MATH 244 (Calculus IV), Fall 2021
Midterm Exam 1

I understand it is against the rules to cheat or engage in other academic misconduct during this test.

Question 1	20	
Question 2	20	
Question 3	20	
Question 4	20	
Question 5	20	
Total	100	

- This is a closed book, closed notes, no calculator exam. You are only allowed one two-sided cheat sheet.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 75 minutes to complete the exam, then 15 more minutes to scan and upload your solutions on Gradescope.

Problem 1.

a. Sketch the region D in the first quadrant bounded by the parabolas $x = y^2$ and $x = 8 - y^2$.

b. Calculate the integral

$$\int\int_D y \, dA.$$

Problem 2.

a. Sketch the region D in the xy-plane defined by

$$D := \{(x, y) : 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}.$$

b. Calculate the integral

$$\int\int_D \frac{\arctan(y/x)}{\sqrt{x^2 + y^2}}\, dA.$$

Problem 3.

Find ~~the surface area~~ the Volume of the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the plane $z = -3$.

Problem 4.

a. Sketch the solid E bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 1$.

b. Find the volume of E.

Problem 5.

a. Sketch the surface whose equation in cylindrical coordinates is given by

$$r^2 + z^2 = 4.$$

b. Set up but **do not evaluate** an iterated integral for

$$\int\int\int_E (x + y + z)\, dV,$$

where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.