MATH 244 (Calculus IV), Fall 2021
Midterm Exam 2

I understand it is against the rules to cheat or engage in other academic misconduct during this test.

Question 1	20	
Question 2	20	
Question 3	20	
Question 4	20	
Question 5	20	
Total	100	

- This is a closed book, closed notes, no calculator exam. You are only allowed one two-sided cheat sheet.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.**
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 75 minutes to complete the exam, then 15 more minutes to scan and upload your solutions on Gradescope.

Problem 1. Evaluate the integral

$$\int\int\int_E z^2 \, dV,$$

where E is the solid hemisphere

$$x^2 + y^2 + z^2 \le 9, y \ge 0.$$

Problem 2. Consider the square R with vertices $(0,0),(1,1),(2,0)$, and $(1,-1)$.

a. Find the image of R under the transformation

$$u = y - x,\ v = y + x.$$

b. Use this transformation to evaluate the integral

$$\int\int_R x\,dA.$$

Problem 3. Consider the vector field

$$\vec{F}(x, y) := (4x^3y^2 - 2xy^3)\vec{i} + (2x^4y - 3x^2y^2 + 4y^3)\vec{j}.$$

a. Is $\vec{F}$ conservative? Explain.

b. If your answer to **a.** is yes, find a potential f for $\vec{F}$.

c. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve parametrized by

$$\vec{r}(t) := (t + \sin(\pi t))\vec{i} + (2t + \cos(\pi t))\vec{j}, \qquad 0 \leq t \leq 1.$$

Problem 4. Evaluate the line integral

$$\oint_C \sqrt{1+x^3}\,dx + 2xy\,dy,$$

where C is the triangle with vertices $(0,0),(1,0)$, and $(1,3)$.

Problem 5. True or False? Justify.

a. The vector field

$$\vec{F}(x,y,z) := e^y\vec{\imath} + (xe^y + e^z)\vec{\jmath} + ye^z\vec{k}$$

is conservative.

b. If

$$\vec{F}(x,y,z) := xz\vec{\imath} + xyz\vec{\jmath} - y^2\vec{k},$$

then there is a vector field $\vec{G}$ such that $\vec{F} = \operatorname{curl}\vec{G}$.

c. If $\vec{F} := P\vec{\imath} + Q\vec{\jmath}$ is a vector field on an open connected set D such that P and Q have continuous first-order partial derivatives and

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

on D, then $\vec{F}$ is conservative.