Problem 4

(a) Let $y = \cos \pi x$.

  (ii) The slope of PQ is

  $$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.4\pi)}{0.5 - 0.4} = \frac{-\cos(0.4\pi)}{0.1} = -3.090170.$$

  (iii) The slope of PQ is

  $$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.49\pi)}{0.5 - 0.49} = -\frac{\cos(0.49\pi)}{0.01} = -3.141076.$$

  (iv) The slope of PQ is

  $$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.499\pi)}{0.5 - 0.499} = -\frac{\cos(0.499\pi)}{0.001} = -3.141586.$$

  (vi) The slope of PQ is

  $$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.6\pi)}{0.5 - 0.6} = \frac{\cos(0.6\pi)}{0.1} = -3.090170.$$

  (vii) The slope of PQ is

  $$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.51\pi)}{0.5 - 0.51} = \frac{\cos(0.51\pi)}{0.01} = -3.141076.$$

  (viii) The slope of PQ is

  $$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.501\pi)}{0.5 - 0.501} = \frac{\cos(0.501\pi)}{0.001} = -3.141586.$$

(b) The slope would be $-\pi$.

(c) The equation of a line with slope $-\pi$ is $y - y_0 = -\pi(x - x_0)$ where the line passes through the point (x_0, y_0). Therefore, since $(0.5, 0)$ is on the line, we get

$$y = \pi(x - 0.5) = \pi x - \pi/2.$$

(d) Plot using Desmos.

Problem 8

(a) (i) $v_{ave} = \frac{s(2)-s(1)}{2-1} = 3 - (-3) = 6\,\text{cm/s}.$

    (ii) $v_{ave} = \frac{s(1.1)-s(1)}{1.1-1} \approx -4.7120\,\text{cm/s}.$

    (iii) $v_{ave} = \frac{s(1.01)-s(1)}{1.01-1} \approx -6.1341\,\text{cm/s}.$

    (iv) $v_{ave} = \frac{s(1.001)-s(1)}{1.001-1} \approx -6.2683\,\text{cm/s}.$

(b) We first give an estimation using a point on the left side of 1, say 0.999. We get $v_{ave} \approx -6.2746$cm/s. So we estimate the instantaneous velocity as

$$v \approx \frac{-6.2683 + (-6.2746)}{2} = -6.2714\,\text{cm/s}.$$