Solution

Worksheet 02

Question 1

(a) By the product rule, we have:

$$F'(x) = (\sin x)' \tan x + \sin x \cdot (\tan x)'$$
$$= \cos x \tan x + \sin x\,(\sec^2 x)$$
$$= \cos x \left(\frac{\sin x}{\cos x}\right) + \frac{\sin x}{\cos^2 x}$$

$$= \sin x + \frac{\sin x}{\cos^2 x} = \sin x\left(1 + \frac{1}{\cos^2 x}\right)$$

So, $F'(x) = \sin x\,(1 + \sec^2 x)$.

(d) By the power rule & the product rule:

$$v'(x) = -(\cos x \sin x)^{-2}\,(\cos x \sin x)'$$

and

$$(\cos x \sin x)' = (\cos x)' \sin x + \cos x\,(\sin x)'$$
$$= -\sin^2 x + \cos^2 x$$
$$= \cos(2x) \quad \text{[trigo. identity]}.$$

So,

$$v'(x) = -\frac{\cos 2x}{(\cos x \sin x)^2} = -\frac{\cos 2x}{(\sin 2x/2)^2}$$

$$= -4\left(\frac{\cos 2x}{\sin 2x}\right)\left(\frac{1}{\sin 2x}\right).$$

So, $v'(x) = -4\cot(2x)\operatorname{cosec}(2x)$.

Question 2.

(a) By the chain rule,

$$(\tan(y))' = \sec^2 y \cdot y'$$

and so using implicit differentiation:

$$\sec^2 y \cdot y' = 2x$$

$$\Rightarrow \quad y' = \frac{2x}{\sec^2 y}.$$

(b) By the product rule,

$$(y\sin y)' = y'\sin y + y\cos y \cdot y'$$

$$= y'(\sin y + y\cos y).$$

So, by using implicit differentiation:

$$y'(\sin y + y\cos y) = 1$$

$$\Rightarrow \quad y' = \frac{1}{\sin y + y\cos y}.$$