

C.I DISTRIBUTION FUNCTION

EXAMPLE 1. Let X be a discrete random variable with $\text{Im } X = \{0, 1, 2, 3, 4, 5\}$ and let p_X be its probability mass function given by $p_X(x) = 1/6$, if $x = 0, 1, 2, 3, 4, 5$.

- a) What is $P(X \leq -1)$?
- b) What is $P(X \leq 2)$?
- c) What is $P(X \leq 3.5)$?

EXAMPLE 2. Three coins are flipped and let X be the number of heads obtained. We know that $P(X = 0) = 1/8$, $P(X = 1) = P(X = 2) = 3/8$, and $P(X = 3) = 1/8$. Find the distribution function of X .

C.II CONTINUOUS RANDOM VARIABLE

EXAMPLE 3. A random variable X has density function

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a) Find the distribution function F_X of X .
- b) Find $P(0.25 \leq X \leq 0.75)$.

Uniform Distribution

EXAMPLE 4. Arrivals of customers at a checkout counter follow a Poisson distribution. It is known that, during a given 30-minute period, one customer arrived at the counter. Find the probability that the customer arrived during the last 5 minutes of the 30-minute period.

EXAMPLE 5. If a parachutist lands at a random point on a line between markers A and B , find the probability that she is closer to A than to B . Find the probability that her distance to A is more than three times her distance to B .

C.III FUNCTIONS OF RANDOM VARIABLES

EXAMPLE 6. Let X be a continuous random variable and let $g(x) = 2x + 3$. Find the distribution function of $Y = g(X)$ and its density function f_Y .

EXAMPLE 7. Let X be a continuous random variable. If $Y = X^2$, find the distribution function of Y and its density function.

EXAMPLE 8. Find the expectation of a continuous random variable $X \sim U(-1, 1)$.

EXAMPLE 9. Find the variance of $X \sim U(a, b)$.

Normal Distribution

EXAMPLE 10. The achievement scores of college entrance examination are normally distributed with mean 75 and standard deviation 10. What fraction of the scores lies between 80 and 90?