Chapter 1

Functions and Limits

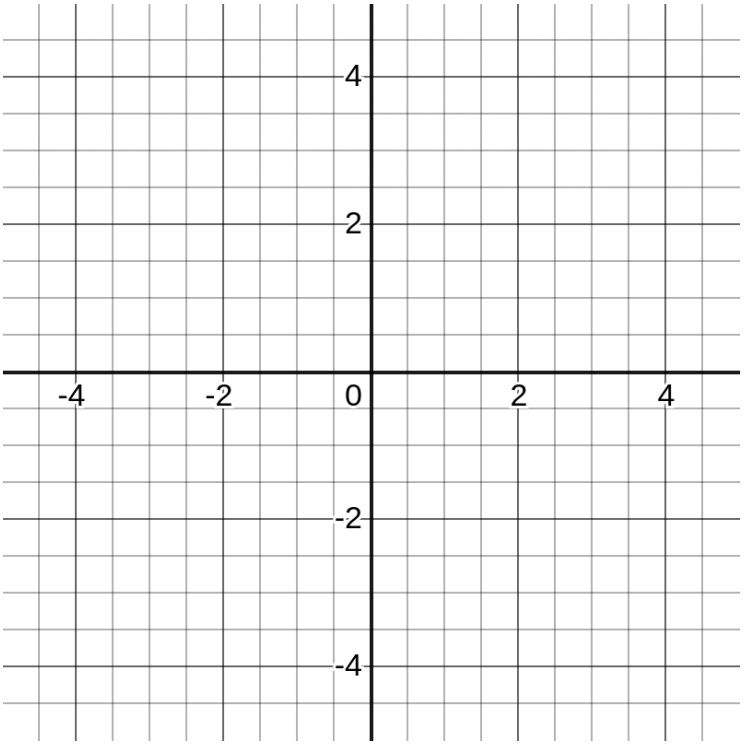
1.2 Mathematical Models: A catalog of Essential Functions

Linear Models.

$$y = f(x) = mx + b$$

.m: the slope

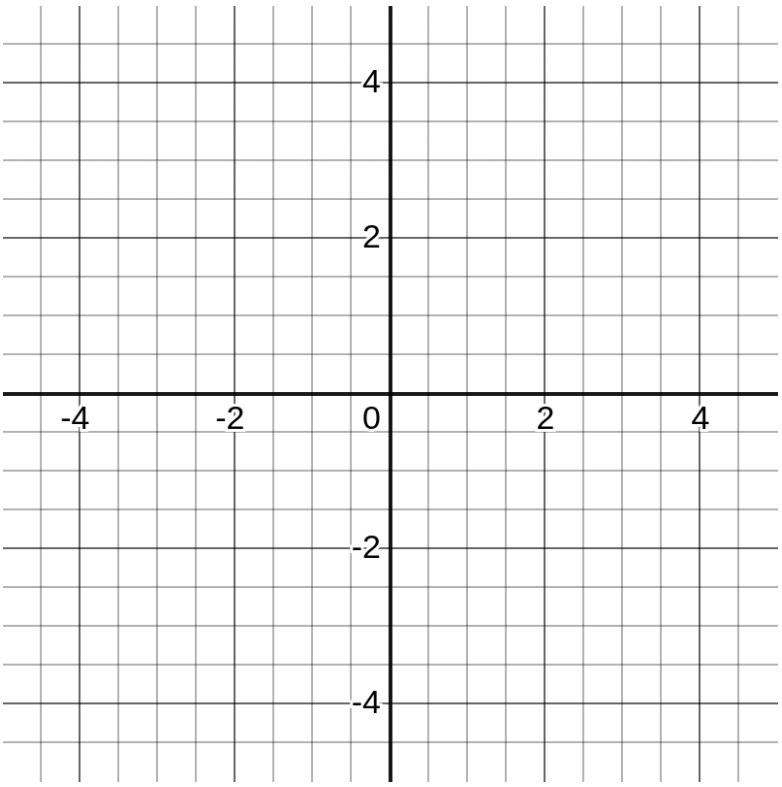
.b: y-intercept



Another formulation (knowing a point):

$$y - y_0 = m(x - x_0)$$

Example. A line passes through the points (0, 1) and (3, 1/2). Find the equation of the line and sketch its graph.



Polynomials.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

: Coefficients

: Leading coefficient

: Degree of polynomial

Domain: All the numbers (real numbers).

Examples.

a) Concrete example.

b) Degree 1.

