**Section 4.3 — Problem 8 — 5 points**

By the Fundamental Theorem of Calculus, we immediately have

$$g'(x) = \cos(x^2).$$

**Section 4.3 — Problem 10 — 5 points**

Again, from the Fundamental Theorem of Calculus, we have

$$h'(u) = \frac{\sqrt{t}}{t+1}.$$

**Section 4.3 — Problem 12 — 5 points**

Using a property of the integral, we can rewrite $R(y)$ as

$$R(y) = -\int_2^y t^3 \sin(t)\, dt.$$

Therefore, using the FTC, we obtain

$$R'(y) = -y^3 \sin(y).$$

**Section 4.3 — Problem 14 — 5 points**

Write $H(x)$ for $\int_1^x \frac{z^2}{z^4+1}\,dz$. Then the function $h(x)$ can be rewritten as

$$h(x) = H(\sqrt{x}).$$

From the Chain Rule, we find that $h'(x) = H'(\sqrt{x})\frac{d}{dx}(\sqrt{x})$. Using the FTC, we know that

$$H'(x) = \frac{x^2}{x^4+1}$$

and therefore, we obtain

$$h'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1}\left(\frac{d}{dx}(\sqrt{x})\right) = \frac{1}{2\sqrt{x}}\frac{x}{x^2+1} = \frac{\sqrt{x}}{2(x^2+1)}.$$

## Section 4.3 — Problem 18 — 5 points

We rewrite the expression of $y$ as

$$y = -\int_1^{\sin x} \sqrt{1+t^2}\, dt.$$

Writing $Y(x) = \int_1^x \sqrt{1+t^2}\, dt$, we can rewrite $y$ as

$$y(x) = Y(\sin(x)).$$

From the Chain Rule, we obtain

$$y'(x) = Y'(\sin(x))\frac{d}{dx}\big(\sin(x)\big).$$

Using the FTC, we see that

$$Y'(x) = \sqrt{1+x^2}$$

and replacing $x$ by $\sin(x)$, we obtain

$$y'(x) = \left(\sqrt{1+\sin^2(t)}\right)\cos(x)$$

## Section 4.3 — Problem 28 — 5 points

We simply the integrand:

$$(4-t)\sqrt{t} = 4\sqrt{t} - t^{3/2}.$$

An antiderivative of this last function is

$$\frac{8}{3}t^{3/2} - \frac{2}{5}t^{5/2}.$$

Therefore, from the FTC, we have

$$\begin{aligned}
\int_0^4 (4-t)\sqrt{t}\,dt &= \left(\tfrac{8}{3}t^{3/2} - \tfrac{2}{5}t^{5/2}\right)\Big|_0^4 \\
&= \left(\tfrac{8}{3}(4)^{3/2} - \tfrac{2}{5}(4)^{5/2}\right) - \left(\tfrac{8}{3}(0)^{3/2} - \tfrac{5}{2}(0)^{5/2}\right) \\
&= \frac{64}{3} - \frac{64}{5} \\
&= \frac{64}{15}(5-3) \\
&= \frac{128}{15}.
\end{aligned}$$

**Section 4.3 — Problem 30 — 5 points**

We rewrite the expression of the integrand as

$$(3u-2)(u+1) = 3u^2 + u - 2.$$

An antiderivative for this integrand is

$$u^3 + \frac{u^2}{2} - 2u.$$

Therefore, from the FTC, we get

$$\begin{aligned}
\int_{-1}^{2} (3u-2)(u+1)\,du &= \Big(u^3 + \tfrac{1}{2}u^2 - 2u\Big)\Big|_{-1}^{2} \\
&= \Big(8 + 2 - 4\Big) - \Big(-1 + \tfrac{1}{2} + 2\Big) \\
&= 6 - \tfrac{3}{2} \\
&= \tfrac{9}{2}.
\end{aligned}$$

**Section 4.3 — Problem 35 — 5 points**

The expression of the integrand can be rewritten as

$$\frac{v^5 + 3v^6}{v^4} = v + 3v^2.$$

An antiderivative for this integrand is

$$\frac{v^2}{2} + v^3.$$

Therefore, form the FTC, we have

$$\begin{aligned}\int_1^2 \frac{v^5 + 3v^6}{v^4}\, dv &= \left(\tfrac{1}{2}v^2 + v^3\right)\Big|_1^2 \\ &= \left(2 + 8\right) - \left(\tfrac{1}{2} + 1\right) \\ &= 10 - \tfrac{3}{2} \\ &= \tfrac{17}{2}.\end{aligned}$$

## **Section 4.3 — Problem 54 — 5 points**

We rewrite $g(x)$ as followed:

$$g(x) = \int_{1-2x}^{0} t \sin t \, dt + \int_{0}^{1+2x} t \sin t \, dt = -\int_{0}^{1-2x} t \sin t \, dt + \int_{0}^{1+2x} t \sin t \, dt.$$

Write

$$G(x) = \int_{0}^{x} t \sin t \, dt$$

so that

$$g(x) = -G(1-2x) + G(1+2x).$$

Using the Chain Rule, we get

$$g'(x) = -G'(1-2x)\frac{d}{dx}(1-2x) + G'(1+2x)\frac{d}{dx}(1+2x).$$

From the FTC, we have

$$G'(x) = x \sin x$$

so that

$$g'(x) = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x).$$

## Section 4.4 — Problem 58 — 5 points

**(a)** The general antiderivative of $a(t)$ is

$$t^2 + 3t + C.$$

Since $v(0) = -4$, we find that $C = -4$. Therefore, we obtain

$$v(t) = t^2 + 3t - 4.$$

**(b)** The distance travelled during the interval is given by

$$\int_0^3 |v(t)|\, dt.$$

The function $v(t) = (t+4)(t-1)$ and therefore

$$|v(t)| = \begin{cases} -(t^2+3t-4) & \text{if } 0 \le t \le 1 \\ t^2+3t-4 & \text{if } 1 < t \le 3. \end{cases}$$

We then obtain

$$\begin{aligned} \int_0^3 |v(t)|\, dt &= \int_0^1 -t^2 - 3t + 4\, dt + \int_1^3 t^2 + 3t - 4\, dt \\ &= \Big(-\frac{t^3}{3} - \frac{3}{2}t^2 + 4t\Big)\Big|_0^1 + \Big(\frac{t^3}{3} + \frac{3}{2}t^2 - 4t\Big)\Big|_1^3 \\ &= \frac{89}{6} \end{aligned}$$

Therefore, the total distance traveled is $\frac{89}{6} \approx 14.8333$ meters.

Total (Points): 50.