

Section 4.5 — Problem 2 — 5 points

Let $u = 2x^2 + 3$, so that

$$\frac{du}{dx} = 4x \quad \Rightarrow \quad \frac{1}{4}du = xdx.$$

The integral becomes

$$\int x(2x^2 + 3)^4 dx = \int u^4 \frac{du}{4} = \frac{u^5}{20} + C$$

and the answer is

$$\frac{(2x^2 + 3)^5}{20} + C.$$

Section 4.5 — Problem 4 — 5 points

Let $u = \sin \theta$. Then

$$\frac{du}{dx} = \cos \theta \quad \Rightarrow \quad du = \cos \theta \, dx.$$

There integral becomes

$$\int \sin^2 \theta \cos \theta \, d\theta = \int u^2 \, du = \frac{u^3}{3} + C$$

and the answer is

$$\frac{\sin^3 \theta}{3} + C.$$

Section 4.5 — Problem 8 — 5 points

Let $u = x^3$, so that

$$\frac{du}{dx} = 3x^2 \quad \Rightarrow \quad \frac{1}{3} du = x^2 dx.$$

Therefore, the integral becomes

$$\int x^2 \sin(x^3) dx = \int \sin(u) \frac{du}{3} = -\frac{1}{3} \cos(u) + C.$$

The answer is then

$$-\frac{1}{3} \cos(x^3) + C.$$

Section 4.5 — Problem 10 — 5 points

Set $u = 1 + \cos t$, then

$$\frac{du}{dt} = -\sin(t) \quad \Rightarrow \quad -du = \sin(t) dt.$$

The integral then becomes

$$\int \sin t \sqrt{1 + \cos t} dt = - \int \sqrt{u} du = -\frac{2}{3} u^{3/2} + C$$

and the final answer is

$$-\frac{2}{3}(1 + \cos t)^{3/2} + C.$$

Section 4.5 — Problem 16 — 5 points

Let $u = \sqrt{x}$, so that

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} dx \quad \rightarrow \quad 2 du = \frac{1}{\sqrt{x}} dx.$$

The integral then becomes

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = \int \sin(u) 2du = -2 \cos(u) + C.$$

The answer is then

$$-2 \cos(\sqrt{x}) + C.$$

Section 4.5 — Problem 20 — 5 points

Let $u = x + 2$, then $du = dx$ and

$$\int x\sqrt{x+2} \, dx = \int (u-2)\sqrt{u} \, du = \int u^{3/2} - 2u^{1/2} \, du = \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C.$$

Then, the answer is

$$\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C.$$

Section 4.5 — Problem 26 — 5 points

Let $u = \tan x$, then

$$\frac{du}{dx} = \sec^2 x \, dx.$$

The integral then becomes

$$\int \frac{\sec^2 x}{\tan^2 x} \, dx = \int u^{-2} \, du = -u^{-1} + C$$

and therefore the final answer is

$$-\frac{1}{\tan x} + C.$$

Section 4.5 — Problem 30 — 5 points

Let $u = x^2 + 1$. Therefore, we have

$$\frac{du}{dx} = 2x \quad \Rightarrow \quad \frac{1}{2}du = x \, dx.$$

The integral becomes

$$\int x^3 \sqrt{x^2 + 1} \, dx = \int x^2 \sqrt{u} \, dx = \frac{1}{2} \int x^2 \sqrt{u} \, du.$$

Since $u = x^2 + 1$, we have $x^2 = u - 1$ and therefore

$$\int x^2 \sqrt{u} \, du = \int (u - 1) \sqrt{u} \, du = \int u^{3/2} - u^{1/2} \, du = \frac{1}{2} \left(\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right) + C$$

and the final answer is

$$\frac{1}{5} (x^2 + 1)^{5/2} - \frac{1}{3} (x^2 + 1)^{3/2} + C.$$

Section 4.5 — Problem 36 — 5 points

Set $u = 3t - 1$ so that $\frac{1}{3}du = dx$ and

$$x = 0 \rightarrow u = -1 \quad \text{and} \quad x = 1 \rightarrow u = 2.$$

After a u -sub, the integral becomes

$$\int_{-1}^2 u^{50} \frac{du}{3} = \frac{1}{3} \left(\frac{u^{51}}{51} \right) \Big|_{-1}^2 = \frac{2^{51} + 1}{153}.$$

Section 4.5 — Problem 38 — 5 points

Let $u = x^2$, so that $\frac{1}{2}du = x dx$. The integral then becomes

$$\int_0^{\sqrt{\pi}} x \cos(x^2) dx = \frac{1}{2} \int_0^{\pi} \cos(u) du = \sin u \Big|_0^{\pi} = 0.$$

Section 4.5 — Problem 46 — Bonus points

The function $f(x) = x^4 \sin x$ is an odd function since

$$f(-x) = (-x)^4 \sin(-x) = -x^4 \sin(x) = f(x).$$

We divide the integral in two

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x \, dx = \int_{-\pi/3}^0 x^4 \sin x \, dx + \int_0^{\pi/3} x^4 \sin x \, dx.$$

The first integral in the RHS, let $u = -x$, so that $-du = dx$ and therefore

$$\int_{-\pi/3}^0 x^4 \sin x \, dx = - \int_{\pi/3}^0 (-u)^4 \sin(-u) \, du.$$

Using the property of the integral:

$$- \int_{\pi/3}^0 (-u)^4 \sin(-u) \, du = \int_0^{\pi/3} (-u)^4 \sin(-u) \, du.$$

Since $f(-u) = -f(u)$, we get that

$$\int_0^{\pi/3} (-u)^4 \sin(-u) \, du = - \int_0^{\pi/3} u^4 \sin u \, du.$$

Recall that the value of the integral does not depend on the name of the variable, therefore, we have

$$\int_0^{\pi/3} u^4 \sin u \, du = \int_0^{\pi/3} x^4 \sin x \, dx$$

and so

$$\int_{-\pi/3}^{\pi/3} x^4 \sin(x) \, dx = - \int_0^{\pi/3} x^4 \sin x \, dx + \int_0^{\pi/3} x^4 \sin x \, dx = 0.$$

TOTAL (POINTS): 50.