

# Chapter 1

## Functions and Limits

1.2 Mathematical Models: A catalog of Essential Functions

## Linear Models.

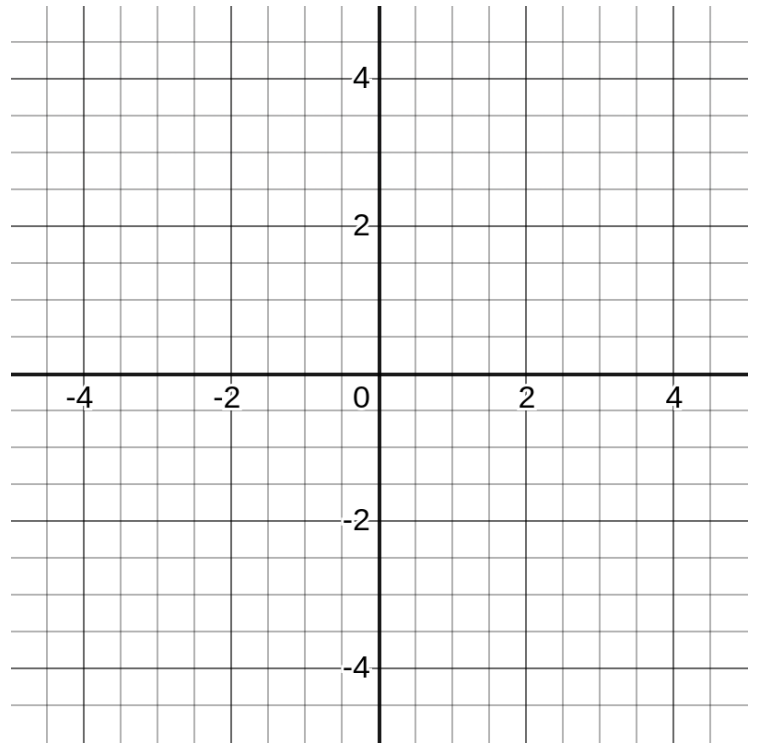
$$y = f(x) = mx + b$$

.m: the slope

.b: y-intercept

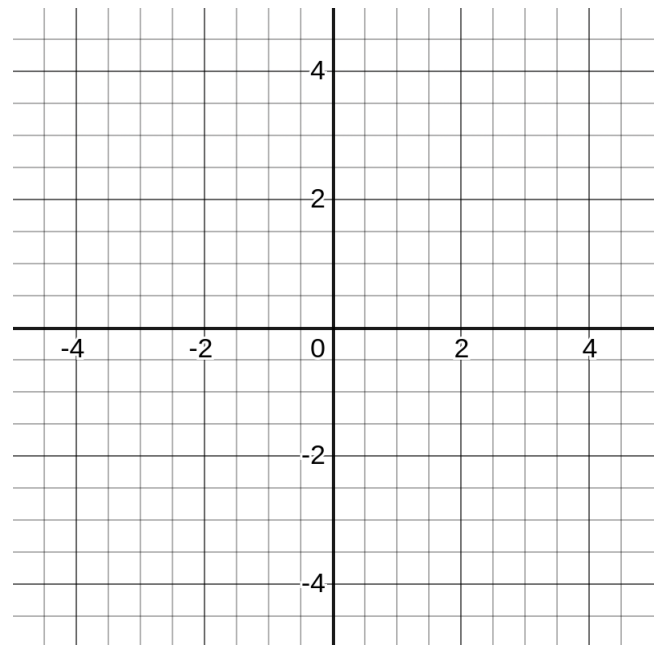
Another formulation (knowing a point):

$$y - y_0 = m(x - x_0)$$



### EXAMPLE 1

- (a) As dry air moves upward, it expands and cools. If the ground temperature is 20°C and the temperature at a height of 1 km is 10°C, express the temperature  $T$  (in °C) as a function of the height  $h$  (in kilometers), assuming that a linear model is appropriate.
- (b) Draw the graph of the function in part (a). What does the slope represent?
- (c) What is the temperature at a height of 2.5 km?



Polynomials.

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

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Domain:

Examples.

a) Concrete example.

b) Degree 1.

