Section 15.2, Problem 6

We first have that

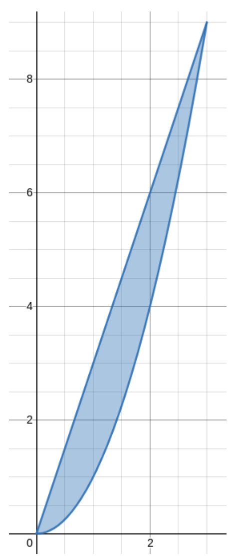
$$\int_0^{e^v} \sqrt{1+e^v}\, dw = \sqrt{1+e^v}\, (w)\Big|_0^{e^v} = e^v\sqrt{1+e^v}.$$

So

$$\int_0^1 \int_0^{e^v} \sqrt{1+e^v}\, dwdv = \int_0^1 e^v\sqrt{1+e^v}\, dv = \frac{2}{3}((1+e)^{3/2} - 2\sqrt{2}) \approx 2.894.$$

Section 15.2, Problem 14

The first thing to do is to draw the region D.



We see that the curves $y = x^2$ and $y = 3x$ intersects at the points $(0, 0)$ and $(3, 9)$.

Type I We have $0 \leq x \leq 3$ and $x^2 \leq y \leq 3x$. So the functions bounding the values of y are x^2 and $3x$. As a type I, the domain is written as

$$D = \{(x, y) \,:\, 0 \leq x \leq 2,\, x^2 \leq y \leq 3x\}.$$

Type II We see that $0 \leq y \leq 9$ and since $x \geq 0$, the curves bounding the values of x are $x = y/3$ and $x = \sqrt{y}$. As a a type II, the domain is written as

$$D = \{(x, y) \,:\, y/3 \leq x \leq \sqrt{y},\, 0 \leq y \leq 9\}.$$

Now the integral is

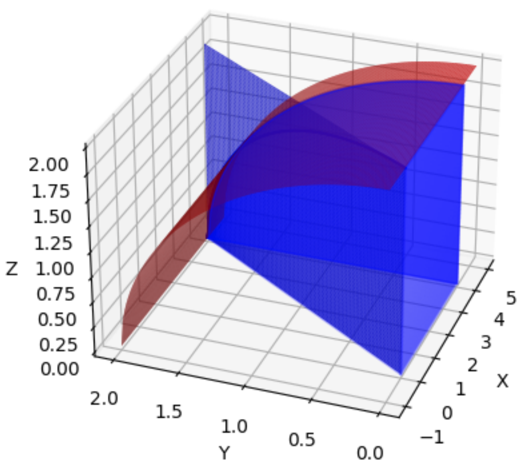
$$\int_0^3 \int_{x^2}^{3x} xy\, dydx = \int_0^3 x\Big(\frac{9x^2 - x^4}{2}\Big)\, dx = \int_0^3 \frac{9x^3 - x^5}{2} = \frac{243}{8}.$$

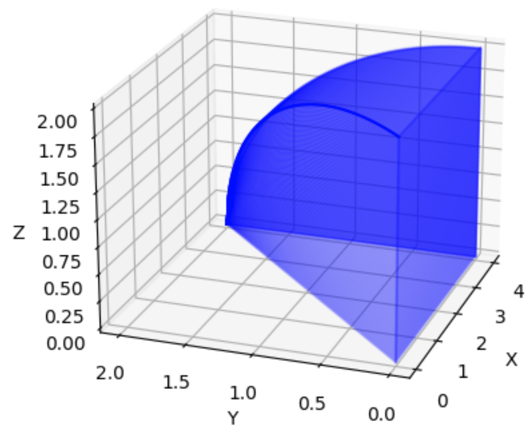
If you chose the other way, then your integral should look like this:

$$\int_0^9 \int_{y/3}^{\sqrt{y}} xy\, dxdy.$$

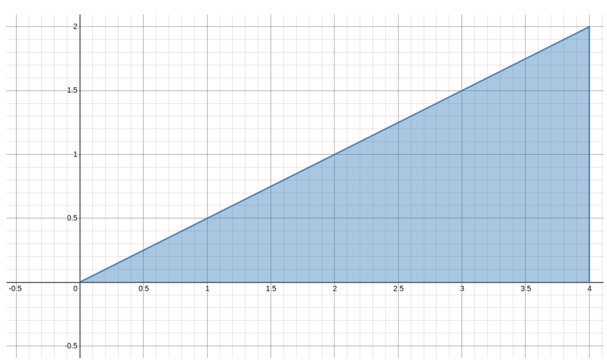
Section 15.2, Problem 30

The solid we are trying to find the volume is represented in the figure below.





