# Chapter 1

# Functions and Limits

1.1 Four Ways of Representing a Function

A **function** $f$ is a rule that assigns to each element $x$ in a set $D$ exactly one element, called $f(x)$, in a set $E$.
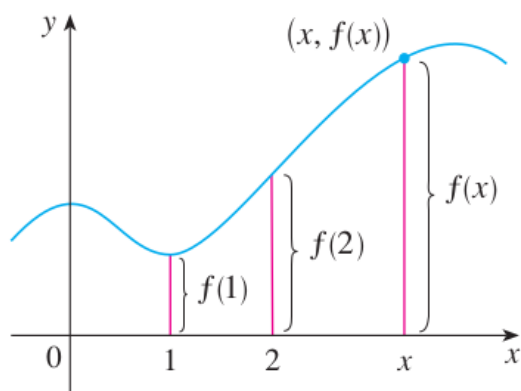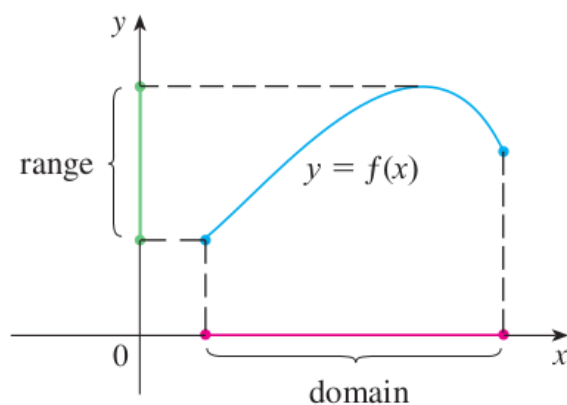
Machine visualization.

$x$ ⟶ $f$ ⟶ $f(x)$
(input)              (output)

Domain:

Range:

Graph of a function.

$y$ | $(x, f(x))$

$f(x)$

$f(2)$

$f(1)$

$0$  $1$  $2$  $x$  $x$
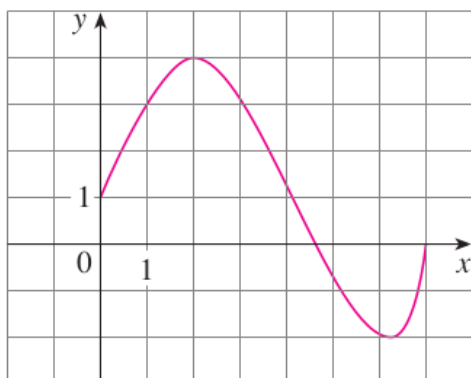
Dependant variable.

$y$

range

$y = f(x)$

$0$

domain

$x$

Independant variable.

**EXAMPLE 1** The graph of a function $f$ is shown in Figure 6.
(a) Find the values of $f(1)$ and $f(5)$.
(b) What are the domain and range of $f$?
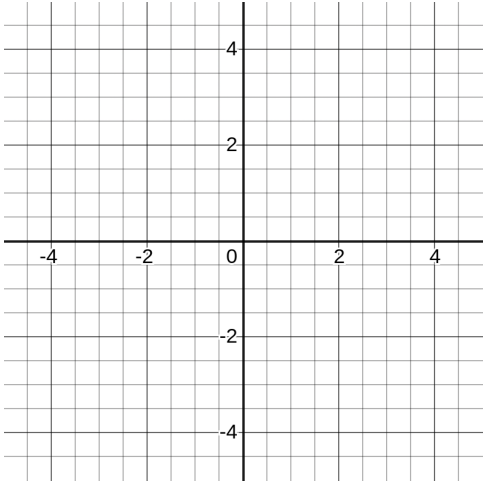
$y$

$1$

$0$  $1$  $x$

**EXAMPLE 2** Sketch the graph and find the domain and range of each function.
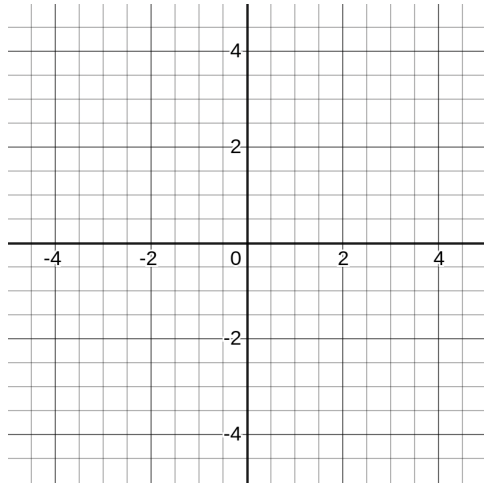
(a) $f(x) = 2x - 1$            (b) $g(x) = x^2$

a)

b)

---

**EXAMPLE 3** If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a + h) - f(a)}{h}$.

Remark:

There are four possible ways to represent a function:

- verbally            (by a description in words)
- numerically         (by a table of values)
- visually            (by a graph)
- algebraically       (by an explicit formula)

---

**EXAMPLE 5**  A rectangular storage container with an open top has a volume of $10 \text{ m}^3$. The length of its base is twice its width. Material for the base costs $10 per square meter; material for the sides costs $6 per square meter. Express the cost of materials as a function of the width of the base.

