Example 4

The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$ where s and t are measured in meters and seconds respectively.

a) Find the velocity at time t.
b) What is the velocity after $2s$.
c) When is the particle at rest?
d) ~~f)~~ Find the acceleration at time t and after 4s.
e) ~~g)~~ Graph the position, velocity, and acceleration functions for $0 \leq t \leq 5$.
f) ~~h)~~ When is the particle speeding up? When is it slowing down?

a) $v(t) = s'(t) = 3t^2 - 12t + 9$.

b) We $v(2) = 3 \cdot 4 - 12 \cdot 2 + 9 = 12 - 24 + 9$
$= -3$

c) This is when $v(t) = 0$

$\Leftrightarrow 3t^2 - 12t + 9 = 0$

$\Leftrightarrow t^2 - 4t + 3 = 0$

$\Leftrightarrow (t-3)(t-1) = 0$

$\Leftrightarrow \underline{t = 3}$ or $\underline{t = 1}$

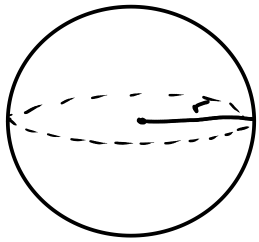
d) $a(t) = v'(t) = 6t - 12$

$a(4) = 6 \cdot 4 - 12 = 12\ m/s^2$

e) See desmos.

Example 5

Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diamter is 50 cm?



V: volume in cm^3

t: time in seconds.

r: radius.

rate of increase of r ?? $\to \frac{dr}{dt}$

rate of volume $\to \frac{dV}{dt}$.

We have the diameter: 50cm $\to$ 25 cm radius.

We have

$$V = \frac{4}{3}\pi r^3$$

Here $V = V(t)$ & $r = r(t)$

So,

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}(r^3)$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{\cancel{3}}\pi\, \cancel{3} r^2 \cdot \frac{dr}{dt}$$

So,

$$\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$$

Now, $r = 25$ cm, $dV/dt = 100$ cm/s²

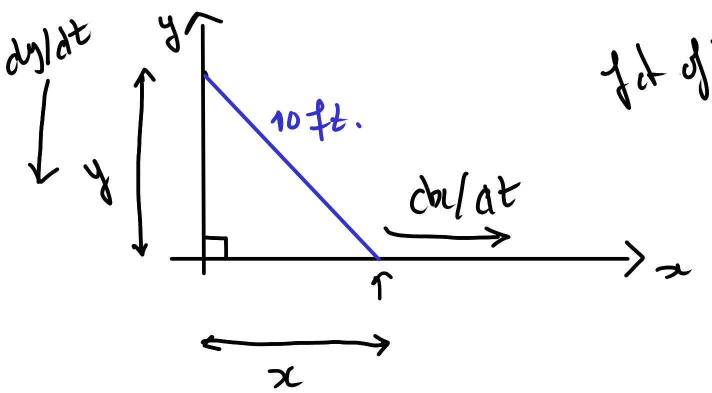
$$\frac{dr}{dt} = \frac{100}{4\pi 25^2} = \frac{100}{100\pi \cdot 25} = \frac{1}{25\pi}$$

So, $\frac{dr}{dt} = \frac{1}{25\pi}$ cm/s².

Example 6

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall.

1) Diagram



ft. of t & x: displacement in the x-direction

ft. of t. & y: displacement in the y-direction

$\frac{dx}{dt}$: rate of change of x

$\frac{dy}{dt}$: rate of change of y.

2) Express a relation between x & y.

Goal Find $\frac{dy}{dt}$?

We have, from our friend Pyth,

$$x^2 + y^2 = 100$$

$$\Rightarrow \quad \frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(100)$$

$$\Rightarrow \quad \frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = 0$$

$$\Rightarrow \quad \cancel{2}x\left(\frac{dx}{dt}\right) + \cancel{2}y \cdot \left(\frac{dy}{dt}\right) = 0$$

$x = \sqrt{100-y^2}$

$$\Rightarrow \quad \frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right)$$

So, $\frac{dy}{dt} = -\frac{\sqrt{100-6^2}}{6} \cdot 1 = -\frac{8}{6} = -\frac{4}{3}$ ft/s.

$\Rightarrow -\frac{6}{\sqrt{100-6^2}} = -\frac{6}{8} = -\frac{3}{4}$ ft/s.

Example 7

A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2\text{m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3m deep.