

FIGURE 23

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for *every* pair of numbers x_1 and x_2 in I with $x_1 < x_2$.

You can see from Figure 23 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

1.1 EXERCISES

1. If $f(x) = x + \sqrt{2-x}$ and $g(u) = u + \sqrt{2-u}$, is it true that $f = g$?

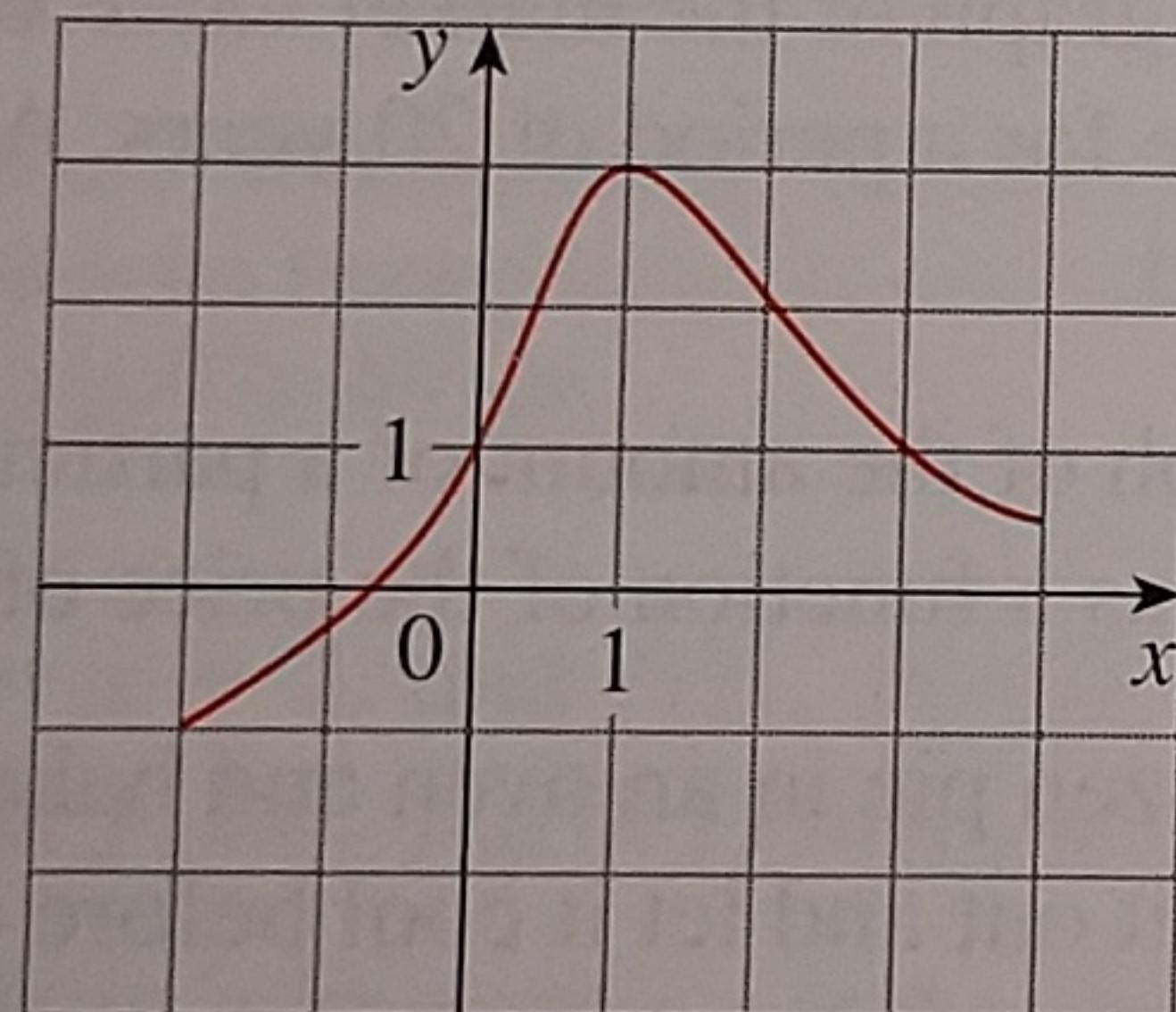
2. If

$$f(x) = \frac{x^2 - x}{x - 1} \quad \text{and} \quad g(x) = x$$

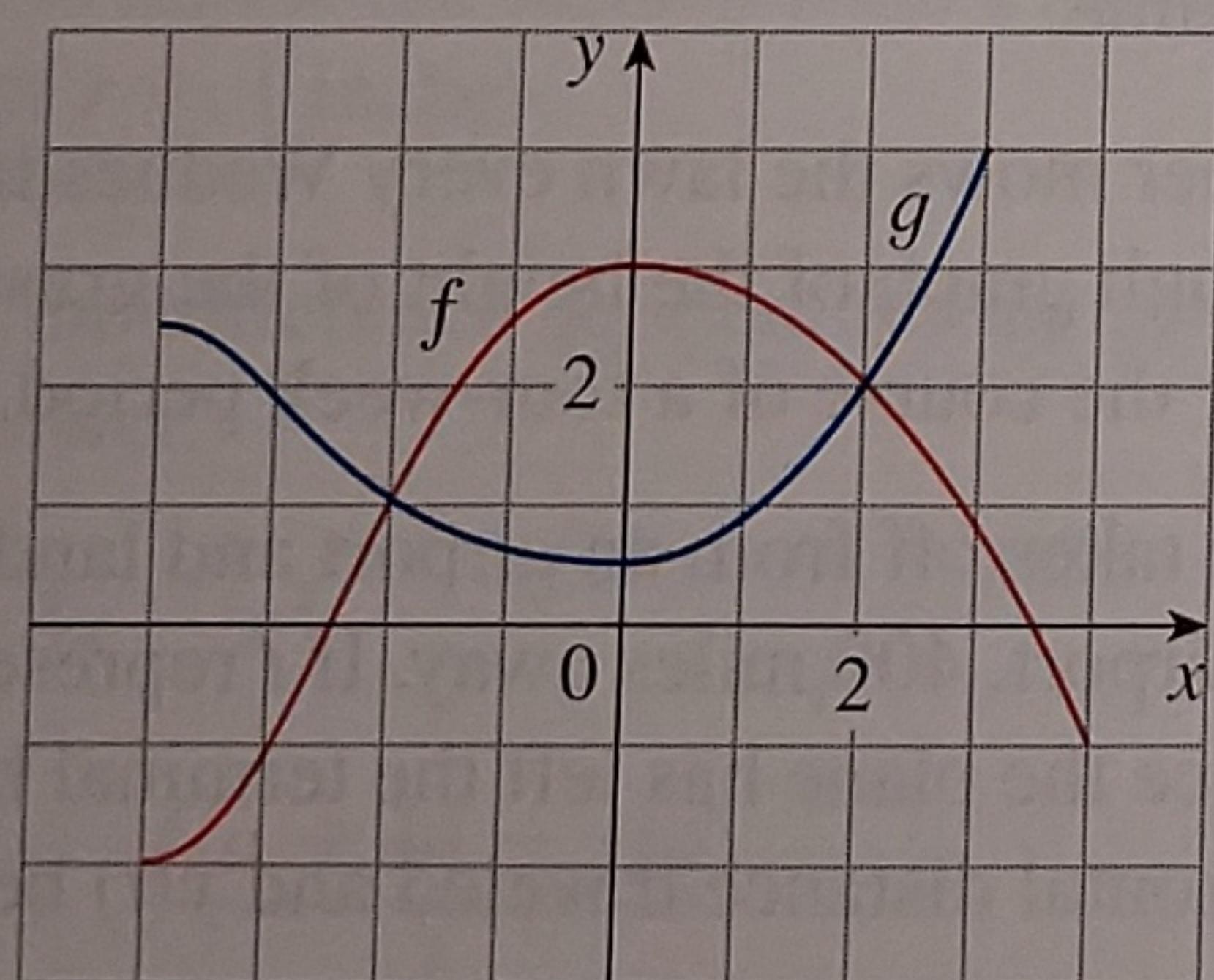
is it true that $f = g$?

3. The graph of a function f is given.

- (a) State the value of $f(1)$.
- (b) Estimate the value of $f(-1)$.
- (c) For what values of x is $f(x) = 1$?
- (d) Estimate the value of x such that $f(x) = 0$.
- (e) State the domain and range of f .
- (f) On what interval is f increasing?



4. The graphs of f and g are given.



- (a) State the values of $f(-4)$ and $g(3)$.
- (b) For what values of x is $f(x) = g(x)$?

- (c) Estimate the solution of the equation $f(x) = -1$.

- (d) On what interval is f decreasing?

- (e) State the domain and range of f .

