F.I Moments

Problem 1. If X is uniformly distributed on (a, b), show that

$$\mathrm{Exp}(X^k) = \frac{b^{k+1} - a^{k+1}}{(b-a)(k+1)} \quad \text{for } k = 1, 2, \ldots.$$

F.II Moment Generating Function

Problem 2. If X has the normal distribution with mean 0 and variance 1, find $\mathrm{E}(X^3)$.

Problem 3. Show that, if X has a normal distribution, then so does $aX + b$, for any $a, b \in \mathbb{R}$ with $a \neq 0$.

Problem 4. Suppose that the waiting time for the first customer to enter a retail shop after 9:00am is a random variable X with an exponential density function given by

$$f(x) = \begin{cases} (1/\theta)e^{-x/\theta} & x > 0, \\ 0 & \text{elsewhere.} \end{cases}$$

a) Find the moment-generating function of X.

b) Use the answer from part (a) to find $\mathrm{Exp}(X)$ and $\mathrm{Var}(X)$.