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# MATH 241 – MIDTERM 02

Fall 2022, 11/23/2022

4:00–5:15pm

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Last name: \_\_\_\_\_

First name: \_\_\_\_\_

Section: \_\_\_\_\_

## Instructions:

- Write your last name, first name and section number above.
- Answer the eight questions on this exam.
- Show all the details of your work.
- No electronic devices are to be used during the exam (this includes calculators).
- The exam is closed book and closed notes.
- **Do not use L'Hôpital's rule anywhere on this exam.**
- You have 75 minutes to complete the exam.
- Turn in your exam when you are done or at the end of the 75-min period.

Sign below to acknowledge you have read the instructions.

Signature: \_\_\_\_\_

May the Force be with you!

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QUESTION 1 (10 pts)

If a spherical snowball melts so that its surface area decreases at a rate of  $1 \text{ cm}^2/\text{min}$ , find, using **Calculus**, the rate at which the diameter decreases when the diameter is 10 cm.

*Note: The surface area of a sphere is  $A = 4\pi r^2$ .*

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QUESTION 2

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(10 pts)

Let  $f(x) = \sqrt{4+x}$ .

(a) (5 points) Find the linearization of the function  $f$  at the point  $a = 0$ .

(b) (5 points) Using the linearization, estimate the value of  $\sqrt{4.1}$ . Explain clearly how you obtained your answer and leave it in decimal form.

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QUESTION 3

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(22 pts)

Let  $f(x) = \frac{3x^2 - 3}{x^2 + 3}$ .

- (a) (4 points) Using **Calculus**, find the vertical asymptotes (if any) and horizontal asymptotes (if any) of the function  $f(x)$ .

- (b) (4 points) The first derivative of  $f$  is  $f'(x) = \frac{24x}{(x^2+3)^2}$ . Find the critical numbers (if any) and the open interval(s) of increase and decrease.

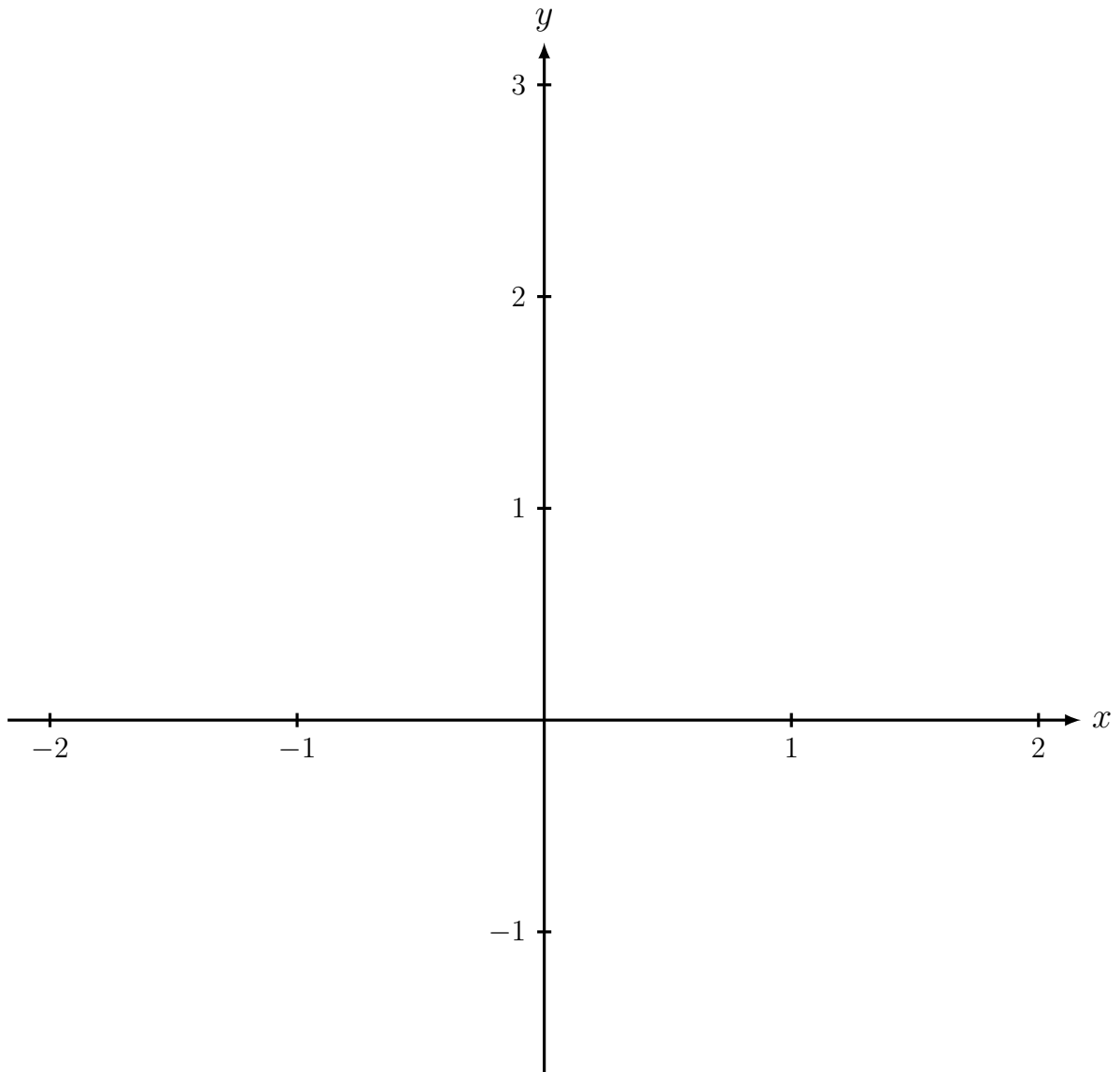
...Question 3 continued...

