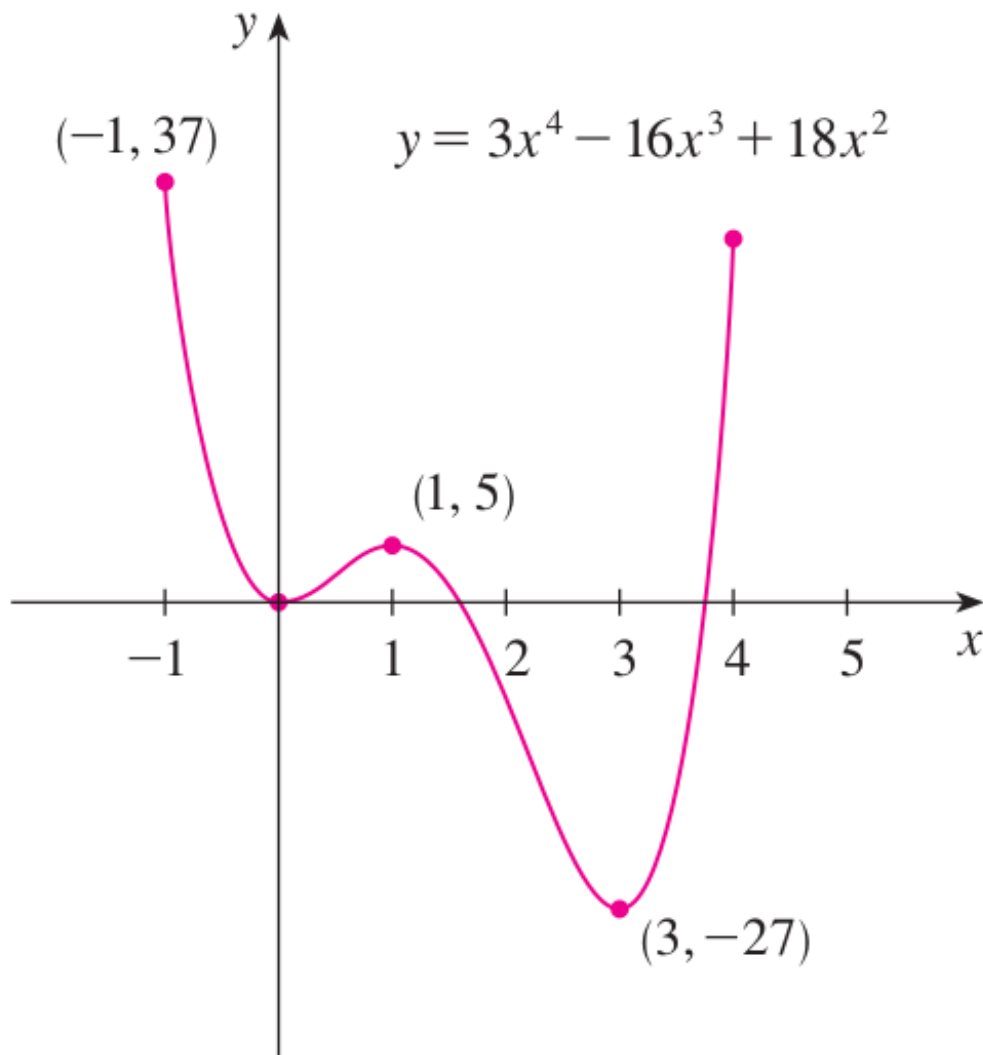


Chapter 3

Applications of Derivatives

3.1 Maximum and Minimum Values

What would be a maximum value or a minimum value of a function?



Suggestions/observations:

- 1)
- 2)
- 3)
- 4)

Important observations:

- a)
- b)

1 Definition Let c be a number in the domain D of a function f . Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geq f(x)$ for all x in D .
- **absolute minimum** value of f on D if $f(c) \leq f(x)$ for all x in D .

2 Definition The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geq f(x)$ when x is near c .
- **local minimum** value of f if $f(c) \leq f(x)$ when x is near c .

