

Section 1.2
Problems Solution

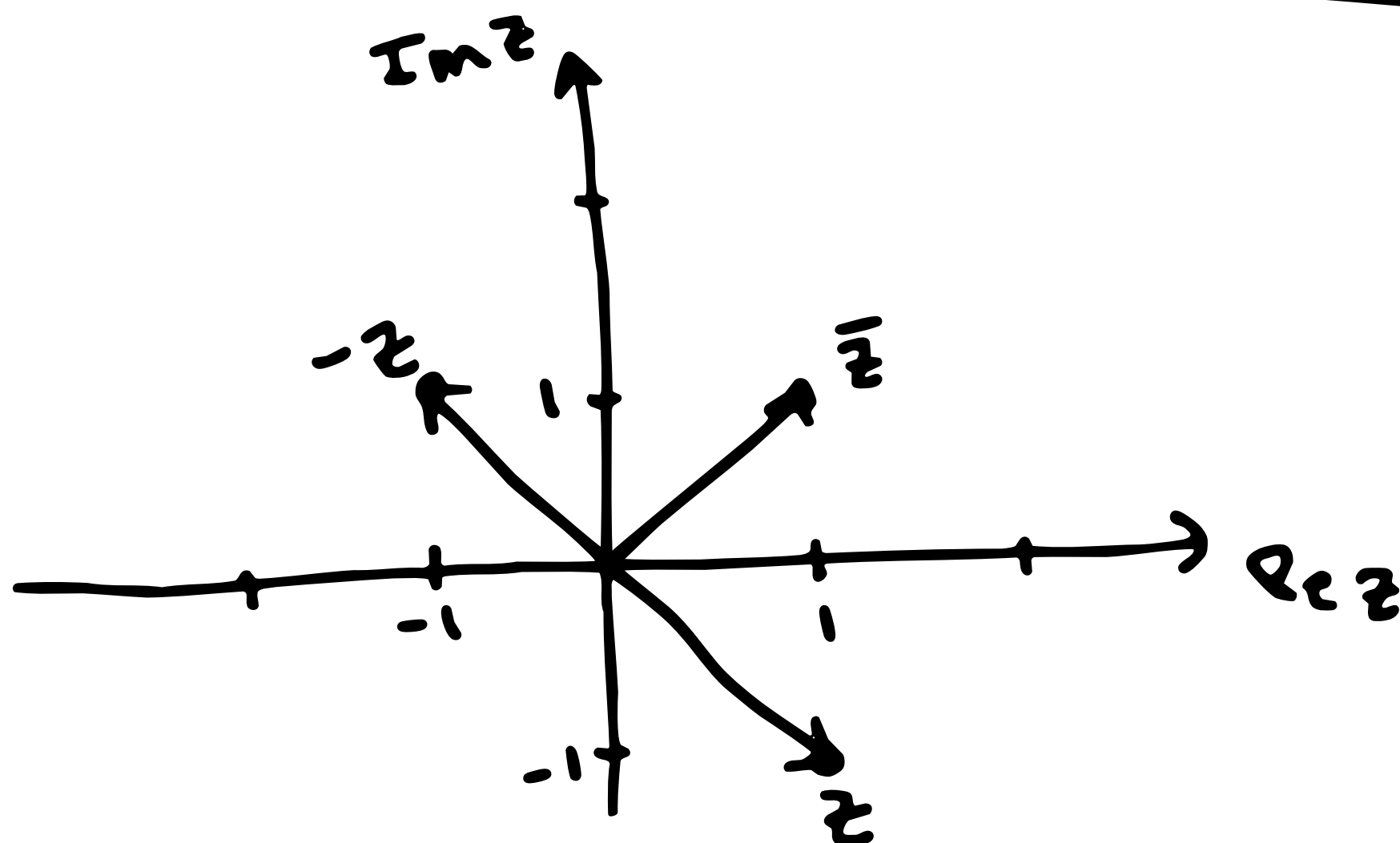
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Problem 1