---

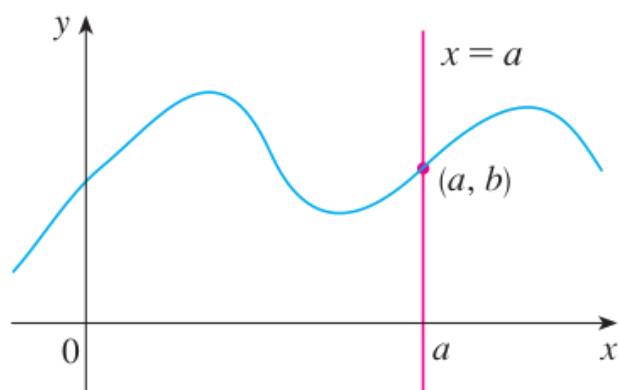Domain of functions given by an explicit formula.

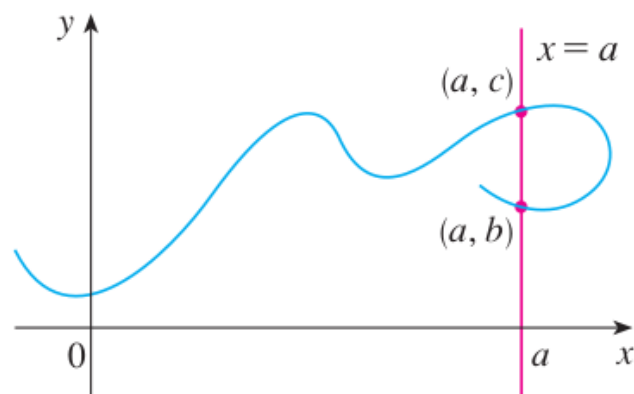**EXAMPLE 6**  Find the domain of each function.

(a)  $f(x) = \sqrt{x + 2}$

(b)  $g(x) = \dfrac{1}{x^2 - x}$

> **The Vertical Line Test** A curve in the $xy$-plane is the graph of a function of $x$ if and only if no vertical line intersects the curve more than once.
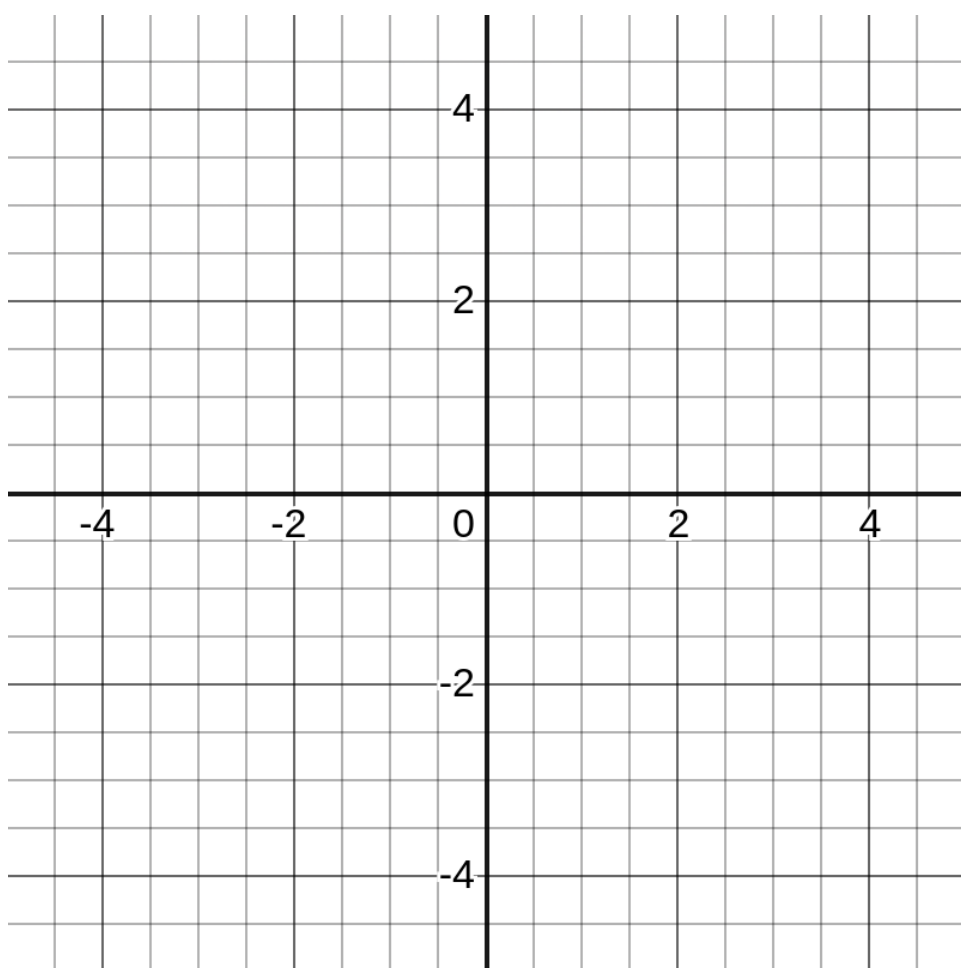


(a) This curve represents a function.



