University of Hawai'i

MATH-241 Calculus I
Midterm 02

Created by Pierre-O. Parisé
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Last name: ______________________________
First name: ______________________________

Question:	1	2	3	4	Total
Points:	10	10	20	10	50
Score:					

Instructions: Make sure to write your complete name on your copy. You must answer all the questions below and write your answers directly on the questionnaire. At the end of the 50 minutes, hand out your copy.

No devises such as a smart phone, cell phone, laptop, or tablet can be used during the exam. You are not allowed to use the lecture notes also.

Make sure to justify clearly your answers. An answer without justification worth no point.

Good luck!

Pierre-Olivier Parisé

Your Signature: ______________________________

QUESTION 1 **(10 pts)**

Use implicit differention to find the expression of y' if y and x are defined implicitly by

$$x^3 y + y^4 = 16.$$

Solution: By implicit differentiation, the left-hand side is

$$3x^2 y + x^3 y' + 4y^3 y'$$

and the right-hand side is 0. So, we get

$$3x^2 y + (x^3 + 4y^3)y' = 0$$

which gives $y' = \frac{-3x^2 y}{x^3+4y^3}$.

Question 2 **(10 pts)**

Suppose $y = \sqrt{2x+1}$ where $x = x(t)$ and $y = y(t)$ are functions of t.

(a) (5 points) If $dx/dt = 3$, find dy/dt when $x = 4$.

> **Solution:** We have $dy/dt = \frac{dx/dt}{\sqrt{2x+1}}$. So, pluging in $x = 4$ and $dx/dt = 3$, we find $dy/dt = 1$.

(b) (5 points) If $dy/dt = 5$, find dx/dt when $x = 12$.

> **Solution:** From the previous formula, we get $dx/dt = (dy/dt)\sqrt{2x+1}$. Pluging in $x = 12$ and $dy/dt = 5$, we get $dy/dt = 25$.

Question 3 **(20 pts)**

The following table shows the signs of the derivative and the second derivative of the function $f(x) = \frac{1}{x^2+2x-3}$. (You don't have to verify the signs, I did it for you!)

x		-3		-1		1	
$f'(x)$	$+$	$\nexists$	$+$	0	$-$	$\nexists$	$-$
$f''(x)$	$+$	$\nexists$	$-$	$-$	$-$	$\nexists$	$+$

(a) (5 points) Find where the function is increasing and decreasing.

Solution: From the table, we see that $f'(x) > 0$ when x belongs to the intervals $(-\infty, -3)$ and $(-3, -1)$. So f is increasing there.

Also form the table, we see that $f'(x) < 0$ when x belongs to the intervals $(-1, 1)$ and $(1, \infty)$. So f is decreasing there.

(b) (5 points) Find where the function is concave upward and concave downward.

Solution: From the table, we see that $f''(x) > 0$ when x belongs to the intervals $(-\infty, -3)$ and $(1, \infty)$. So f is concave upward there.

Also from the table, we see that $f''(x) < 0$ when x belongs to the interval $(-3, 1)$. So f is concave downward there.

(c) (5 points) Find the critical points of the function. Find the local extremum(s) of the function.

Solution: The critical points are where $f'(x) = 0$ or $f'(x)$ doesn't exist. So from the table, this occurs when $x = -3$, $x = -1$, and $x = 1$. At the points $x = -3$ and $x = 1$, we get the values $+\infty$ and $-\infty$ respectively. So these can't be maximum values.