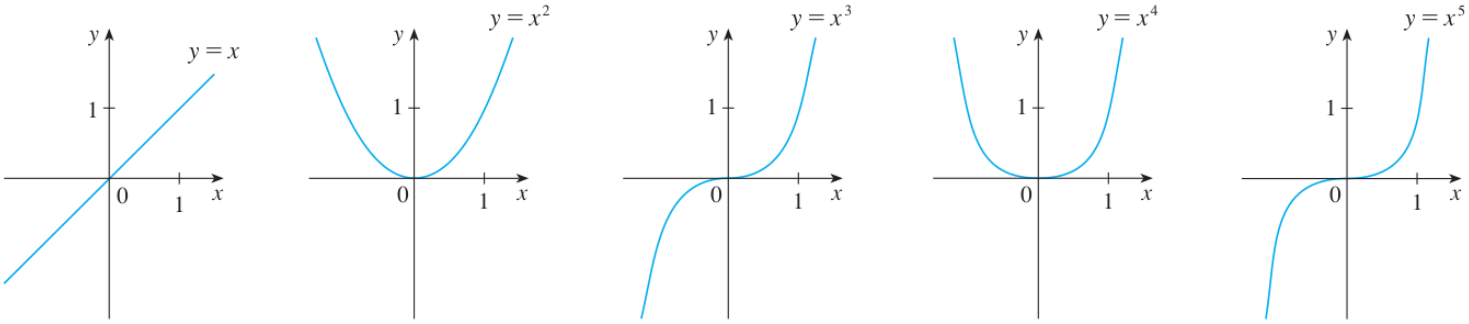
c) Degree 2.

d) Degree 3.

Power Functions.

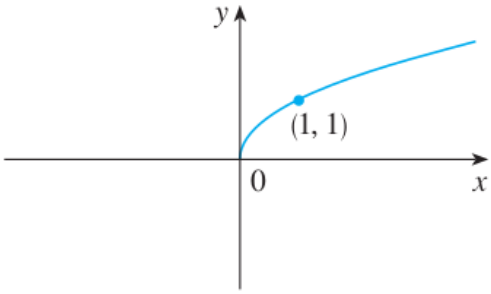
$$f(x) = x^a$$

i) a is a positive integer or is zero.

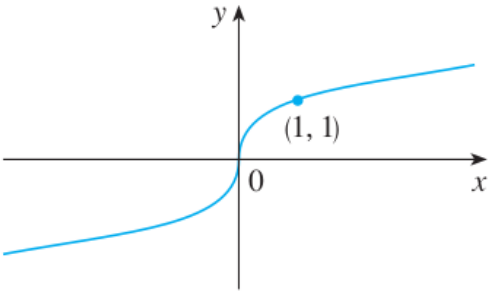


Domain: All the numbers (real numbers).

ii) a is the reciprocal of a positive integer.



(a) $f(x) = \sqrt{x}$

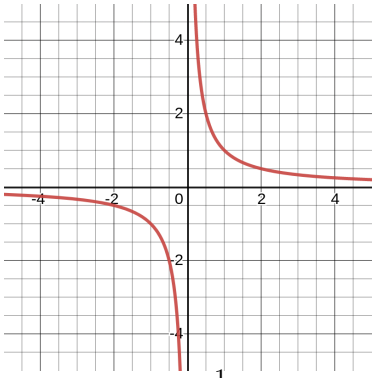


(b) $f(x) = \sqrt[3]{x}$

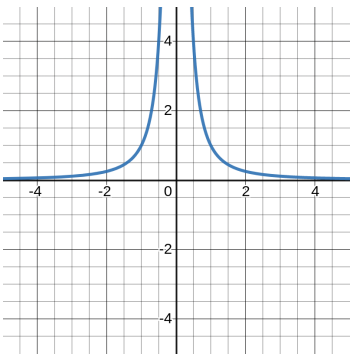
Domain: For odd integer ---> all the numbers (Real numbers).

For even integer ----> Positive numbers or zero.

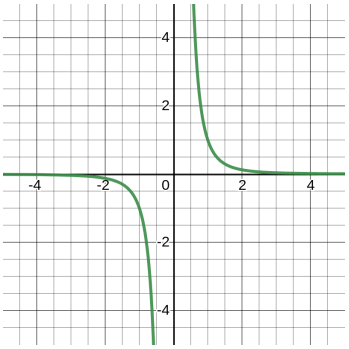
iii) When a is a negative integer.



$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x^2}$



$f(x) = \frac{1}{x^3}$

Domain: All the numbers except zero.

Rational Functions.

$$f(x) = \frac{P(x)}{Q(x)}$$

P: polynomial

Q: polynomial

Domain: all the numbers except the number x such that Q(x) = 0.

Example. Find the domain of the function $f(x) = \dfrac{2x^4 - x^2 + 1}{x^2 - 4}$.

Algebraic Functions.

