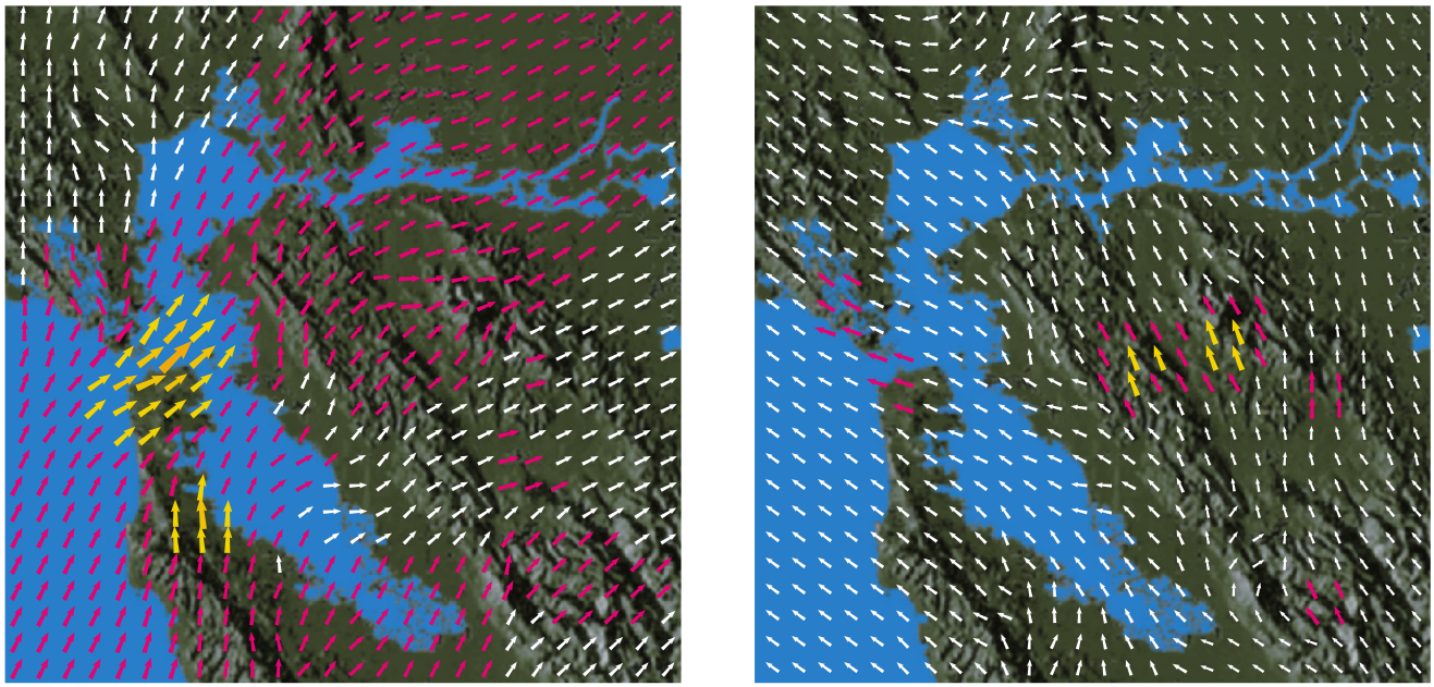
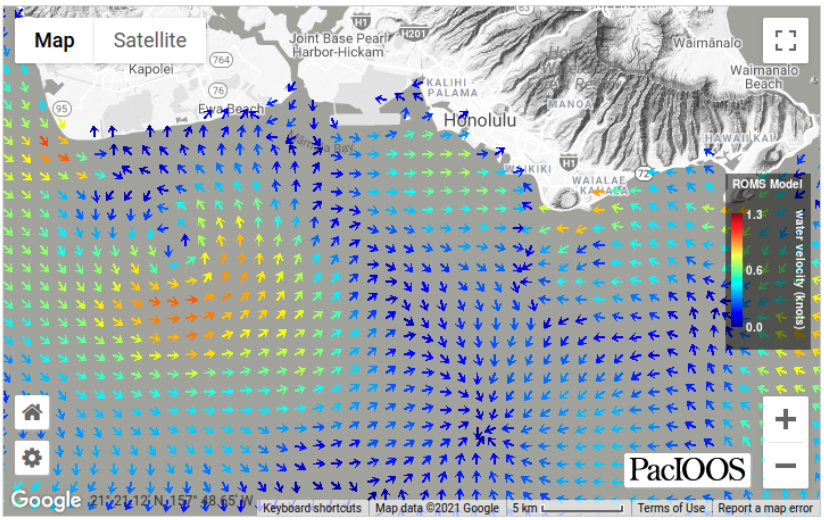
Section 16.1 Vector Fields.

