Section 2.2, Problem 26

By definition, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Let's simplify the difference quotient:

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{\frac{(x+h)^2-1}{2(x+h)-3} - \frac{x^2-1}{2x-3}}{h(2x-3)(2x+2h-3)} \\
&= \frac{(x^2+2xh+h^2-1)(2x-3) - (x^2-1)(2x+2h-3)}{h(2x-3)(2x+2h-3)} \\
&= \frac{(x^2-1)(2x-3) + (2xh+h^2)(2x-3) - (x^2-1)(2x-3) - 2h(x^2-1)}{h(2x-3)(2x+2h-3)} \\
&= \frac{4x^2h - 6xh + 2xh^2 - 3h^2 - 2x^2h + 2h}{h(2x-3)(2x+2h-3)} \\
&= \frac{2x^2h + 2xh^2 - 6xh - 3h^2 + 2h}{h(2x-3)(2x+2h-3)} \\
&= \frac{2x^2 + 2xh - 6x - 3h + 2}{(2x-3)(2x+2h-3)}.
\end{aligned}$$

Now we just have to take the limit as $h \to 0$:

$$f'(x) = \lim_{h\to 0} \frac{2x^2 + 2xh - 6x - 3h + 2}{(2x-3)(2x+2h-3)} = \frac{2x^2 - 6x + 2}{(2x-3)^2}.$$

Section 2.2, Problem 48

We have to take a look at the slope of the tangent lines in each graph.

We can see that the curve 'c' is positive where the slopes of the tangents to the graph of the curve 'd' are postive. Also, we see that the curve 'c' is negative when the slopes of the tangents to the graph of the curve 'd' are negative. So the curve 'c' represents the derivative of 'd'.

We remark that the sign of the y-coordinate of the points of the curve 'b' is the same as the slopes of the tangents to the curve 'c'. So curve 'b' is the derivative of the curve 'c'.

Finally, we see that the sign of the slopes of the tangents to the curve 'b' are always positive or zero and this is the same sign as the y-coordinate of the points on the curve 'a'. So the curve 'a' is the derivative of the curve 'b'.

In summary, we have

$$f \leftrightarrow d,\ f' \leftrightarrow c,\ f'' \leftrightarrow b \text{ and } f''' \leftrightarrow a.$$

$\boxed{\textbf{Section 2.3, Problem 2}}$

Don't get confused, π is a constant and so π^2 is a constant. Then $f'(x) = 0$.

Section 2.3, Problem 18

Using the quotient rule, we obtain

$$y' = \frac{(\sqrt{x}+x)'x^2 - (\sqrt{x}+x)(x^2)'}{x^4}.$$

We have, from the power rule,

$$(\sqrt{x}+x)' = 1/2\sqrt{x}+1 \quad \text{and} \quad (x^2)' = 2x$$

and so replacing that in y', we obtain

$$y' = \frac{(1/2\sqrt{x}+1)x^2 - (\sqrt{x}+x)2x}{x^4} = \frac{x^{3/2}/2 + x^2 - 2x^{3/2} - 2x^2}{x^4} = \frac{-3x^{3/2}/2 - x^2}{x^4}.$$

Finally, we get $y' = -3x^{-5/2}/2 - x^{-2}$.

There is another approach. By letting $x \neq 0$, we can rewrite the expression as

$$y = x^{-3/2} + x^{-1}$$

and by the sum and quotient rules, we obtain

$$y' = -3x^{-5/2}/2 - x^{-2}.$$

Section 2.3, Problem 54

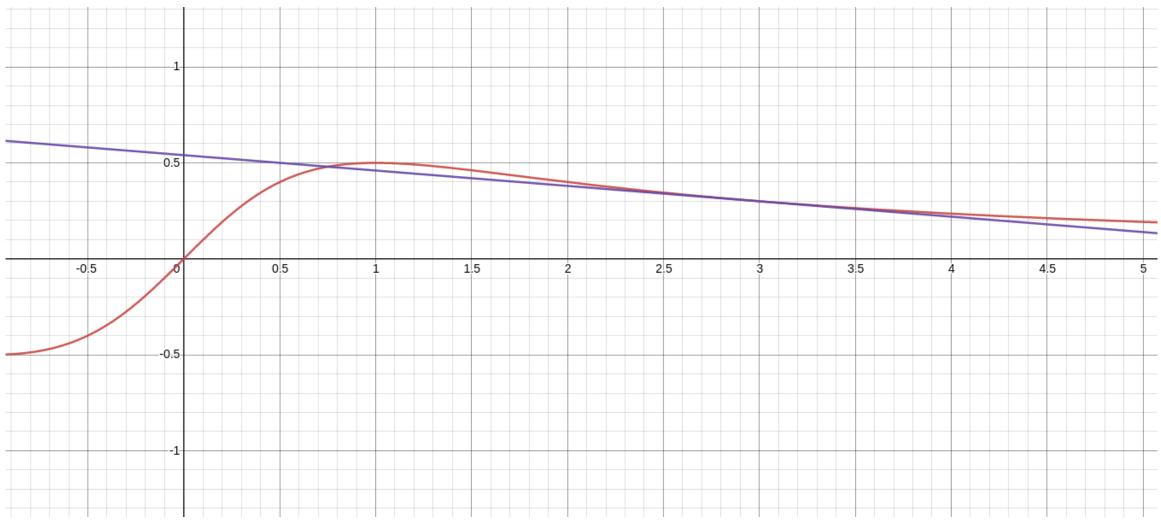
(a) We first find the derivative. We get

$$y' = \frac{1 + x^2 - 2x^2}{1 + x^2} = \frac{1 - x^2}{(1 + x^2)^2}.$$

The equation of the tangent line is given by the equation $y - 0.3 = y'(3)(x - 3)$. So, pugging in the numbers, with $y' = -8/10 = -4/5$, we obtain

$$y - 0.3 = (-4/5)(x - 3) \iff y = -0.8x + 2.4.$$

(b) Using Desmos, we get the following picture.



Section 2.3, Problem 72

Since the function $f(x) = x$ is not zero at $x = 2$, we can use the quotient rule. We obtain

$$\frac{d}{dx}\Big(\frac{h(x)}{x}\Big) = \frac{h'(x)x - h(x)}{x^2}$$

and then, at $x = 2$, we get

$$\frac{d}{dx}\Big(\frac{h(x)}{x}\Big)\Big|_{x=2} = \frac{h'(2) \times 2 - h(2)}{4} = \frac{(-3) \times 2 - 4}{4} = -5/2.$$