Section 5.1 — Problem 8 — 10 points

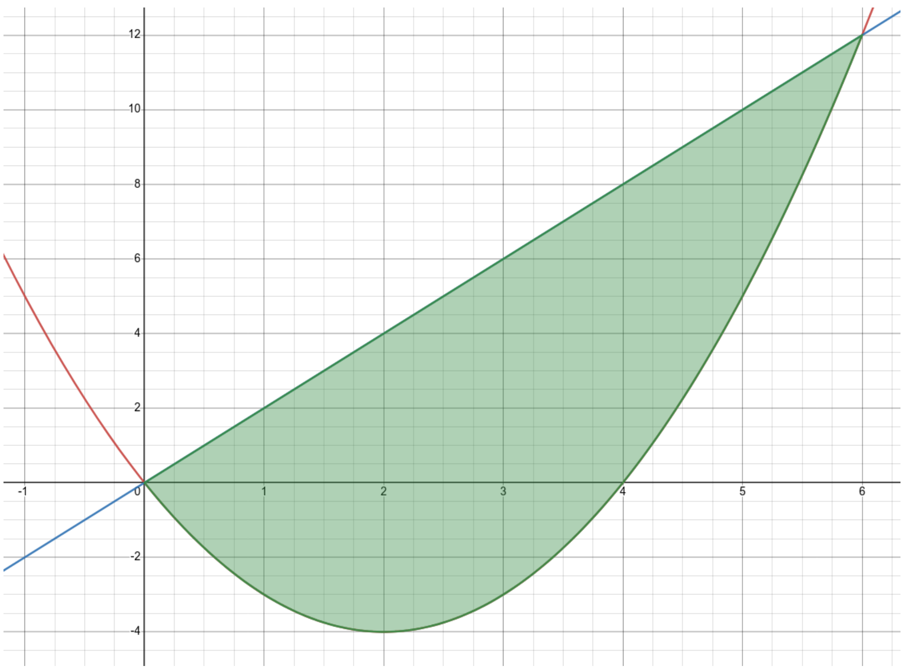
The intersections between $x^2 - 4x$ and $2x$ is given by the solutions to

$$x^2 - 4x = 2x \iff x^2 - 6x = 0 \iff x = 6 \text{ or } x = 0.$$

To have

$$x^2 - 4x \leq 6x \iff x(x-6) \leq 0$$

the value of x should be between 0 and 6 ($0 \leq x \leq 6$). Therefore, the region is enclosed by the curve $6x$ (top/ceiling) and $x^2 - 4x$ (bottom/floor) from $x = 0$ to $x = 6$.



Therefore, the area of the region is given by

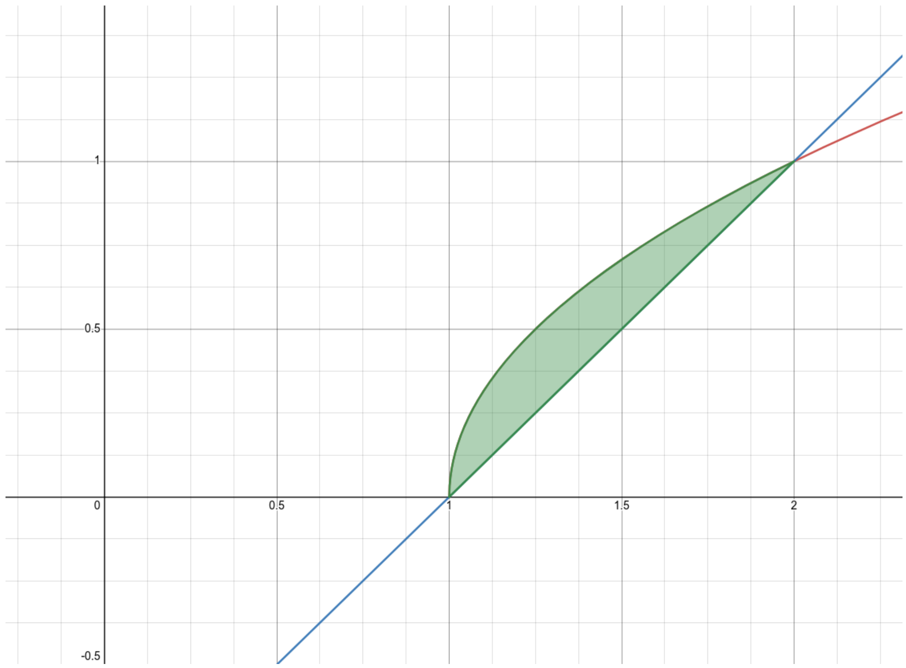
$$\int_0^6 2x - (x^2 - 4x)\,dx = \int_0^6 6x - x^2\,dx = \left(3x^2 - \frac{x^3}{3}\right)\Big|_0^6 = 36.$$

Section 5.1 — Problem 18 — 10 points

The second curve is $y = x - 1$. The intersections between the two curves are

$$\sqrt{x-1} = x - 1 \iff x - 1 = (x-1)^2 \iff (x-2)(x-1) = 0$$

and therefore $x = 1$ or $x = 2$. Here is the region between the two curves.