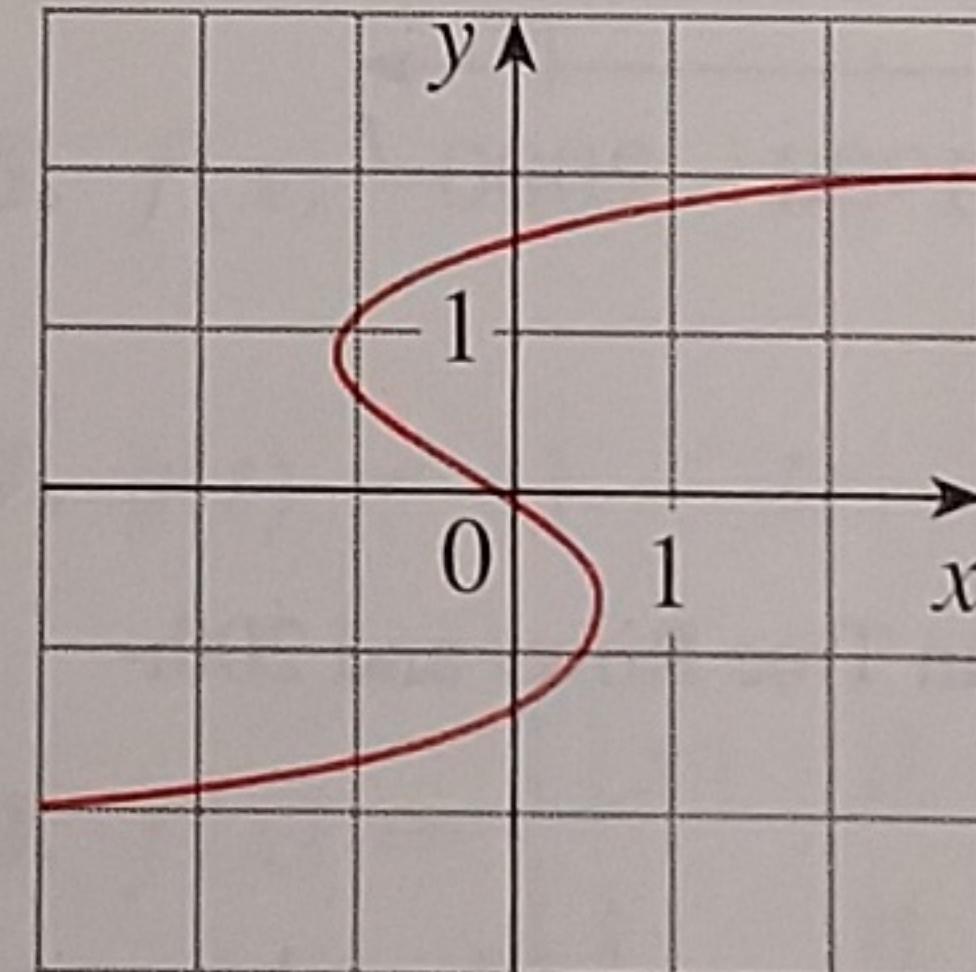
- (f) State the domain and range of g .

5. Figure 1 was recorded by an instrument operated by the California Department of Mines and Geology at the University Hospital of the University of Southern California in Los Angeles. Use it to estimate the range of the vertical ground acceleration function at USC during the Northridge earthquake.

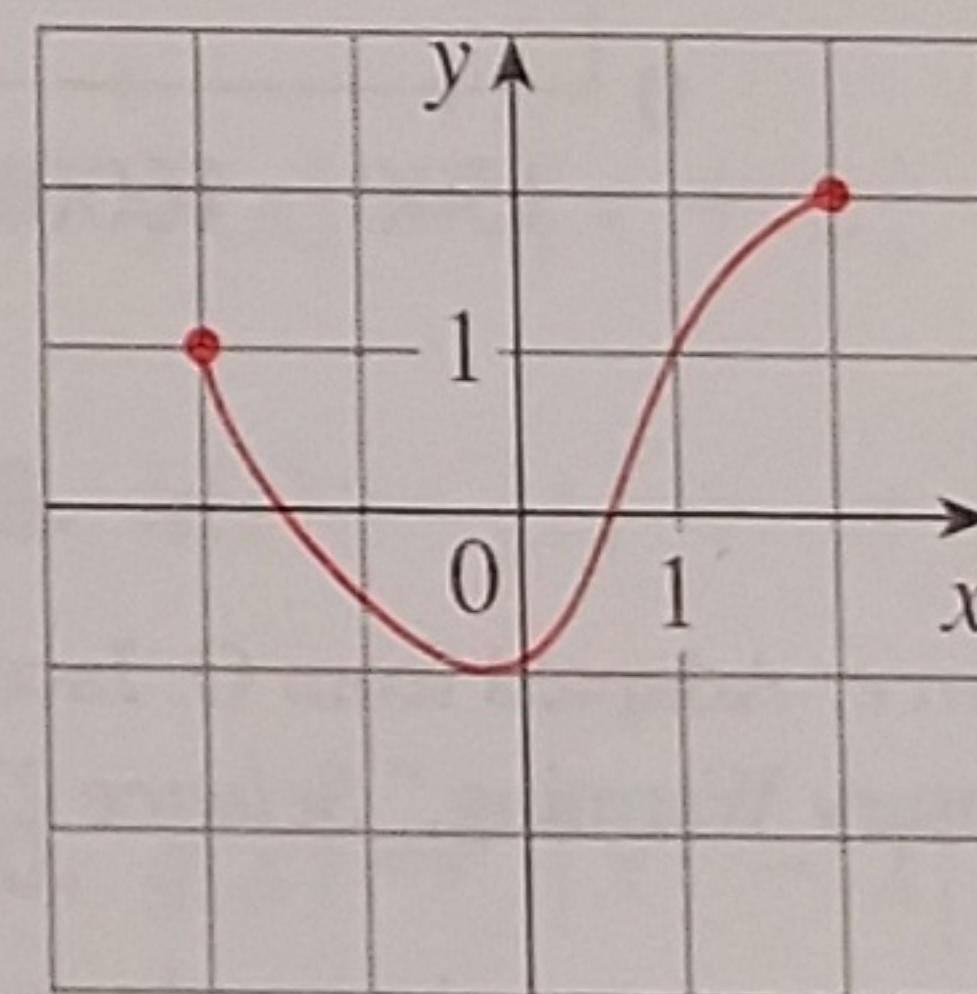
6. In this section we discussed examples of ordinary, everyday functions: Population is a function of time, postage cost is a function of weight, water temperature is a function of time. Give three other examples of functions from everyday life that are described verbally. What can you say about the domain and range of each of your functions? If possible, sketch a rough graph of each function.

- 7–10** Determine whether the curve is the graph of a function of x . If it is, state the domain and range of the function.

