Chapter 2

Derivatives

2.3 Differentiation Formulas

Constant Function.

$$f(x) = c \Rightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c-c}{h}$$

$$= 0$$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

Power Functions. $f(x) = x^n$

n = 1. $f(x) = x$

$$\lim_{h \to 0} \frac{x+h-x}{h} = 1 \Rightarrow \frac{d}{dx}(x) = 1$$

n = 2. $f(x) = x^2$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{2x\cancel{h} + \cancel{h^2}}{\cancel{h}} = 2x$$

h

$$\Rightarrow \frac{d}{dx}(x^2) = 2x$$

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

EXAMPLE. Find the derivative of $y = x^4 - 6x^2 + 4.$

Multiplication by a constant.

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Sum.

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Difference.

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Product.

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Caution!!!

$$\frac{d}{dx}(fg) \neq \frac{d}{dx}(f)\frac{d}{dx}(g).$$

Example.

Example. Find the derivative of the function $f(x) = (5x^2 - 2)(x^3 + 3x)$.

Quotient.

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Caution !!

$$\frac{d}{dx}\left(\frac{f}{g}\right) \neq \frac{\frac{d}{dx}(f)}{\frac{d}{dx}(g)} \longrightarrow$$

Example.

EXAMPLE 8 Let $y = \dfrac{x^2 + x - 2}{x^3 + 6}$. Compute the derivative.

General Power rule.

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Case n = 0:

Example. Find the derivative of the function $f(x) = x^{2/3}$.

EXAMPLE 13 At what points on the hyperbola $xy = 12$ is the tangent line parallel to the line $3x + y = 0$?

Summary of Differentiation Formulas.

Table of Differentiation Formulas

$$\frac{d}{dx}(c) = 0 \qquad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf' \qquad (f+g)' = f' + g' \qquad (f-g)' = f' - g'$$

$$(fg)' = fg' + gf' \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$