Section 1.6 M444

Problems Solution Pierre Parisé

Problem 10 $z_1 = 1+i, \quad z_2 = 1-i, \quad z_3 = 2+5i.$

(a) $e^{z_1} e^{z_2} e^{z_3} = e^{1+i} e^{1-i} e^{2+5i}$

$= e^{4+5i}$

$= e^4(\cos 5 + i \sin 5)$

$= \boxed{e^4 \cos 5 + i e^4 \sin 5}$

(b) $\dfrac{1}{e^{z_1}} = \dfrac{1}{e^{1+i}} = e^{-1-i} = e^{-1}(\cos(-1) + i\sin(-1))$

$= \boxed{e^{-1}\cos(1) - e^{-1}\sin(1)}$

(c) $(e^{z_1} e^{z_2})^{10} = (e^{z_1+z_2})^{10} = e^{10z_1 + 10z_2}$

$= e^{10+10i+10-10i} = \boxed{e^{20}}$

(d) $\dfrac{e^{z_1}+e^{z_2}}{e^{z_3}} = \dfrac{e^1(\cos 1 + i\sin 1) + e^1(\cos(1) - i\sin(1))}{e^2 e^{5i}}$

$= \dfrac{2e^1\cos(1)}{e^2 e^{5i}} = 2e^{-1}e^{-5i}$

$= \boxed{2e^{-1}(\cos 5 - i\sin 5)}$

Problem 15 b

Let $z = x+iy$. Then

$$e^{z^2} = e^{(x+iy)^2}.$$

Now, $(x+iy)^2 = x^2 - y^2 + 2xyi$

$$\Rightarrow \quad e^{z^2} = e^{x^2-y^2} e^{i2xy}$$

$$= e^{x^2-y^2}\cos 2xy + i e^{x^2-y^2}\sin 2xy$$

Thus,

$$u(x,y) = e^{x^2-y^2}\cos(2xy)$$

and

$$v(x,y) = e^{x^2-y^2}\sin(2xy).$$

Problem 16b

Let $z = x+iy$ so that $\bar{z} = x+iy$. We have

$$\overline{e^{z}} = \overline{e^{x} e^{iy}} = \overline{e^{x}}\,\overline{e^{iy}} = e^{x} e^{-iy} = e^{x-iy} = e^{\bar{z}}. \quad \square$$