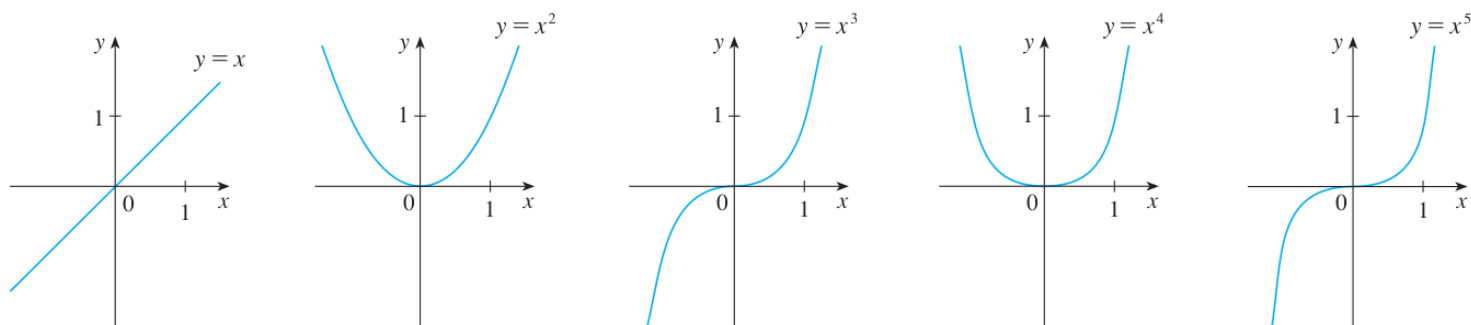
c) Degree 2.

d) Degree 3.

# Power Functions.

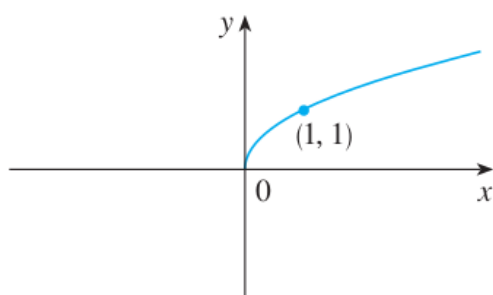
$$f(x) = x^a$$

i)  $a$  is a positive integer or is zero.

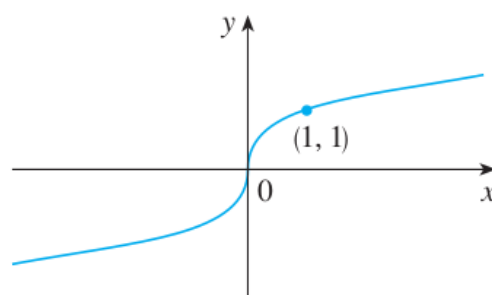


Domain:

ii)  $a$  is the reciprocal of a positive integer.



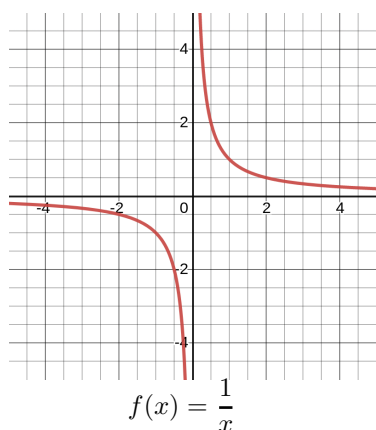
(a)  $f(x) = \sqrt{x}$



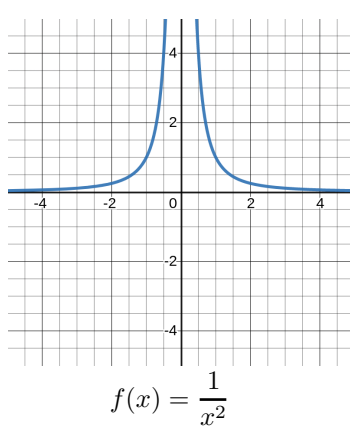
(b)  $f(x) = \sqrt[3]{x}$

Domain:

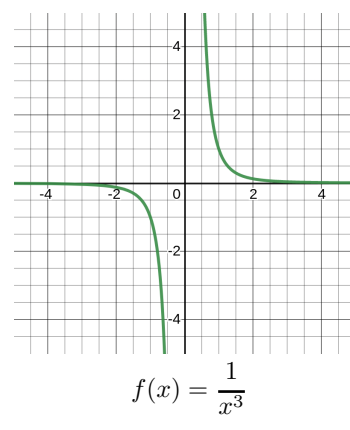
iii) When  $a$  is a negative integer.



$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x^2}$



$f(x) = \frac{1}{x^3}$

Domain:

## Rational Functions.

$$f(x) = \frac{P(x)}{Q(x)}$$

P:

Q:

Domain:

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**Example.** Find the domain of the function  $f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$ .

