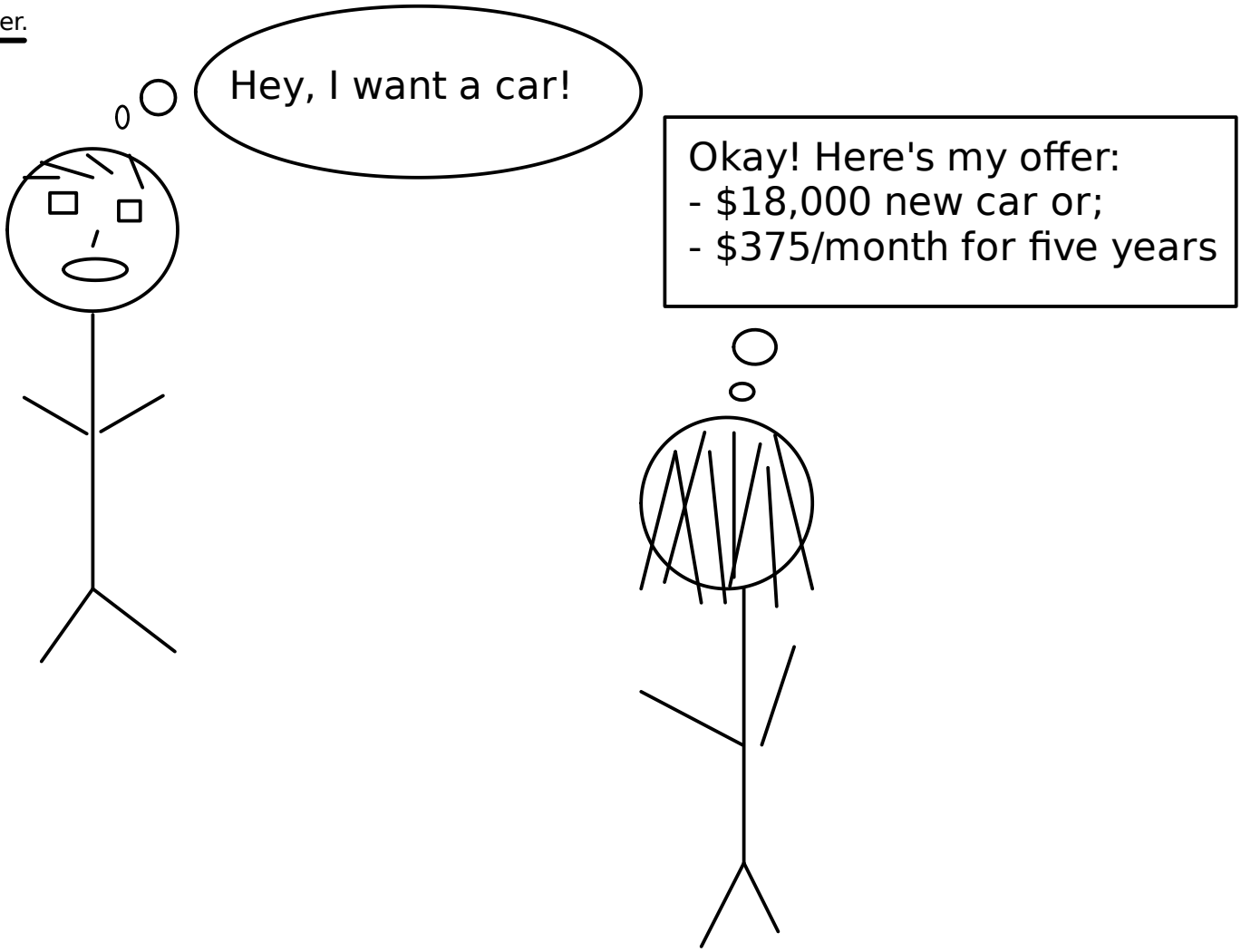
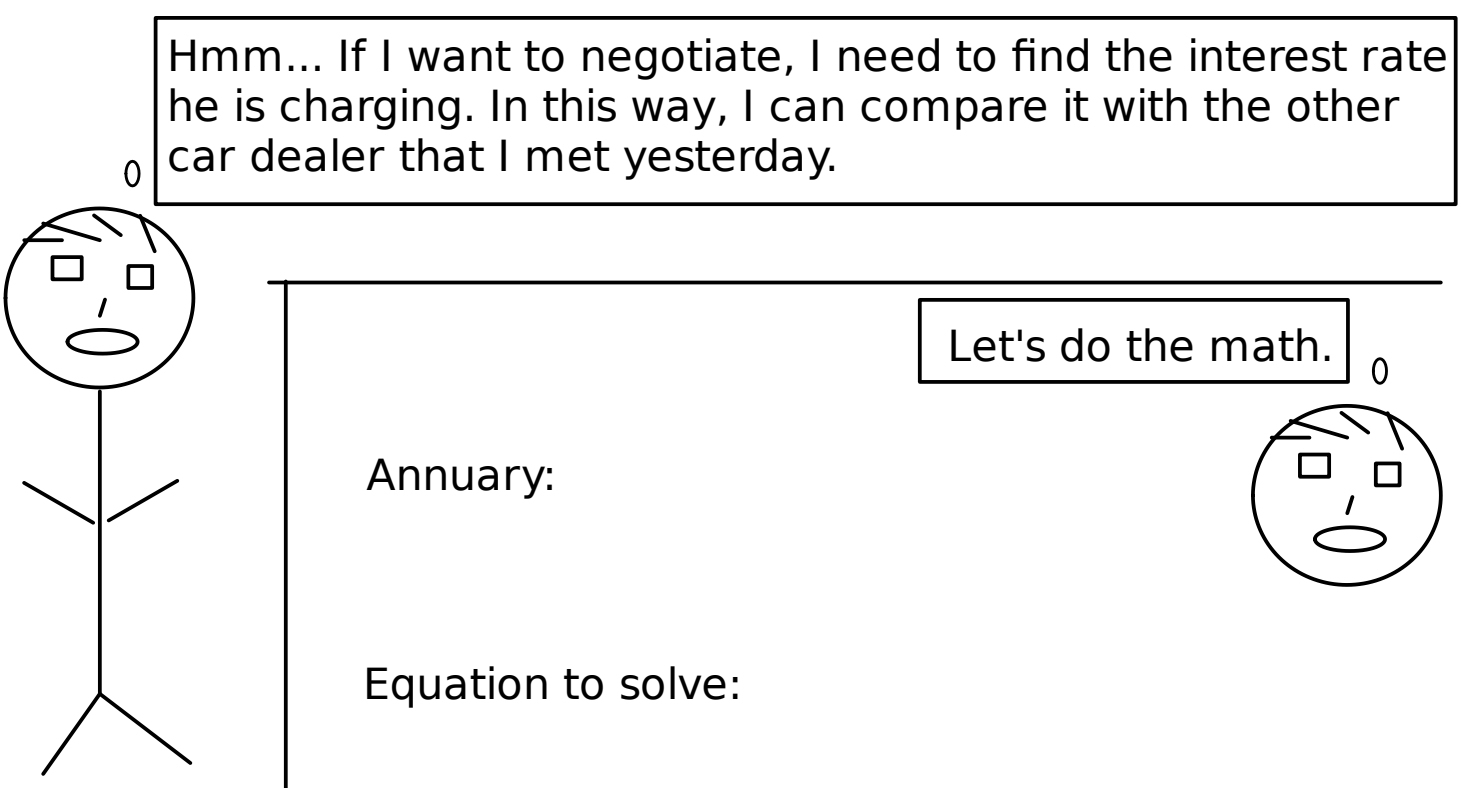
Chapter 3
Applications of Derivatives

