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**Problem 18**

As  $x \rightarrow -2^-$ , we have  $f(x) \rightarrow -\infty$  and as  $x \rightarrow -2^+$ , we have  $f(x) \rightarrow \infty$ . So we have an infinite discontinuity.

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**Problem 36**

The function  $x + \sin x$  is continuous because it is the sum of two continuous functions. Now,  $\sin x$  is continuous and therefore the composition  $\sin(x + \sin x)$  is continuous. In particular, the function  $x \mapsto \sin(x + \sin x)$  is continuous at  $x = \pi$ . Using the continuity, this means that

$$\lim_{x \rightarrow \pi} \sin(x + \sin x) = \sin(\pi + \sin \pi) = \sin(\pi + 0) = 0.$$

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**Problem 56**

Let  $f(x) = \sin x - x^2 + x$ . We have  $a = 1$  and  $b = 2$ .

We will verify if the hypothesis of the Intermediate Value Theorem are verified.

- The function  $f$  is a sum of continuous function on all of  $(-\infty, \infty)$ , therefore  $f$  is continuous on all of  $(-\infty, \infty)$ . In particular, the function  $f$  is continuous on  $(1, 2)$ .
- $f(1) = \sin(1) - 1^2 + 1 = \sin(1) > 0$  because for any  $0 < x < \pi$ , we have  $\sin(x) > 0$ .
- $f(2) = \sin(2) - 4 + 2 = \sin(2) - 2 < 0$  because  $\sin(1) < 1 < 2$ .

All the hypothesis of the IVP are satisfied. We therefore conclude that there is come  $c$ , between 1 and 2, such that  $f(c) = 0$ . This means that

$$\sin(c) - c^2 + c = 0 \quad \Longleftrightarrow \quad \sin(c) = c^2 - c$$

for some  $c$  such that  $1 < c < 2$ .

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**Problem 58 (a)**

We have  $f(0) = 3$  and  $f(-1) = -1 - 1 - 2 + 3 = -1$ . So, we have  $f(-1) < 0$  and  $f(0) > 0$ . So, by the intermediate Theorem, with  $N = 0$ , there is a number  $c \in (-1, 0)$  such that  $f(c) = 0$ .