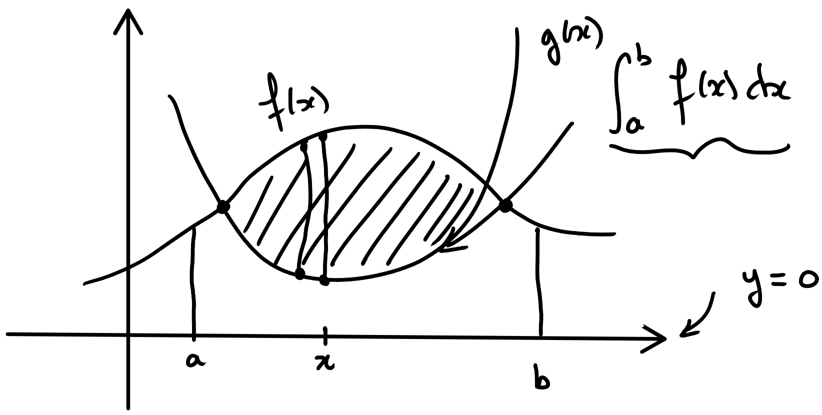
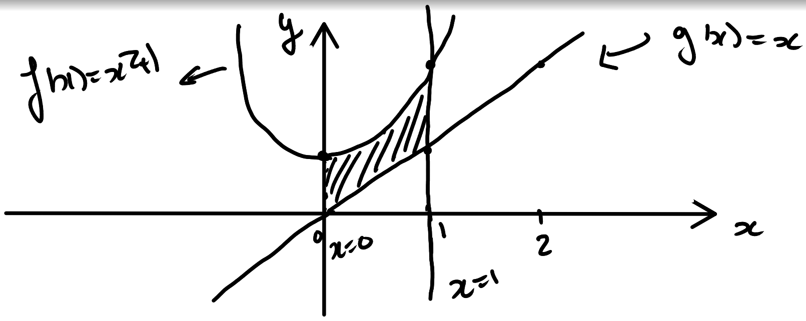



$\underbrace{\int_a^b f(x)\,dx}$ = Area under the curve $y = f(x)$

## Example 2

Compute the region bounded from above by the curve $f(x) = x^2 + 1$, bounded from below by the curve $g(x) = x$, and bounded on the sides by $x = 0$ and $x = 1$.

① Graphs.




② Area.

$$A = \int_0^1 \underbrace{f(x) - g(x)}_{\geq 0} \, dx = \int_0^1 x^2 + 1 - x \, dx$$

$$= \frac{x^3}{3} + x - \frac{x^2}{2} \Big|_0^1$$

$$\boxed{= \frac{5}{6} u^2}$$

## Example 3

Find the area of the region enclosed by the parabola $y = x^2$ and $y = 2x - x^2$.

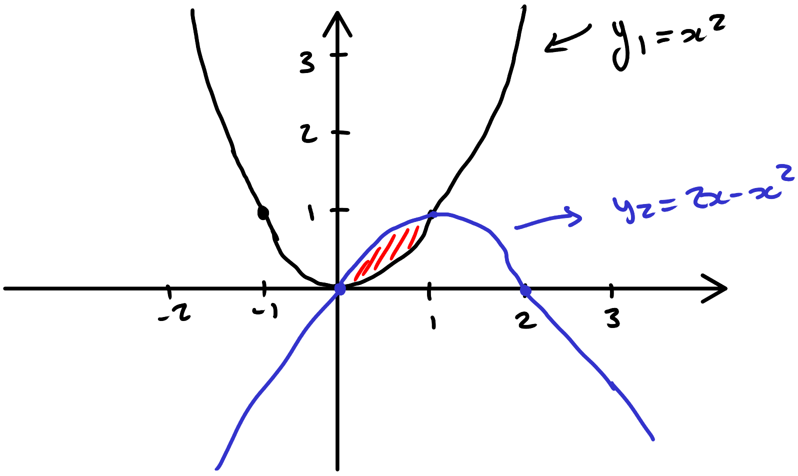
① Graphs.

$y_2 = 2x - x^2$

$= (2-x)x$

$= 0$

if $x = 2$ & $x = 0$




② Find the intersections between $y_1$ & $y_2$

We want to find when $y_1 = y_2$

$y_1 = y_2$ iff $x^2 = 2x - x^2$

iff $0 = 2x - 2x^2$

iff $0 = 2x(1-x)$

iff $0 = x$ or $0 = 1 - x$

iff $x = 0$ or $x = 1$.