$$-z = -1 + i$$

$$\bar{z} = 1 + i$$

$$|z| = \sqrt{2}$$



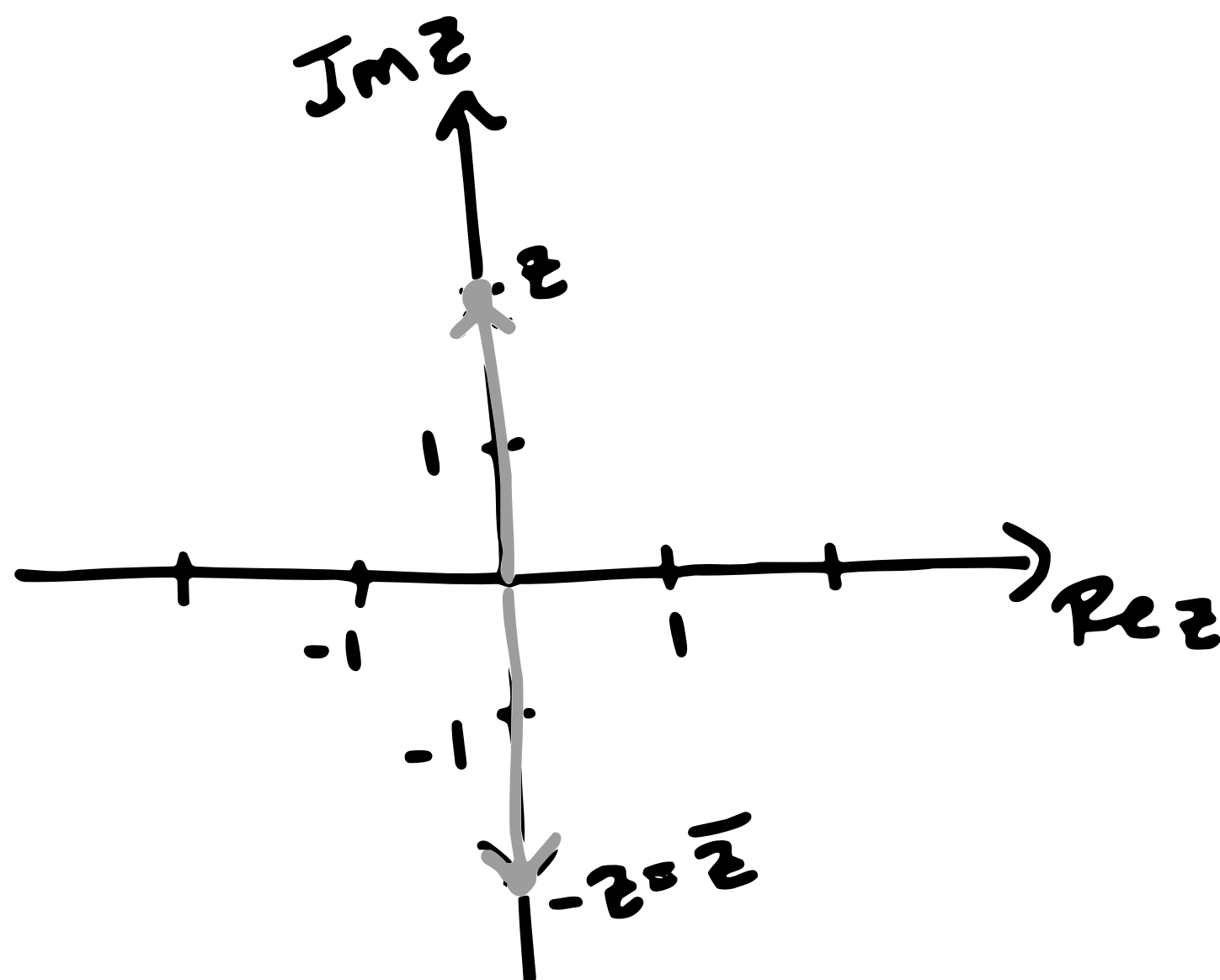
Problem 6

$$z = (1+i)^2 = 2i$$

$$-z = -2i$$

$$\bar{z} = -2i$$

$$|z| = 2$$



Problem 12

$$\left| \frac{1+i}{(1-i)(1+3i)} \right| = \frac{|1+i|}{|1-i||1+3i|} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{10}} = \boxed{\frac{1}{\sqrt{10}}}$$

Problem 13

$$\left| \frac{i}{2-i} \right| = \frac{|i|}{|2-i|} = \frac{1}{\sqrt{5}} = \boxed{\frac{1}{\sqrt{5}}}$$

