

### Example 1

Find the general indefinite integral of

$$\int (10x^4 - 2 \sec^2 x) dx.$$

$$\begin{aligned}\int 10x^4 - 2 \sec^2 x \, dx &= \int 10x^4 \, dx + \int (-2 \sec^2 x) \, dx \\&= 10 \int x^4 \, dx - 2 \int \sec^2 x \, dx \\&= 10 \frac{x^5}{5} + \underbrace{C_1} - 2 \tan x + \underbrace{C_2} \\&= 10 \frac{x^5}{5} - 2 \tan x + C \\&\quad \text{where } C = C_1 + C_2.\end{aligned}$$

$$\boxed{\int 10x^4 - 2 \sec^2 x \, dx = 2x^5 - 2 \tan x + C}$$

## Example 2

Evaluate the indefinite integral  $\int \frac{\cos \theta}{\sin^2 \theta} d\theta$ .

$$\begin{array}{cc} \cot \theta & \csc \theta \\ \uparrow & \uparrow \\ \frac{\cos \theta}{\sin \theta} & \cdot \frac{1}{\sin \theta} \end{array}$$

$$\int \frac{\cos \theta}{\sin^2 \theta} d\theta = \int \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta} d\theta = \int \cot \theta \cdot \csc \theta d\theta$$

$$\boxed{= -\csc \theta + C}$$

### Example 3

$\downarrow$ 
 $t^2 \cdot t^{1/2} = t^{2+1/2} = t^{5/2}$ 
 $\frac{t^{5/2}}{t^2} = t^{5/2-2}$

Evaluate  $\int \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt.$

$$\begin{aligned}
 \int \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt &= \int \frac{2\cancel{t^2}}{\cancel{t^2}} + \frac{\cancel{t^2}\sqrt{t}}{\cancel{t^2}} - \frac{1}{t^2} dt \\
 &= \int 2 + \sqrt{t} - \frac{1}{t^2} dt \\
 &= 2 \int 1 dt + \int \underbrace{\sqrt{t}}_{t^{1/2}} dt - \int \frac{1}{t^2} dt \quad \rightarrow t^{-2} \\
 &= 2t + C_1 + \frac{t^{3/2} \times C_2}{3/2} - \frac{t^{-1} \times C_3}{-1} \\
 &= 2t + \frac{2}{3} t^{3/2} + \frac{1}{t} + \underbrace{C_1 + C_2 + C_3}_{=C} \\
 &= 2t + \frac{2}{3} t^{3/2} + \frac{1}{t} + C
 \end{aligned}$$

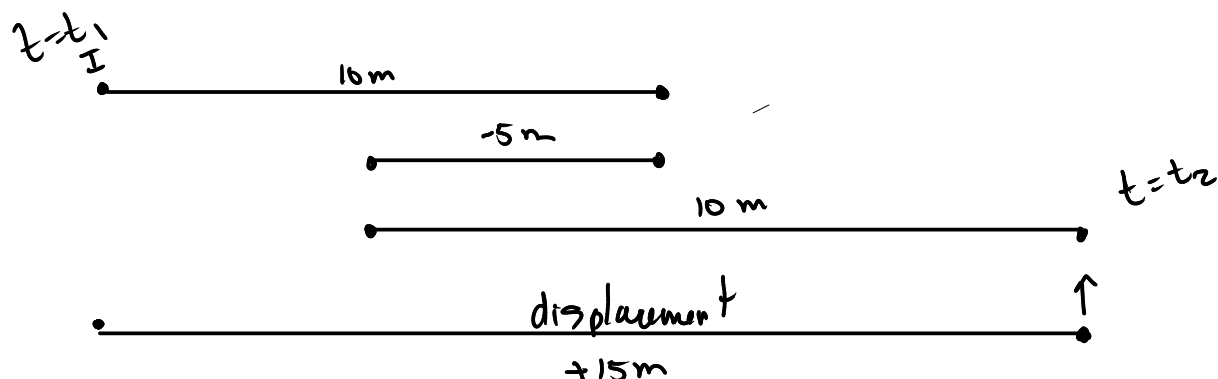
### Example 4

A particle moves along a line so that its velocity at time  $t$  is  $v(t) = t^2 - t - 6$  (measured in meters per second).

