Chapter 4
Integrals

4.5 The Substitution Rule

Example to start. Find the indefinite integral of $2x\sqrt{1+x^2}$, that is compute

$$\int 2x\sqrt{1+x^2}\,dx.$$

Another example. Compute the indefinite integral

$$\int x\sqrt{1+x^2}\,dx.$$

> **Substitution Rule.** If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then
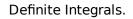> $$\int f'(g(x))g'(x)\,dx = \int f(u)\,du.$$

Relation between du and dx:

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EXAMPLE 1 Find $\displaystyle\int x^3 \cos(x^4 + 2)\,dx$.

EXAMPLE 2 Evaluate $\int \sqrt{2x + 1} \, dx$.

EXAMPLE 3 Find $\int \dfrac{x}{\sqrt{1 - 4x^2}} \, dx$.

EXAMPLE 5 Find $\displaystyle\int \sqrt{1 + x^2}\; x^5 \, dx.$

EXAMPLE 7 Evaluate $\displaystyle\int_1^2 \frac{dx}{(3 - 5x)^2}$.