Chapter 3: Applications of differentiation
Week 9

Pierre-Olivier Parisé
Calculus I (MATH-241 01/02)

University of Hawai'i
Fall 2021

Upcoming this week

1. 3.7 Optimisation problems (Part 2)
2. 3.8 Newton's method
3. 3.9 Antiderivatives

Example 1

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

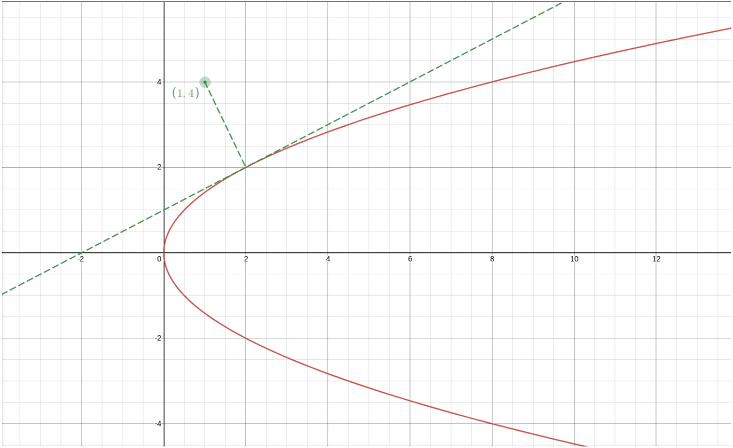

Figure: Drawing of the situation

Example 2

Find the area of the largest rectangle that can be inscribed in a semicircle of radius 2.

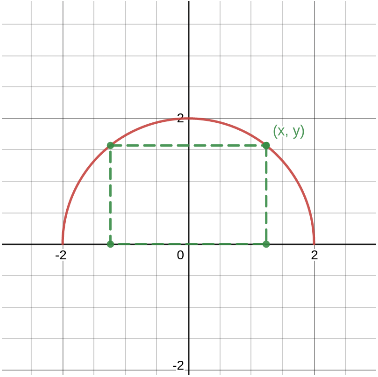

Figure: Drawing of the situation

Exercises: 2, 3, 5, 7, 9, 12-14, 22, 29, 30, 31, 41, 50, 72.

Finding zeros of a function may be laborous.

- For a linear function $f(x) = ax + b$, it is easy to find the zero: $x = -b/a$.
- For a polynomial of degree 2 $f(x) = ax^2 + bx + c$, we have the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- For polynomials of degree 3 and 4, there are complicated formulas. Cubic equation

For more general polynomial, we don't have a formula involving simple operations (we know that no such formula exists for polynomial with a degree greater than or equal to 5!!)

(a) Evariste Galois

(b) Niels Henrik Abel

This is why we need a numerical method to approximate the solutions to an equation

$$f(x) = 0$$

Recall:

- the tangent line approximate the function pretty well around a point.
- the x-intercept of a line is pretty easy to find.

Example 3

Find an approximation to the root of

$$x^5 - 2x^4 - 5 = 0.$$

Illustration of the method

Newton's method

Let a be a solution to the equation $f(x) = 0$. Let x_1 be an initial condition. If x_n is the n-th approximation to a given by Newton's method and if $f'(x_n) \neq 0$, then the $(n+1)$-th approximation to a using Newton's method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Example 4

Use Newton's method to find $\sqrt[6]{2}$ correct to eight decimal places.

Exercises: 5, 6-8, 11-12, 15-16.

Question 5

Can you find a function $F(x)$ such that $F'(x) = 3x^2$?

Definition 6

A function F is called an antiderivative of f on an interval if $F'(x) = f(x)$ for all x in I.

Remark: When you find an antiderivative F, the function $F(x) + C$ where C is a constant is also an antiderivative.

Example 7

Find all the antiderivative of each of the following functions.

a) $f(x) = \sin x$.
b) $f(x) = x^3$.
c) $f(x) = x^{-3}$.

Function	Antiderivative
$cf(x)$	$cF(x) + C$
$f(x) + g(x)$	$F(x) + G(x) + C$
x^n $(n \neq 1)$	$\frac{x^{n+1}}{n+1} + C$
$\cos x$	$\sin x + C$
$\sin x$	$-\cos x + C$
$\sec^2 x$	$\tan x + C$
$\sec x \tan x$	$\sec x$

Table: Table of some functions and their antiderivatives

Example 8

A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6cm/s$ and its initial displacement is $s(0) = 9cm$. Find its position function $s(t)$.

Exercises: 1-20, 21-22, 33-36, 46, 53-58.