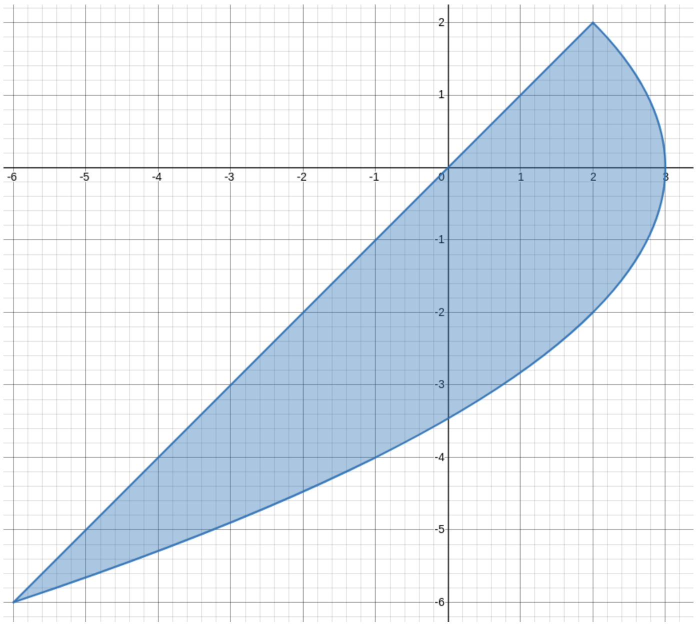
P.-O. Parisé

Section 5.1, Problem 12

The sketch of the region is the following: The intersections between the two curves are



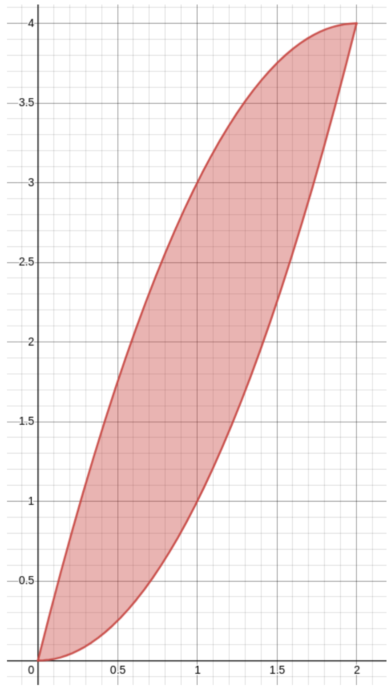
$$4y + y^2 = 12 \iff y^2 + 4y - 12 = 0 \iff y = -6 \text{ or } y = 2.$$

We now have

$$A(S) = \int_{-6}^{2} x_R - x_L \, dy = \int_{-6}^{2} 3 - y^2/4 - y \, dy = 64/3.$$

Section 5.1, Problem 14

The sketch of the region is the following: The intersections between the two curves are



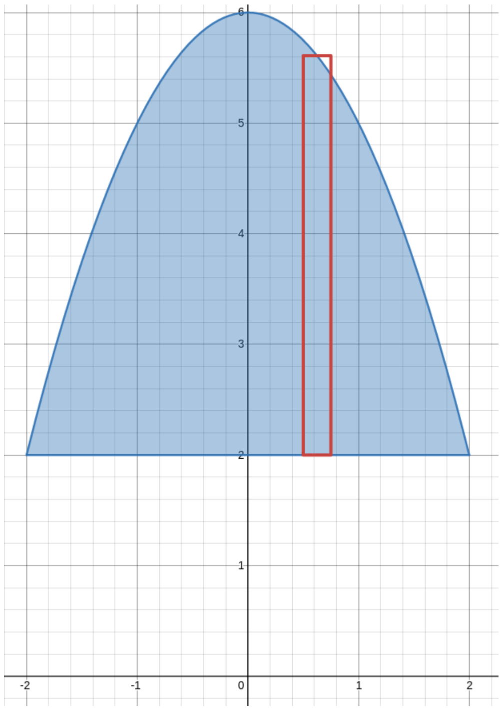
$$x^2 = 4x - x^2 \iff x^2 - 2x = 0 \iff x = 0 \text{ or } x = 2.$$

