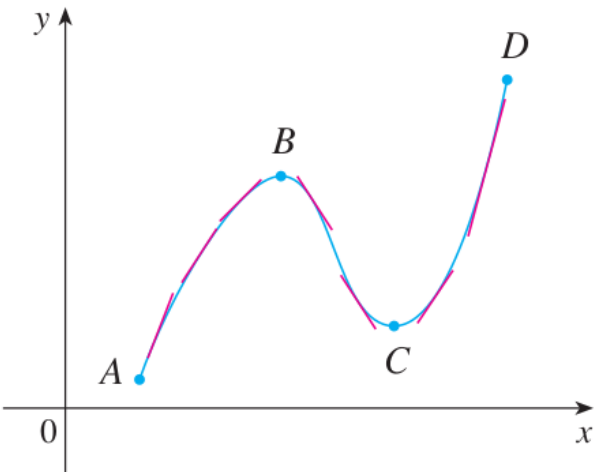


Chapter 3

Applications of Derivatives

3.3 How Derivatives affect the Shape of a Graph

What does f' tell us about f .



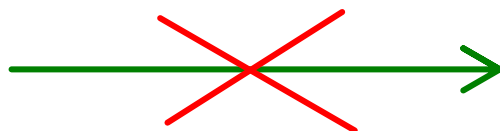
	A	B	C	D
$f'(x)$				
$f(x)$				

Increasing/Decreasing Test

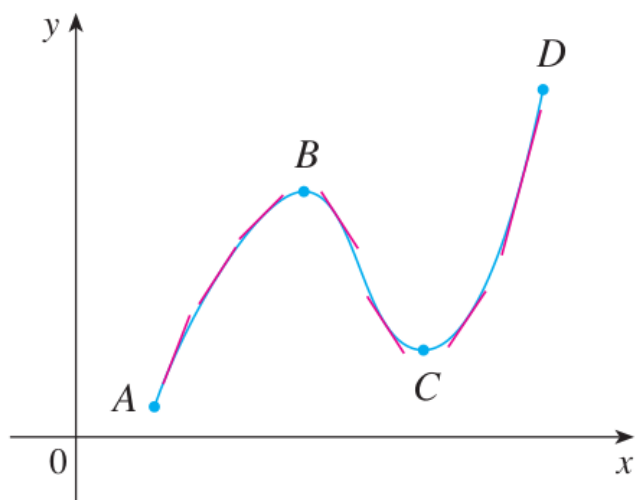
- (a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- (b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

EXAMPLE 1 Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing.

CRITICAL POINTS



EXTREME VALUES
(MAX OR MIN)

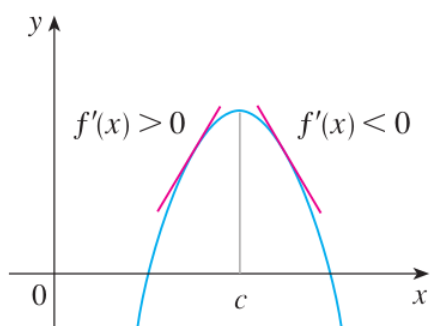


	A	B	C	D
$f'(x)$	+	0	-	0
$f(x)$				

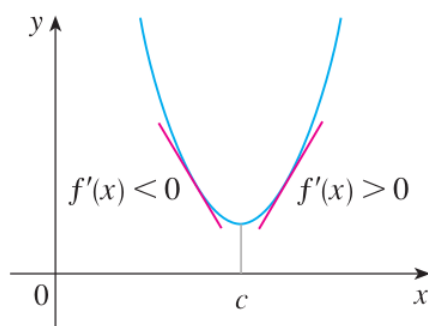
Blue arrows indicate the function's behavior: an upward arrow between A and B, a downward arrow between B and C, and an upward arrow between C and D.

The First Derivative Test Suppose that c is a critical number of a continuous function f .

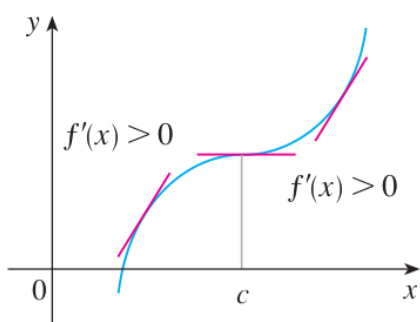
- (a) If f' changes from positive to negative at c , then f has a local maximum at c .
- (b) If f' changes from negative to positive at c , then f has a local minimum at c .
- (c) If f' is positive to the left and right of c , or negative to the left and right of c , then f has no local maximum or minimum at c .



