

Assigned date: 09/13/2021 9am
Due date: 09/20/2021 5pm

Last name: _____
First name: _____
Section: _____

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|-----------|----|----|----|----|----|----|----|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| Points: | 10 | 10 | 20 | 10 | 15 | 20 | 15 | 100 |
| Score: | | | | | | | | |

Instructions: You must answer all the questions below and upload your solutions (in a PDF format) to Gradescope (go to www.gradescope.com with the Entry code GEK6Y4). Be sure that after you scan your copy, it is clear and readable. You must name your file like this: `LASTNAME_FIRSTNAME.pdf`. A homework may not be corrected if it's not readable and if it's not given the good name. No other type of files will be accepted (no PNG, no JPG, only PDF) and no late homework will be accepted. Good luck!

QUESTION 1

(10 points)

Is the following function f continuous at the given point a ?

(a) (5 points) $f(x) = \begin{cases} x^2 & x \neq 2 \\ 0 & x = 2. \end{cases}$ at $a = 2$.

(b) (5 points) $f(x) = \frac{x^2+x}{\sin x+1}$ at $a = 0$.

QUESTION 2

(10 points)

Where are the following functions continuous?

(a) (5 points) $f(x) = \frac{x^2+x^4}{1+\cos x}$.

(b) (5 points) $f(x) = \frac{x^3}{\sqrt{x^2+x-2}}$.

QUESTION 3

(20 points)

Suppose f and g are two continuous function at the point $x = a$. Find the value of following limits. State the appropriate rule that you used to get your answer and show all your work.

- (a) (5 points) Find the value of $\lim_{x \rightarrow a} f(x)g(x)$ if $f(a) = 2$ and $g(a) = 3$.
- (b) (5 points) Find the value of $\lim_{x \rightarrow a} f(x)$ if $\sqrt{f(a)} = 2$.
- (c) (5 points) Find the value of $\lim_{x \rightarrow a} f(x)$ if $\lim_{x \rightarrow a} [f(x)g(x) + g(x)] = 2$ and $g(a) = -1$.
- (d) (5 points) Find the value of $\lim_{x \rightarrow \pi} f(\cos(x))$ if $f(-1) = 0$.

QUESTION 4

(10 points)

For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2. \end{cases}$$

QUESTION 5

(15 points)

Let $f(x) = 3 + 4x^2 - 2x$.

- (a) (5 points) Use the definition of the derivative with the limit (see section 2.1 of the lecture notes) to find the slope of the tangent to the curve $y = f(x)$ at the point where $x = a$. No solution will be credited for using the derivative rules from section 2.3.
- (b) (5 points) Find the general equation of the tangent line at a point $(a, f(a))$.
- (c) (5 points) Graph the curve and the tangent line at the point $(1, 5)$ on a common picture.

QUESTION 6

(20 points)

Find the equation of the tangent line and the normal line to the curve $y = x + \sqrt{x}$ at the point $P = (1, 2)$. (You may use the derivative rules for this question).

QUESTION 7 (15 points)

You may use the derivative rules for this question.

- (a) (5 points) Find $h'(0)$ if $h(x) = f(x)g(x)$ and if $f(0) = 1$, $g(0) = 2$, $f'(0) = -1$, and $g'(0) = -2$.
- (b) (5 points) Find $f'(x)$ if $f(x) = \frac{x}{x+\frac{1}{x}}$.
- (c) (5 points) $G''(r)$ if $G(r) = \sqrt{r} + \sqrt[3]{r}$.