Examples.

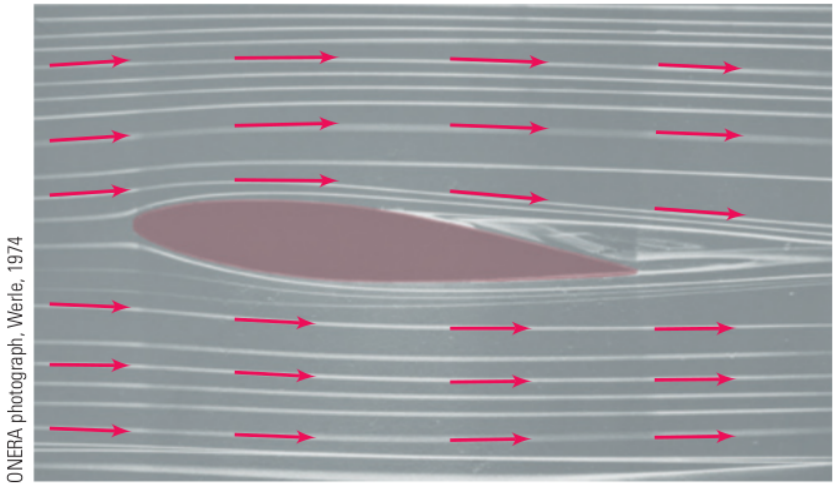


(a) 6:00 PM, March 1, 2010 (b) 6:00 AM, March 1, 2010

FIGURE 1 Velocity vector fields showing San Francisco Bay wind patterns



Map took from
http://www.pacioos.hawaii.edu/currents/model-oahu/

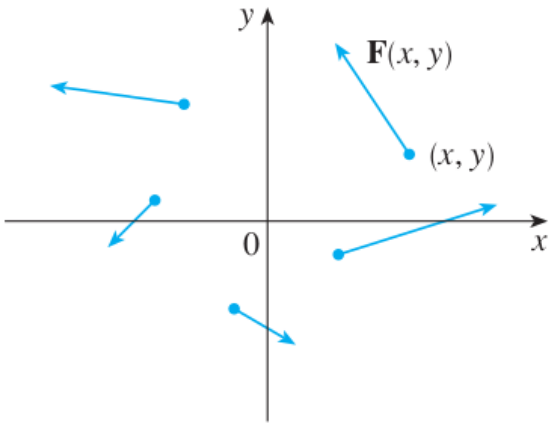


(b) Airflow past an inclined airfoil

Vector Fields in 2D.

1 Definition Let D be a set in $\mathbb{R}^2$ (a plane region). A **vector field on** $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$.

Representation.



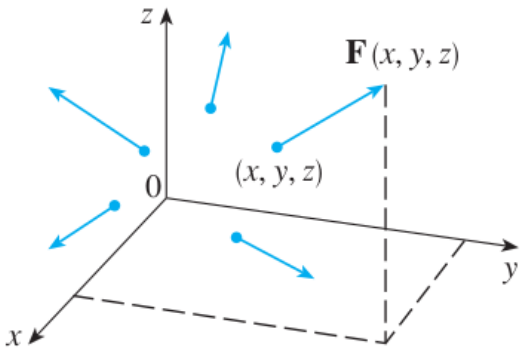
Component Functions.

Remark:

Vector Fields in 3D.

2 Definition Let E be a subset of $\mathbb{R}^3$. A **vector field on** $\mathbb{R}^3$ is a function $\mathbf{F}$ that assigns to each point (x, y, z) in E a three-dimensional vector $\mathbf{F}(x, y, z)$.

Representation.



Component Functions.

Remark:

EXAMPLE 1 A vector field on $\mathbb{R}^2$ is defined by $\mathbf{F}(x, y) = -y\,\mathbf{i} + x\,\mathbf{j}$. Describe $\mathbf{F}$ by sketching some of the vectors $\mathbf{F}(x, y)$.

EXAMPLE 2 Sketch the vector field on $\mathbb{R}^3$ given by $\mathbf{F}(x, y, z) = z\,\mathbf{k}$.

EXAMPLE 4 Newton's Law of Gravitation.

Gradient Fields.

Gradient.

EXAMPLE 6 Find the gradient vector field of $f(x, y) = x^2y - y^3$. Plot the gradient vector field together with a contour map of f. How are they related?

Conservative Vector Fields.