So, we have

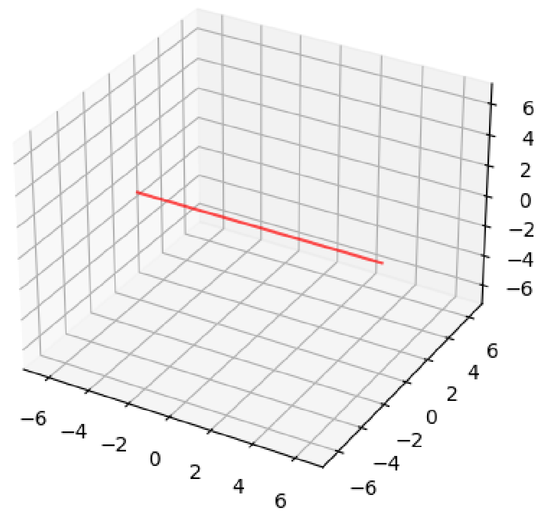
$$A(S) = \int_0^2 (4x - x^2) - (x^2)\, dx = 8/3.$$

$\boxed{\text{Section 5.2, Problem 8}}$

Here is the sketch of the region to rotate about the x-axis and a gif of the rotation[1]:



(a) Region to rotate



(b) Rotation of the region

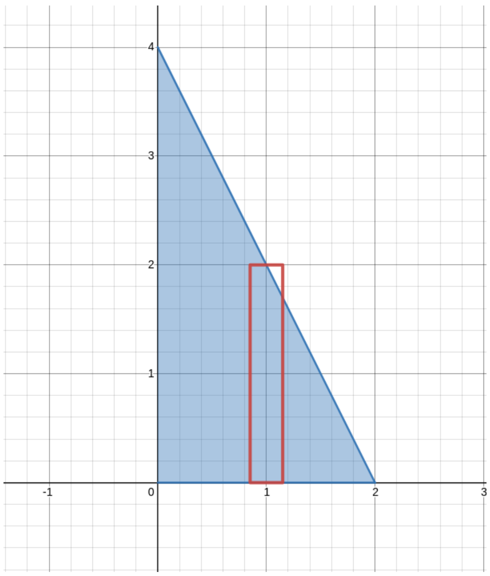
We will use the washer method. The inner radius is $r_{in} = y = 2$ and the outer radius is $r_{out} = y = 6 - x^2$. The values of x start at $x = -2$ and ends at $x = 2$. So, by the washer method, we obtain

$$V(S) = \int_{-2}^{2} \pi r_{out}^2 - \pi r_{in}^2 \, dx = \pi \int_{-2}^{2} ((6 - x^2)^2 - 2^2) \, dx = 384/5.$$

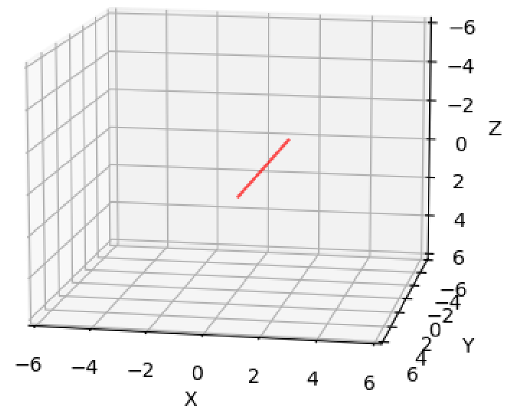
[1]The gif will work if you open the pdf with Adobe Acrobat Reader.

Section 5.3, Problem 16

Here is the sketch of the region to rotate about the axis $x = 3$:



(a) Region to rotate



(b) Rotation of the region

We will use the cylindrical shells method. The radius is $x+1$ and the height is y and the limits are $0 \le x \le 2$. Thus, we get

$$V(S) = \int_0^2 2\pi(x+1)y\,dx = 2\pi \int_0^2 (x+1)(4-2x)\,dx = 20/3.$$

$\boxed{\textbf{Section 5.3, Problem 30}}$

The radius is y and the height is $x = \sqrt{y-1}$ and the limits are $1 \le y \le 5$. So, the solid is obtained by rotating the following region around the x axis: