Chapter 1

Functions and Limits

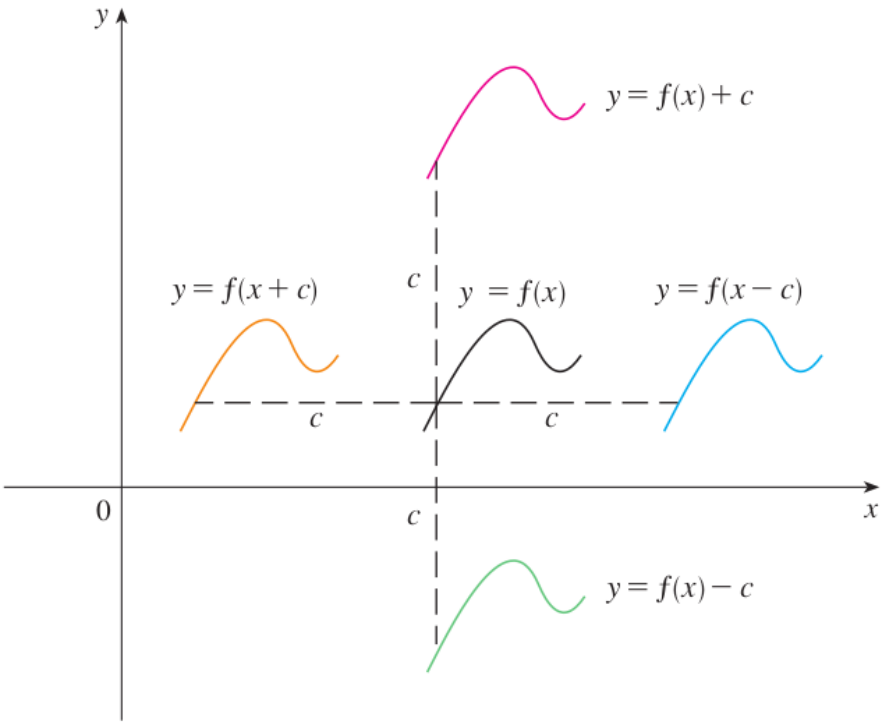
1.3 New Functions from Old Functions

Transformations of Functions.

Translation.

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

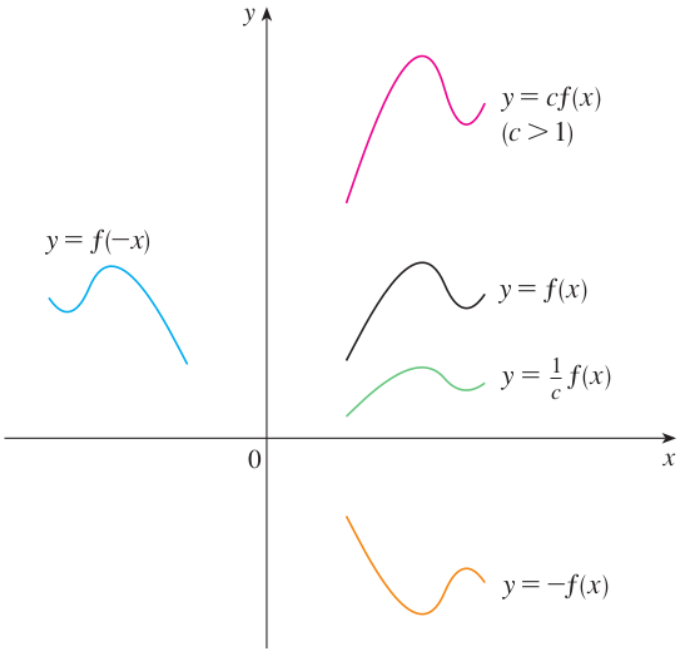
$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left



