# Chapter 2 Derivatives

2.3 Differentiation Formulas

### Constant Function.

$$f(x) = c \Rightarrow \lim_{h \to 0} \frac{f(x_0 + h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

**Derivative of a Constant Function** 

$$\frac{d}{dx}(c) = 0$$

$$n = 1.$$
  $f(x) = x$ 

$$\frac{x+h-x}{h} = :$$

$$\lim_{x \to \infty} \frac{1}{x^2 + x^2} = 1 \quad \Rightarrow \frac{d}{dx}(x) = 1$$

$$n=2.$$
  $f(x)=x^{7}$ 

$$\lim_{h\to 0} \frac{(2h^2)^2 - x^2}{h} = \lim_{h\to 0} \frac{2\pi x^2 + 4x^2}{h} = 2x$$

= 0

$$\Rightarrow \frac{d}{dx}(x^2) = 2x$$

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

**EXAMPLE**. Find the derivative of

$$y = x^4 - 6x^2 + 4.$$

## Multiplication by a constant.

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

## Sum.

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

# Difference.

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

## Product.

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Caution!!!

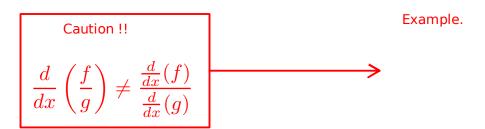
$$\frac{d}{dx}(fg) \neq \frac{d}{dx}(f)\frac{d}{dx}(g).$$

Example.

Example. Find the derivative of the function  $f(x) = (5x^2 - 2)(x^3 + 3x)$ .

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$



**EXAMPLE 8** Let  $y = \frac{x^2 + x - 2}{x^3 + 6}$ . Compute the derivative.

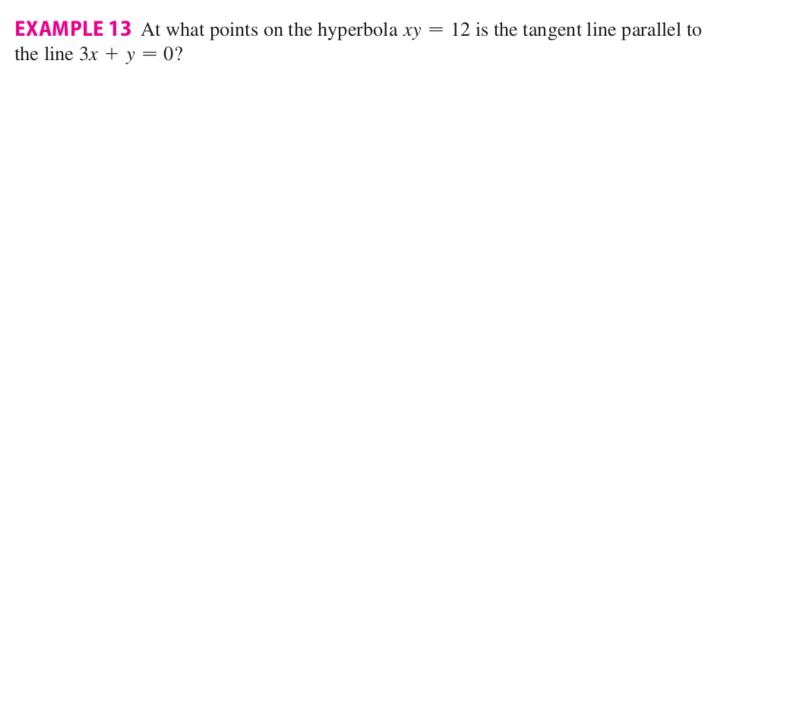
General Power rule.

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

Case n = 0:

Example. Find the derivative of the function  $\ f(x)=x^{2/3}$  .



#### Summary of Differentiation Formulas.

#### **Table of Differentiation Formulas**

$$\frac{d}{dx}(c) = 0 \qquad \qquad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf' \qquad (f+g)' = f' + g' \qquad (f-g)' = f' - g'$$

$$(fg)' = fg' + gf' \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$