# Chapter 2 Derivatives

2.3 Differentiation Formulas

$$f(x) = c - D \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = 0$$

**Derivative of a Constant Function** 

$$\frac{d}{dx}(c) = 0$$

#### Power Functions.

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

**EXAMPLE 4** Find the points on the curve  $y = x^4 - 6x^2 + 4$  where the tangent line is horizontal.

Groad: Find where 
$$y'$$
 is zero.

 $y' = \lim_{h \to 0} \frac{(x+h)^4 - (e(x+h)^2 + 4 - x^4 + 6x^2 - 4)}{h}$ 
 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4 - (e(x+h)^2 + 6x^2 + 4 - 4)}{h}$ 
 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4 + \lim_{h \to 0} - (e(x+h)^2 - x^2)}{h}$ 
 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4 + \lim_{h \to 0} - (e(x+h)^2 - x^2)}{h}$ 
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 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4 + \lim_{h \to 0} - (e(x+h)^4 - x^4)}{h$ 

Multiplication by a constant.

**The Constant Multiple Rule** If c is a constant and f is a differentiable function,

$$\overbrace{\frac{d}{dx}}[cf(x)] = c \frac{d}{dx}f(x)$$

Sum.

**The Sum Rule** If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Difference.

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

# Product.

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Caution!!!

$$\frac{d}{dx}(fg) \neq \frac{d}{dx}(f)\frac{d}{dx}(g).$$

Example.

Example. Find the derivative of the function  $f(x) = (5x^2 - 2)(x^3 + 3x)$ .

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

Caution!! Example. 
$$\frac{d}{dx}\left(\frac{f}{g}\right) \neq \frac{\frac{d}{dx}(f)}{\frac{d}{dx}(g)}$$

**EXAMPLE 8** Let  $y = \frac{x^2 + x - 2}{x^3 + 6}$ . Compute the derivative.

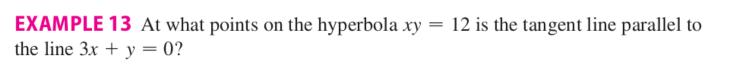
General Power rule.

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

Case n = 0:

Example. Find the derivative of the function  $\ f(x)=x^{2/3}$  .



## Summary of Differentiation Formulas.

### **Table of Differentiation Formulas**

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf'$$

$$(f+g)' = f' + g'$$

$$(f-g)' = f' - g'$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$