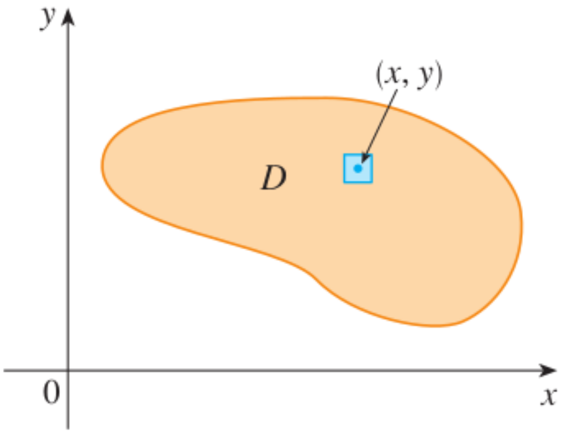
15.4 Applications of Double Integrals.

Density, mass and charge.




$$\boxed{m = \iint_D \rho(x, y)\, dA}$$

$$\boxed{Q = \iint_D \sigma(x, y)\, dA}$$

**EXAMPLE 1** Charge is distributed over the triangular region $D$ in Figure 3 so that the charge density at $(x, y)$ is $\sigma(x, y) = xy$, measured in coulombs per square meter ($\mathrm{C/m^2}$). Find the total charge.

Moments and center of mass.

Moment about the x-axis

$$M_x = \iint_D y\rho(x,y)\,dA$$

Moment about the y-axis

$$M_y = \iint_D x\rho(x,y)\,dA$$

**5** The coordinates $(\bar{x}, \bar{y})$ of the center of mass of a lamina occupying the region $D$ and having density function $\rho(x, y)$ are

$$\boxed{\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x, y)\,dA} \qquad \boxed{\bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x, y)\,dA}$$

where the mass $m$ is given by

$$m = \iint_D \rho(x, y)\,dA$$

**EXAMPLE 2** Find the mass and center of mass of a triangular lamina with vertices $(0, 0)$, $(1, 0)$, and $(0, 2)$ if the density function is $\rho(x, y) = 1 + 3x + y$.

**EXAMPLE 3** The density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass of the lamina.

Moment of Intertia.

Inertia about the x-axis

$$\boxed{I_x = \iint_D y^2 \rho(x,y)\, dA}$$

Inertia about the y-axis

$$\boxed{I_y = \iint_D x^2 \rho(x,y)\, dA}$$

Inertia about the origin

$$\boxed{I_0 = \iint_D (x^2 + y^2) \rho(x,y)\, dA}$$

**EXAMPLE 4** Find the moments of inertia $I_x$, $I_y$, and $I_0$ of a homogeneous disk $D$ with density $\rho(x, y) = \rho$, center the origin, and radius $a$.