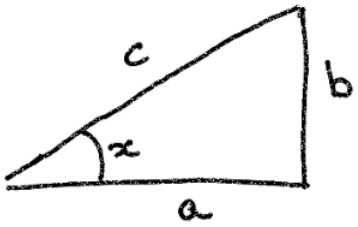
An algebraic function f is a function that can be expressed only in term of the basic operations :

- summation ;
- substraction ;
- multiplication ;
- division ;
- extracting roots (i.e. taking $\sqrt[n]{\cdot}$).

Domain: Depends on the components of the function.

Examples. Find the domain of the following function $g(x) = \dfrac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$.

Trigonometric Functions.

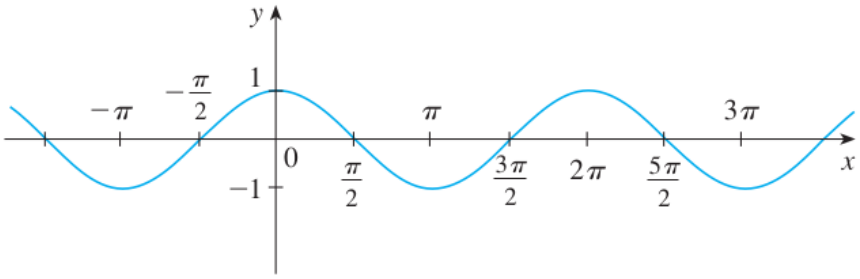


$\cos x = \frac{a}{c}$ $\quad \sec x = \frac{1}{\cos x}$

$\sin x = \frac{b}{c}$ $\quad \operatorname{cosec} x = \frac{1}{\sin x}$

$\tan x = \frac{\sin x}{\cos x}$ $\quad \operatorname{cotan} x = \frac{1}{\tan x}$.

i) Cosine function.



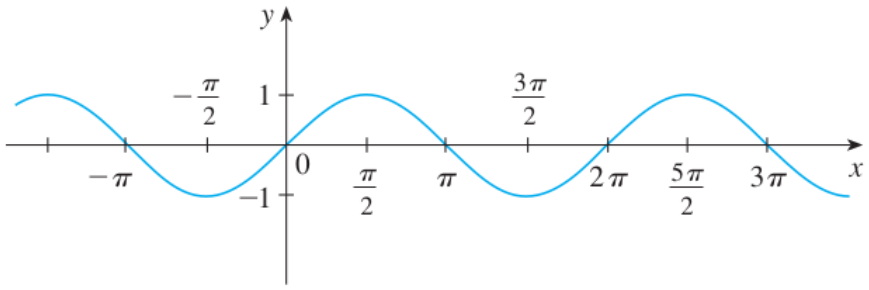
Domain: All of the numbers

Range: the interval [-1, 1]

Zeros: $x = \frac{(2k+1)\pi}{2},\ k = \ldots, -2, -1, 0, 1, 2, \ldots$

Other: $\cos(-x) = \cos(x)$

ii) Sine Function.



Domain: All the numbers

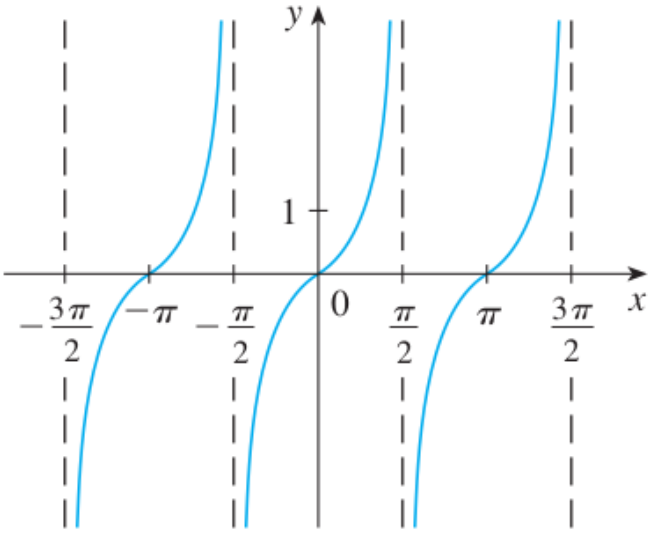
Range: [-1, 1]

Zeros: $x = k\pi,\ k = \ldots, -2, -1, 0, 1, 2, \ldots$

Other:

- $\sin(-x) = -\sin(x)$
- $\sin^2(x) + \cos^2(x) = 1$
- See trigonometric sheet

iii) Tangent Function.



Domain: $(-\infty, \infty) - \{\ldots, -3\pi/2, -\pi/2, \pi/2, 3\pi/2\}$

Range: all numbers

Zeros: same as the cos (x).

Other:

EXAMPLE 5 What is the domain of the function $f(x) = \dfrac{1}{1 - 2\cos x}$?