Section 2.1 — Problem 6 — 10 points

The equation of the tangent line at $(2, 3)$ is

$$y - 3 = f'(2)(x - 2).$$

We have to find $f'(2)$. We have $f(x) = x^3 - 3x + 1$, and therefore

$$\begin{aligned} f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} &= \lim_{h\to 0} \frac{(2+h)^3 - 3(2+h) + 1 - 3}{h} \\ &= \lim_{h\to 0} \frac{8 + 12h + 6h^2 + h^3 - 6 - 3h + 1 - 3}{h} \\ &= \lim_{h\to 0} \frac{9h + 2h^2 + h^3}{h} \\ &= \lim_{h\to 0} 9 + 2h + h^2 \\ &= 9. \end{aligned}$$

Therefore, we obtain $f'(2) = 9$. Therefore, the equation of the tangent line is

$$y = 9x - 18 + 3 = 9x - 15.$$

Section 2.1 — Problem 34 — 10 points

The value of $f'(a)$ is given by

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

Evaluating f at $a + h$ and at a in this expression, we can do some calculations:

$$\begin{aligned}
\lim_{h\to 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h\to 0} \frac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h} \\
&= \lim_{h\to 0} \frac{a^2 - (a+h)^2}{(a+h)^2 a^2 h} \\
&= \lim_{h\to 0} \frac{a^2 - a^2 - 2ah - h^2}{(a+h)^2 a^2 h} \\
&= \lim_{h\to 0} -\frac{2ah + h^2}{(a+h)^2 a^2 h} \\
&= \lim_{h\to 0} -\frac{2a + h}{(a+h)^2 a^2} \\
&= -\frac{2a}{a^4} \\
&= -\frac{2}{a^3}.
\end{aligned}$$

Therefore, we get $f'(a) = -2/a^3$.

Section 2.1 — Problem 44 — 10 points

The velocity at $t = 4$ is given by $f'(4)$. This is given by

$$\begin{aligned}
\lim_{h\to 0}\frac{f(4+h)-f(4)}{h} &= \lim_{h\to 0}\frac{10+\frac{45}{5+h}-10-\frac{45}{5}}{h}\\
&= \lim_{h\to 0}\frac{\frac{45}{5+h}-9}{h}\\
&= \lim_{h\to 0}\frac{45-45-9h}{(5+h)h}\\
&= \lim_{h\to 0}-\frac{9h}{(5+h)h}\\
&= \lim_{h\to 0}-\frac{9}{5+h}.
\end{aligned}$$

Evaluating the last limit with the Quotient Rule, we get $f'(4) = -9/5$.

Section 2.1 — Problem 60 — 5 points

By definition, we have

$$\begin{aligned} f'(0) &= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h\to 0} \frac{h^2 \sin(1/h)}{h} \\ &= \lim_{h\to 0} h \sin(1/h). \end{aligned}$$

The last limit exists because

$$-h \le h \sin(1/h) \le h$$

for any $h > 0$ and

$$h \le h \sin(1/h) \le -h$$

when $h < 0$. We can simplify this by using the absolute value:

$$0 \le |h \sin(1/h)| \le |h|$$

because $0 \le |\sin(1/h)| \le 1$. Using the Squeeze Theorem, we conclude that

$$\lim_{h\to 0} h \sin(1/h) = 0.$$

Therefore, $f'(0)$ exists and $f'(0) = 0$.

Section 2.2 — Problem 12 — 5 points

That $t = 0$, the slope of the tangent line is positive and quite small. When we move towards $t = 5$, the slope increases and attain a maximum around $t = 6$. Then the slope decreases as we more towards $t = 10$. The slope becomes really small (close to zero) when we reach $t = 15$. The graph show look like this:

Section 2.2 — Problem 32 — 10 points

(a) By definition, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{x+h+\frac{1}{x+h}-x-\frac{1}{x}}{h} \\ &= \lim_{h\to 0} \frac{h+\frac{1}{x+h}-\frac{1}{x}}{h} \\ &= \lim_{h\to 0} \frac{(x+h)xh+x-x-h}{(x+h)xh} \\ &= \lim_{h\to 0} \frac{x^2h+xh^2-h}{(x+h)xh} \\ &= \lim_{h\to 0} \frac{x^2+xh-1}{(x+h)x} \end{aligned}$$

Then use the Quotient Rule to evaluate the last limit. We get

$$f'(x) = \frac{x^2-1}{x^2} = 1 - \frac{1}{x^2}.$$

The domain of f' is $(-\infty, 0) \cup (0, \infty)$.

(b) Here are the graphs of f and f'. Desmos was used to draw the figure.

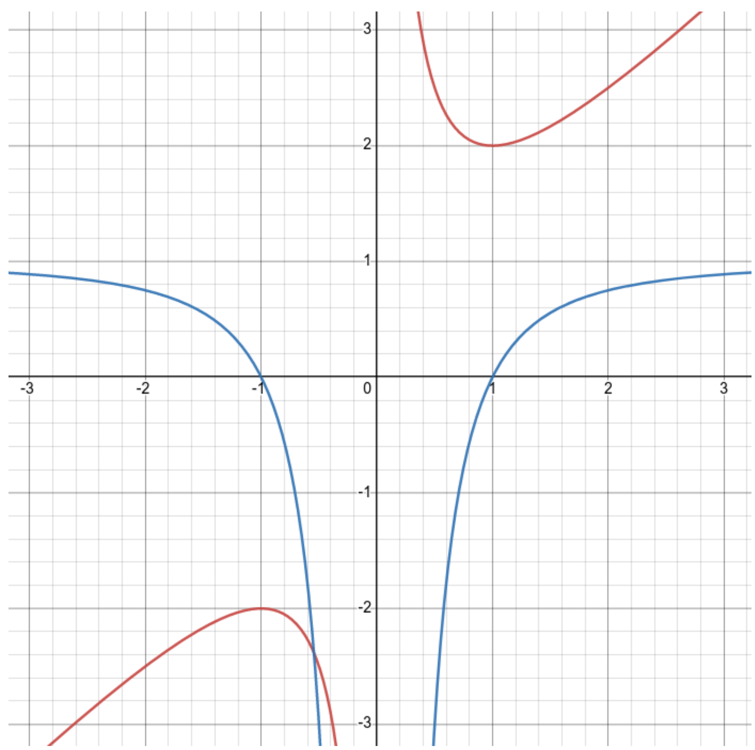

Figure 1: In red, graph of $f(x)$ and, in blue, the graph of $f'(x)$

TOTAL (POINTS): 50.