Chapter 1

Functions and Limits

1.1 Four Ways of Representing a Function

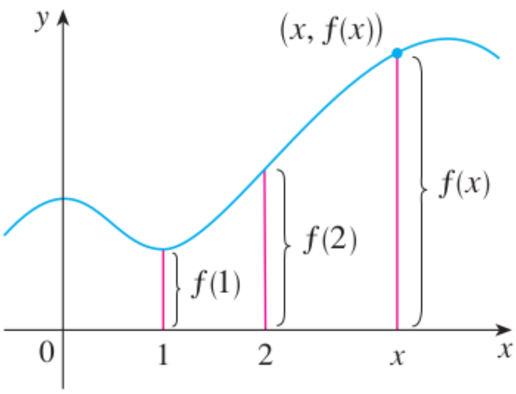
A **function** f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.

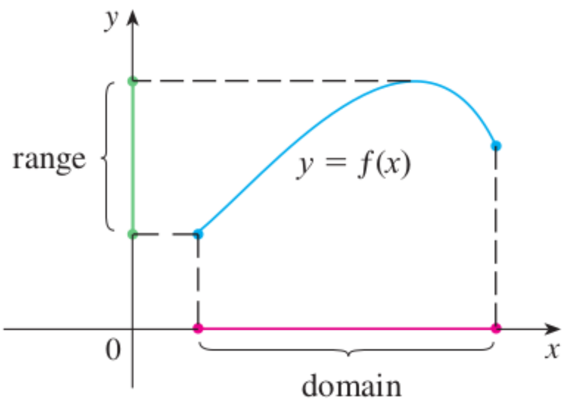
Machine visualization.

Domain: Every admissible inputs

Range: Every possible outputs

Graph of a function.





Dependant variable.

- Usually represents the output.
- Its value depends on the input.

Independant variable.

- Usually represents the input.
- Its value does not depend on anything else.

EXAMPLE 1 The graph of a function f is shown in Figure 6.
(a) Find the values of $f(1)$ and $f(5)$.
(b) What are the domain and range of f?

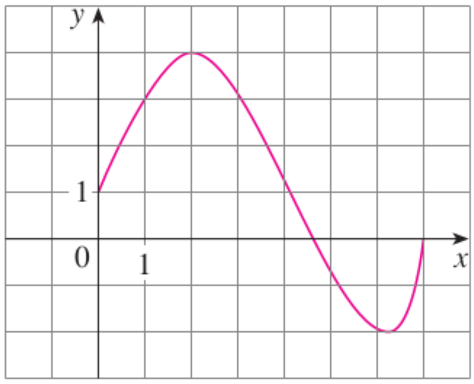
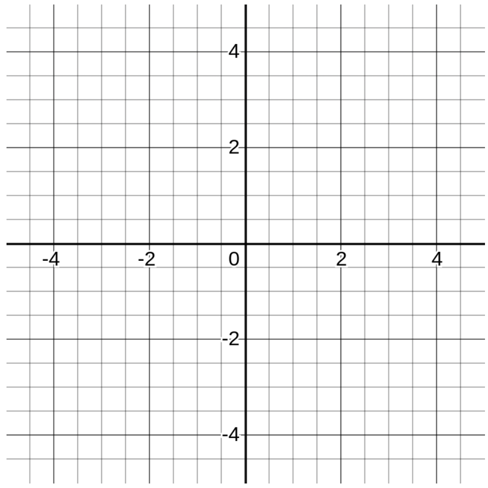


Figure 6

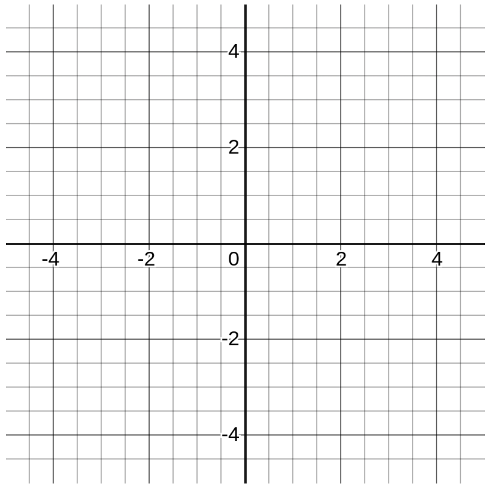
EXAMPLE 2 Sketch the graph and find the domain and range of each function.

(a) $f(x) = 2x - 1$ (b) $g(x) = x^2$

a)



b)



EXAMPLE 3 If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a+h) - f(a)}{h}$.

Remark: The fraction $\dfrac{f(a+h) - f(a)}{h}$ is called the DIFFERENCE QUOTIENT.

Representations of functions.