To find the domain of integration D, we have to project the surfaces $y^2 + z^2 = 4$ and $x = 2y$ on the XY-place. For the first surface, we obtain $y = \pm 2$ (two horizontal lines in the XY-plane) and $x = 2y$ (a line with slope $1/2$). So the domain of integration is the following region: So, the



domain D is

$$D = \{(x, y) \,:\, 2y \le x \le 4,\, 0 \le y \le 2\}.$$

The function to integrate is $z = \sqrt{4 - y^2}$. Thus, the volume of the solid S is given by

$$V(S) = \int_0^2 \int_{2y}^4 \sqrt{4 - y^2}\, dxdy = \int_0^2 (4 - 2y)\sqrt{4 - y^2}\, dy.$$

The integral with $2y\sqrt{4-y^2}\,dy$ is done by a change of variable and we get

$$\int_0^2 2y\sqrt{4-y^2}\,dy = \frac{16}{3}.$$

The integral with $4\sqrt{4-y^2}\,dy$ is done by a trigonomoetric substitution and we get

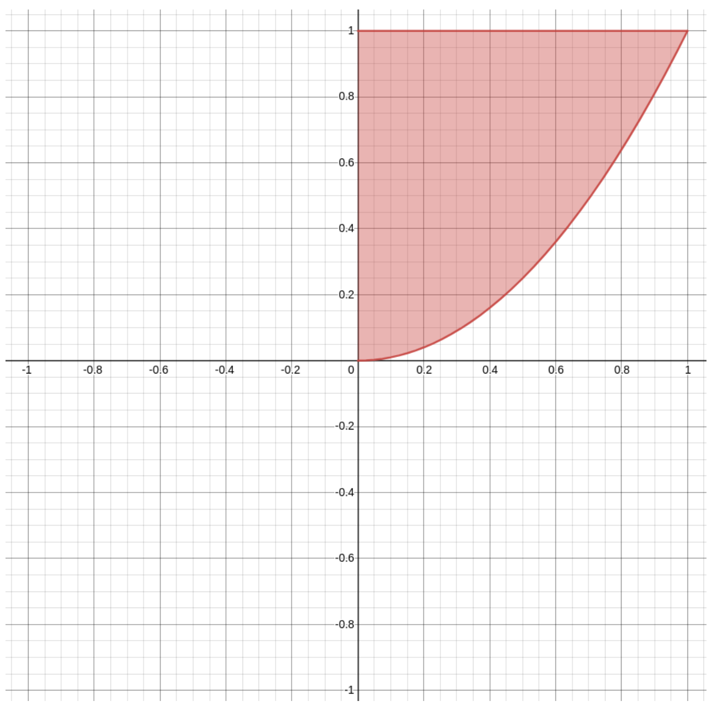
$$\int_0^2 4\sqrt{4-y^2}\,dy = 4\pi.$$

Thus, the volume of the solid is

$$V(S) = 4\pi - \frac{16}{3} \approx 7.233037.$$

Section 15.2, Problem 52

From the limits in the integrals, we see that $0 \le x \le 1$ and that $x^2 \le y \le 1$. So the region of integration looks like this: So the region D is the region bounded by the curves $x = 0$, $y = x^2$,



and $y = 1$. Since $x \ge 0$, the region D is also the region bounded by the curves $x = 0$, $x = \sqrt{y}$, and $y = 1$. So we can say that

$$D = \{(x, y) \,:\, 0 \le x \le \sqrt{y}, 0 \le y \le 1\}.$$

Thus, the integral now becomes

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y}\sin y\,dxdy = \int_0^1 \sqrt{y}\sin(y)\Big(\sqrt{y} - 0\Big)\,dy = \int_0^1 y\sin y\,dy.$$

After an integration by parts, we get the value of the integral:

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y} \sin y \, dx dy = \sin(1) - \cos(1) \approx 0.301168$$

Section 15.3, Problem 12

Let $x = r\cos\theta$ and $y = r\sin\theta$ (we change from cartesian to polar coordinates). The equation of the circle of radius 2 centered at the origin is simply $r = 2$. Also, in polar coordinates, we have $dA = r dr d\theta$ and so

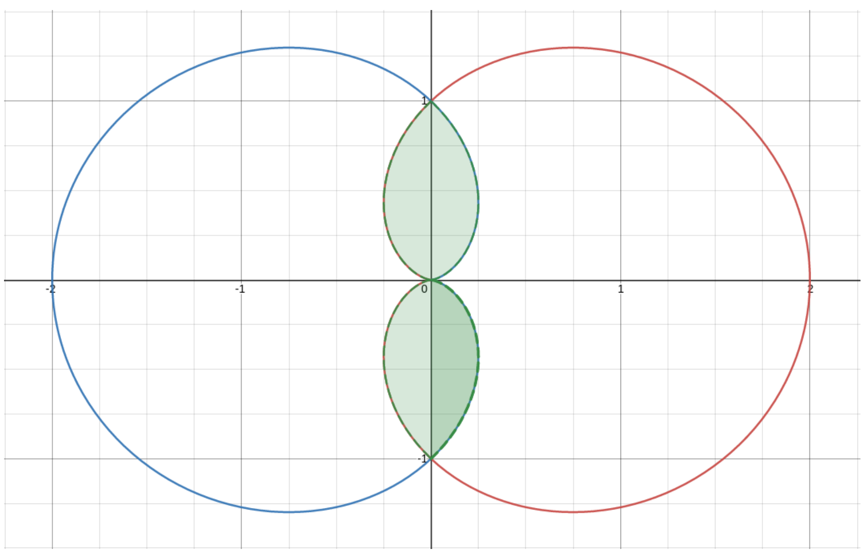
$$\iint_D \cos\sqrt{x^2+y^2}\, dA = \int_0^{2\pi} \int_0^2 (\cos r) r \, dr d\theta = \Big(\int_0^{2\pi} d\theta\Big)\Big(\int_0^2 r\cos r \, dr\Big).$$

After an integration by parts, the value of the integral is

$$\iint_D \cos\sqrt{x^2+y^2}\, dA = 2\pi(-1 + 2\sin(2) + \cos(2)).$$

Section 15.3, Problem 16

We draw the region between the two cardioids. Here are the two regions (in green) enclosed within the two cardiods: The two cardiods meet when $1 + \cos\theta = 1 - \cos\theta$. After rearranging, we



have to solve the equation $2\cos\theta = 0$. This occurs only when θ is $\pi/2 + k\pi$. We choose the values $\theta = \pi/2$ and $\theta = -\pi/2$. So, the polar coordinates of the two points of intersection are

$$(1, \pi/2) \quad \text{and} \quad (1, -\pi/2)$$

which corresponds to the following points in the cartesian plane:

$$(0, 1) \quad \text{and} \quad (0, -1).$$

We have now to setup the integral. Let D denote the region enclosed by the two cardioids. The area is given by

$$A(D) = \iint_D dA.$$

In polar coordinates, we have $dA = rdrd\theta$. Due to the symmetry of the domain, we can only compute the area of the petal with a positive (or zero) y coordinate (above the x-axis) and then multiply our result by 2. Call this region D_1.

The argument θ will vary from 0 to π. However, we have to split the interval $[0, \pi]$ into the intervals $[0, \pi/2]$ and $[\pi/2, \pi]$ because the cardioids intersect at $\theta = \pi/2$. We can apply again the symmetry argument because the region D_1 is symmetric with respect to the y axis and multiply by 2 to get the area of D_1. Denote half of the petal by D_2. So we have

$$A(D_2) = \int_0^{\pi/2} \int_0^{1-\cos(\theta)} r\, drd\theta = \int_0^{\pi/2} \frac{(1-\cos\theta)^2}{2}\, d\theta = \frac{3\pi}{8} - 1.$$

Thus,

$$A(D) = 2A(D_1) = 4A(D_2) = \frac{3\pi}{2} - 4.$$