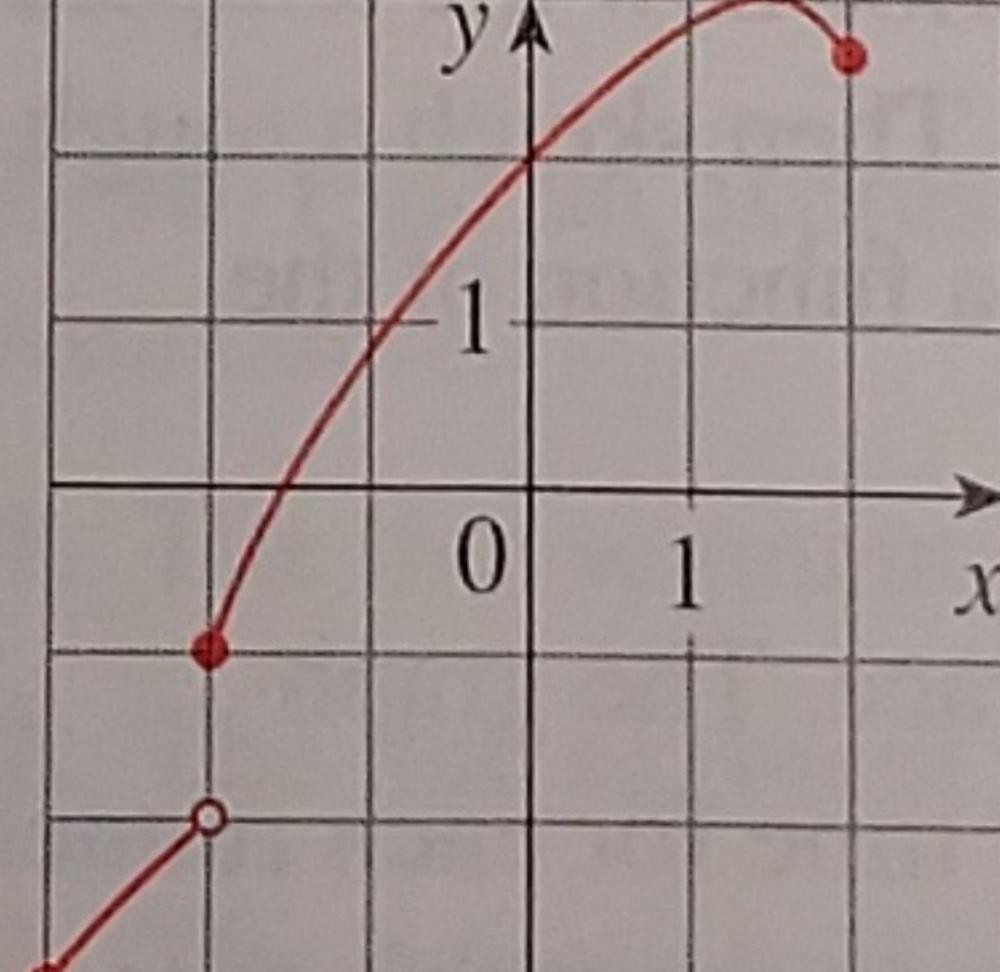
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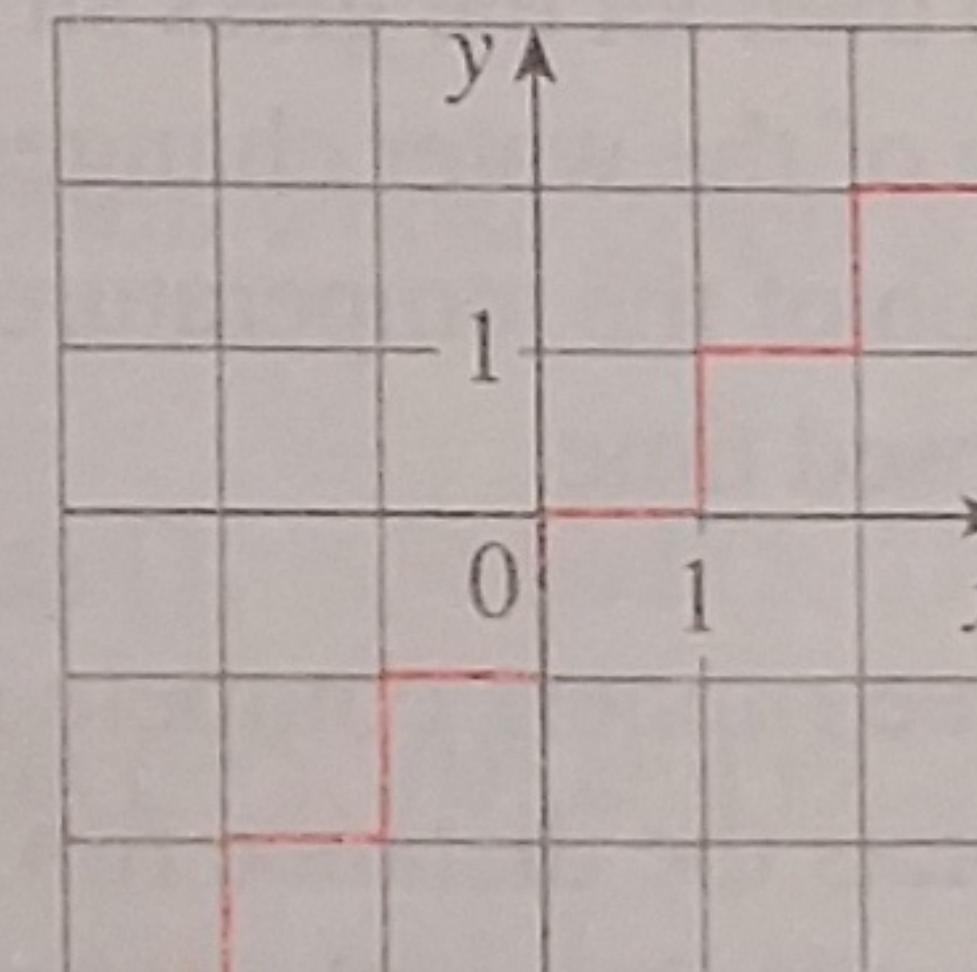
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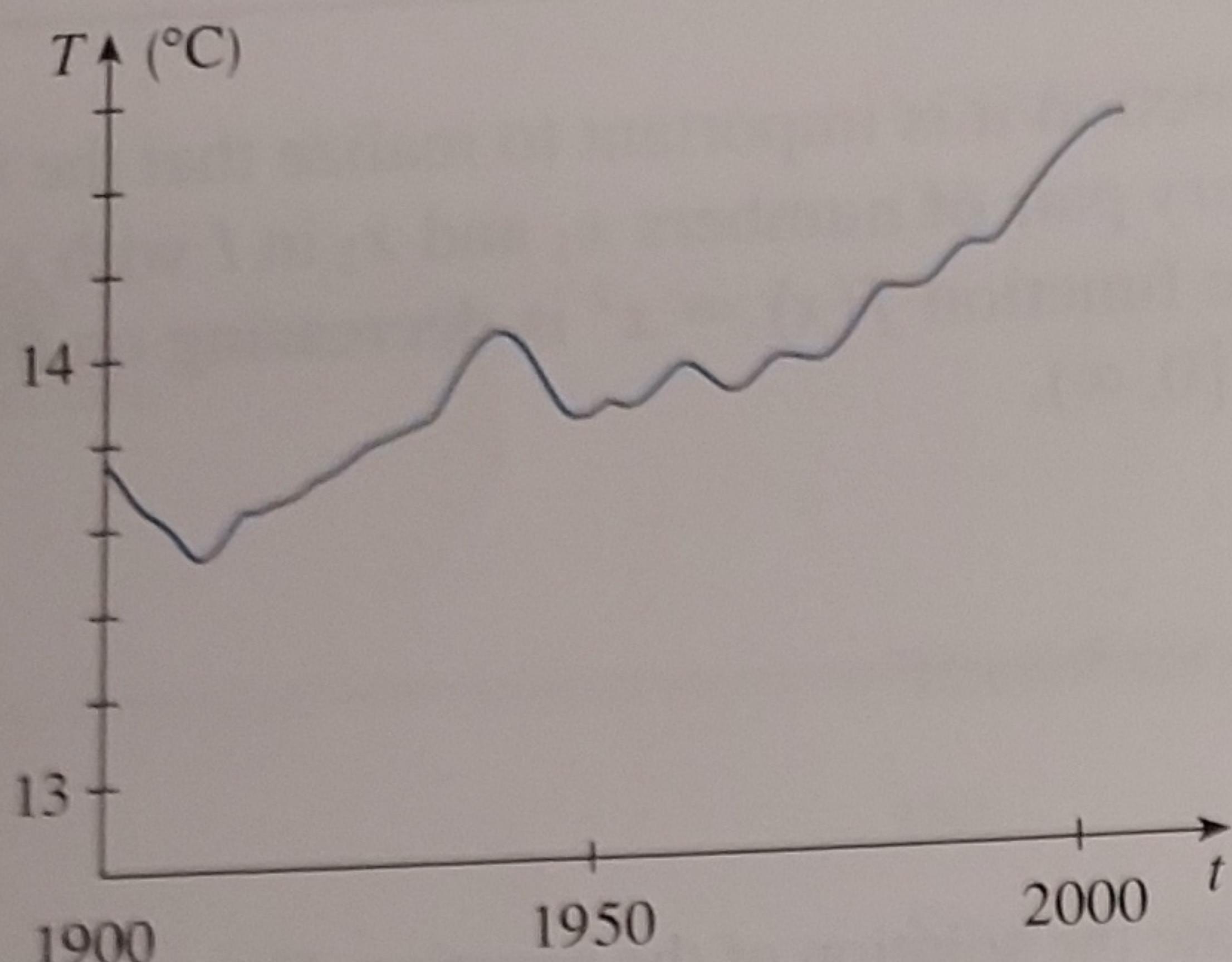
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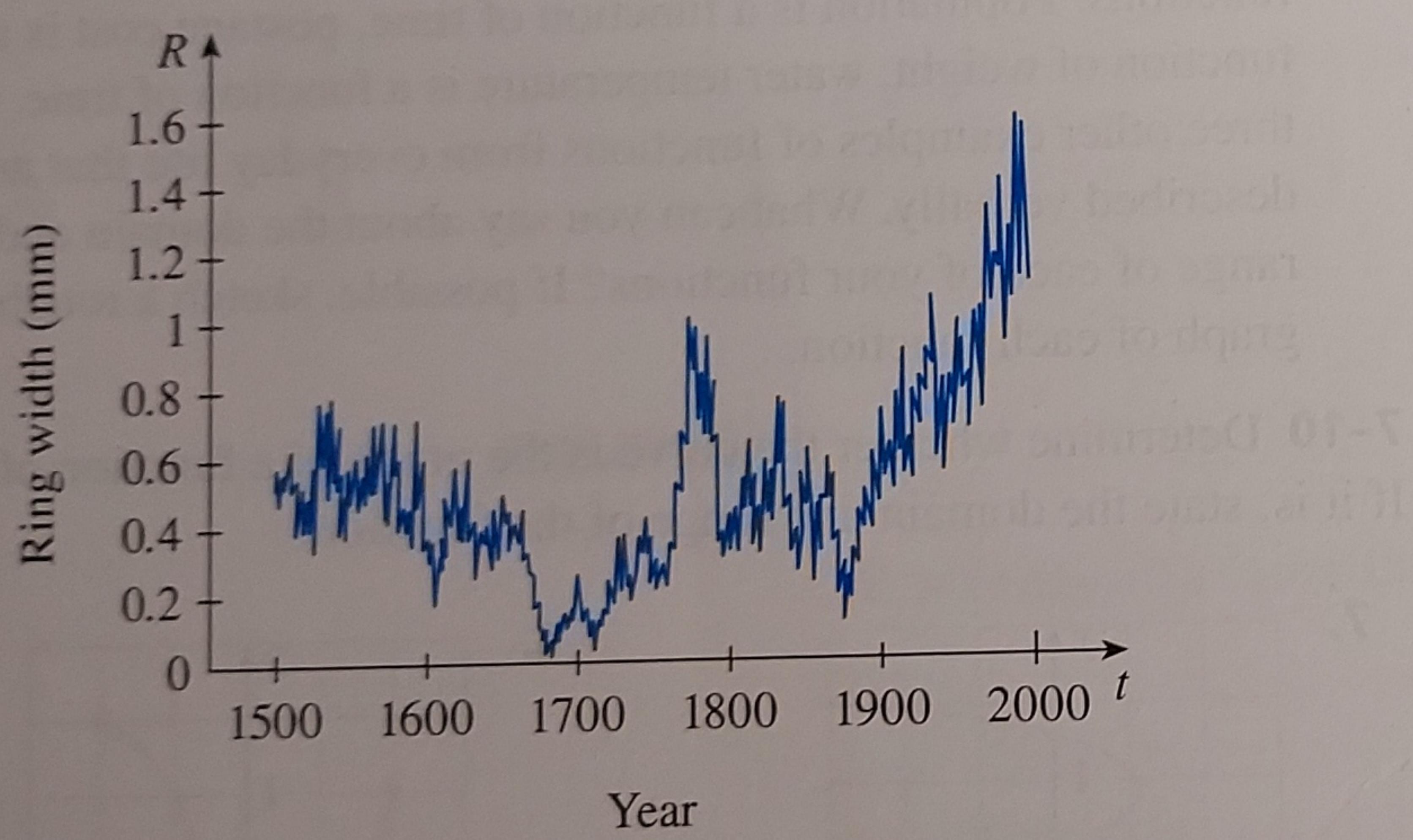


