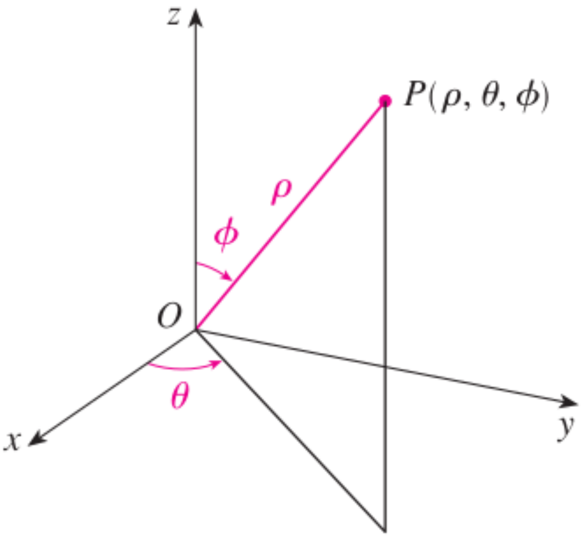
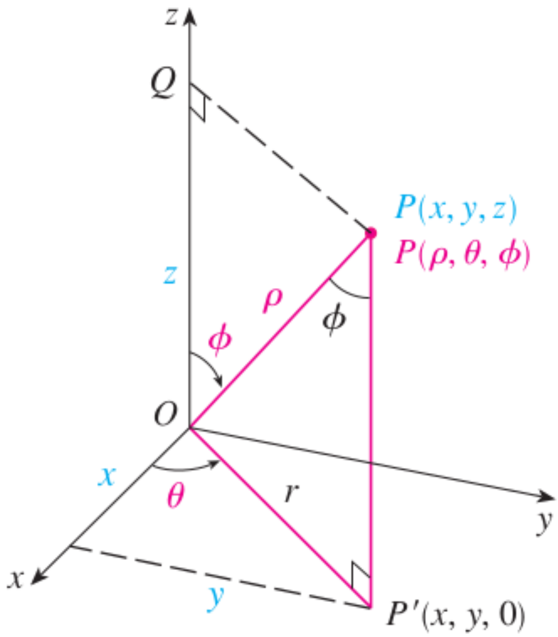
15.8 Integrals in spherical coordinates.



Basic settings.

Relationships with cartesian coordinates.

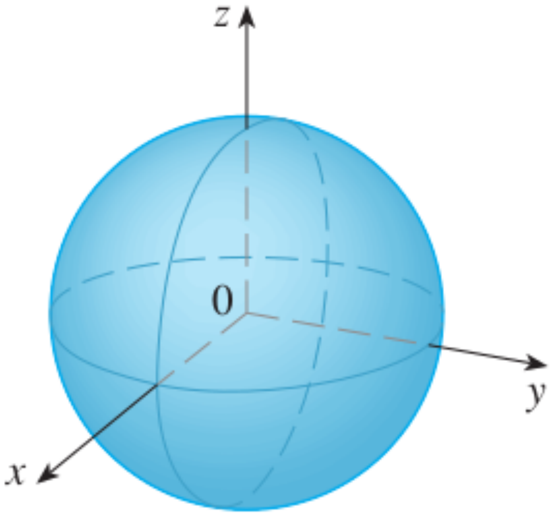


EXAMPLE 1 The point $(2, \pi/4, \pi/3)$ is given in spherical coordinates. Plot the point and find its rectangular coordinates.

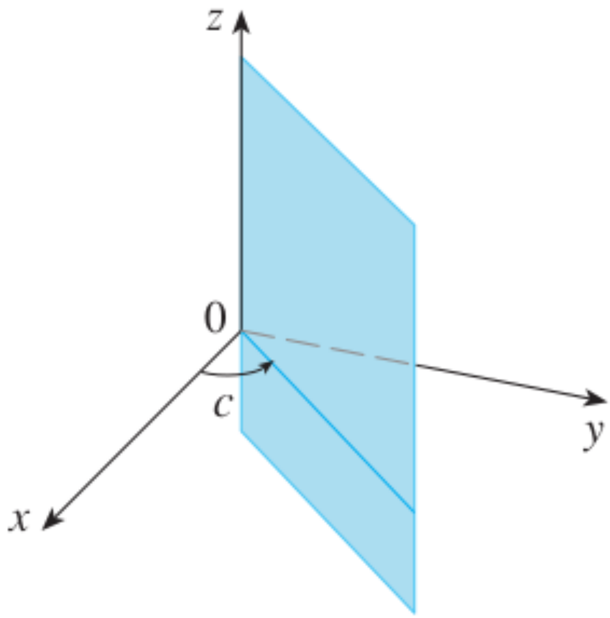
EXAMPLE 2 The point $(0, 2\sqrt{3}, -2)$ is given in rectangular coordinates. Find spherical coordinates for this point.

