Section 1.5, Problem 4

a) 3.

b) 1.

c) Doesn't exists because the left-hand side limit is not equal to the right-hand side limit.

d) 3.

e) 4.

f) Doesn't exists because the function is not defined at this point (there is an empty dot on the graph of f).

Section 1.5, Problem 16

An expression for the function can be

$$f(x) = \begin{cases} -x+1 & \text{if } x < 0 \text{ and } 0 < x < 3 \\ 3x-1 & \text{if } x > 3 \\ -1 & x = -1 \\ 1 & x = 3. \end{cases}$$

Section 1.5, Problem 30

We see that the limit on the numerator is 6 and the limit of the denominator is 0^- meaning that the value approaches 0 from the left. So we have a number divided by a very small negative quantity. We then get

$$\lim_{x \to 5^-} \frac{x+1}{x-5} = 6/0^- = -\infty.$$

Section 1.5, Problem 38

We will use a table. We can also simply use the limit rules. The limit on the numerator is $4-4=0$. The limit on the denominator is $4-8+4=0$. So we have a $0/0$, which is problematic!

We will instead use a table. Let $f(x) = (x^2-2x)/(x^3-4x+4)$.

x	$f(x)$
1.0	-1.0
1.5	-3
1.9	-19
1.99	199
1.999	-1999
1.9999	-19999

We see clearly from the table that $\lim_{x\to 2^-} f(x) = -\infty$.

Section 1.6, Problem 8

Let's call L the limit. We have

$$\begin{aligned}
L &= \left(\lim_{t\to 2} \frac{t^2-2}{t^3-3t+5}\right)^2 && \text{(Power Rule)}\\
&= \left(\frac{\lim_{t\to 2} t^2-2}{\lim_{t\to 2} t^3-3t+5}\right)^2 && \text{(Quotient Rule)}\\
&= \left(\frac{\lim_{t\to 2} t^2-\lim_{t\to 2} 2}{\lim_{t\to 2} t^3-\lim_{t\to 2} 3t+\lim_{t\to 2} 5}\right)^2 && \text{(Sum \& Difference Rules)}\\
&= \left(\frac{(\lim_{t\to 2} t)^2-\lim_{t\to 2} 2}{(\lim_{t\to 2} t)^3-3\lim_{t\to 2} t+\lim_{t\to 2} 5}\right)^2 && \text{(Product \& Power rules)}\\
&= \left(\frac{2^2-2}{2^3-6+5}\right)^2 = \frac{4}{49}.
\end{aligned}$$

So the limit is $L = 4/49$.

Section 1.6, Problem 26

We have

$$\frac{1}{t} - \frac{1}{t^2+t} = \frac{1}{t} - \frac{1}{(t+1)t} = \frac{t+1-1}{t(t+1)} = \frac{1}{t+1}.$$

So the limit is

$$\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim_{t\to 0}\frac{1}{1+t} = 1.$$

Section 1.6, Problem 60

a) Since $\lim_{x\to 0} x^2$ exists and $\lim_{x\to 0} f(x)/x^2$ also exists, from the properties of the limits, we have

$$\left(\lim_{x\to 0} x^2\right)\left(\lim_{x\to 0}\frac{f(x)}{x^2}\right) = \lim_{x\to 0} x^2\left(\frac{f(x)}{x^2}\right) = \lim_{x\to 0} f(x).$$

But $\lim_{x\to 0} x^2 = 0$ and $\lim_{x\to 0} f(x)/x^2 = 5$, we get $\lim_{x\to 0} f(x) = 0 \times 5 = 0$.

b) We use the same strategy. Since $\lim_{x\to 0} x$ exists and $\lim_{x\to 0} f(x)/x^2$ also exists, we get

$$\left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0} \frac{f(x)}{x^2}\right) = \lim_{x\to 0} x\left(\frac{f(x)}{x^2}\right) = \lim_{x\to 0} \frac{f(x)}{x}.$$

But $\lim_{x\to 0} x = 0$ and $\lim_{x\to 0} f(x)/x = 5$, we get $\lim_{x\to 0} f(x)/x = 0 \times 5 = 0$.