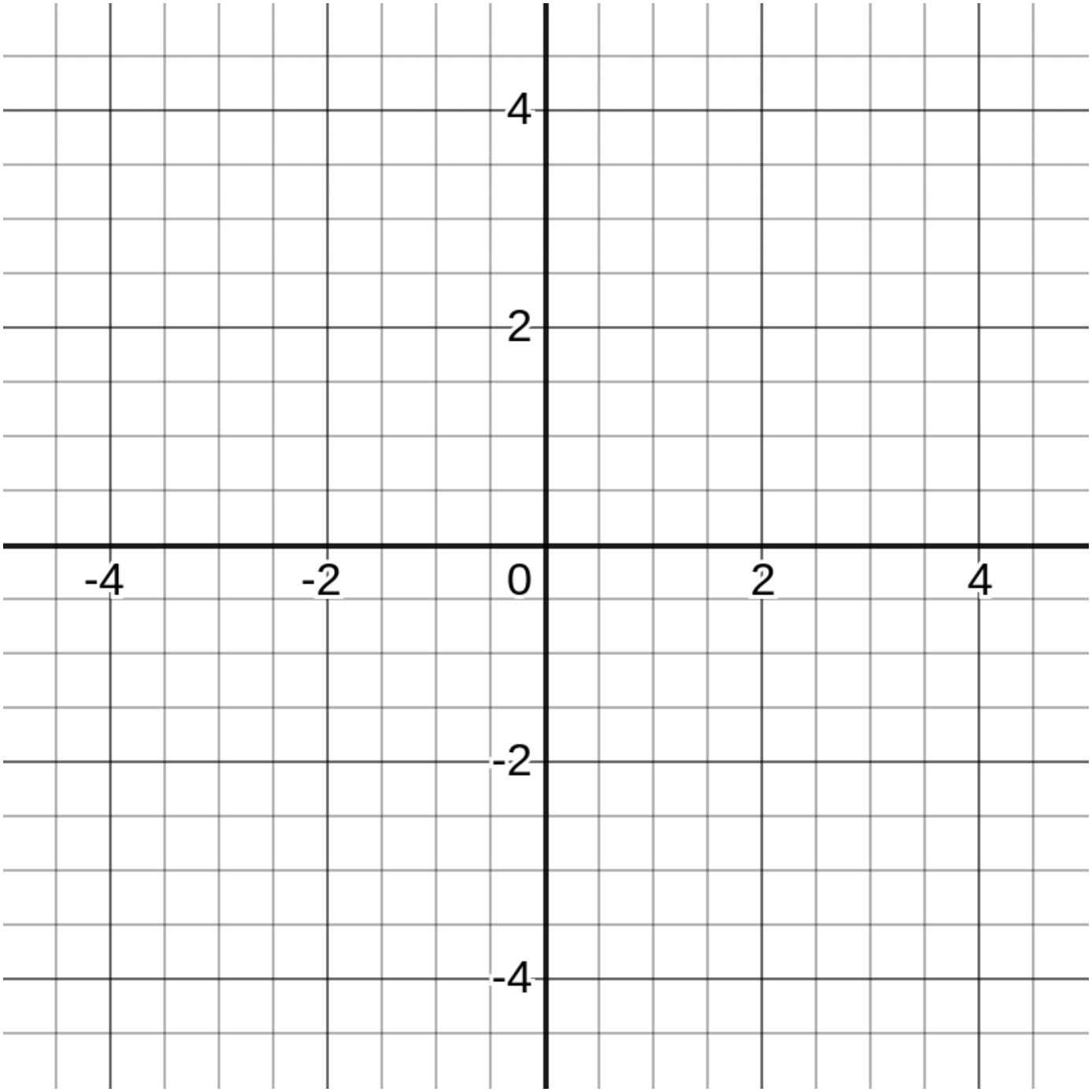
Stretching and reflecting.