Important solids' equations.

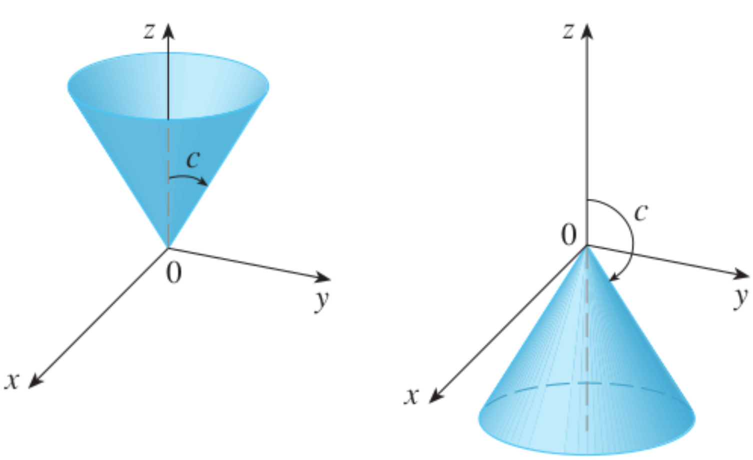
Sphere.



Half planes.



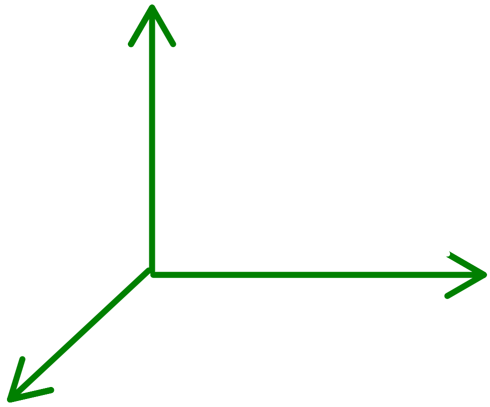
Cones.

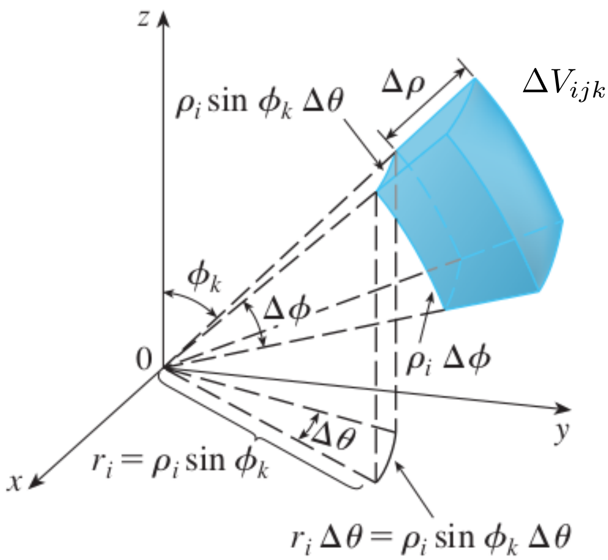


$0 < c < \pi/2$

$\pi/2 < c < \pi$

Evaluating integrals.





$$\boxed{\Delta V_{ijk} = \rho_i^2 \sin(\phi_k)\Delta\rho\Delta\theta\Delta\phi}$$

$$\boxed{V \approx \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ijk}, y_{ijk}, z_{ijk})\Delta V_{ijk}}$$

Formula for the change of variable (in polar coordinate).

$$\boxed{\iiint_E f(x,y,z)\,dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))\ \rho^2\sin(\phi)\,d\rho\,d\theta\,d\phi}$$

$E = \{(\rho,\theta,\phi)\,|\,a \le \rho \le b,\ \alpha \le \theta \le \beta,\ c \le \phi \le d\}$

EXAMPLE 3 Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$, where B is the unit ball:

$$B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$$

EXAMPLE 4 Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.