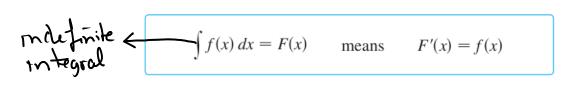
Chapter 4 Integrals

4. Indefinite Integrals and the Net Change Theorem

Previously on Calc I:

We introduce a notation for the antiderivatives:



Stheldse General A.D.

Example.

a)
$$\int x^2 dx = \frac{x^3}{3} + C$$

b)
$$\int \cos x \, dx = \sin x + C$$

c)
$$\int \sec^2 x \, dx = \int anx + C$$

Table of Indefinite integrals

$$\int cf(x) dx = c \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Remark: We adopt the convention that the general indefinite integral is valid on a given interval. Thus we write $\sqrt{\frac{1}{1+\frac{1}{2}}}$

 $\left(\int \frac{1}{x^2} \, dx \right) = -\frac{1}{x} + C \qquad \qquad x^{-7} \quad -\infty \quad \underbrace{x^{-7}}_{-7,1}$

with the understanding that it is valid on the interval $(0,\infty)$ or on the interval $(\underline{-\infty},0)$.

EXAMPLE 1 Find the general indefinite integral

$$\int \frac{10x^{4} - 2 \sec^{2}x}{dx} dx = \int \frac{10x^{4} dx}{dx} + \int \frac{-2 \sec^{2}x}{dx} dx$$

$$= \int \frac{10x^{4} dx}{x^{5}/5} + C_{1} - 2 \int \frac{x}{x^{7}} dx$$

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$$= \int \frac{x}{x^{7}} + C$$

EXAMPLE 2 Evaluate
$$\int \frac{\cos \theta}{\sin^2 \theta} d\theta$$
.

$$\int \frac{\cos \Theta}{\sin^2 \Theta} d\Theta = \int \frac{\cos \Theta}{\sin \Theta} \cdot \frac{1}{\sin \Theta} d\Theta$$

$$= \int \cot \Theta \cdot \csc \Theta d\Theta$$

EXAMPLE 4 Find $\int_0^{12} (x - 12 \sin x) dx$.

$$\int_{0}^{12} x - |2\sin x \, dx = \int_{0}^{12} x \, dx - |2| \int_{0}^{12} \sin x \, dx$$

$$= \frac{x^{2}}{2} \Big|_{0}^{12} - |2| \left(-\cos x \right) \Big|_{0}^{12}$$

$$= \frac{|2^{2}}{2} - \frac{\sqrt{2}}{2} - |2| \left(-\cos |2| - (-\cos x) \right)$$

$$= \frac{|44}{2} - |2| \left(-\cos |2| + 1 \right)$$

$$= \frac{72}{2} + |2\cos |2| - |2|$$

$$= \frac{|60| + |2\cos |2|}{2}$$

EXAMPLE 5 Evaluate
$$\int_{1}^{9} \frac{2t^{2} + t^{2}\sqrt{t} - 1}{t^{2}} dt$$
. $f(t)$ is confinuous an $(1, 9)$

$$\frac{2t^{2} + t^{2}\sqrt{t} - 1}{t^{2}} = 2 + \sqrt{t} - \frac{1}{t^{2}} = 2 + \sqrt{t} - t^{-2}$$

$$\int_{1}^{9} \frac{2t^{2} + t^{2}\sqrt{t} - 1}{t^{2}} dt = \int_{1}^{9} 2 + \sqrt{t} - t^{-2} dt$$

$$= 2\int_{1}^{9} 1 dt + \int_{1}^{9} \sqrt{t} dt - \int_{1}^{9} t^{-2} dt$$

$$= 2 \frac{t^{0+1}}{9} + \frac{t}{1/2+1} \Big|_{1}^{9} - \frac{t^{-2+1}}{2+1} \Big|_{1}^{9}$$

$$= 2t \Big|_{1}^{9} + \frac{2t^{3/2}}{3} + \frac{t^{-1}}{9} \Big|_{1}^{9}$$

$$= 2 \cdot 9 - 2 \cdot 1 + 2 \cdot \frac{9^{3/2}}{3} - \frac{2 \cdot 1}{3} + 9^{-1} - 1$$

$$= 32 \frac{4}{9} \Big|_{1}^{9}$$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x) \, dx = F(b) - F(a)$$

a) Displacement:

b) Total distance traveled:

c) Acceleration:

EXAMPLE 6 A particle moves along a line so that its velocity at time t is

 $v(t) = t^2 - t - 6$ (measured in meters per second).

- (a) Find the displacement of the particle during the time period $1 \le t \le 4$.
- (b) Find the distance traveled during this time period.