# SECTION 1.7: TRIG FUNCTIONS AND HYP(E) FUNCTIONS

## TRIG FCTS

If $\theta \in \mathbb{R}$, then

$$e^{i\theta} = \cos\theta + i\sin\theta$$

and

$$e^{-i\theta} = \cos\theta - i\sin\theta .$$

So,

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

and

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

**DEF 1.7.1** For $z \in \mathbb{C}$, we define

$$\cos(z) := \frac{e^{iz} + e^{-iz}}{2}$$

and

$$\sin(z) := \frac{e^{iz} - e^{-iz}}{2i} .$$

## Example 1.7.2 Compute

(a) $\cos(2+i\pi)$ (b) $\sin(i5\pi/4)$

## Solutions

$$(a)\ \cos(2+i\pi) = \frac{1}{2}\left(e^{i(2+i\pi)} + e^{-i(2+i\pi)}\right)$$

$$= \frac{1}{2}\left(e^{-\pi+2i} + e^{\pi-2i}\right)$$

$$= \frac{1}{2}\left(e^{-\pi}(\cos 2 + i\sin 2) + e^{\pi}(\cos 2 - i\sin 2)\right)$$

$$= \frac{1}{2}\left(e^{-\pi}+e^{\pi}\right)\cos 2 + \frac{i}{2}\left(e^{-\pi}-e^{\pi}\right)\sin 2$$

$$= \frac{1}{2}\left(e^{\pi}+e^{-\pi}\right)\cos 2 - i\,\frac{1}{2}\left(e^{\pi}-e^{-\pi}\right)\sin 2$$

$$= \cosh(\pi)\cos(2) - i\sinh(\pi)\sin(2).$$

here, $\cosh(x) = \dfrac{e^{x}+e^{-x}}{2}$ and $\sinh(x) = \dfrac{e^{x}-e^{-x}}{2}$.