Chapter 3
Applications of Derivatives

3.5 Summary of Curve Sketching

EXAMPLE 1 Use the guidelines to sketch the curve $y = \dfrac{2x^2}{x^2-1}$. $= f(x)$

Ⓐ Domain. $x^2-1 = (x-1)(x+1) = 0 \iff x=-1, x=1$

is $(-\infty,-1) \cup (-1,1) \cup (1,\infty)$.

Ⓑ Y-interc. & x-interc.

$f(0) = 0$ & $f(x)=0 \iff 2x^2 = 0$

$\Rightarrow x=0$

Ⓒ Symmetries.

- Odd or even: $f(-x) = \dfrac{2(-x)^2}{(-x)^2-1} = \dfrac{2x^2}{x^2-1} = f(x)$

$\rightarrow$ even.

Ⓓ HA & VA.

HA: $\displaystyle\lim_{x\to\infty} \frac{2x^2}{x^2-1} = \frac{2}{1} = 2 \rightarrow y=2$ HA. $x\to\infty$

$\displaystyle\lim_{x\to-\infty} \frac{2x^2}{x^2-1} = \frac{2}{1} = 2 \rightarrow y=2$ HA $x\to-\infty$

VA: $x=1$ $\displaystyle\lim_{x\to1^-} \frac{2x^2}{x^2-1} = \frac{2}{0^-} = -\infty$

VA

$\displaystyle\lim_{x\to1^+} \frac{2x^2}{x^2-1} = \frac{2}{0^+} = +\infty$

$x=-1$ $\displaystyle\lim_{x\to-1^-} \frac{2x^2}{x^2-1} = \frac{2}{0^+} = +\infty$

VA

$\displaystyle\lim_{x\to-1^+} \frac{2x^2}{x^2-1} = \frac{2}{0^-} = -\infty$

(E) Interv. increase / decr.

$$f'(x) = -\frac{4x}{(x^2-1)^2} \rightarrow \quad \text{C.N.}: 0, -1, 1$$

$(x^2-1)^2 \geq 0 \rightarrow f'(x) > 0$ when $-4x > 0$

when $x < 0$

$\rightarrow f$ is $\uparrow$ on $(-\infty, 0)$

$(x^2-1)^2 \geq 0 \rightarrow f'(x) < 0$ when $-4x < 0$

when $x > 0$

$\rightarrow f$ is $\searrow$ on $(0, \infty)$.

(F) Max & Min.

$f \nearrow$ on $(-\infty, 0)$ & $f \searrow$ on $(0, \infty)$

$\Rightarrow x = 0$ is a loc. Max.

& $f(0) = 0$.

(G) Concavity.

$$f''(x) = \frac{12x^2+4}{(x^2-1)^3} \rightarrow f''(x) > 0$$

$\rightarrow$ concave up on

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(H) Graph.

		-1		0		1	
f'	+		+		−		−
f''	+		+		+		+
f	$\nearrow$	$\nexists$ VA		0		$\nexists$	

Guidelines for Sketching a Curve.

A. Find the domain of the function.

B. Find the y-intercept and x-intercept, that is $f(0)$ and when $y = 0$.

C. Search for symmetries in the function (facultative)

- If $f(x) = f(-x)$, then the function is even.
- If $-f(x) = f(-x)$, then the function is odd.
- If $f(x + p) = f(x)$, then the function repeats itself after a period p (it is periodic).

D. Find the asymptotes of the function:

- The Horizontal asymptotes.
- The Vertical asymptotes.

E. Find the intervals of increase and decrease.

F. Find the local maximum and minimum values.

G. Find the concavity and the points of inflections.