Problem 15

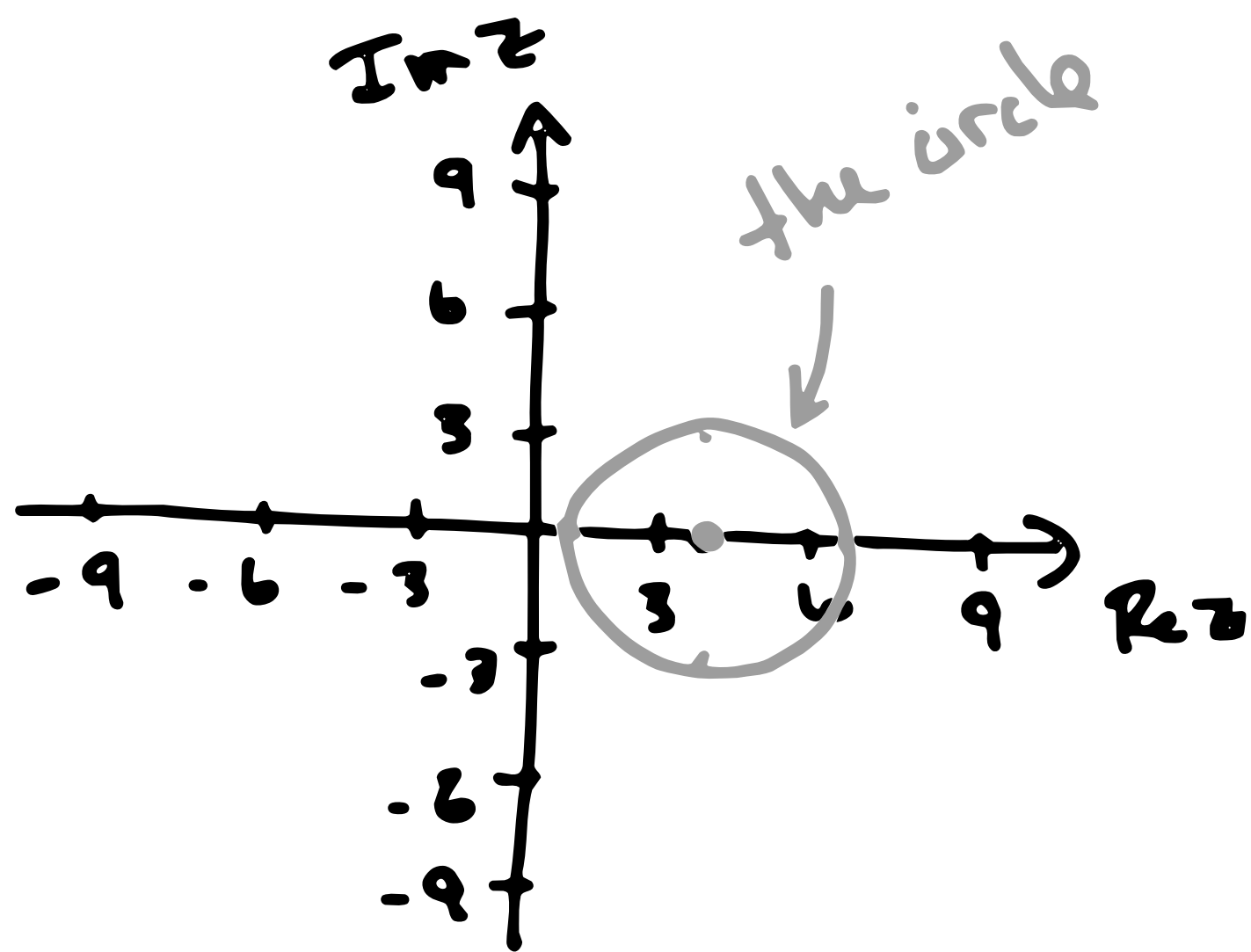
$$|z-4| = 3$$

$$\Leftrightarrow |x-4+iy| = 3$$

$$\Leftrightarrow \sqrt{(x-4)^2 + y^2} = 3$$

$$\Leftrightarrow (x-4)^2 + y^2 = 9$$

↳ circle centered at $z_0 = 4$ and of radius 3.

