

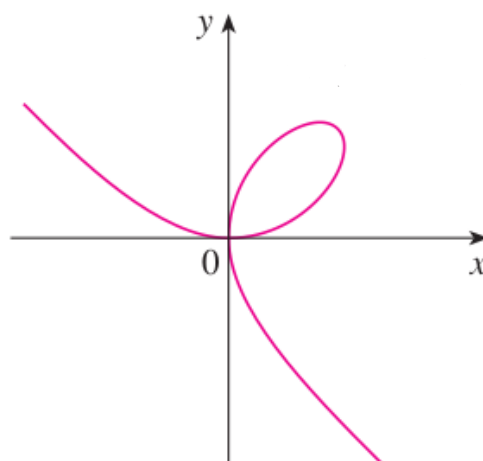
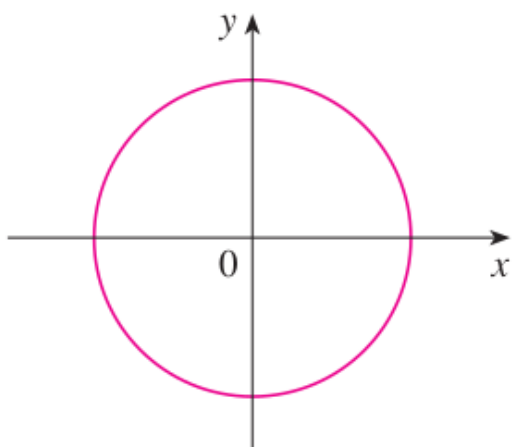
# Chapter 2

## Derivatives

### 2.6 Implicit Differentiation

Functions defined implicitly.

Geometry of curves.



In Natural Science.

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

- P:
- V:
- T:
- R, a, b are constants depending on the gas.

Main steps for implicit differentiation:

- 1) Take the derivative on each side of the relation.
- 2) Use the chain rule and other rules to make the computations.
- 3) Isolate the derivative  $dy/dx$ .

**EXAMPLE 1**

(a) If  $x^2 + y^2 = 25$ , find  $\frac{dy}{dx}$ .

(b) Find an equation of the tangent to the circle  $x^2 + y^2 = 25$  at the point  $(3, 4)$ .

### EXAMPLE 2

- (a) Find  $y'$  if  $x^3 + y^3 = 6xy$ .
- (b) Find the tangent to the folium of Descartes  $x^3 + y^3 = 6xy$  at the point  $(3, 3)$ .
- (c) At what point in the first quadrant is the tangent line horizontal?

**EXAMPLE 3** Find  $y'$  if  $\sin(x + y) = y^2 \cos x$ .