

# MATH 302

## CHAPTER 8

### SECTION 8.3: UNIT STEP FUNCTION

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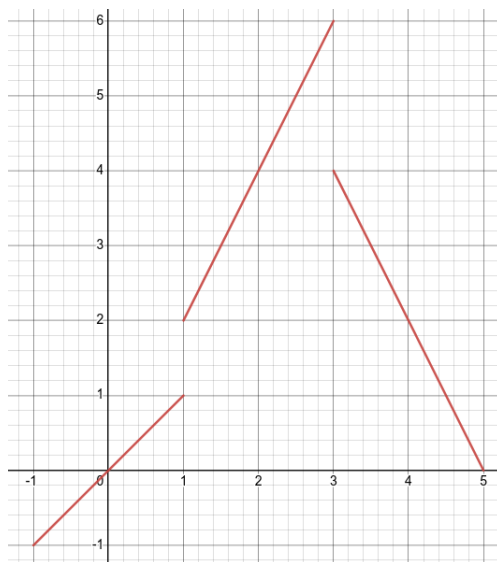
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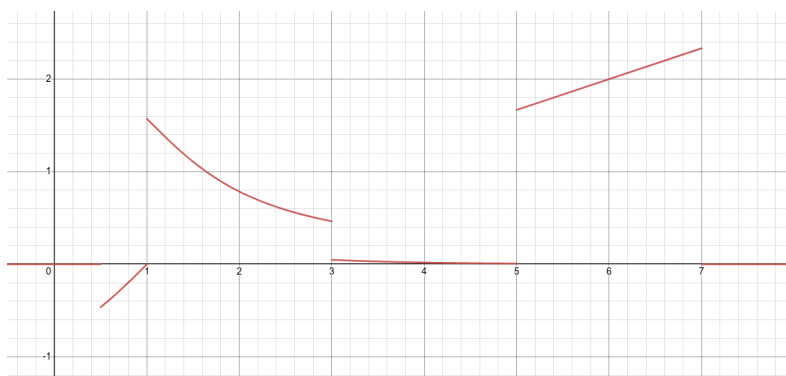
# PIECEWISE CONTINUOUS FUNCTIONS

A piecewise continuous function  $f$  is

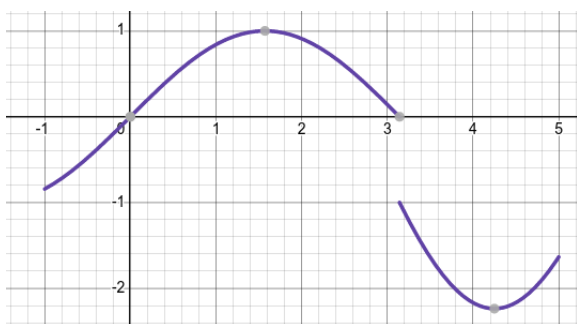
- a function defined on a finite number of intervals  $[t_0, t_1]$ ,  $[t_1, t_2]$ ,  $\dots$ ,  $[t_{n-1}, t_n]$ ;
- such that it is continuous on each interval  $(t_0, t_1)$ ,  $(t_1, t_2)$ ,  $\dots$ ,  $(t_{n-1}, t_n)$ .



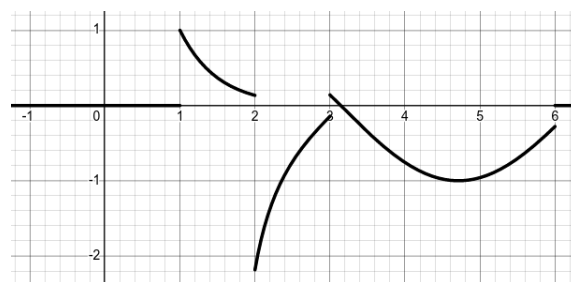
(a) A function  $f(x)$



(b) A function  $k(x)$



(c) A function  $g(x)$



(d) A function  $h(x)$

**EXAMPLE 1.** Find the Laplace transform of

$$f(t) = \begin{cases} t & 0 < t \leq 1 \\ 2t & 1 < t \leq 3 \\ 10 - 3t & 3 < t \leq 5 \\ 0 & 5 < t. \end{cases}$$