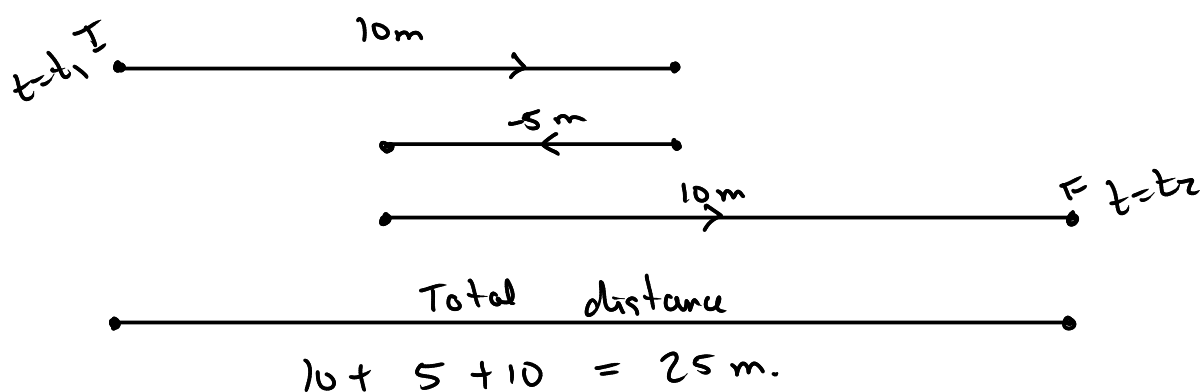
- a) Find the displacement of the particle during the time period  $1 \leq t \leq 4$ .
- b) Find the distance traveled during this time period.

two concepts.

① Displacement =  $\int_{t_1}^{t_2} v(t) dt$



② Distance =  $\int_{t_1}^{t_2} |v(t)| dt$



a) Displacement =  $\int_1^4 t^2 - t - 6 dt = \left( \frac{t^3}{3} - \frac{t^2}{2} - 6t \right) \Big|_1^4$

$$= \frac{4^3}{3} - \frac{4^2}{2} - 6 \cdot 4 - \left( \frac{1}{3} - \frac{1}{2} - 6 \right)$$
$$= -4.5 \text{ m.}$$

b) Total distance =  $\int_1^4 |t^2 - t - 6| dt$

We have to find an explicit expression for  $|t^2 - t - 6|$ :

$$|t^2 - t - 6| = \begin{cases} t^2 - t - 6 & \text{if } t^2 - t - 6 \geq 0 \\ -(t^2 - t - 6) & \text{if } t^2 - t - 6 < 0. \end{cases}$$

When is  $v(t) \geq 0$

$$t^2 - t - 6 \geq 0 \quad \text{if} \quad (t-6)(t+1) \geq 0$$

$$\text{if} \quad t-6 \geq 0 \quad \text{and} \quad t+1 \geq 0$$

$$\text{or} \quad t-6 \leq 0 \quad \text{and} \quad t+1 \leq 0$$

$$\text{if} \quad t \geq 6 \quad \text{and} \quad t \geq -1$$

or

$$t \leq 6 \quad \text{and} \quad t \leq -1$$

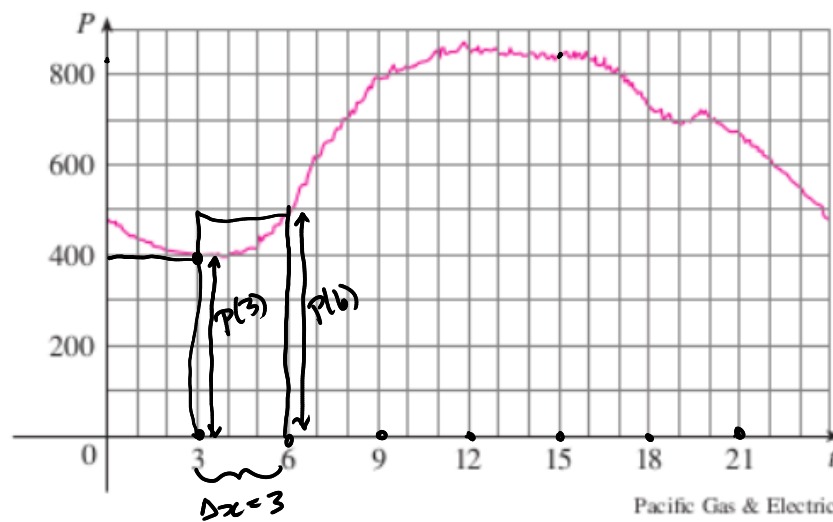
$$\text{if} \quad \boxed{t \geq 6 \quad \text{or} \quad t \leq -1}$$

Since  $1 \leq t \leq 4$ , then  $v(t) \leq 0$

$$\begin{aligned} \text{So, } \int_1^4 |v(t)| dt &= \int_1^4 |t^2 - t - 6| dt = \int_1^4 -(t^2 - t - 6) dt \\ &= -\int_1^4 t^2 - t - 6 dt \\ &= -(-4.5) \quad \boxed{= 4.5 \text{ m.}} \end{aligned}$$

### Example 5

The figure shows the power consumption in the city of San Francisco for a day in September ( $P$  is measured in megawatts;  $t$  is measured in hours starting at midnight). Estimate the energy used on that day.