We have $\sqrt{x-1} \geq x - 1$ for $1 \leq x \leq 2$. The area of the region is therefore

$$\int_1^2 \sqrt{x-1} - (x-1)\, dx = \int_1^2 \sqrt{x-1}\, dx - \int_1^2 x - 1\, dx$$

For the first integral, use a change of variable. Set $u = x - 1$, then $du = dx$ and

$$\int_1^2 \sqrt{x-1}\, dx = \int_0^1 \sqrt{u}\, du = \left(\frac{2}{3}u^{3/2}\right)\Big|_0^1 = \frac{2}{3}.$$

Also, we have

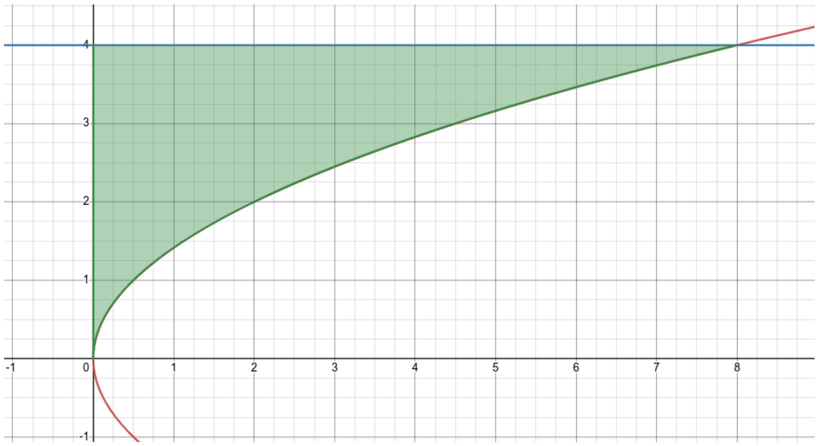
$$\int_1^2 (x-1)\, dx = \left(\frac{x^2}{2} - x\right)\Big|_1^2 = (2-2) - (1/2 - 1) = 1/2.$$

Therefore, the area is

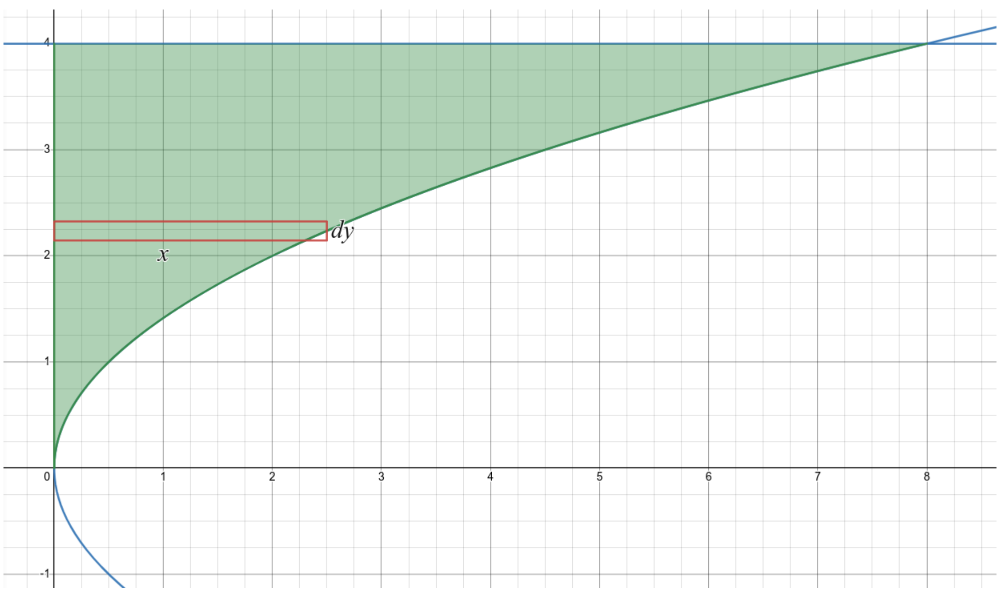
$$\int_1^2 \sqrt{x-1} - (x-1)\, dx = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$

Section 5.2 — Problem 6 — 15 points

The curve is a parabola, $x = y^2/2$. The y values are bounded by $y = 0$ (because $x = 0$ implies that $y = 0$) and $y = 4$. Therefore, the region is given by $0 \le x \le y^2/2$ and $0 \le y \le 4$.



