Math 241 – Midterm 02
Fall 2022, 11/16/2022
6:00-7:15pm

Last name: _____

First name: _____

Section: _____

Instructions:

- Write your last name, first name and section number above.

- Answer the eight questions on this exam.

- Show all the details of your work.

- No electronic devices are to be used during the exam (this includes calculators).

- The exam is closed book and closed notes.

- **Do not use L'Hôpital's rule anywhere on this exam**.

- You have 75 minutes to complete the exam.

- Turn in your exam when you are done or at the end of the 75-min period.

Sign below to acknowledge you have read the instructions.

Signature: _____

May the Force be with you!

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If a spherical snowball melts so that its surface area decreases at a rate of $1\,\mathrm{cm}^2/\mathrm{min}$, find, using **Calculus**, the rate at which the diameter decreases when the diameter is 10 cm.

Note: The surface area of a sphere is $A = 4\pi r^2$.

Let $f(x) = \sqrt{1 + x}$.

(a) (5 points) Find the linearization of the function f at the point $a = 0$.

(b) (5 points) Using the linearization, estimate the value of $\sqrt{1.1}$. Explain clearly how you obtained your answer and leave it in decimal form.

Let $f(x) = \dfrac{3x^2 - 3}{x^2 + 3}$.

(a) (4 points) Using **Calculus**, find the vertical asymptotes (if any) and horizontal asymptotes (if any) of the function $f(x)$.

(b) (4 points) The first derivative of f is $f'(x) = \dfrac{24x}{(x^2+3)^2}$. Find the critical numbers (if any) and the open interval(s) of increase and decrease.

...Question 3 continued...

(c) (6 points) The second derivative of f is $f''(x) = \frac{-72(x^2-1)}{(x^2+3)^3}$. Find the x-coordinate of the inflection points (if any) and the open interval(s) of concavity.

(d) (4 points) Using one of the derivative tests, find the local maximum(s) and/or local minimum(s) of the function.

(e) (4 points) Sketch the graph of the function f in the axes below. Note that the y-intercept is -1 and the x-intercepts are $x = -1$ and $x = 1$.
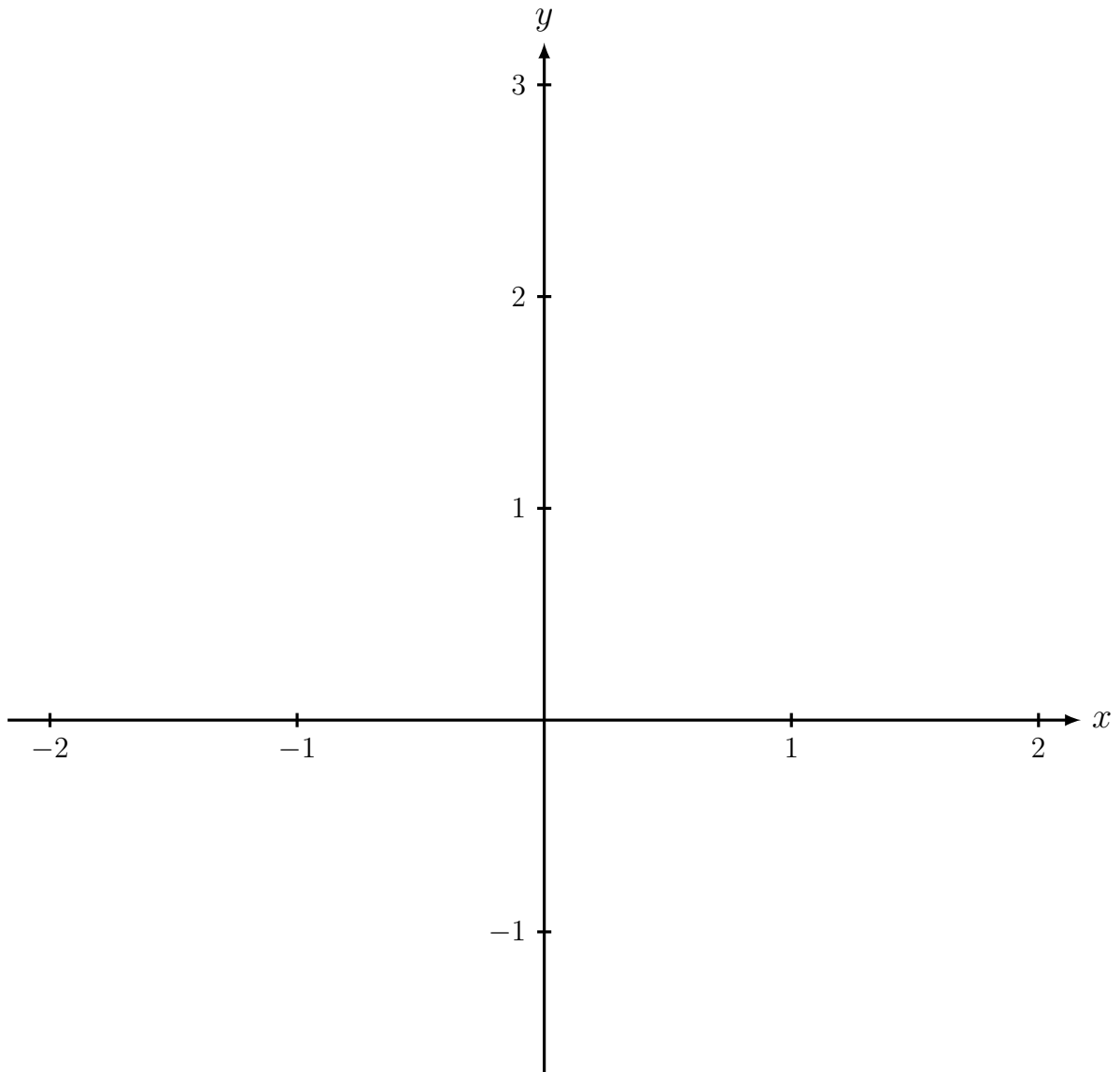
Compute the following limits. If the limit does not exist, write explicitly DNE. Make sure to write all the details of your calculations.

