## Chapter 3 Applications of Derivatives

3.5 Summary of Curve Sketching

**EXAMPLE 1** Use the guidelines to sketch the curve  $y = \frac{2x^2}{x^2 - 1}$ . =  $\pm (x)$ 

(A) Domain. 
$$x^{2}-1=(x-1)(x+1)=0 \ge x=-1, x=1$$
is  $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$ .

B Y-interc. d x- interc.

$$f(0) = 0$$

$$f(x) = 0$$

(C) Symmetries.

• Odd or even: 
$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2 - 1} = f(x)$$

Lo even.

D HAD VA.

HA: 
$$\lim_{\chi \to \infty} \frac{2\chi^2}{\chi^2 - 1} = \frac{2}{1} = 2$$
 -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = \frac{2}{1} = 2$  -Dy=z HA  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = \frac{2}{1} = 2$  -Dy=z HA  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = \frac{2}{1} = 2$  -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = \frac{2}{1} = 2$  -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = \frac{2}{1} = 2$  -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = 2$  -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = 2$  -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = 2$  -Dy=z HA.  $\lim_{\chi \to -\infty} \frac{2\chi^2}{\chi^2 - 1} = 2$  -Dy=z HA.

$$\lim_{x \to 1^{+}} \frac{2x^{2}}{x^{2}-1} = \frac{2}{0^{+}} = +\infty$$

$$\lim_{x \to -1^{-}} \frac{2x^{2}}{x^{2}-1} = \frac{2}{0^{+}} = +\infty$$

$$\lim_{x \to -1^{+}} \frac{2x^{2}}{x^{2}-1} = \frac{2}{0^{+}} = -\infty$$

$$\lim_{x \to -1^{+}} \frac{2x^{2}}{x^{2}-1} = \frac{2}{0^{-}} = -\infty$$

$$f'(x) = -\frac{4x}{(x^2-1)^2}$$
 — C.N.: O, -1, 1

$$(x^2-1)^2 \ge 0$$
 ->  $f'(x) > 0$  when  $-4x > 0$  when  $> 1 < 0$ 

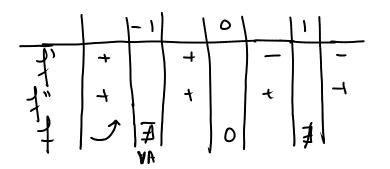
$$(x^{2}-1)^{2} \ge 0$$
 ->  $f'(x) < 0$  When -4x < 0

When >2>0

$$f(0) = 0$$

$$f''(x) = \frac{12x^2+4}{(x^2-1)^3}$$
 —  $f''(x) > 0$ 

—  $f''(x) = 0$  concave up on



## Guidelines for Sketching a Curve.

- **A.** Find the <u>domain</u> of the function.
- **3.** Find the <u>y-intercept</u> and <u>x-intercept</u>, that is f(0) and when y=0.
- **C.** Search for <u>symmetries</u> in the function (facultative)
  - If f(x) = f(-x), then the function is even.
  - If -f(x) = f(-x), then the function is odd.
  - If f(x+p) = f(x), then the function repeats itself after a period p (it is periodic).
- **D.**Find the <u>asymptotes</u> of the function:
  - The Horizontal asymptotes.
  - The Vertical asymptotes.
- **E**. Find the <u>intervals of increase and decrease</u>.
- **F.** Find the <u>local maximum</u> and <u>minimum</u> values.
- **G**. Find the <u>concavity</u> and the <u>points of inflections</u>.