Section 2.8, Problem 12

We differentiate with respect t the equation in x and y to get

$$x'y + xy' = 0..$$

Replacing x by 4, y by 2, and $y' = 3$, we then obtain $x' = -6$ cm/s.

$\boxed{\textbf{Section 3.1, Problem 30}}$

The derivative is $f'(x) = 3x^2 + 12x$. So the critical numbers are the solutions to the equation $3x^2 + 12x = 0$. The solutions are $x = 0$. The critical numbers are $x = 0$ (the derivative exists everywhere).

Section 3.1, Problem 50

The derivative of the function is $f'(t) = 6t(t^2 - 4)^3$. So the critical points within $(-2, 3)$ are $t = 0$ and $t = 2$.

We have $f(-2) = 0$, $f(0) = -64$, $f(2) = 0$, and $f(3) = 125$. So the maximum is

$$M = 125$$

and the minimum is

$$m = -64.$$

Section 3.1, Problem 54

We have to find the critical points inside the interval $(0, 2)$. The derivative of f is

$$f'(t) = \frac{(1+t^2)/2\sqrt{t} - \sqrt{t}(2t)}{(1+t^2)^2} = \frac{1+t^2-4t^2}{2\sqrt{t}(1+t^2)^2} = \frac{1-3t^2}{2\sqrt{t}(1+t^2)^2}.$$

The zeros of the derivative are at $t = \pm\sqrt{1/3}$. We have to discard $-\sqrt{1/3}$ because it's not in the interval. The derivative exists at every point in $(0, 2)$.

Now the maximum and the minimum will be given by the max of the values $f(0) = 0$, $f(\sqrt{1/3}) \approx 0.5698$, and $f(2) \approx 0.2828$. So the maximum

$$M = 0.5698$$

and the minimum

$$m = 0.2828.$$

Section 3.2, Problem 12

The function is a polynomial, so it is differentiable and continuous on $[-2, 2]$.

So, there must be a number c such that

$$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} = 4.$$

The derivative of the function is $f'(x) = 3x^2 - 3$. So, replacing x by c, we have to solve the equation

$$3c^2 - 3 = 5 \iff c^2 = \frac{2}{3} \iff c = \pm\sqrt{2/3}.$$