- (c) (6 points) The second derivative of  $f$  is  $f''(x) = \frac{-72(x^2-1)}{(x^2+3)^3}$ . Find the  $x$ -coordinate of the inflection points (if any) and the open interval(s) of concavity.

- (d) (4 points) Using one of the derivative tests, find the local maximum(s) and/or local minimum(s) of the function.

...Question 3 continued...

- (e) (4 points) Sketch the graph of the function  $f$  in the axes below. Note that the  $y$ -intercept is  $-1$  and the  $x$ -intercepts are  $x = -1$  and  $x = 1$ .



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QUESTION 4

(10 pts)

Compute the following limits. If the limit does not exist, write explicitly DNE. Make sure to write all the details of your calculations.

(a) (5 points)  $\lim_{x \rightarrow \infty} \frac{3x - 2}{2x + 1}.$

(b) (5 points)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 1}}{3x - 5}.$

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QUESTION 5

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(15 pts)

A box with an open top is to be constructed from a square piece of cardboard of side length 3 ft by cutting out a square from each of the four corners and bending up the sides. **Using calculus**, find the largest volume that such a box can have. Make sure to justify clearly your answer.



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QUESTION 6

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(15 pts)

Find the most general antiderivative of the following functions.

(a) (5 points)  $f(x) = 4\sqrt{x} - 6x^2 + 3$ .