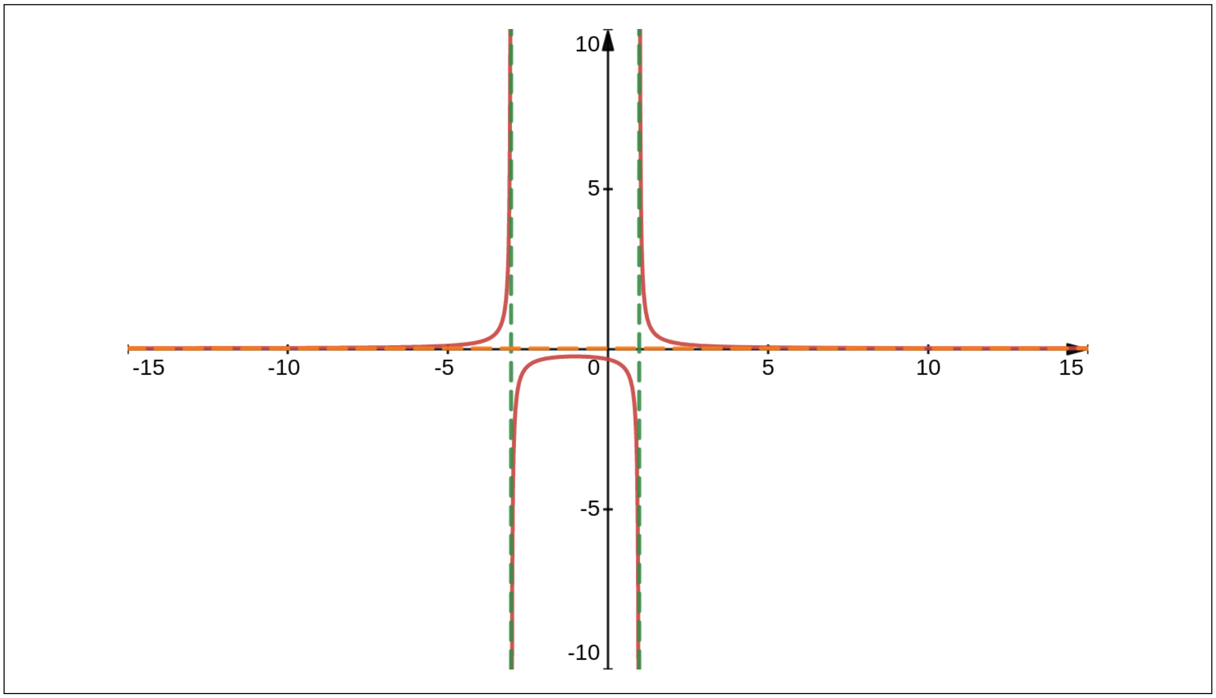
At $x = -1$, we see that f goes from increasing to decreasing. So $f(-1) = -1/4$ is a local maximum.

(d) (5 points) Find the Vertical asymptotes, the horizontal asymptotes and sketch the graph of the function.

Solution: The vertical asymptotes are $x = -3$ and $x = 1$ because $\lim_{x\to -3^{\pm}} f(x) = \pm\infty$ and $\lim_{x\to 1^{\pm}} f(x) = \mp\infty$.

The horizontal asymptote is $y = 0$. Because $\lim_{x\to\pm\infty} f(x) = 0$.

The graph of the function looks like this.


10
5
-5
-10
-15
-10
-5
0
5
10
15

Question 4 (10 pts)

A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

Solution: Let x denote the length of the side of the squares cut from the cardboard. The height of the box is then x and the length of the sides is $3-2x$. So the volume of the box is

$$V(x) = x(3-2x)^2.$$

Here, x is at most $3/2$. So, the domain of V is $[0, 3/2]$.

We take the derivative of V. We find $V'(x) = (3-2x)^2 - 4x(3-2x)$ which is, after simplification, equalled to $V'(x) = 9 - 24x + 12x^2$. We can rewrite $V'(x)$ as $V'(x) = 3(3-2x)(1-2x)$. From there, we see that $V'(x) = 0$ if $x = 3/2$ or $x = 1/2$.

We now analyse the sign of $V'(x)$ around $x = 1/2$. We have that $V'(x) > 0$ if $x < 1/2$ and $V'(x) < 0$ if $x > 1/2$. Thus, $V(1/2)$ is a local maximum.

Finally, we have $V(0) = 0$, $V(1/2) = 2$, and $V(3/2) = 0$. Thus, the largest volume of the box is $2\,\mathrm{ft}^2$.