The rotation is about the y-axis. We therefore draw a small horizontal rectangle with height dy and width x.



After rotation, the radius of the disk created is x and the height is dy. Therefore, the volume is

$$\int_0^4 \pi(\text{radius})^2 dy = \int_0^4 \pi x^2\, dy.$$

But now, $x = y^2/2$, and therefore

$$\int_0^4 \pi x^2\, dy = \int_0^4 \pi \frac{y^4}{4}\, dy = \pi \left(\frac{y^5}{20}\right)\Big|_0^4 = \frac{256\pi}{5}.$$

Section 5.3 — Problem 18 — 15 points

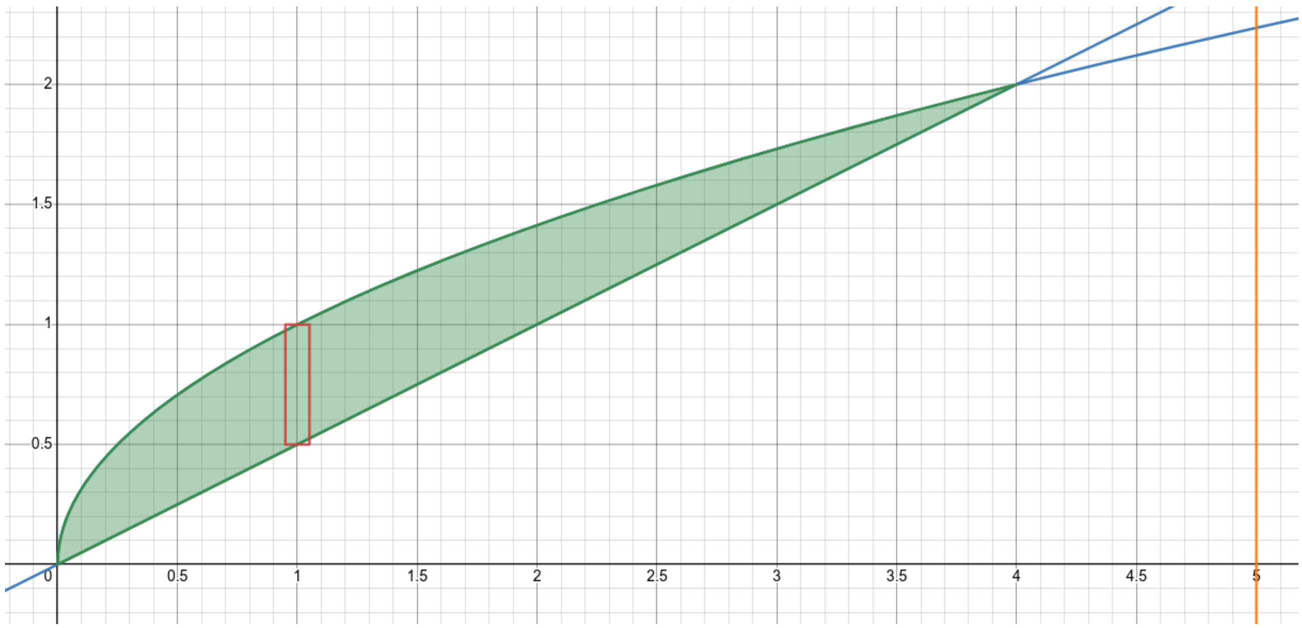
The region is bounded by the curves

$$y = \sqrt{x} \quad \text{and} \quad x = 2y.$$

Therefore, the curve meets when

$$\sqrt{x} = \frac{x}{2} \iff x = \frac{x^2}{4} \iff \frac{1}{4}x(x-4) = 0 \iff x = 0 \text{ or } x = 4.$$

A sketch of the region is presented below with a typical rectangle to generate the spherical shell:



After rotating about the line $x = 5$, we obtain a cylindrical shell with

- height: $\sqrt{x} - \frac{x}{2}$;
- radius: $5 - x$;
- thickness: dx.

Therefore, the volume is given by

$$\int_a^b 2\pi(\text{radius})(\text{height})\, dx = \int_0^4 2\pi(5-x)\Big(\sqrt{x} - \frac{x}{2}\Big)\, dx$$

The value of this integral is the volume of the solid of revolution. Therefore, the volume of the solid of revolution is $\frac{136}{15}\pi$.

TOTAL (POINTS): 50.