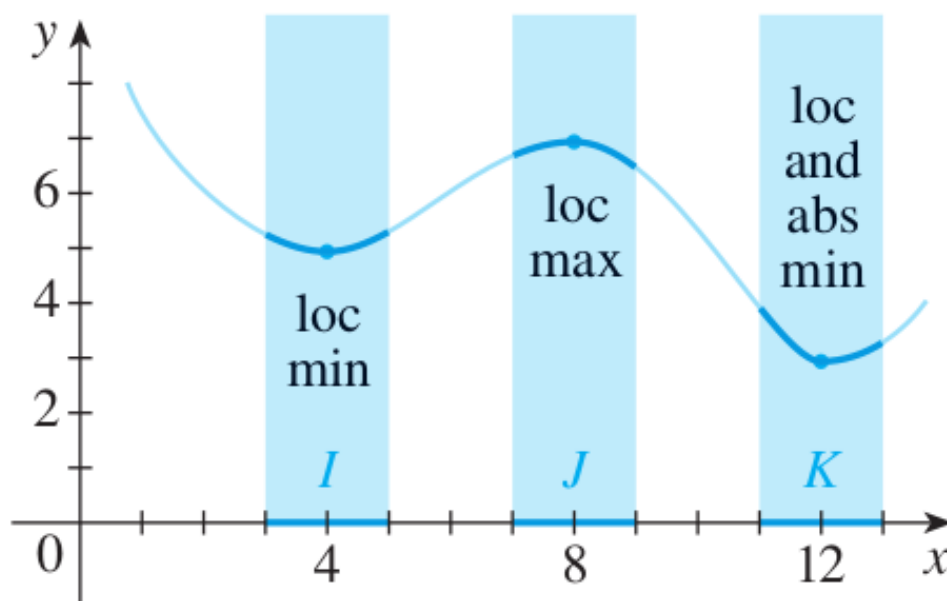


Illustration of the local and absolute max and min.

Terminology.

- 1) Global maximum or global minimum
- 2) Extreme values for abs. max. and abs. min.

Extreme Values Theorem.

Which conditions guarantee that extreme values exist?

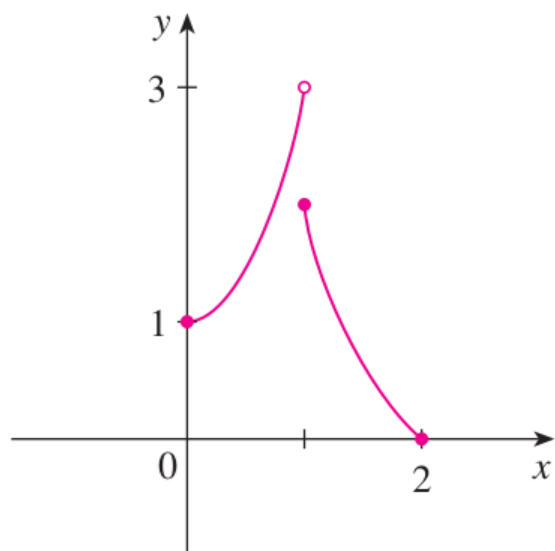


FIGURE 9

This function has minimum value $f(2) = 0$, but no maximum value.

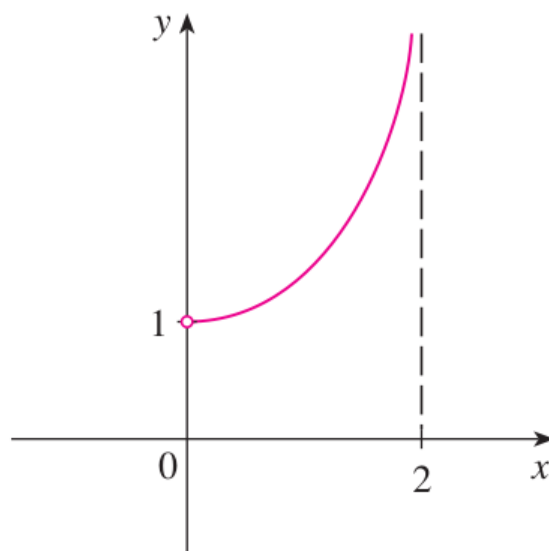
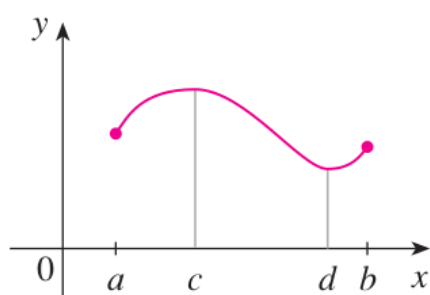


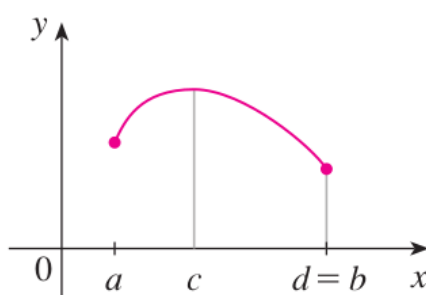
FIGURE 10

This continuous function g has no maximum or minimum.

