Chapter 4
Integrals

4.5 The Substitution Rule

Example to start. Find the indefinite integral of $2x\sqrt{1+x^2}$, that is compute

$$\int 2x\sqrt{1+x^2}\,dx.$$

Another example. Compute the indefinite integral

$$\int x\sqrt{1+x^2}\,dx.$$

4 The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$$

Relation between du and dx:

EXAMPLE 1 Find $\int x^3 \cos(x^4 + 2)\, dx$.

EXAMPLE 2 Evaluate $\int \sqrt{2x + 1}\, dx$.

EXAMPLE 3 Find $\int \frac{x}{\sqrt{1 - 4x^2}}\, dx$.

EXAMPLE 5 Find $\int \sqrt{1 + x^2}\, x^5\, dx$.

EXAMPLE 7 Evaluate $\int_1^2 \frac{dx}{(3 - 5x)^2}$.