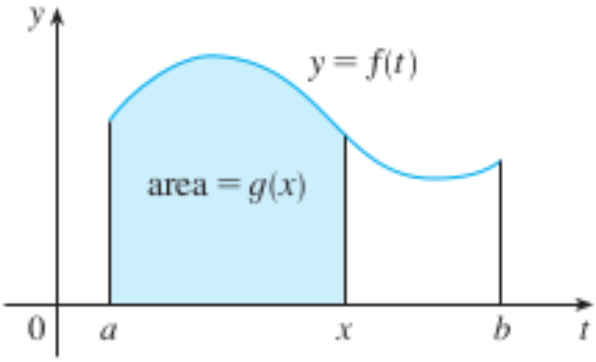
Chapter 4
Integrals

4.3 The Fundamental Theorem of Calculus

Area Function.



EXAMPLE 1 If f is the function whose graph is shown in Figure 2 and $g(x) = \int_0^x f(t)\,dt$, find the values of $g(0)$, $g(1)$, $g(2)$, $g(3)$, $g(4)$, and $g(5)$. Then sketch a rough graph of g.

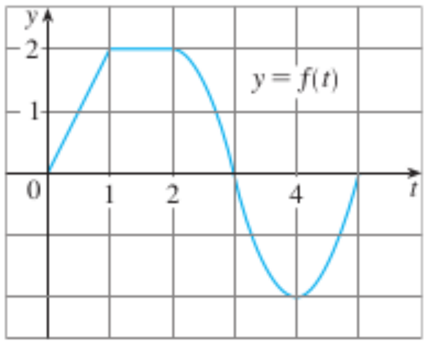
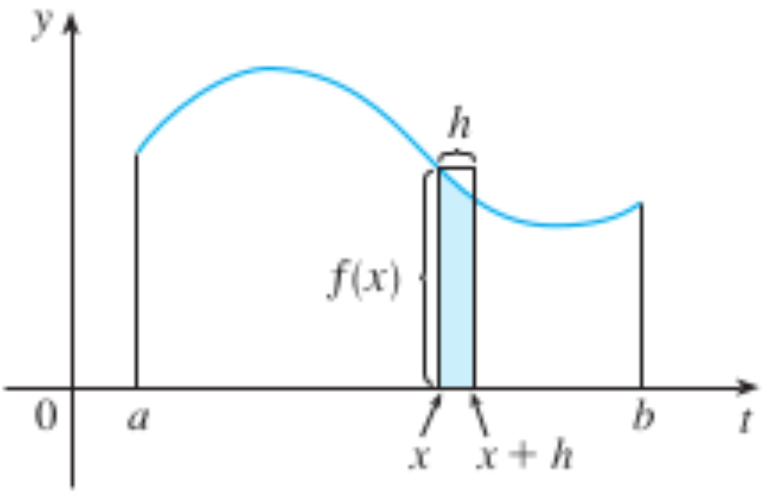


FIGURE 2



The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leqslant x \leqslant b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

EXAMPLE 2 Find the derivative of the function $g(x) = \int_0^x \sqrt{1 + t^2}\,dt$.

Example. Find $\dfrac{d}{dx}\Big(\displaystyle\int_1^{x^4} \sec(t)\,dt\Big)$.

Example. Find the derivative of the function $f(x) = \int_{\sin x}^{1} \sqrt{1+t^2}\, dt$

Second part of the Fundamental Theorem of Calculus.

Example. Compute the integral $\int_a^b x\,dx$ where a and b are two numbers such that a < b.

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function F such that $F' = f$.

Example. Evaluate the integral $\int_{-2}^{1} x^3\,dx$.

Example. Find the value of the integral $\int_0 \big(3x^2 - \sin(\pi x) + \cos(x) - x\sin(x)\big)\, dx$.

EXAMPLE 8 What is wrong with the following calculation?

$$\int_{-1}^{3} \frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\Bigg]_{-1}^{3} = -\frac{1}{3} - 1 = -\frac{4}{3}$$

Differentiation and Integration as Inverse Processes.

The Fundamental Theorem of Calculus Suppose f is continuous on $[a, b]$.

1. If $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$.

2. $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f, that is, $F' = f$.