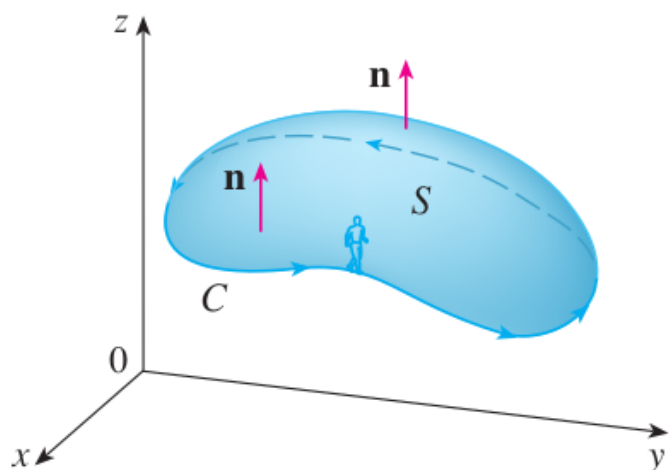


16.8 Stokes' Theorem.

Another story of orientation.



Stokes' Theorem Let S be an oriented piecewise-smooth surface that is bounded by a simple, closed, piecewise-smooth boundary curve C with positive orientation. Let \mathbf{F} be a vector field whose components have continuous partial derivatives on an open region in \mathbb{R}^3 that contains S . Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

Green's Theorem as a special case of Stokes' Theorem.

EXAMPLE 1 Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = -y^2\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$. (Orient C to be counterclockwise when viewed from above.)

EXAMPLE 2 Use Stokes' Theorem to compute the integral $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = xz \mathbf{i} + yz \mathbf{j} + xy \mathbf{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane. (See Figure 4.)

Computing surface integrals when the surface is difficult.

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1. A hemisphere H and a portion P of a paraboloid are shown. Suppose \mathbf{F} is a vector field on \mathbb{R}^3 whose components have continuous partial derivatives. Explain why

$$\iint_H \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \iint_P \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$$