③ Area between $y_1$ & $y_2$

$\underset{f(x)}{y_2} \geq \underset{g(x)}{y_1}$

$$A = \int_0^1 y_2 - y_1 \, dx = \int_0^1 2x - x^2 - x^2 \, dx$$

$$= \int_0^1 2x - 2x^2 \, dx$$

$$= x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$\boxed{= \frac{1}{3} \text{ unit}^2}$$

## Example 4

Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

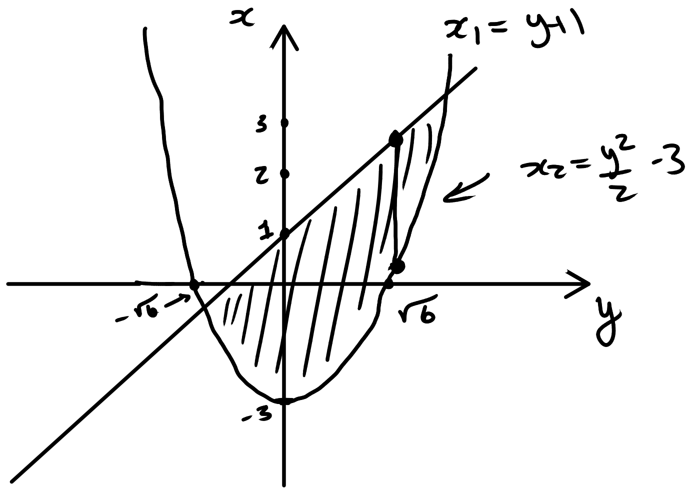
$y = \pm\sqrt{2x+6}$

① Graphs.

$x_1 = y + 1$

$x_2 = \frac{y^2}{2} - 3$

$x_2 = 0$ if $y = \pm\sqrt{6}$




② Intersections

$x_2 = x_1$ if $\frac{y^2}{2} - 3 = y + 1$ if $y = 4$ & $y = -2$.

$y^2 - 2y - 8 = (y-4)(y+2)$

③ Area

$\underbrace{x_1}_{f(x)} \geq \underbrace{x_2}_{g(x)}$

$$A = \int_{-2}^{4} x_1 - x_2 \, dy = \int_{-2}^{4} y + 1 - \left(\frac{y^2}{2} - 3\right) dy$$

$$= \int_{-2}^{4} y - \frac{y^2}{2} + 4 \, dy$$

$$= \frac{y^2}{2} - \frac{y^3}{6} + 4y \Big|_{-2}^{4}$$

$$\boxed{= 18 \text{ units}^2}$$

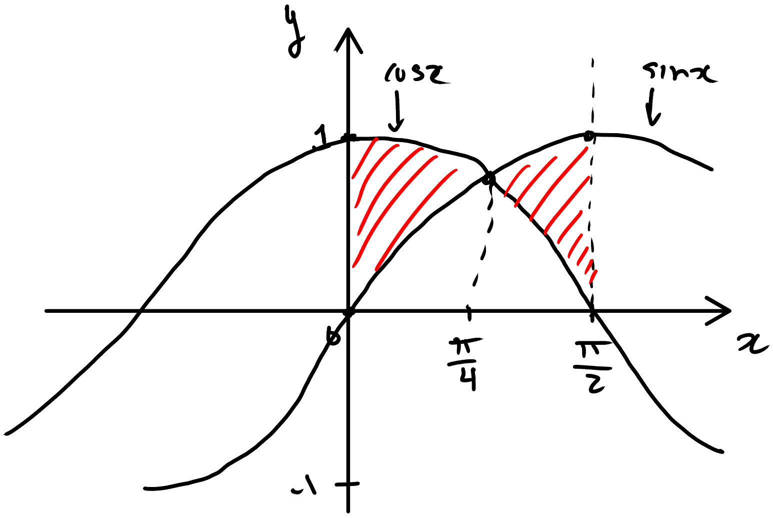
## Example 6

Find the area of the region bounded by the curve $y = \sin x$ and $y = \cos x$ from $x = 0$ and $x = \pi/2$.

① Graphs.

$y_1 = \sin x$

$y_2 = \cos x$




Notice that $\cos x \geq \sin x$ on $[0, \pi/4]$
& $\sin x \geq \cos x$ on $[\pi/4, \pi/2]$

② Intersection.

$\cos x = \sin x$ iff $1 = \tan x$ iff $x = \frac{\pi}{4} + k\pi$

$\downarrow$ because

$x = \frac{\pi}{4}$ $\quad 0 \leq x \leq \frac{\pi}{2}$

③ Tot. Area

$$A = \int_0^{\pi/2} |\cos x - \sin x| \, dx = \int_0^{\pi/4} \underbrace{\cos x - \sin x}_{\geq 0} \, dx + \int_{\pi/4}^{\pi/2} \underbrace{\sin x - \cos x}_{\geq 0} \, dx$$

$$\boxed{\simeq 0.828427 \text{ units}^2}$$