11. Shown is a graph of the global average temperature T during the 20th century. Estimate the following.
- The global average temperature in 1950
 - The year when the average temperature was 14.2°C
 - The year when the temperature was smallest; the year it was largest
 - The range of T



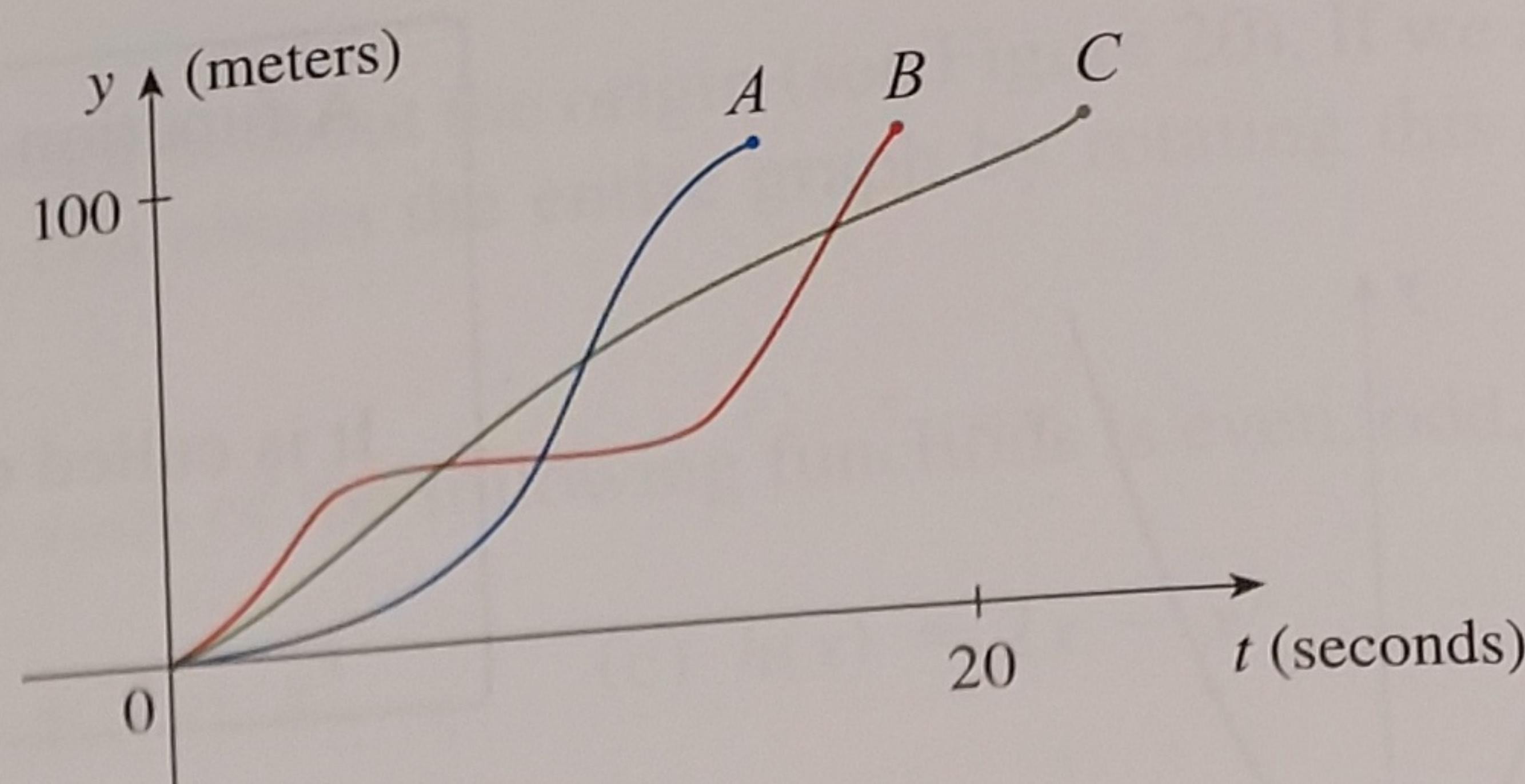
Source: Adapted from *Globe and Mail* [Toronto], 5 Dec. 2009. Print.

12. Trees grow faster and form wider rings in warm years and grow more slowly and form narrower rings in cooler years. The figure shows ring widths of a Siberian pine from 1500 to 2000.
- What is the range of the ring width function?
 - What does the graph tend to say about the temperature of the earth? Does the graph reflect the volcanic eruptions of the mid-19th century?

