MATH-241 Calculus I
Midterm 01

Created by Pierre-O. Parisé
Fall 2021, 09/22/2021

Last name: Correction
First name: —
Section: —

Question:	1	2	3	4	Total
Points:	10	10	10	20	50
Score:	—	—	—	—	—

Instructions: Make sure to write your complete name on your copy. You must answer all the questions below and write your answers directly on the questionnaire. At the end of the 50 minutes, hand out your copy.

No devises such as a smart phone, cell phone, laptop, or tablet can be used during the exam. You are not allowed to use the lecture notes also.

Good luck!

Pierre-Olivier Parisé

Question 1 (10 pts)

Find the domain of the following functions.

(a) (5 points) $f(x) = \sqrt{x+1}$

(b) (5 points) $f(x) = \frac{1}{\cancel{\frac{1}{4} - \sin(x)}}$. $1 - \sin x$

(a) In the $\sqrt{\cdot}$, we can't have negative value.

So, $x+1 \geq 0 \iff x \geq -1$. So,

$$\operatorname{dom} f = [-1, \infty).$$

(b) The denominator must be $\neq 0$. So,

$$1 - \sin x = 0 \iff 1 = \sin x$$

$$\iff x = (4n+1)\frac{\pi}{2}.$$

So, $\operatorname{dom} f = \mathbb{R} \setminus \{\ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \ldots\}$.

Question 2 **(10 pts)**

With the limit rules, find the value of the following limits. Make sure to write explicitly the rules that you are using.

(a) (5 points) $\lim_{x\to 2}\sqrt{u^4+3u-6}$.

(b) (5 points) $\lim_{x\to 1}(x^4-3x)(x^2+5x+3)$.

(a)
$$\lim_{x\to 2}\sqrt{u^4+3u-6} = \sqrt{\lim_{x\to 2} u^4+3u-6} \quad \text{[root Rule]}$$
$$= \sqrt{\lim_{x\to 2} x^4 + \lim_{x\to 2} 3u - \lim_{x\to 2} 6} \quad \text{[sum rule]}$$
$$= \sqrt{\left(\lim_{x\to 2} x\right)^4 + 3\lim_{x\to 2} u - 6} \quad \text{[Power rule]}$$
$$= \sqrt{2^4 + 3\cdot 2 - 6} \quad \left[\lim_{x\to a} x = a\right]$$
$$= \sqrt{16+6-6} = \sqrt{16} = 4$$

(b)
$$\lim_{x\to 1}(x^4-3x)(x^2+5x+3)$$
$$= \lim_{x\to 1}(x^4-3x)\cdot\lim_{x\to 1}(x^2+5x+3) \quad \text{[Product rule]}$$
$$= \left[\lim_{x\to 1} x^4 - \lim_{x\to 1} 3x\right]\left[\lim_{x\to 1} x^2 + \lim_{x\to 1} 5x + \lim_{x\to 1} 3\right]$$
[Sum & Difference rule]
$$= (1^4 - 3\cdot 1)(1^2+5\cdot 1+3) \quad \text{[Power rules]}$$
$$= (-2)\cdot(9)$$
$$= -18.$$

Question 3 **(10 pts)**

Find the value of the following limit. Be careful, you may not apply the limit rules directly. You have to make some modifications to the expression in order to apply the limit rules.

$$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x^2}.$$

We have

$$\frac{1-\sqrt{x}}{1-x^2} = \frac{1-\sqrt{x}}{1-x^2} \cdot \frac{(1+\sqrt{x})}{(1+\sqrt{x})}$$

$$= \frac{1-x}{(1-x^2)(1+\sqrt{x})}.$$

Also, $1-x^2 = (1-x)(1+x)$. So,

$$\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x^2} = \lim_{x\to 1} \frac{\cancel{1-x}}{\cancel{(1-x)}(1+x)(1+\sqrt{x})}$$

$$= \lim_{x\to 1} \frac{1}{(1+x)(1+\sqrt{x})}$$

$$= \frac{1}{\lim (1+x)(1+\sqrt{x})} \quad \text{[Quotient law]}$$

[Product & Sum rules] $$= \frac{1}{(\lim 1 + \lim x)(\lim 1 + \lim \sqrt{x})}$$

$$= \frac{1}{(1+1)(1+1)} \quad \text{[root law]}$$

$$= \frac{1}{4}.$$

QUESTION 4 **(20 pts)**

Find the equation of the tangent line to the curve $y = 3 + 4x^2 - 2x^3$ at the point $(2, 3)$. You may use the rules to compute derivatives.

1) Find the derivative

$y' = 8x - 6x^2$ (by the rules of derivatives).

2) Find the slope

We have $y'(2) = 16 - 6 \cdot 4 = -8$.

3) Find the equation of the tangent line.

$T(x) = ax + b$.

We have $a = y'(2) = -8$.

We also have $T(2) = 3$. So:

$$3 = -8 \cdot 2 + b \Rightarrow b = 19.$$

Thus, the equation of the tangent line at $(2,3)$ is

$$T(x) = -8x + 19.$$

$\square$