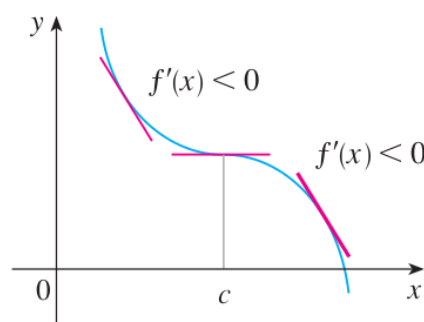
(a)



(b)



(c)

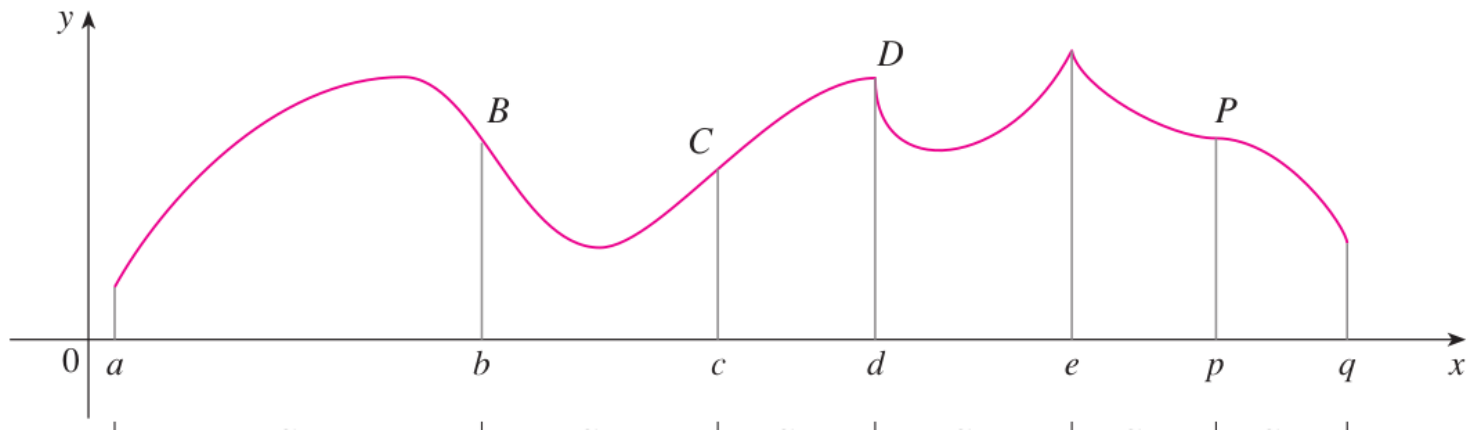


(d)

EXAMPLE 3 Find the local maximum and minimum values of the function

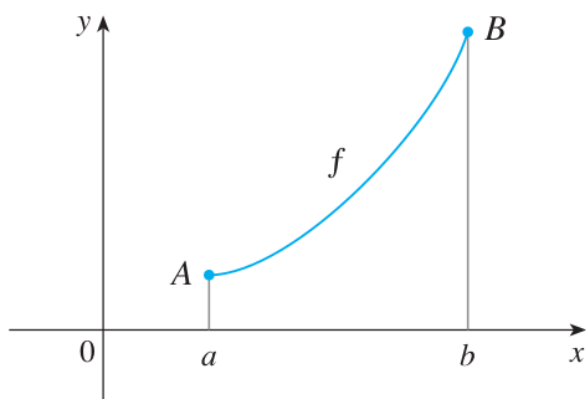
$$g(x) = x + 2 \sin x \quad 0 \leq x \leq 2\pi$$

What does f'' tell us about f ?

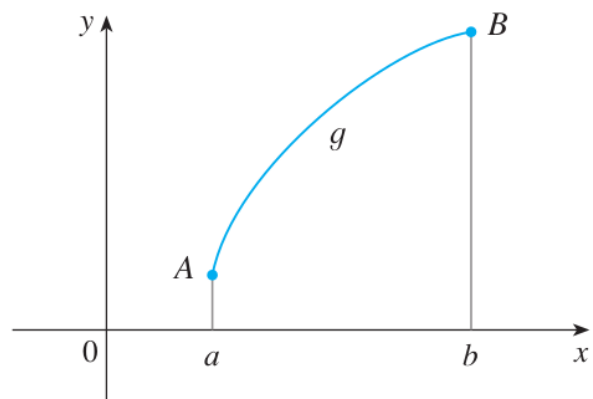


Two important definitions:

- 1) **Definition** If the graph of f lies above all of its tangents on an interval I , then it is called **concave upward** on I . If the graph of f lies below all of its tangents on I , it is called **concave downward** on I .
- 2) **Definition** A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P .



(a)



(b)

Concavity Test

- (a) If $f''(x) > 0$ for all x in I , then the graph of f is concave upward on I .
- (b) If $f''(x) < 0$ for all x in I , then the graph of f is concave downward on I .

Example. Find the interval(s) of concavity of the function $f(x) = x^3 - 3x^2 - 9x + 4$.

The Second Derivative Test Suppose f'' is continuous near c .

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c .

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c .

REMARK!

EXAMPLE 7 Sketch the graph of the function $f(x) = x^{2/3}(6 - x)^{1/3}$.