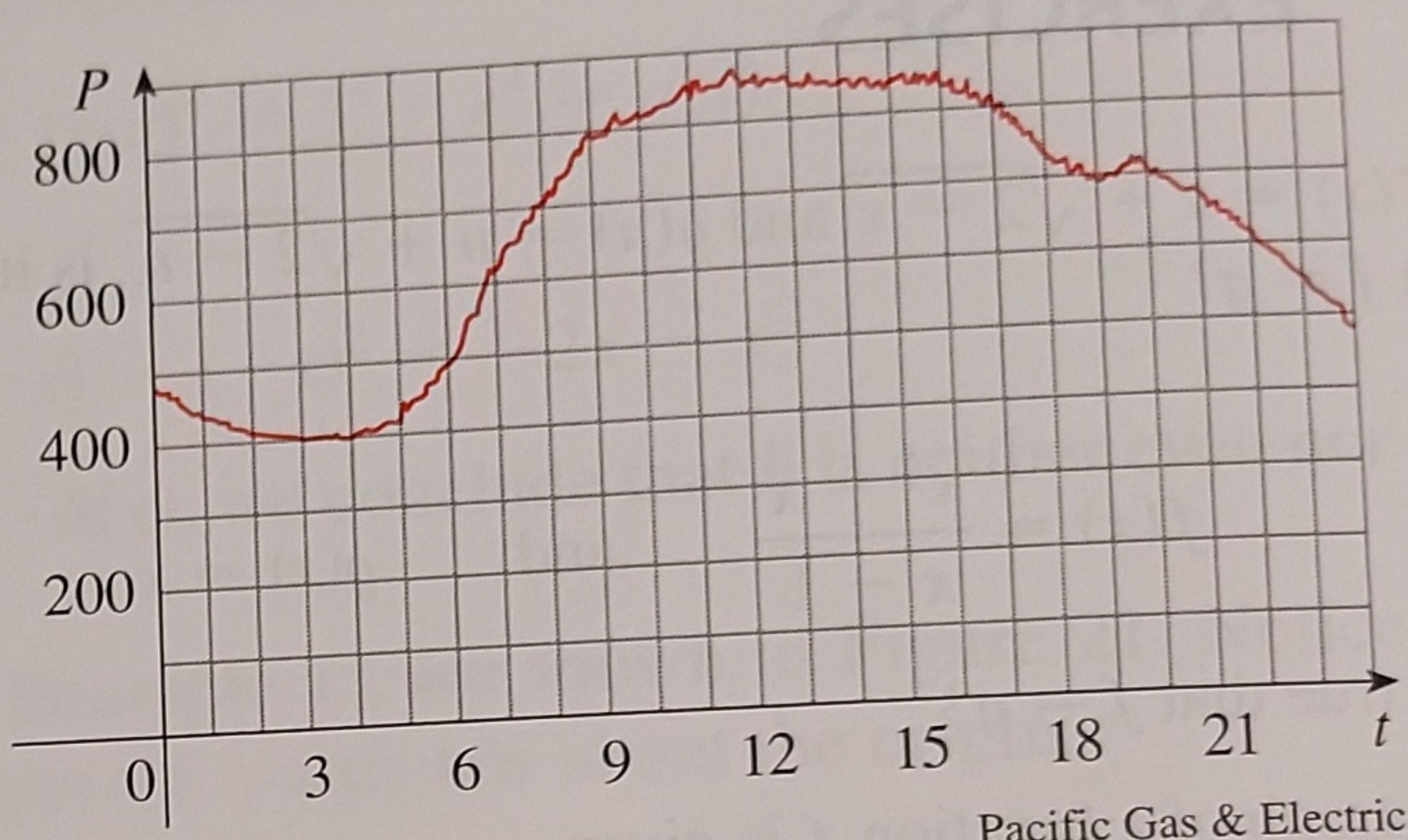


Source: Adapted from G. Jacoby et al., "Mongolian Tree Rings and 20th-Century Warming," *Science* 273 (1996): 771–73.

13. You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.
14. Three runners compete in a 100-meter race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race?



15. The graph shows the power consumption for a day in September in San Francisco. (P is measured in megawatts; t is measured in hours starting at midnight.)
- What was the power consumption at 6 AM? At 6 PM?
 - When was the power consumption the lowest? When was it the highest? Do these times seem reasonable?



16. Sketch a rough graph of the number of hours of daylight as a function of the time of year.
17. Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.
18. Sketch a rough graph of the market value of a new car as a function of time for a period of 20 years. Assume the car is well maintained.
19. Sketch the graph of the amount of a particular brand of coffee sold by a store as a function of the price of the coffee.
20. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.
21. A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.
22. An airplane takes off from an airport and lands an hour later at another airport, 400 miles away. If t represents the time in minutes since the plane has left the terminal building, let $x(t)$ be the horizontal distance traveled and $y(t)$ be the altitude of the plane.
- Sketch a possible graph of $x(t)$.
 - Sketch a possible graph of $y(t)$.

- (c) Sketch a possible graph of the ground speed.
 (d) Sketch a possible graph of the vertical velocity.
- 23.** Temperature readings T (in °F) were recorded every two hours from midnight to 2:00 PM in Atlanta on June 4, 2013. The time t was measured in hours from midnight.

t	0	2	4	6	8	10	12	14
T	74	69	68	66	70	78	82	86

- (a) Use the readings to sketch a rough graph of T as a function of t .
 (b) Use your graph to estimate the temperature at 9:00 AM.
- 24.** Researchers measured the blood alcohol concentration (BAC) of eight adult male subjects after rapid consumption of 30 mL of ethanol (corresponding to two standard alcoholic drinks). The table shows the data they obtained by averaging the BAC (in g/dL) of the eight men.
- (a) Use the readings to sketch the graph of the BAC as a function of t .
 (b) Use your graph to describe how the effect of alcohol varies with time.

t (hours)	BAC	t (hours)	BAC
0	0	1.75	0.022
0.2	0.025	2.0	0.018
0.5	0.041	2.25	0.015
0.75	0.040	2.5	0.012
1.0	0.033	3.0	0.007
1.25	0.029	3.5	0.003
1.5	0.024	4.0	0.001

Source: Adapted from P. Wilkinson et al., "Pharmacokinetics of Ethanol after Oral Administration in the Fasting State," *Journal of Pharmacokinetics and Biopharmaceutics* 5 (1977): 207–24.

- 25.** If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.
- 26.** A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r+1$ inches.

- 27–30** Evaluate the difference quotient for the given function. Simplify your answer.

27. $f(x) = 4 + 3x - x^2$, $\frac{f(3+h) - f(3)}{h}$

28. $f(x) = x^3$, $\frac{f(a+h) - f(a)}{h}$

29. $f(x) = \frac{1}{x}$, $\frac{f(x) - f(a)}{x - a}$

30. $f(x) = \frac{x+3}{x+1}$, $\frac{f(x) - f(1)}{x-1}$

- 31–37** Find the domain of the function.

31. $f(x) = \frac{x+4}{x^2-9}$

32. $f(x) = \frac{2x^3-5}{x^2+x-6}$

33. $f(t) = \sqrt[3]{2t-1}$

34. $g(t) = \sqrt{3-t} - \sqrt{2+t}$

35. $h(x) = \frac{1}{\sqrt{x^2-5x}}$

36. $f(u) = \frac{u+1}{1+\frac{1}{u+1}}$

37. $F(p) = \sqrt{2-\sqrt{p}}$

- 38.** Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4-x^2}$.

- 39–40** Find the domain and sketch the graph of the function.

39. $f(x) = 1.6x - 2.4$

40. $g(t) = \frac{t^2-1}{t+1}$

- 41–44** Evaluate $f(-3)$, $f(0)$, and $f(2)$ for the piecewise defined function. Then sketch the graph of the function.

41. $f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 1-x & \text{if } x \geq 0 \end{cases}$

42. $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x < 2 \\ 2x-5 & \text{if } x \geq 2 \end{cases}$

43. $f(x) = \begin{cases} x+1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

44. $f(x) = \begin{cases} -1 & \text{if } x \leq 1 \\ 7-2x & \text{if } x > 1 \end{cases}$

- 45–50** Sketch the graph of the function.

45. $f(x) = x + |x|$

46. $f(x) = |x+2|$

47. $g(t) = |1-3t|$

48. $h(t) = |t| + |t+1|$

49. $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

50. $g(x) = ||x|-1|$

- 51–56** Find an expression for the function whose graph is the given curve.

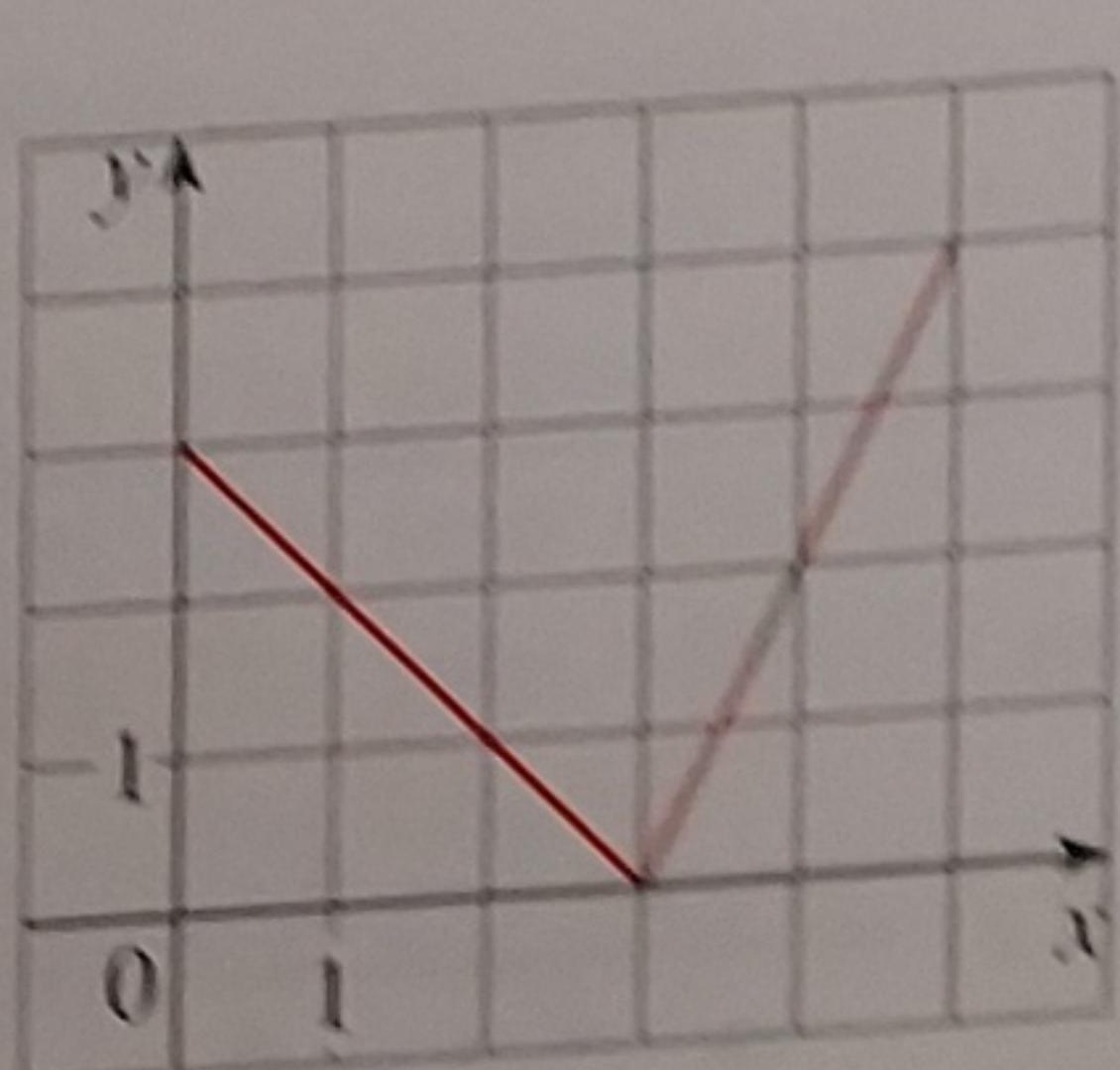
- 51.** The line segment joining the points $(1, -3)$ and $(5, 7)$

