Math 644

Chapter 5

Section 5.3: Removable Singularities

Contents

Created by: Pierre-Olivier Parisé

Spring 2023

Riemann's Removable Singularity Theorem

Theorem 1. Suppose f is analytic in $\Omega = \{z : 0 < |z-a| < \delta\}$ and suppose

$$\lim_{z \to a} (z-a) f(z) = 0.$$

Then f extends to be analytic in $\{z : |z-a| < \delta\}$.

Proof.

Note:

- Important case: If f is bounded and analytic in a punctured neighborhood of a, then f extends to be analytic in a neighborhood of a.

DEFINITION 2. A compact set $E \subset \mathbb{C}$ has **one-dimensional Hausdorff measure equal to** 0 if for every $\varepsilon > 0$ there are finitely many disks D_j with radius r_j so that

$$E \subset \cup_j D_j \quad \text{and} \quad \sum_j r_j < \varepsilon.$$

THEOREM 3. Suppose $E \subset \mathbb{C}$ is a compact set with one-dimensional Hausdorff measure 0. If f is bounded and analytic on $U \backslash E$, where U is open and $E \subset U$, then f extends to be analytic in U.

Formula For the Inverse