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## Algebraic Functions.

An algebraic function  $f$  is a function that can be expressed only in term of the basic operations :

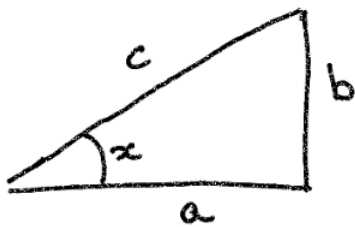
- summation ;
- subtraction ;
- multiplication ;
- division ;
- extracting roots (i.e. taking  $\sqrt[n]{\cdot}$ ).

Domain:

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**Examples.** Find the domain of the following function  $g(x) = \frac{x^4 - 16x^2}{x + \sqrt{x}} + (x - 2)\sqrt[3]{x + 1}$ .

## Trigonometric Functions.



$$\cos x = \frac{a}{c}$$

$$\sin x = \frac{b}{c}$$

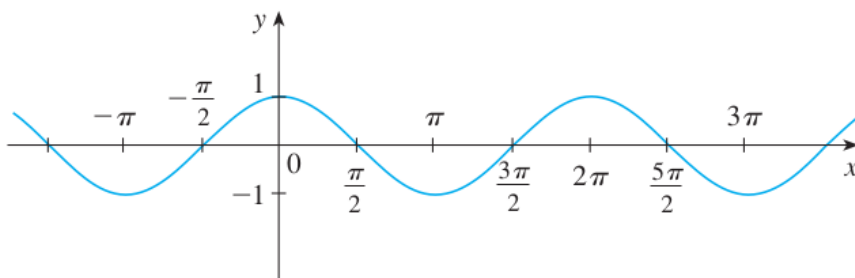
$$\tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$\cotan x = \frac{1}{\tan x}$$

i) Cosine function.



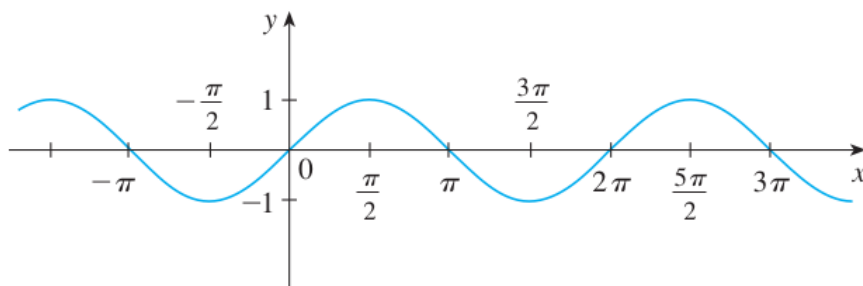
Domain:

Range:

Zeros:

Other:

ii) Sine Function.



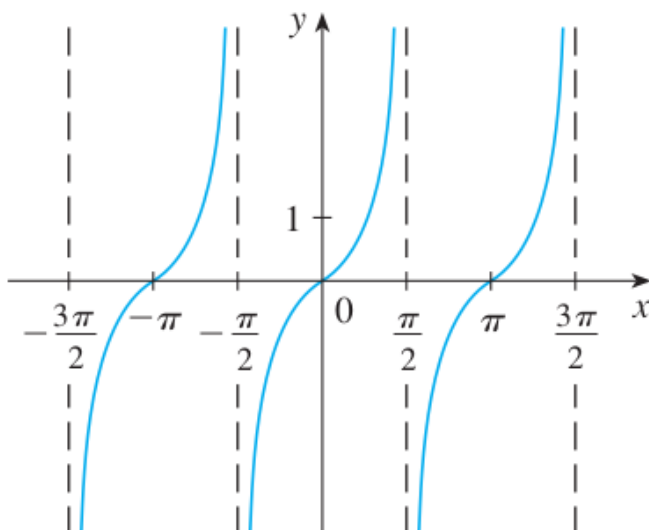
Domain:

Range:

Zeros:

Other:

iii) Tangent Function.



Domain:

Range:

Zeros:

Other:

**EXAMPLE 5** What is the domain of the function  $f(x) = \frac{1}{1 - 2 \cos x}$ ?

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**EXAMPLE 6** Classify the following functions as one of the types of functions that we have discussed.

~~(a)  $f(x) = 5^x$~~

(b)  $g(x) = x^5$

(c)  $h(x) = \frac{1 + x}{1 - \sqrt{x}}$

(d)  $u(t) = 1 - t + 5t^4$