- 52.** The line segment joining the points $(-5, 10)$ and $(7, -10)$

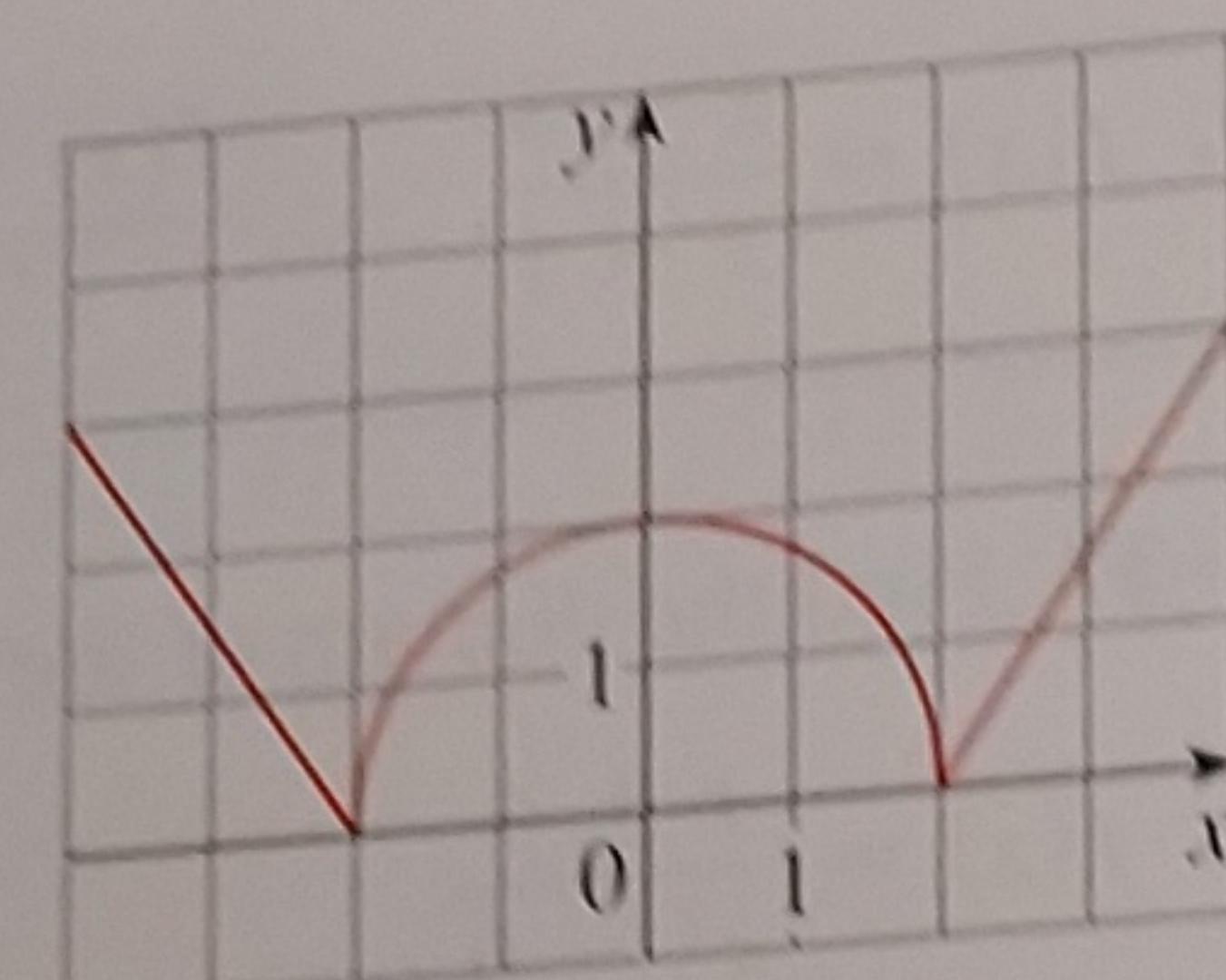
- 53.** The bottom half of the parabola $x + (y-1)^2 = 0$

- 54.** The top half of the circle $x^2 + (y-2)^2 = 4$

55.



56.

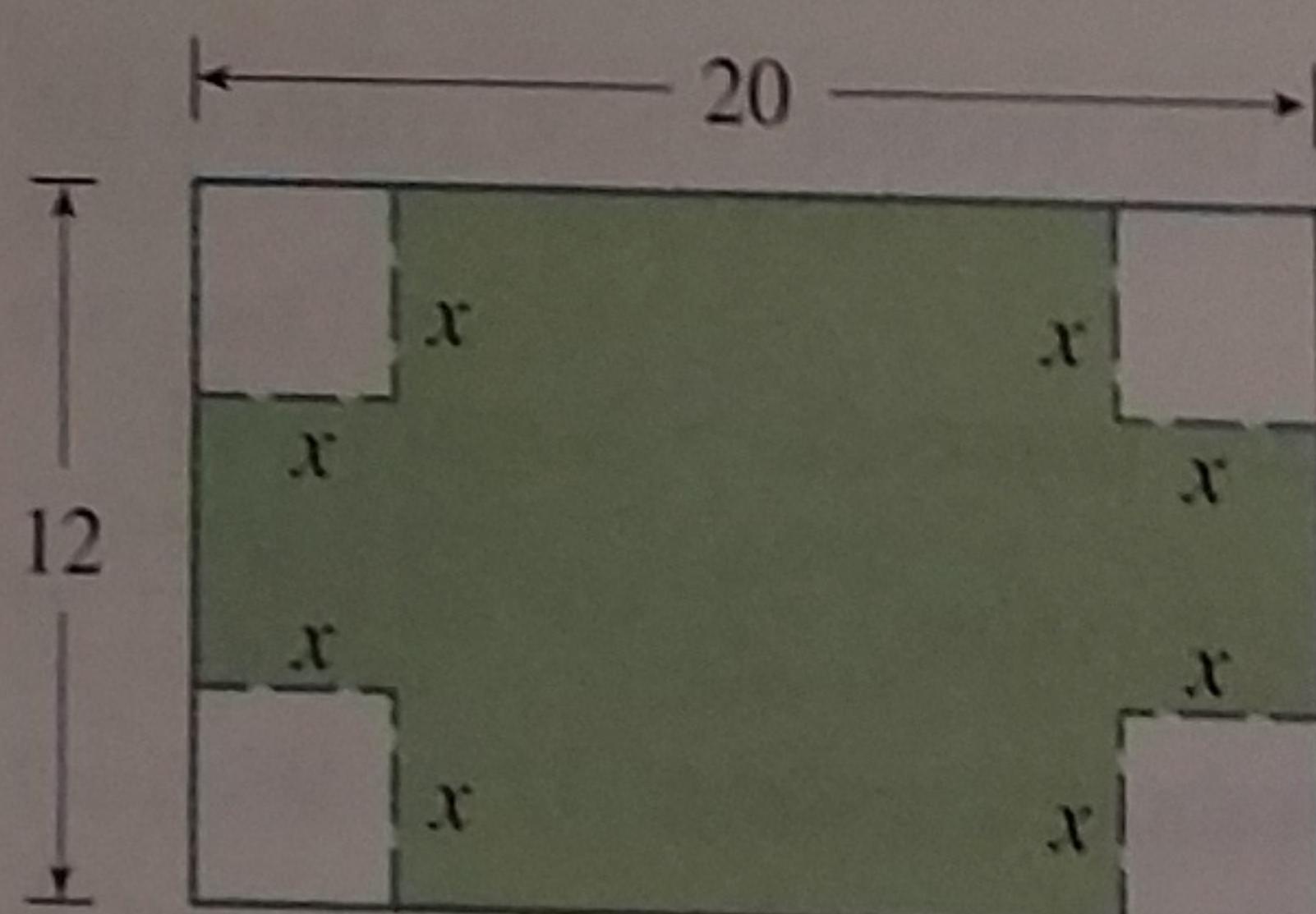


- 57–61** Find a formula for the described function and state its domain.

57. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.
58. A rectangle has area 16 m^2 . Express the perimeter of the rectangle as a function of the length of one of its sides.
59. Express the area of an equilateral triangle as a function of the length of a side.
60. A closed rectangular box with volume 8 ft^3 has length twice the width. Express the height of the box as a function of the width.
61. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.
62. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window.



63. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x .



64. A cell phone plan has a basic charge of \$35 a month. The plan includes 400 free minutes and charges 10 cents for each additional minute of usage. Write the monthly cost C as a function of the number x of minutes used and graph C as a function of x for $0 \leq x \leq 600$.

65. In a certain state the maximum speed permitted on freeways is 65 mi/h and the minimum speed is 40 mi/h. The fine for violating these limits is \$15 for every mile per hour above the maximum speed or below the minimum speed. Express the amount of the fine F as a function of the driving speed x and graph $F(x)$ for $0 \leq x \leq 100$.

66. An electricity company charges its customers a base rate of \$10 a month, plus 6 cents per kilowatt-hour (kWh) for the first 1200 kWh and 7 cents per kWh for all usage over 1200 kWh. Express the monthly cost E as a function of the amount x of electricity used. Then graph the function E for $0 \leq x \leq 2000$.

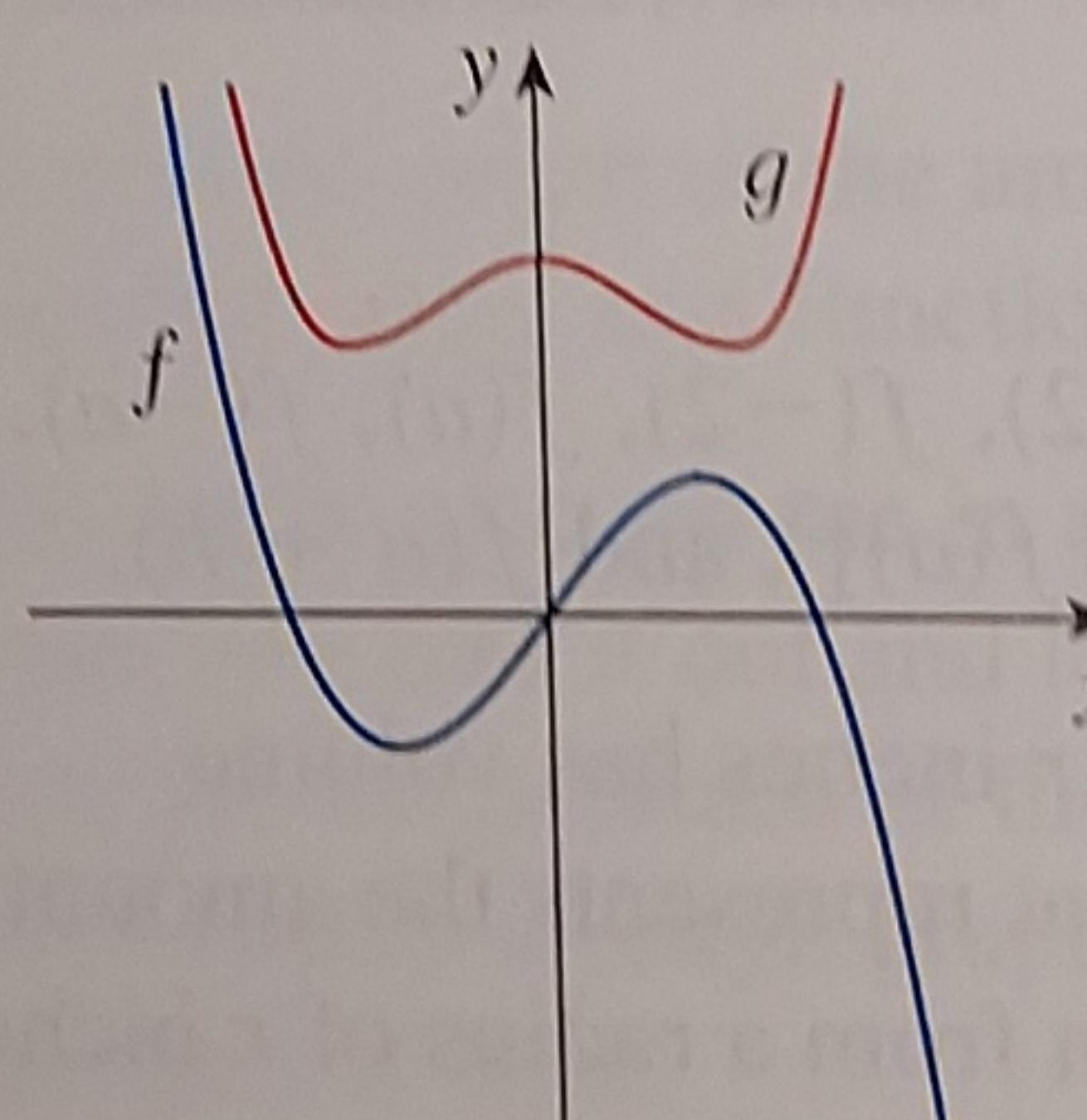
67. In a certain country, income tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$10,000 is taxed at a rate of 10%, up to an income of \$20,000. Any income over \$20,000 is taxed at 15%.

- (a) Sketch the graph of the tax rate R as a function of the income I .
(b) How much tax is assessed on an income of \$14,000?
On \$26,000?
(c) Sketch the graph of the total assessed tax T as a function of the income I .

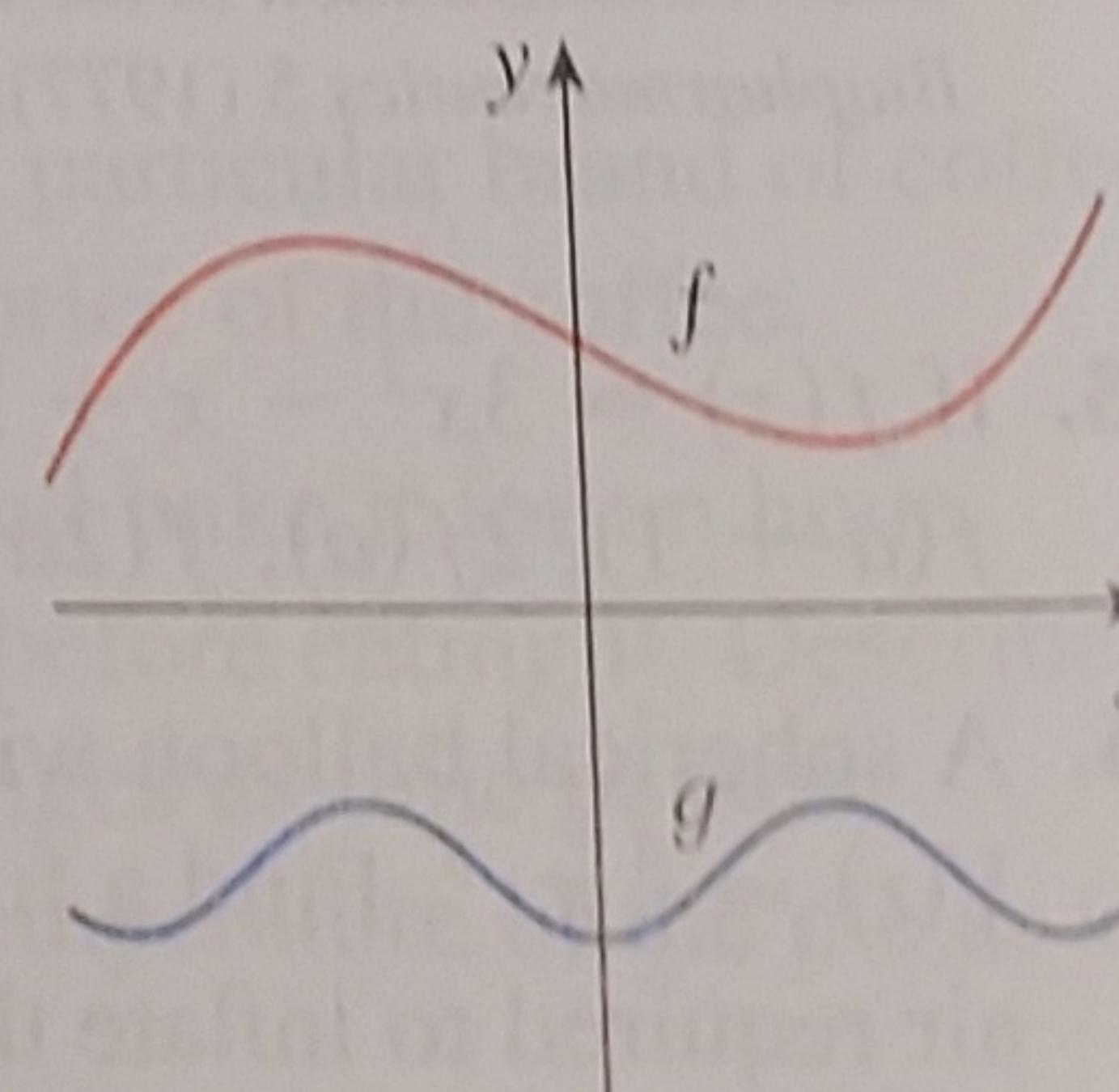
68. The functions in Example 10 and Exercise 67 are called *step functions* because their graphs look like stairs. Give two other examples of step functions that arise in everyday life.