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

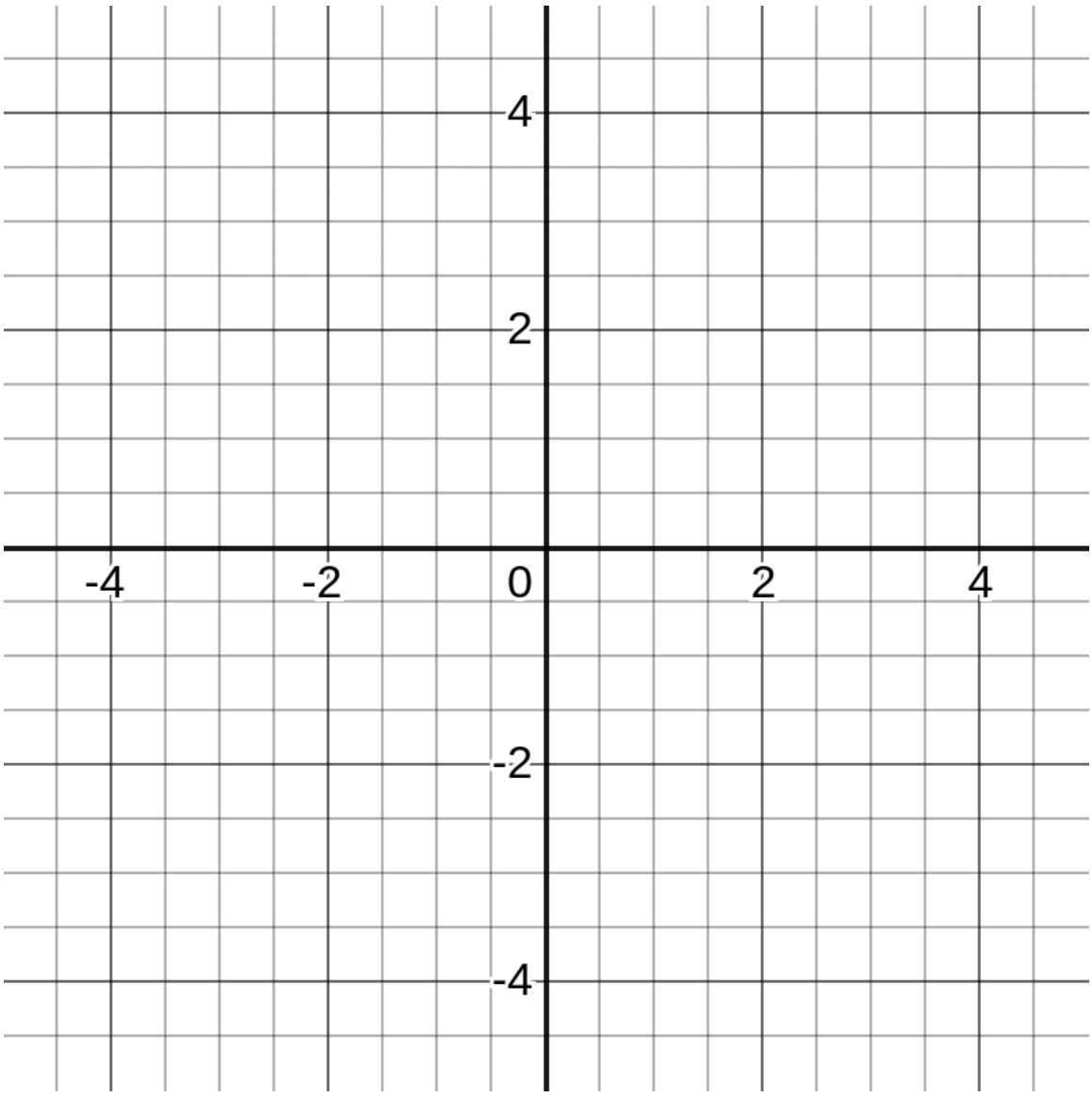
$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c
$y = (1/c)f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c
$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c
$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c
$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis



EXAMPLE 1 Given the graph of $y = \sqrt{x}$, use transformations to graph $y = \sqrt{x} - 2$, $y = \sqrt{x - 2}$, $y = -\sqrt{x}$, $y = 2\sqrt{x}$, and $y = \sqrt{-x}$.



EXAMPLE 5 Sketch the graph of the function $y = |x^2 - 1|$.



Combinaisons of Functions.

Adding.

$$(f+g)(x) = f(x) + g(x)$$

Domain = $\mathrm{Dom}(f) \cap \mathrm{Dom}(g)$

Substracting.

$$(f-g)(x) = f(x) - g(x)$$

Domain = $\mathrm{Dom}(f) \cap \mathrm{Dom}(g)$

Multiplying.

$$(fg)(x) = f(x)g(x)$$

Domain = $\mathrm{Dom}(f) \cap \mathrm{Dom}(g)$

Dividing.

$$(f/g)(x) = f(x)/g(x)$$

Domain = everything in $\mathrm{Dom}\,(f)$ which is not in $\mathrm{Dom}(g)$.

Example. Find the domain of the function $h(x) = \sqrt{x} + \sqrt{2-x}$.

Example Find the domain of the function $h(x) = \dfrac{x^2}{x-1}$.

Composite of two functions (Composition).

Definition Given two functions f and g, the **composite function** $f \circ g$ (also called the **composition** of f and g) is defined by

$$(f \circ g)(x) = f(g(x))$$

Domain = every x in the $\mathrm{Dom}(g)$ such that $g(x)$ is in $\mathrm{Dom}(f)$.

EXAMPLE 6 If $f(x) = x^2$ and $g(x) = x - 3$, find the composite functions $f \circ g$ and $g \circ f$.

EXAMPLE 9 Given $F(x) = \cos^2(x + 9)$, find functions f, g, and h such that $F = f \circ g \circ h$.

Example. Find the domain of the function $h(x) = \sqrt{x+2}$.