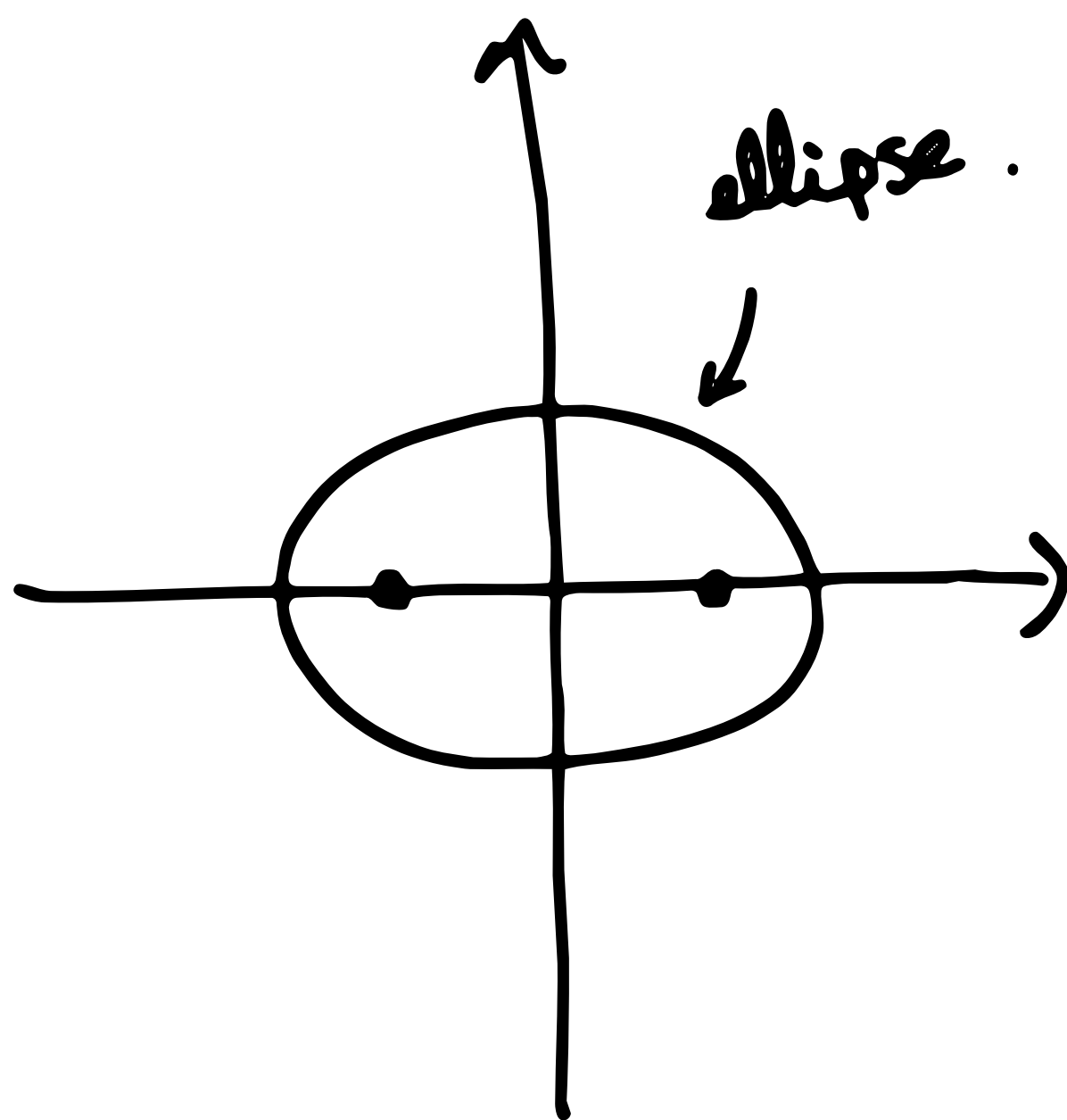


Problem 18

$$|z+1| + |z-1| = 4$$

Geometric interpretation:

Sum of the distances from z to -1 and from z to 1 is always a constant equal to 4
 \Rightarrow ellipse! foci -1 & 1