(b) This curve doesn't represent a function.

Example. The parabola $x = y^2 - 2$ is not the graph of a function. Show it using the Vertical Line Test.

<u>Piece-wise Functions.</u>

The functions in the following four examples are defined by different formulas in different parts of their domains. Such functions are called **piecewise defined functions**.

**EXAMPLE 7**  A function $f$ is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

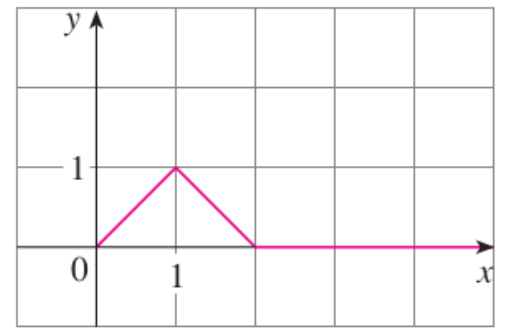Evaluate $f(-2)$, $f(-1)$, and $f(0)$ and sketch the graph.

Absolute Value.

$$|a| = a \quad \text{if } a \geqslant 0$$
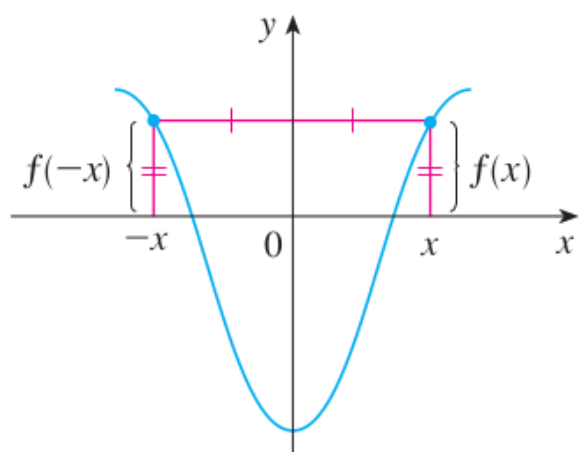$$|a| = -a \quad \text{if } a < 0$$

Properties:

**EXAMPLE 8**  Sketch the graph of the absolute value function $f(x) = |x|$.

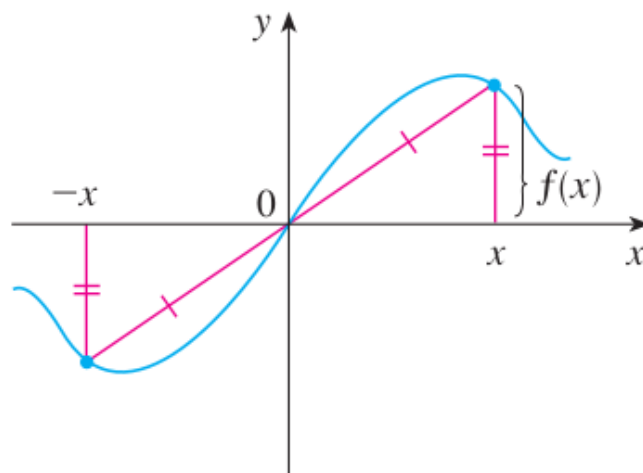**EXAMPLE 9** Find a formula for the function $f$ graphed in Figure 17.

Symmetries.

Even functions | Odd functions.



**EXAMPLE 11** Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$      (b) $g(x) = 1 - x^4$      (c) $h(x) = 2x - x^2$
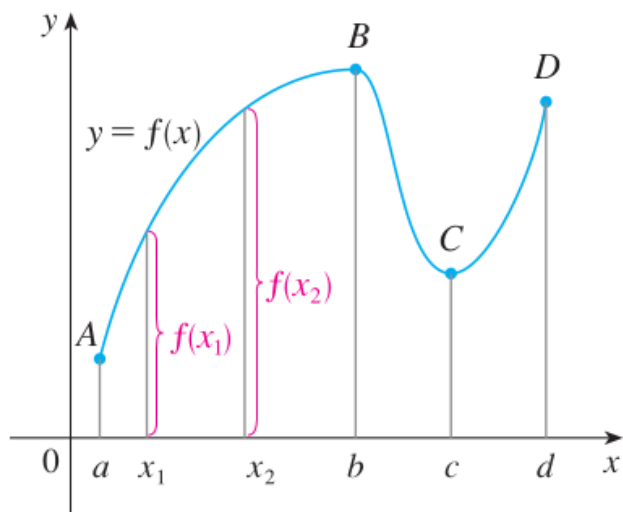
Increasing/Decreasing Functions.

A function $f$ is called **increasing** on an interval $I$ if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on $I$ if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$



- From A to B:

- From B to C:

. From C to D:

---

Example. Where the function $f(x) = x^2$ is increasing? Where is it decreasing?