

L.I Mathematical Statements

PROBLEM 1. Are the following statements? If it is, say if it is true or false. If it is not, explain briefly why.

- a) $|-12| = -12$.
- b) $x < 0$.
- c) Is that an odd integer?
- d) If $a = 2$ and $b = 4$, then $a + b = 6$.

L.II Logic and Mathematical Language

PROBLEM 2. Give the converse and the contrapositive of the following conditional statements.

- a) If it is Saturday, then Angela sleeps in.
- b) If it rains outside, then I will use my umbrella.
- c) If I went surfing, then the surf was bigger than 4 feet high.

PROBLEM 3. Write useful negations of the following statements in English. You can use symbols to simplify the statement.

- a) It is raining and Charlie is cold.
- b) If it is raining, then Charlie is cold.
- c) For every real number x , there exists a real number y such that $x + y = 0$.
- d) $|a| > 0$ if and only if $a \neq 0$.

PROBLEM 4.

- a) By constructing the truth table of $P \Rightarrow Q$ and $Q \Rightarrow P$, show when a conditional statement and its converse do not have the same truth values.
- b) By constructing the truth table of $P \Rightarrow Q$ and $(\neg Q) \Rightarrow (\neg P)$, show a conditional statement and its contrapositive always have the same truth values.

L.III Methods of Proof

PROBLEM 5. Suppose that a and b are integers. Prove each of the following.

- a) If a and b are both odd, then $a + b$ is even.
- b) If a is even and b is odd, then $a + b$ is even.

PROBLEM 6. A rational number is a number q that can be put in the form of a fraction, that is there exist two integers n and m such that $q = n/m$. Show that $\sqrt{2}$ is not rational.

PROBLEM 7. Prove that there exist integers m and n such that $2m + 3n = 12$.

PROBLEM 8. For all even integers a and b , a/b is an integer.