE 8 Let $y = \dfrac{x^2 + x - 2}{x^3 + 6}$. Compute the derivative.

General Power rule.

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Case n = 0:

Example. Find the derivative of the function $f(x) = x^{2/3}$.

EXAMPLE 13 At what points on the hyperbola $xy = 12$ is the tangent line parallel to the line $3x + y = 0$?

Summary of Differentiation Formulas.

Table of Differentiation Formulas

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf' \qquad (f+g)' = f' + g' \qquad (f-g)' = f' - g'$$

$$(fg)' = fg' + gf' \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$