(a) (5 points) $\displaystyle\lim_{x\to\infty}\frac{3x-2}{2x+1}$.

(b) (5 points) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{3x-5}$.

A box with an open top is to be constructed from a square piece of cardboard of side length 3 ft by cutting out a square from each of the four corners and bending up the sides. **Using calculus**, find the largest volume that such a box can have. Make sure to justify clearly your answer.

(15 pts)

Find the most general antiderivative of the following functions.
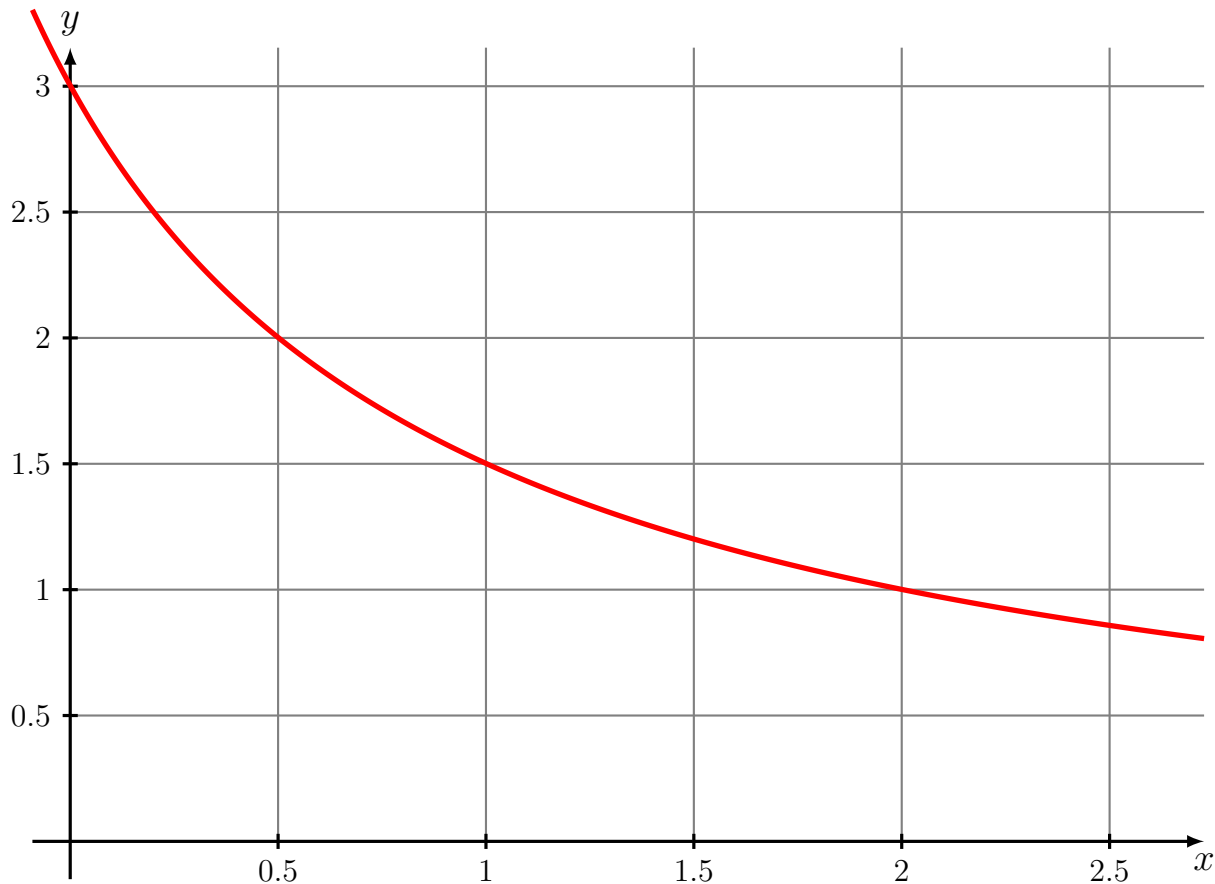
(a) (5 points) $f(x) = 4\sqrt{x} - 6x^2 + 3$.

(b) (5 points) $f(x) = \cos(x) + 2\sec^2(x)$.

(c) (5 points) $f(x) = x\sqrt{x} + \dfrac{x^2 + x}{x}$.

The graph of the function $f(x) = \dfrac{3}{1+x}$ is given below.

(a) (4 points) Estimate $\displaystyle\int_0^2 f(x)\,dx$ using two rectangles and right endpoints.

(b) (2 points) Draw the two rectangles from part (a) on the above picture of the graph of $f(x)$.

(c) (2 points) Is your answer over or under approximating the actual value of the integral?

Answer the following questions.

(a) (5 points) Using a comparison property for the definite integral, find a lower bound and an upper bound for the value of the following definite integral:

$$\int_0^1 \sqrt{1 + 3x}\, dx.$$

(b) (5 points) Using a geometric approach, find the value of the following definite integral:

$$\int_0^1 |2x - 1|\, dx.$$

Do not write on this page.