Math 241

Chapter 2

Section 2.7: Rates of Change

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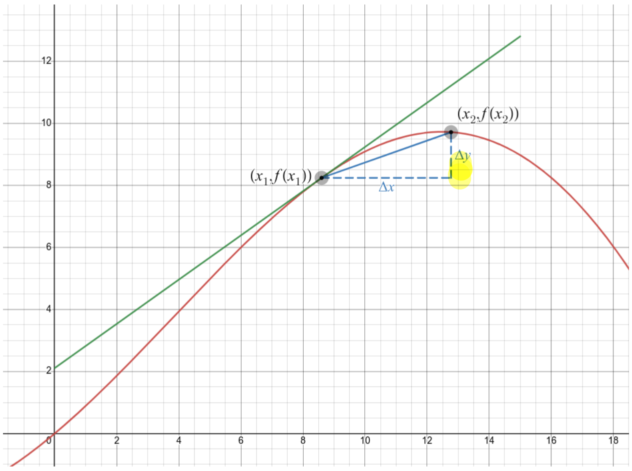
Let $y = f(x)$ be a function.

- If x goes from x_1 to x_2, then the change in x is
$$\Delta x = x_2 - x_1.$$
- When x changes from x_1 to x_2, then y changes from $f(x_1)$ to $f(x_2)$ and the change in y is
$$\Delta y = y_2 - y_1 = f(x_2) - f(x_1).$$
- The **average rate of change** at x_1 is therefore $\frac{\Delta y}{\Delta x}$.
- The **instanteneous rate of change** at x_1 is
$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.$$



https://www.desmos.com/calculator/ajsf8ggdwy

Remark: The name of the variables may be different. We can use the variables x, t (or other letters) for the independent variable and y, s (or other letters) for the dependent variable.

EXAMPLE 1. The position s of an object is given by the function $s = f(t) = t^2$

a) Compute the average rate of change at $t_1 = 1$ if $t_1 = 1$ and $t_2 = 2$.

b) Compute the instanteneous rate of change at $t_1 = 1$.

(a) $\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{2^2 - 1^2}{2-1} = \frac{4-1}{1} = \boxed{3}$ u/unit of time

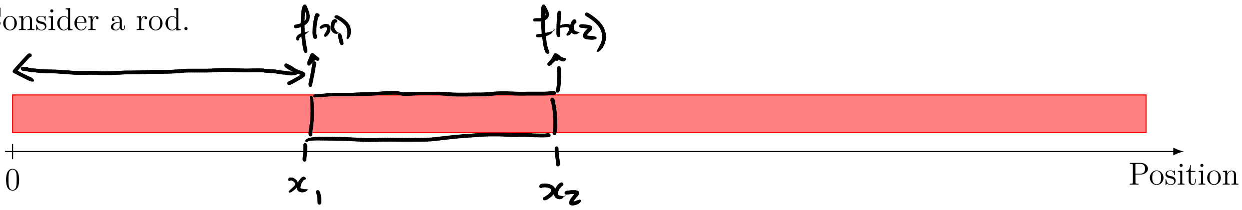
(b) $s'(t) = 2t \Rightarrow \boxed{s'(1) = 2}$

Remarks: Let the position s of an object be given by $s = f(t)$ where f is a function of time t.

- The average velocity at x_1 is the average rate of change in s.
- The instanteneous velocity at x_1 is the instanteneous rate of change at x_1.

Linear Density

Consider a rod.



- The position on the rod from the extremity 0 is given by x.
- The mass of the part of the rod from 0 to x is given by

$$m = f(x).$$

<u>Question:</u> How is the mass distributed along the rod?

- The **average linear density** between x_1 and x_2 is the average rate of change in the mass between x_1 and x_2. $\rightarrow \Delta m / \Delta x$
- The **linear density** at x_1 is the instantaneous rate of change in the mass at x_1.

LD $\dfrac{dm}{dx}$

EXAMPLE 2. A rod as in the figure above has a mass m given by $f(x) = x^3$. $\rightarrow$ g

a) Find the average linear density between $x_1 = 1$ and $x_2 = 2$. x: cm.

b) Find the linear density at $x_1 = 1$.

a) $\dfrac{\Delta m}{\Delta x} = \dfrac{2^3 - 1^3}{2-1} = \dfrac{8-1}{1} = \boxed{7 \text{ g/cm}}$.

b) $\dfrac{dm}{dx} = 3x^2 \Rightarrow \boxed{m'(1) = 3} \rightarrow$ g/cm.

Suppose there is a virus spreading in a population. We are interested in describing:

- The rate at which the virus spreads from one individual to another at a specific moment in time
- If we know the quantity of infected individuals, denoted by $Q(t)$.

In this case, the rate at which the virus spreads between two days, day t_1 and t_2 respectively, is given by the average rate of change in Q:

$$\frac{\Delta Q}{\Delta t}.$$

Therefore, the rate at which the virus spreads at day t_1 is given by the instantaneous rate of change (when $\Delta t \to 0$):

$$\frac{dQ}{dt}!$$

EXAMPLE 3. Suppose a virus is spreading in the population of deers on Moloka'i. Suppose the number of infected deer at day t is given by $Q(t) = (50/\pi)\sin(\pi t) + 60$. Let $t = 0$ be the first day we observed the presence of the virus and the model is valid up to $t = 5$.

a) Find at which rate the virus spreads in the population at day $t = 3$.

b) Estimates the number of deers infected at day $t = 6$.

a) $\frac{dQ}{dt} = \frac{50}{\pi}\cos(\pi t)\,\pi \Rightarrow Q'(3) = -50$ deers/day.

b) $Q'(5)$ & extrapolate.

$Q'(t) = \frac{50}{\pi}\cos(\pi t)\,\pi \Rightarrow Q'(5) = -50$

$Q(5) = 60$

$Q(6) \approx 60 + (-50)\cdot 1 = $ <u>10 deers infected</u>.