15.7 Triple integrals cylindrical coordinates.

Polar coordinates.

$P(r, \theta) = P(x, y)$

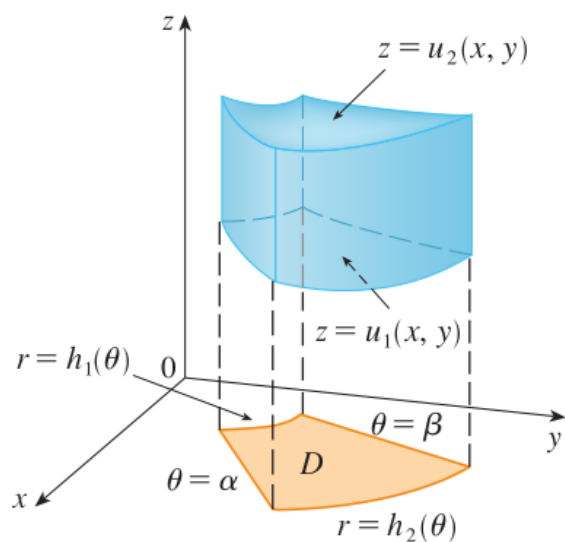## Cylindrical coordinates.

$P(r, \theta, z)$

$(r, \theta, 0)$
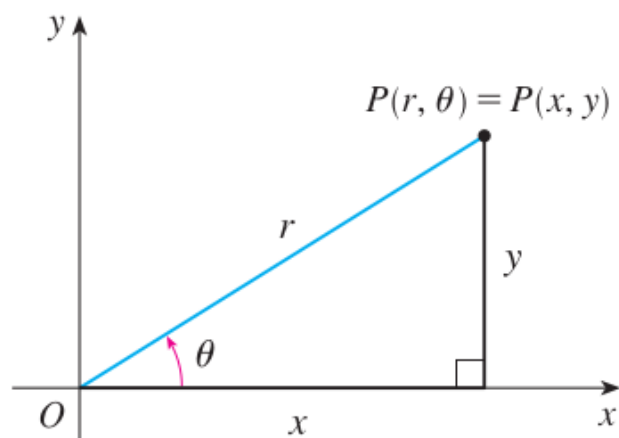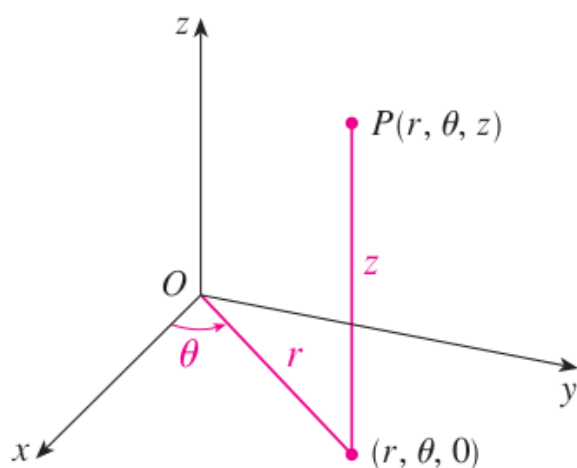
EXAMPLE 1
(a) Plot the point with cylindrical coordinates $(2, 2\pi/3, 1)$ and find its rectangular coordinates.
(b) Find cylindrical coordinates of the point with rectangular coordinates $(3, -3, -7)$.

EXAMPLE 2 Describe the surface whose equation in cylindrical coordinates is $z = r$.

Evaluating triple integrals.

Labels in figure:
$z = u_2(x, y)$
$z = u_1(x, y)$
$r = h_1(\theta)$
$\theta = \beta$
$\theta = \alpha$
D
$r = h_2(\theta)$

$$\iiint\limits_{E} f(x, y, z)\, dV = \int_{\alpha}^{\beta} \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta,\, r\sin\theta)}^{u_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta,\, r\sin\theta,\, z)\, r\, dz\, dr\, d\theta$$

EXAMPLE 3 A solid E lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. (See Figure 8.) The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.

EXAMPLE 4 Evaluate $\displaystyle\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\, dz\, dy\, dx.$