Power :  $P = \frac{E_f - E_i}{\Delta t}$  (average).  $\Delta t \rightarrow 0 \rightarrow P = \frac{dE}{dt}$ .

So, to obtain the net energy used in a day,

$$\int_0^{24} \frac{dE}{dt} dt = E(24) - E(0)$$

$$\Rightarrow \int_0^{24} P(t) dt = E(24) - E(0).$$

Here, we don't have an expression for  $E$  or  $P$ , but we have a table (graph) of the values of  $P$ .

$$\int_0^{24} P(t) dt \approx \sum_{i=1}^8 P(0 + i \Delta t) \Delta t$$

$$\Delta t = 3$$

$$= P(3) \cdot 3 + P(6) \cdot 3 + P(9) \cdot 3 + P(12) \cdot 3 \\ + P(15) \cdot 3 + P(18) \cdot 3 + P(21) \cdot 3 + P(24) \cdot 3$$

$$= 15735 \text{ MW} \cdot \text{h}$$

### Example 6

Find the indefinite integral  $\int \underbrace{2x\sqrt{1+x^2}} dx$ .

$$\int \sqrt{u} du = \frac{u^{3/2}}{3/2} + C = \frac{2}{3} u^{3/2} + C$$

$$u = 1 + x^2 \rightarrow \left( \frac{2}{3} (1+x^2)^{3/2} + C \right)' = \frac{2}{3} \cdot \frac{3}{2} (1+x^2)^{1/2} \cdot (1+x^2)' + \cancel{C}'$$
$$= \underbrace{(1+x^2)^{1/2}} \cdot 2x$$

$$\text{So, } \int 2x \sqrt{1+x^2} dx = \frac{2}{3} (1+x^2)^{3/2} + C.$$

### Example 7

Find the indefinite integrals:

a)  $\int \underline{x^3} \underbrace{\cos(x^4 + 2)} \underline{dx}$ .

b)  $\int \sqrt{2x+1} dx$ .

c)  $\int \sqrt{1+x^2} x^5 dx$ .

a)  $f(x) = \cos x$

$u = g(x) = x^4 + 2$

$$\rightarrow \frac{du}{dx} = 4x^3 \rightarrow du = 4x^3 dx$$

$$\rightarrow \frac{du}{4} = \underline{x^3 dx}$$

$$\begin{aligned} \text{So, } \int x^3 \cos(x^4 + 2) dx &= \int \underbrace{\cos(x^4 + 2)}_{\downarrow} \underbrace{x^3 dx}_{\downarrow} \\ &= \int \cos(u) \frac{du}{4} \\ &= \frac{\sin(u)}{4} + C \end{aligned}$$

$$\boxed{= \frac{\sin(x^4 + 2)}{4} + C}$$

b)  $f(x) = \sqrt{x}$

$u = g(x) = 2x+1$

$$\rightarrow \frac{du}{dx} = 2 \rightarrow du = 2dx \rightarrow \frac{du}{2} = dx$$

$$\begin{aligned} \int \sqrt{2x+1} dx &= \int \sqrt{u} \frac{du}{2} = \frac{1}{2} \frac{u^{3/2}}{3/2} + C \\ &= \frac{u^{3/2}}{3} + C \end{aligned}$$

$$\boxed{= \frac{(2x+1)^{3/2}}{3} + C}$$



$$c) f(x) = \sqrt{x}$$

$$u = g(x) = \underline{1+x^2} \rightarrow \frac{du}{dx} = 2x \rightarrow du = 2x dx \rightarrow \frac{du}{2} = x dx$$

$$\int \sqrt{1+x^2} x^5 dx = \int \sqrt{1+x^2} \underbrace{x^4}_{(x^2)^2} \cdot \underbrace{x dx}_{du/2}$$

$$\text{Here, } u = 1+x^2 \rightarrow \underline{u-1} = x^2$$

$$= \int \sqrt{u} (u-1)^2 \frac{du}{2}$$

$$= \int u^{1/2} (u^2 - 2u + 1) \frac{du}{2}$$

$$= \frac{1}{2} \int u^{5/2} - 2u^{3/2} + u^{1/2} du$$

$$= \frac{1}{2} \left( \frac{u^{7/2}}{7/2} - 2 \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} \right) + C$$

$$= \frac{u^{7/2}}{7} - \frac{2}{5} u^{5/2} + \frac{u^{3/2}}{3} + C.$$

### Example 9

Compute the value of  $\int_0^4 \sqrt{2x+1} \, dx$ .

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### Example 10

Compute the value of the definite integrals.

a)  $\int_0^4 \sqrt{2x+1} \, dx.$

b)  $\int_1^2 \frac{dx}{(3-5x)^2}.$

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### Example 11

Compute the value of

- $\int_{-1}^1 x^2 dx.$

- $\int_{-1}^1 x^3 dx.$

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