

# MATH 241

## CHAPTER 5

### SECTION 5.1: AREA BETWEEN CURVES

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# NON INTERSECTING REGIONS

Desmos: <https://www.desmos.com/calculator/o7vvfgfwzy>

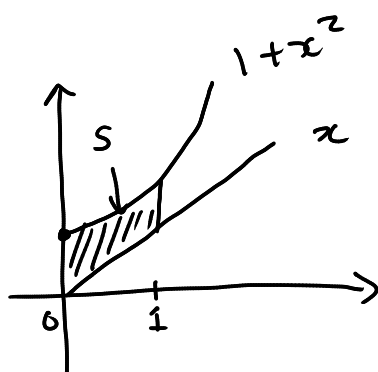
Given two functions  $f(x)$  and  $g(x)$  such that

$$g(x) \leq f(x) \quad a \leq x \leq b,$$

the area of the region  $S$  enclosed by  $f(x)$ ,  $g(x)$ ,  $x = a$  and  $x = b$  is

$$\text{AREA}(S) = \int_a^b f(x) - g(x) dx.$$

**EXAMPLE 1.** Find the area of the region bounded above by  $y = x^2 + 1$ , bounded below by  $y = x$ , and bounded on the sides by  $x = 0$  and  $x = 1$ .



$$\text{Area}(S) = \int_a^b f(x) - g(x) dx$$

$$= \int_0^1 x^2 + 1 - x dx$$

$$= \left( \frac{x^3}{3} + x - \frac{x^2}{2} \right) \Big|_0^1$$

$$= \frac{1}{3} + 1 - \frac{1}{2} - \left( \frac{0^3}{3} + 0 - \frac{0^2}{2} \right)$$

$$= \boxed{\frac{5}{6}}$$

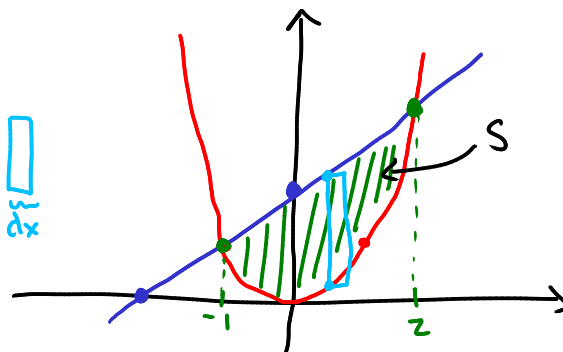
## INTERSECTING REGIONS

**EXAMPLE 2.** Find the area of the region enclosed by the functions  $y = x^2$  and  $y = x + 2$ .

① Sketch & Intersects

$$\begin{aligned} x^2 &= x + 2 \\ \Rightarrow x^2 - x - 2 &= 0 \\ \Rightarrow (x+1)(x-2) &= 0 \\ \Rightarrow x &= -1 \text{ or } x = 2 \end{aligned}$$

$$x+2 - x^2 \quad \left\{ \int dx \right.$$



② Integrate

$$\begin{aligned} \text{Area}(S) &= \int_{-1}^2 (x+2 - x^2) dx \\ &= \left( \frac{x^2}{2} + 2x - \frac{x^3}{3} \right) \Big|_{-1}^2 \\ &= \left( \frac{4}{2} + 4 - \frac{8}{3} \right) - \left( \frac{1}{2} - 2 + \frac{1}{3} \right) \\ &= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} \\ &= 8 - 3 - \frac{1}{2} \\ &= \boxed{\frac{9}{2}} \end{aligned}$$

General Steps:

1. Find the intersection between the two curves.
2. Draw a picture.
3. Set up the definite integral.
4. Evaluate the definite integral.

**EXAMPLE 3.** Find the area of the region enclosed by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ .



**EXAMPLE 4.** Find the area enclosed by the line  $y = x - 1$  and the parabola  $y^2 = 2x + 6$ .