Math 241

Chapter 5

Section 5.1: Area Between Curves

Contents

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Non Intersecting Regions

Desmos: https://www.desmos.com/calculator/o7vvfgfwzy

Given two functions $f(x)$ and $g(x)$ such that

$$g(x) \leq f(x) \quad a \leq x \leq b,$$

the area of the region S enclosed by $f(x)$, $g(x)$, $x = a$ and $x = b$ is

$$\text{Area}\,(S) = \int_a^b f(x) - g(x)\,dx.$$

Example 1. Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$.

Intersecting Regions

Example 2. Find the area of the region enclosed by the functions $y = x^2$ and $y = x + 2$.

General Steps:

1. Find the intersection between the two curves.
2. Draw a picture.
3. Set up the definite integral.
4. Evaluate the definite integral.

EXAMPLE 3. Find the area of the region enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Regions Bounded By Functions of y

Example 4. Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.