3.8 Newton's Method

The car Dealer.





Roots of polynomials.

- for quadratic polynomial $f(x) = ax^2 + bx + c$, the roots are given by:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{a}$$

- There are formulas for cubics and quartics (horribly long...).

- For polynomials of degree greater than 4, there is no general formula!

Niels Henrik Abel

- 1802-1829
- Died from Turberculosis

Evariste Galois

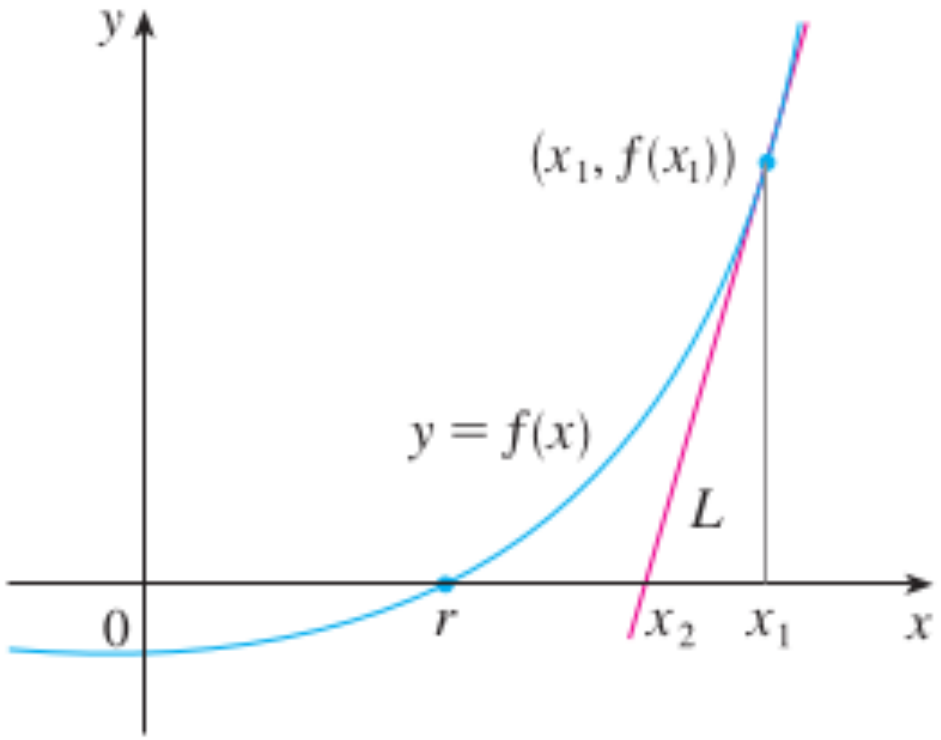
- 1811-1932
- Died in a duel for the mysterious mistress...

The urgent need of Newton's method!

KEY IDEAS:

- The tangent line approximate well the function.
- Replace the fonction with its tangent line.
- Intersect the tangent line with the x-axis.

Data:



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

EXAMPLE 1 Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$.

$MANY^{MANY}\,APPLICATIONS$!!!

- Finding solutions to general equations such as

$$\cos(x) = x$$

- At the core of many numerical methods in ingeenering.