Section 1.6 — Problem 12 — 5 points

Unfortunately, we can't use the quotient rule because we have an indetermination $0/0$. Therefore, we have to check if we can rewrite the expression in the limit into another way so we can use the limit rules.

For $x \neq -3$, we have

$$\frac{x^2 + 3x}{x^2 - x - 12} = \frac{x(x+3)}{(x-4)(x+3)} = \frac{x}{x-4}.$$

Since $\lim_{x \to -3} x - 4 = -7 \neq 0$, we can use the quotient rule and get

$$\lim_{x \to -3} \frac{x^2 + 3x}{x^2 - x - 12} = \lim_{x \to -3} \frac{x}{x-4} = \frac{\lim_{x \to -3} x}{\lim_{x \to -3} x - 4} = \frac{3}{7}.$$

Section 1.6 — Problem 22 — 10 points

If we would use the quotient rule, we would get 0/0. Since this is undefined, we most remove this undetermination.

For $x \neq 2$, we have

$$\frac{\sqrt{4u+1}-3}{u-2} = \left(\frac{\sqrt{4u+1}-3}{u-2}\right)\left(\frac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3}\right) = \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3} = 4\frac{u-2}{(u-2)(\sqrt{4u+1}+3)}$$

and simplifying $u-2$, we obtain

$$\frac{\sqrt{4u+1}-3}{u-2} = \frac{4}{\sqrt{4u+1}+3}$$

Now, using the power rule and the sum rule, we see that

$$\lim_{u\to 2}\sqrt{4u+1}+3 = \sqrt{\lim_{u\to 2} 4u+1}+3 = \sqrt{8+1}+3 = 6.$$

Since 6 is different from zero, we can use the quotient rule! We therefore obtain

$$\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2} = \lim_{u\to 2}\frac{4}{\sqrt{4u+1}+3} = \frac{\lim_{u\to 2} 4}{\lim_{u\to 4}\sqrt{4u+1}+3} = \frac{4}{6} = \frac{2}{3}.$$

Section 1.6 — Problem 36 — 10 points

Since $-1 \le \sin A \le 1$ for any real number A, we know that $-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$. Therefore, multiplying by $\sqrt{x^3+x^2}$, we obtain

$$-\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) \le \sqrt{x^3+x^2}.$$

Using the power rule and the sum rule, we have

$$\lim_{x\to 0}\sqrt{x^3+x^2} = 0 = \lim_{x\to 0} -\sqrt{x^3+x^2}.$$

Therefore, by the Squeeze Theorem, we can conclude that

$$\lim_{x\to 0}\sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) = 0.$$

Section 1.6 — Problem 42 — 10 points

We have to check if the limit from the left is the same as the limit from the right.

For the limit from the left, we will approach -6 with numbers x less than -6. Therefore, $x+6<0$ for $x<-6$ and $|x+6| = -(x+6) = -x-6$. Therefore, we get

$$\lim_{x\to -6^-} \frac{2x+12}{|x+6|} = \lim_{x\to -6^-} 2\frac{x+6}{-(x+6)} = \lim_{x\to -6^-} -2 = -2.$$

For the limit from the right, we will approach -6 with numbers x greater than -6. This means $x+6>0$ (when $x>-6$) and therefore $|x+6| = x+6$. We then get

$$\lim_{x\to -6^+} 2\frac{x+6}{|x+6|} = \lim_{x\to -6^+} 2\frac{x+6}{x+6} = \lim_{x\to -6^+} 2 = 2.$$

The limit from the left is -2 and the limit from the right is 2. Since they are different, we conclude that the limit

$$\lim_{x\to -6} \frac{2x+12}{|x+6|}$$

does not exist.

Section 1.8 — Problem 36 — 5 points

The function $x + \sin x$ is continuous because it is the sum of two continuous functions. Now, $\sin x$ is continuous and therefore the composition $\sin(x + \sin x)$ is continuous. In particular, the function $x \mapsto \sin(x + \sin x)$ is continuous at $x = \pi$. Using the continuity, this means that

$$\lim_{x \to \pi} \sin(x + \sin x) = \sin(\pi + \sin \pi) = \sin(\pi + 0) = 0.$$

Section 1.8 — Problem 56 — 10 points

Let $f(x) = \sin x - x^2 + x$. We have $a = 1$ and $b = 2$.

We will verify if the hypothesis of the Intermediate Value Theorem are verified.

- The function f is a sum of continuous function on all of $(-\infty, \infty)$, therefore f is continuous on all of $(-\infty, \infty)$. In particular, the function f is continuous on $(1, 2)$.
- $f(1) = \sin(1) - 1^2 + 1 = \sin(1) > 0$ because for any $0 < x < \pi$, we have $\sin(x) > 0$.
- $f(2) = \sin(2) - 4 + 2 = \sin(2) - 2 < 0$ because $\sin(1) < 1 < 2$.

All the hypothesis of the IVP are satisfied. We therefore conclude that there is come c, between 1 and 2, such that $f(c) = 0$. This means that

$$\sin(c) - c^2 + c = 0 \quad \iff \quad \sin(c) = c^2 - c$$

for some c such that $1 < c < 2$.

Total (Points): 50.