A.I Sample Space

PROBLEM 1. Draw Venn diagrams to verify de Morgan's laws.

PROBLEM 2. Write down the sample space for each of the following experiments and write if it is finite, discrete, or continuous.

a) Number of committees of 2 people taken from a group of 3 people.

b) Number of people at the beach every day.

c) The magnitude of the wind speed on a given day.

A.II Event Space

PROBLEM 3. Suppose three 2-sided fair coins are flipped.

a) Describe the sample space S of this experiment.

b) Let $A = \{$the first two tosses are head$\}$. Express A in terms of atomic events.

c) Let $B = \{$ the last two tosses are head$\}$. Express A in terms of atomic events.

d) Find $A \cap B$ and interpret it.

PROBLEM 4. Let $S = \{$⚀, ⚁, ⚂, ⚃, ⚄, ⚅$\}$ be the sample space from the experiment of tossing a 6-faced die. Construct an event space with exactly 4 events. Can you construct an event space with 6 events?

PROBLEM 5. Let S be the sample space and let $\mathcal{A}$ be an event space for S.

a) If A, B and C are events, show that $A \cup B \cup C$ is an event.

b) If A and B are events, then show that $A \cap B$ is an event. *[Hint: Use de Morgan's laws to rewrite $A \cap B$.]*

A.III Axioms of Probability

PROBLEM 6. An unfair coin is tossed two times. So $S = \{(h,h),(h,t),(t,t),(t,h)\}$ and assume $\mathcal{A}$ is all the subsets of S. Assume that a probability measure P is defined by

$$P(\{(h,h)\}) = \frac{1}{9}, P(\{(h,t)\}) = P(\{(t,h)\} = \frac{2}{9}, P(\{(t,t)\}) = \frac{4}{9}.$$

a) Let $A = \{$The result of the first toss is tail$\}$. Find $P(A)$.

b) Let $A = \{$At least one of the tosses is tail$\}$. Find $P(A)$.

PROBLEM 7. Let $(S, \mathcal{A}, P)$ be a probability space. Show the following assertions:

a) $P(\overline{A}) = 1 - P(A)$. In particular, we have $P(\varnothing) = 0$.

b) If A, B, and C are admissible events such that $A \cap B = A \cap C = B \cap C = \varnothing$, then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$.

c) If A and B are admissible events with $A \subset B$, then $P(A) \leq P(B)$.

A.IV Computing Probabilities in the Finite Case

PROBLEM 8. If two dice are rolled, what is the probability that the sum of the upturned faces will equal 7?

PROBLEM 9. If 3 balls are "randomly drawn" from a bowl containing 6 orange balls and 5 blue balls, what is the probability that one of the drawn balls is orange and the other two blue?

PROBLEM 10. In the game Wazabi, there are special 6-faced dice. On three faces of the die, there is a "W", on two other faces, there is a card symbol, and on one face, there is a cubed dice symbol. To determine which player starts on the first turn, each player throws four of these special dice and observes the upturned face of the dice. The player with the most "W" starts. What is the probability that a player obtains exactly three "W"'s?

A.V Probability Space For Infinite Sample Spaces

PROBLEM 11. Suppose we toss a fair coin infinitely many times. Let B_i denote the event "the i-th toss lands heads". Interpret the event $B = \cup_{i=1}^{\infty} B_i$ and find $P(B)$. *[Hint: Use the continuity of probability measures.]*