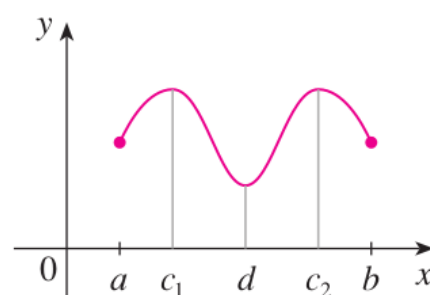
3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



attained inside



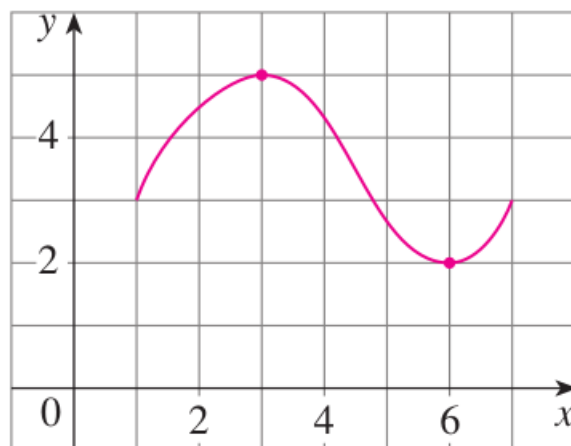
attained on the boundary



Attained multiple times

Fermat's Theorem.

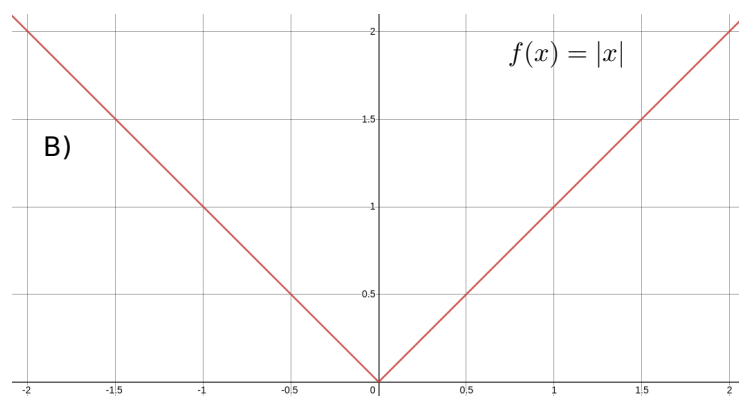
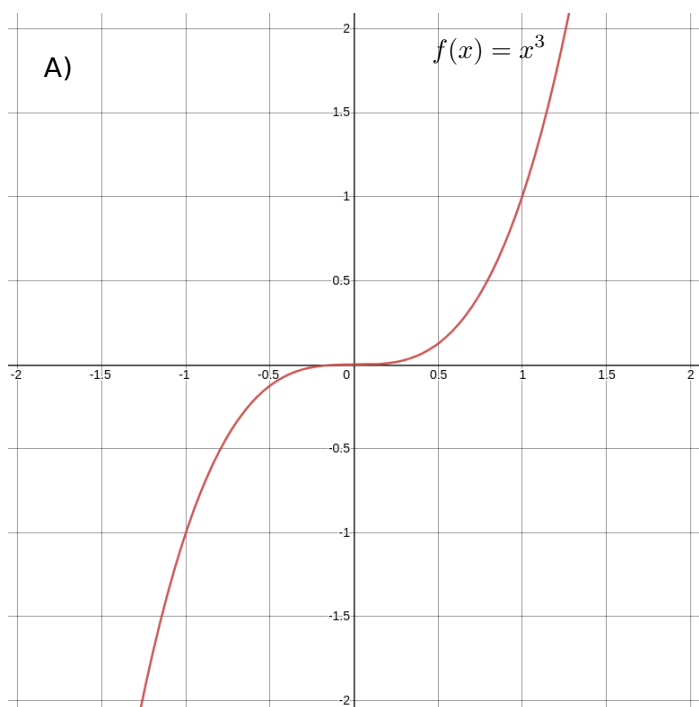
An observation:



4 Fermat's Theorem If f has a local maximum or minimum at c , and if $f'(c)$ exists, then $f'(c) = 0$.

Interested in the proof: see page 207 in the textbook.

BE CAREFUL!!



A)

B)

6 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

EXAMPLE 7 Find the critical numbers of $f(x) = x^{3/5}(4 - x)$.

Finding Extremum Values on closed intervals.

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b) .
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

EXAMPLE 8 Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4$$