Chapter 4
Integrals

4. Indefinite Integrals and the Net Change Theorem

Indefinite Integral.

Previously on Calc I:

Fondamental Theorem of Calculus $\int_a^b f(x)dx = F(b) - F(a).$

We introduce a notation for the antiderivatives:

$$\int f(x)\,dx = F(x) \qquad \text{means} \qquad F'(x) = f(x)$$

Example.

a) $\displaystyle\int x^2\,dx =$

b) $\displaystyle\int \cos x\,dx =$

c) $\displaystyle\int \sec^2 x\,dx =$

Table of Indefinite integrals

$\int cf(x)\,dx = c\int f(x)\,dx$	$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$
$\int k\,dx = kx + C$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$

Remark: We adopt the convention that the general indefinite integral is valid on a given interval. Thus we write

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$$

with the understanding that it is valid on the interval $(0,\infty)$ or on the interval $(-\infty, 0)$.

EXAMPLE 1 Find the general indefinite integral

$$\int (10x^4 - 2\sec^2 x)\, dx$$

EXAMPLE 2 Evaluate $\displaystyle\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$.

EXAMPLE 4 Find $\int_0^{12} (x - 12 \sin x)\, dx$.

EXAMPLE 5 Evaluate $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\, dt$.

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

a) Displacement:

b) Total distance traveled:

c) Acceleration:

EXAMPLE 6 A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

(a) Find the displacement of the particle during the time period $1 \leqslant t \leqslant 4$.

(b) Find the distance traveled during this time period.