

Chapter 2

Derivatives

2.8 Related Rates.

EXAMPLE 1 Air is being pumped into a spherical balloon so that its volume increases at a rate of $100 \text{ cm}^3/\text{s}$. How fast is the radius of the balloon increasing when the diameter is 50 cm ?

Key Steps.

- 1) Identify the given information and the unknown.
- 2) Introduce notation.
- 3) Restate the given information and the unknown with the new notation.
- 4) Connect the variables together with an equation.
- 5) Apply the chain rule to find the related rates.

EXAMPLE 2 A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

EXAMPLE 3 A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of $2 \text{ m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3 m deep.

EXAMPLE 5 A man walks along a straight path at a speed of 4 ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

1. Read the problem carefully.
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and the required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary, use the geometry of the situation to eliminate one of the variables by substitution (as in Example 3).
6. Use the Chain Rule to differentiate both sides of the equation with respect to t .
7. Substitute the given information into the resulting equation and solve for the unknown rate.