

Chapter 1

Functions and Limits

1.4 The Tangent and Velocity Problems

The Tangent problem.

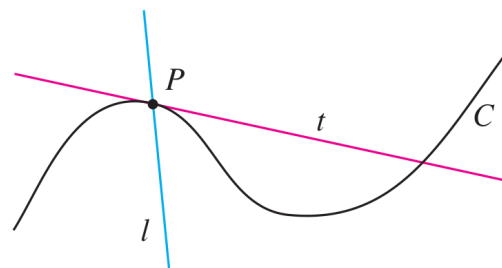
Example. What is the tangent to a circle?

Illustration: <https://www.desmos.com/calculator/itwxbbdwoe>

In Geometry, a **TANGENT LINE** at a given point on a circle is a line that touches the circle only at that point.

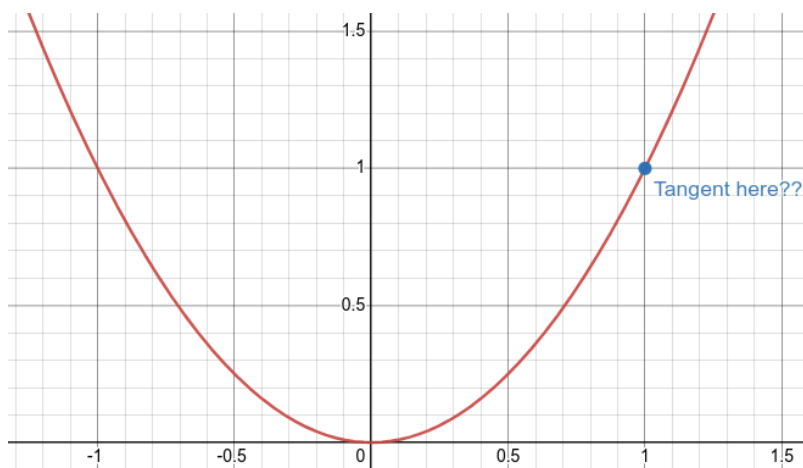
Problems with this definition:

- 1) Not all curves are circle!
- 2) For other curves, the tangent line may intersect at several points!



EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

Go play around with this problem: <https://www.desmos.com/calculator/5eyhh9tfkg>



Main concept: The SLOPE of the tangent line is the LIMIT of the slopes of the secant lines.

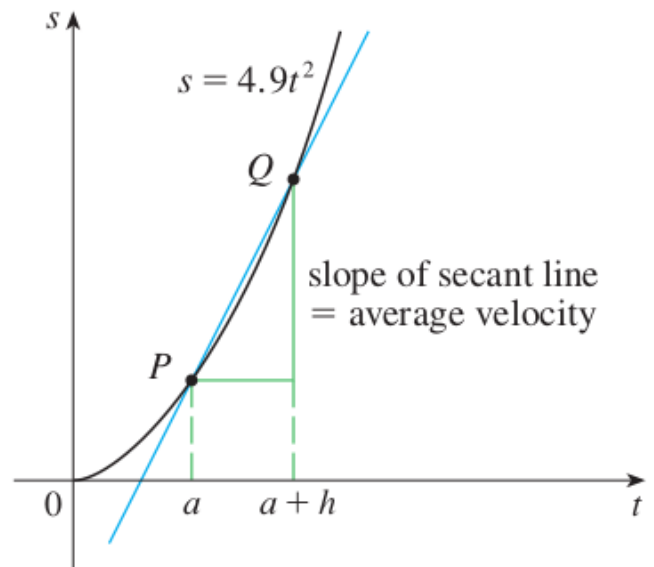
The Velocity Problem.

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.

$$\text{Galileo: } s(t) = 4.9t^2$$

Average velocity.

Relation to the secant line.



Instantaneous Velocity.

Relation to the tangent line.