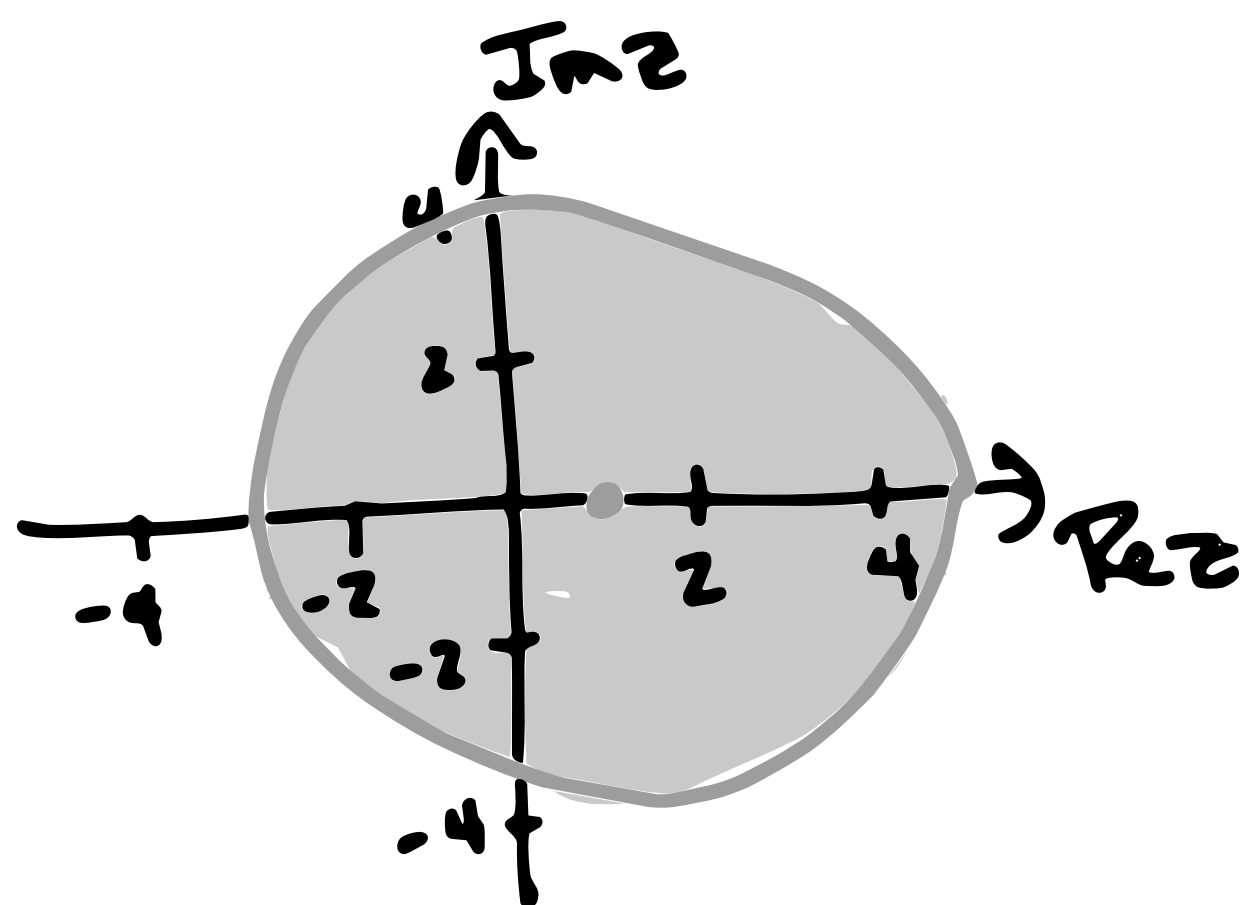
Problem 21

$$|z-1| = \sqrt{(x-1)^2 + y^2}$$

So, we get

$$(x-1)^2 + y^2 \leq 16$$

↳ Region bounded by the circle of radius 4 and centered at $z_0 = 1$



Problem 3a

Assume $|z - 3i| \leq 1$. Then

$$|z - 4| = |z - 3i + 3i - 4|$$

$$\leq |z - 3i| + |3i - 4|$$

$$\leq 1 + \sqrt{25}$$

$$= 1 + 5 = 6$$

$$\Rightarrow \boxed{|z - 4| \leq 6}$$