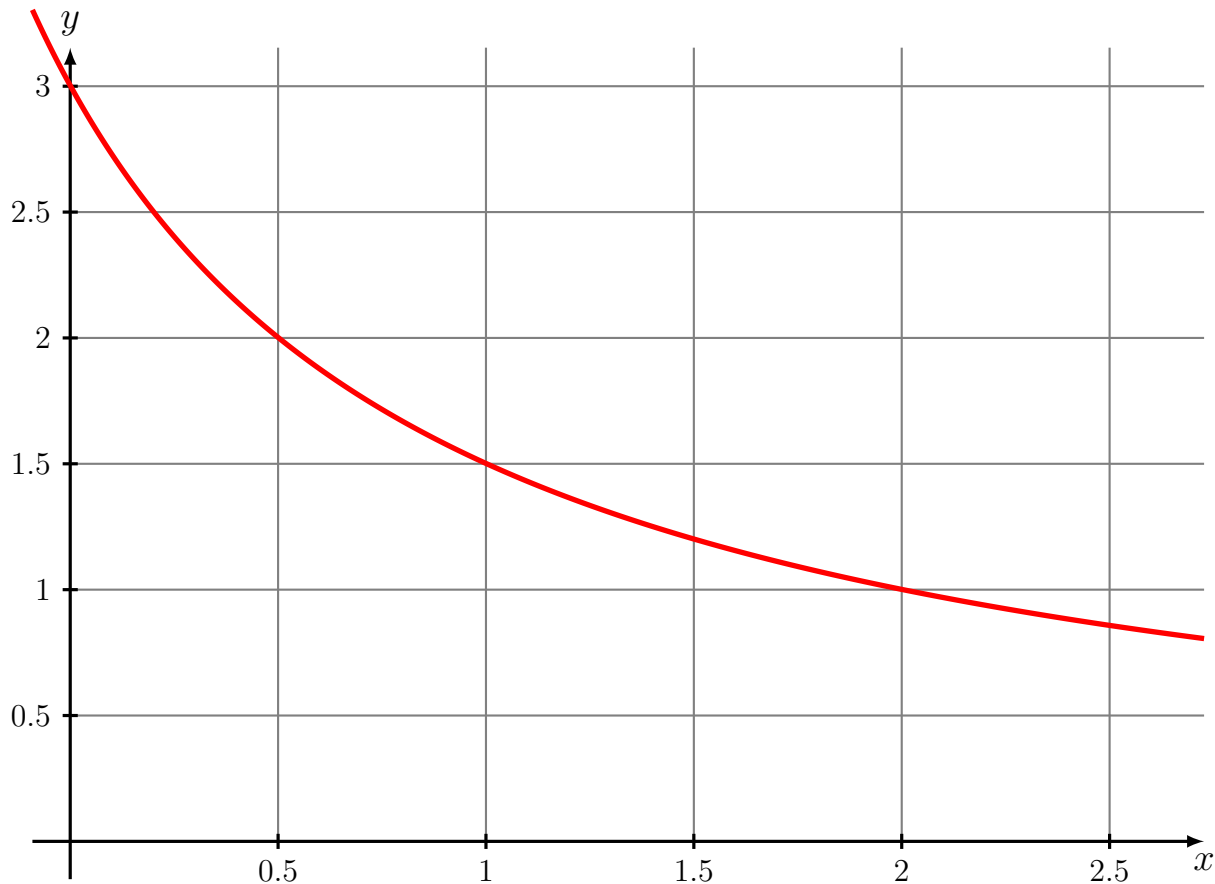
(b) (5 points)  $f(x) = \cos(x) + 2\sec^2(x)$ .

(c) (5 points)  $f(x) = x\sqrt{x} + \frac{x^2 + x}{x}$ .

QUESTION 7

(8 pts)

The graph of the function  $f(x) = \frac{3}{1+x}$  is given below.



(a) (4 points) Estimate  $\int_0^2 f(x) dx$  using two rectangles and right endpoints.

(b) (2 points) Draw the two rectangles from part (a) on the above picture of the graph of  $f(x)$ .

(c) (2 points) Is your answer over or under approximating the actual value of the integral?

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QUESTION 8

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(10 pts)

Answer the following questions.

- (a) (5 points) Using a comparison property for the definite integral, find a lower bound and an upper bound for the value of the following definite integral:

$$\int_0^1 \sqrt{1+8x} \, dx.$$

- (b) (5 points) Using a geometric approach, find the value of the following definite integral:

$$\int_0^2 |x-2| \, dx.$$

**Do not write on this page.**

*For official use only:*

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	10	22	10	15	15	8	10	100
Score:									