There are four possible ways to represent a function:

- verbally (by a description in words)
- numerically (by a table of values)
- visually (by a graph)
- algebraically (by an explicit formula)

EXAMPLE 5 A rectangular storage container with an open top has a volume of 10 m^3. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

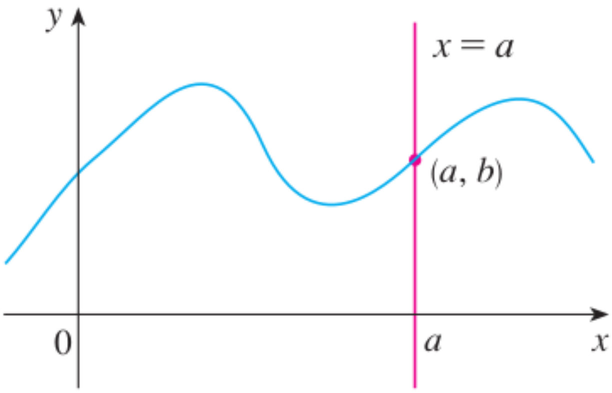
Domain of functions given by an explicit formula.

EXAMPLE 6 Find the domain of each function.

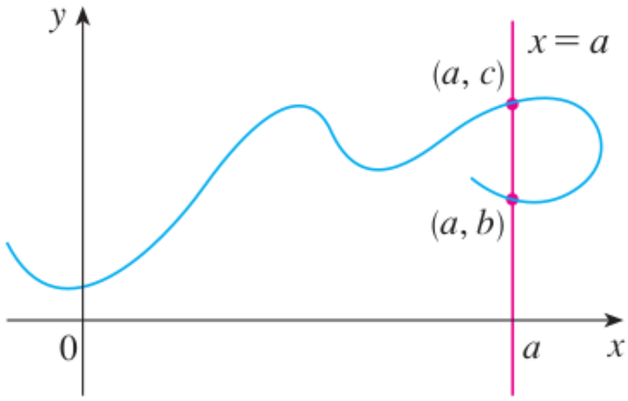
(a) $f(x) = \sqrt{x + 2}$ (b) $g(x) = \dfrac{1}{x^2 - x}$

Which curves are graphs of functions?

The Vertical Line Test A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

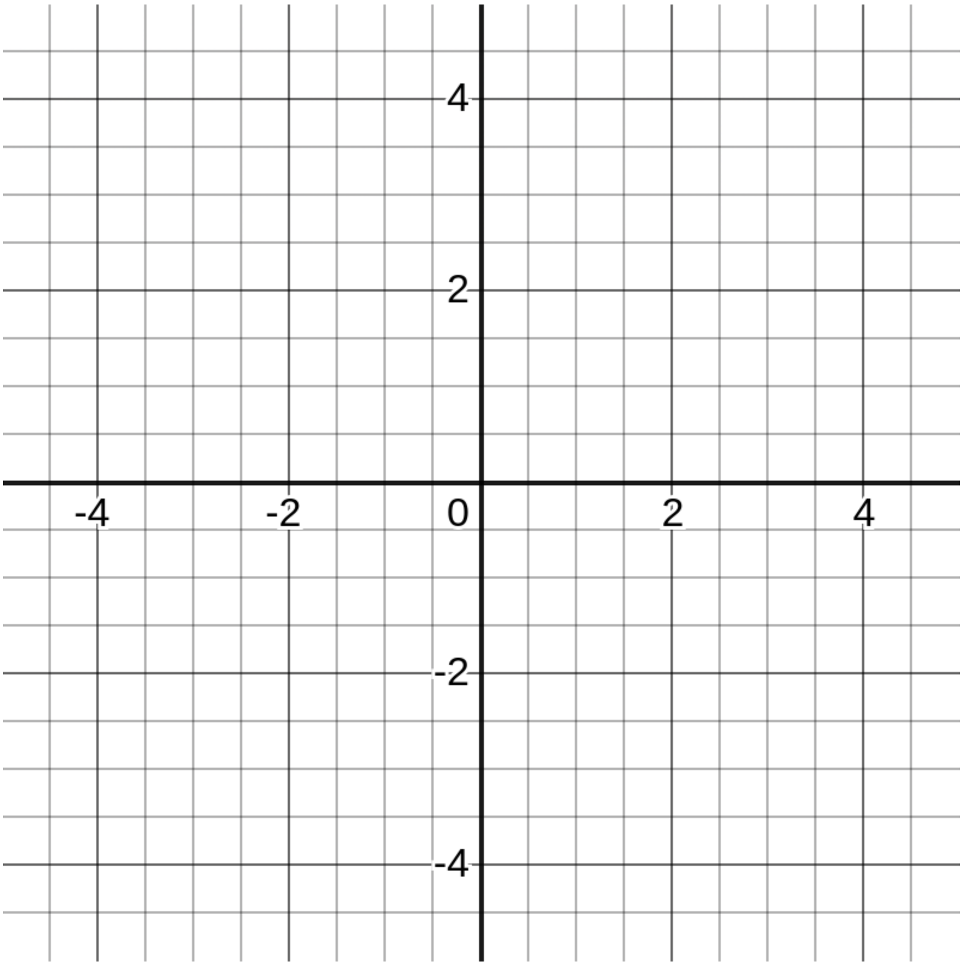


(a) This curve represents a function.



(b) This curve doesn't represent a function.

Example. The parabola $x = y^2 - 2$ is not the graph of a function. Show it using the Vertical Line Test.



Piece-wise Functions.

The functions in the following four examples are defined by different formulas in different parts of their domains. Such functions are called **piecewise defined functions**.

EXAMPLE 7 A function f is defined by

$$f(x) = \begin{cases} 1 - x & \text{if } x \leqslant -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-2)$, $f(-1)$, and $f(0)$ and sketch the graph.

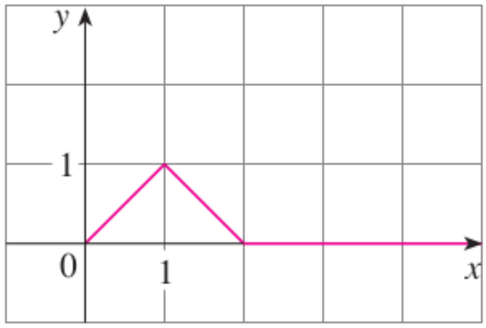
Absolute Value.

$$|a| = a \qquad \text{if } a \geqslant 0$$
$$|a| = -a \quad \text{if } a < 0$$

Properties:

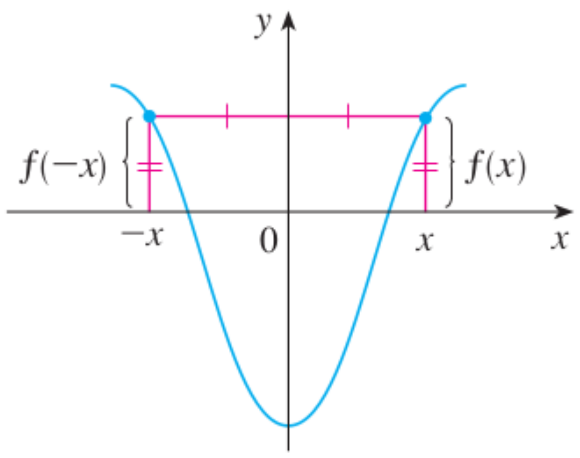
EXAMPLE 8 Sketch the graph of the absolute value function $f(x) = |x|$.

EXAMPLE 9 Find a formula for the function f graphed in Figure 17.

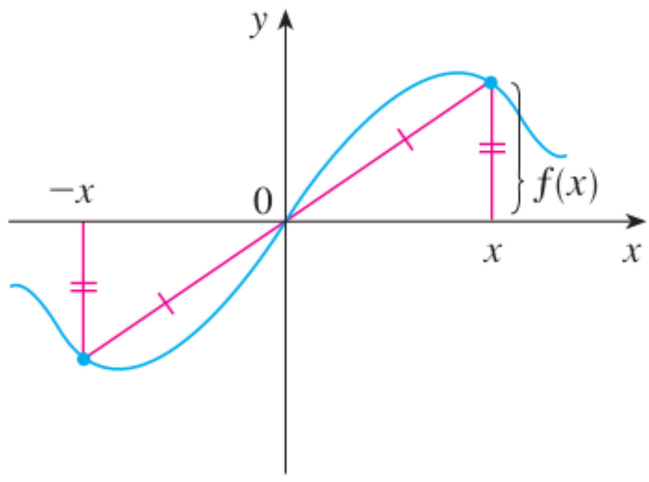


Symmetries.

Even functions



Odd functions.



EXAMPLE 11 Determine whether each of the following functions is even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$ (b) $g(x) = 1 - x^4$ (c) $h(x) = 2x - x^2$

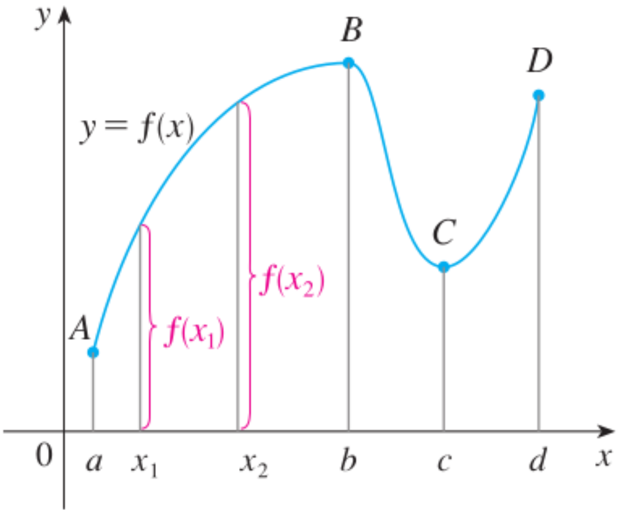
Increasing/Decreasing Functions.

A function f is called **increasing** on an interval I if

$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

It is called **decreasing** on I if

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$



- From A to B:
- From B to C:
- From C to D:

Example. Where the function $f(x) = x^2$ is increasing? Where is it decreasing?