## UNIT STEP FUNCTION

To make the work easier with piecewise continuous function, we introduce the **unit step function**:

$$u(t) := \begin{cases} 0 & t < 0 \\ 1 & t \geq 0. \end{cases}$$

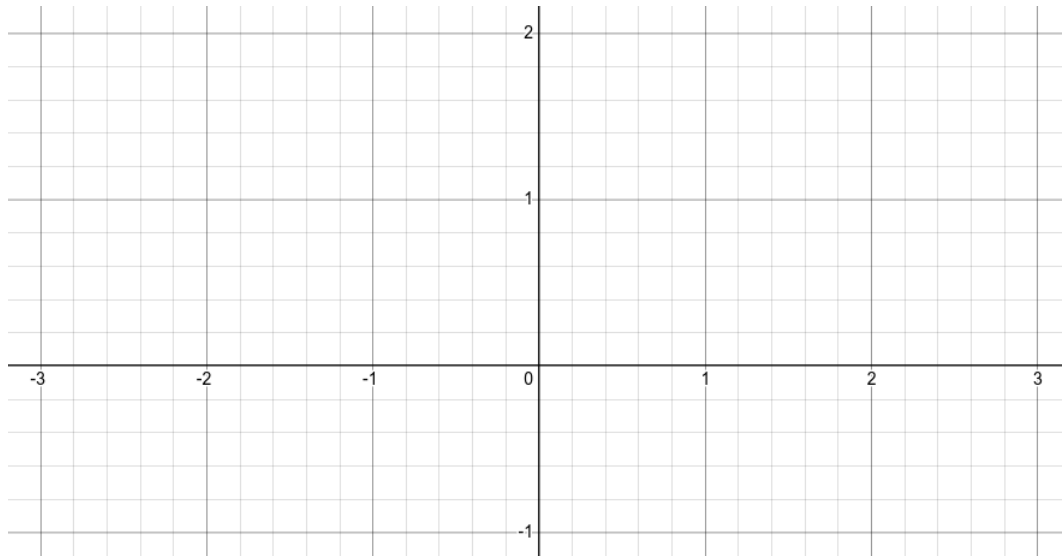


Figure 2: Plot of  $u(t)$

### Basic Operations

- Translation by  $a$  units:

$$u(t - a) = \begin{cases} 0 & t < a \\ 1 & t \geq a. \end{cases}$$

- Multiplication by  $c$ :

$$cu(t) = \begin{cases} 0 & t < 0 \\ c & t \geq 0. \end{cases}$$

- Activation of a function  $f(t)$  at time  $a$ :

$$f(t)u(t - a) = \begin{cases} 0 & t < a \\ f(t) & t \geq a. \end{cases}$$

- Destruction of a function  $f(t)$  at time  $b$  and activation of a function  $g(t)$  at time  $b$ :

$$f(t)u(t - a) + (g(t) - f(t))u(t - b) = \begin{cases} 0 & t < a \\ f(t) & a \leq t < b \\ g(t) & b \leq t. \end{cases}$$

**EXAMPLE 2.** Rewrite the function  $f(t)$  in Example 1 using the unit step function.

Let  $a \geq 0$  be a real number and  $f$  be a function with a Laplace transform  $F(s)$ .

- $L(u(t-a)) = \frac{e^{-sa}}{s}$ .
- $L(u(t-a)f(t)) = e^{-sa}F(f(t+a))$ .
- $L(u(t-a)f(t-a)) = e^{-sa}F(s)$ .

**EXAMPLE 3.** Find the Laplace transform of

$$f(t) = \begin{cases} \sin(t) & , 0 \leq t < \pi/2 \\ \cos(t) - 3\sin(t) & , \pi/2 \leq t < \pi \\ 3\cos(t) & , t \geq \pi. \end{cases}$$



**EXAMPLE 4.** Find

$$L^{-1}\left(\frac{1}{s^2} - e^{-s}\left(\frac{1}{s^2} + \frac{2}{s}\right) + e^{-4s}\left(\frac{4}{s^3} + \frac{1}{s}\right)\right)$$