- 69–70** Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

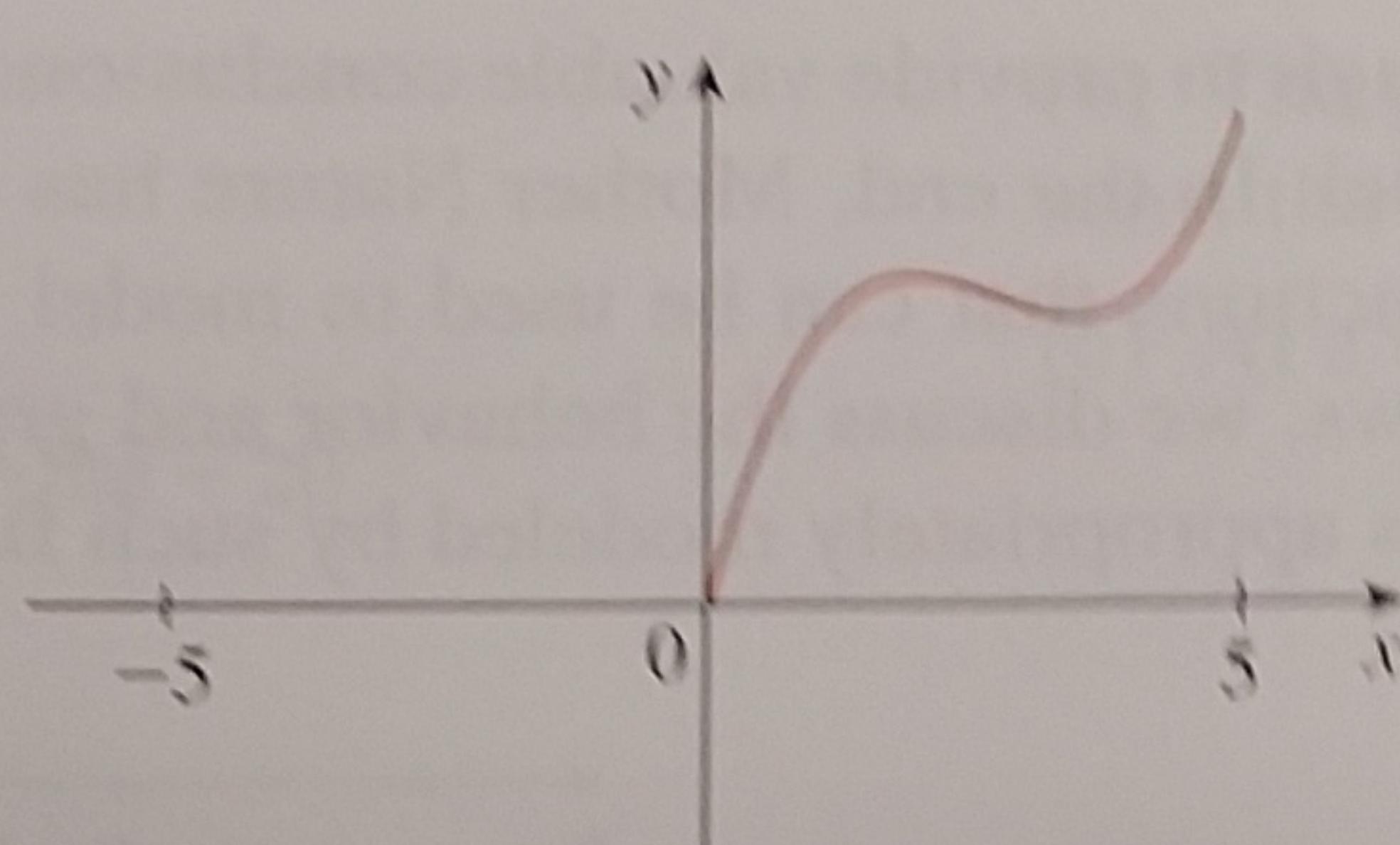
69.



70.



71. (a) If the point $(5, 3)$ is on the graph of an even function, what other point must also be on the graph?
(b) If the point $(5, 3)$ is on the graph of an odd function, what other point must also be on the graph?
72. A function f has domain $[-5, 5]$ and a portion of its graph is shown.
(a) Complete the graph of f if it is known that f is even.
(b) Complete the graph of f if it is known that f is odd.



73–78 Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

73. $f(x) = \frac{x}{x^2 + 1}$

74. $f(x) = \frac{x^2}{x^4 + 1}$

75. $f(x) = \frac{x}{x + 1}$

76. $f(x) = x|x|$

77. $f(x) = 1 + 3x^2 - x^4$

78. $f(x) = 1 + 3x^3 - x^5$

79. If f and g are both even functions, is $f + g$ even? If f and g are both odd functions, is $f + g$ odd? What if f is even and g is odd? Justify your answers.

80. If f and g are both even functions, is the product fg even? If f and g are both odd functions, is fg odd? What if f is even and g is odd? Justify your answers.

1.2 Mathematical Models: A Catalog of Essential Functions

A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real-world phenomenon such as the size of a population, the demand for a product, the speed of a falling object, the concentration of a product in a chemical reaction, the life expectancy of a person at birth, or the cost of emission reductions. The purpose of the model is to understand the phenomenon and perhaps to make predictions about future behavior.

Figure 1 illustrates the process of mathematical modeling. Given a real-world problem, our first task is to formulate a mathematical model by identifying and naming the independent and dependent variables and making assumptions that simplify the phenomenon enough to make it mathematically tractable. We use our knowledge of the physical situation and our mathematical skills to obtain equations that relate the variables. In situations where there is no physical law to guide us, we may need to collect data (either from a library or the Internet or by conducting our own experiments) and examine the data in the form of a table in order to discern patterns. From this numerical representation of a function we may wish to obtain a graphical representation by plotting the data. The graph might even suggest a suitable algebraic formula in some cases.

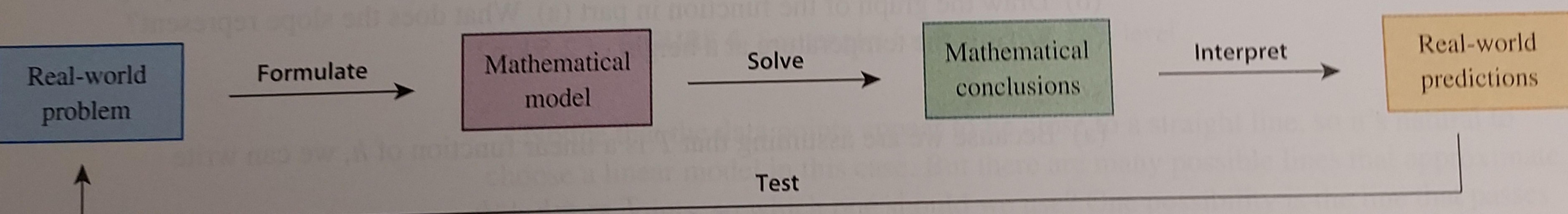


FIGURE 1
The modeling process

The second stage is to apply the mathematics that we know (such as the calculus that will be developed throughout this book) to the mathematical model that we have formulated in order to derive mathematical conclusions. Then, in the third stage, we take those mathematical conclusions and interpret them as information about the original real-world phenomenon by way of offering explanations or making predictions. The final step is to test our predictions by checking against new real data. If the predictions don't compare well with reality, we need to refine our model or to formulate a new model and start the cycle again.

A mathematical model is never a completely accurate representation of a physical situation—it is an *idealization*. A good model simplifies reality enough to permit math-