
Problem 8

The function $f(z) = \frac{e^{iz}}{z - 1/4 - 4i/5}$ is analytic on $\mathbb{C} \setminus \{1/4 + 4i/5\}$. A complex number $z = x + iy$ is in the interior of the triangle if and only if $x + y \leq 1$ and $x, y \geq 0$.

Here, we have

$$\frac{1}{4} + \frac{4}{5} = \frac{5 + 16}{20} = \frac{21}{20} > 1$$

and hence $z = 1/4 + 4i/5$ is not inside the triangle. The function f is therefore analytic on an open set containing the curve and the inside of the triangle and from Cauchy's Theorem, we get

$$\int_{\gamma} \frac{e^{iz}}{z - \frac{1}{4} - \frac{4}{5}i} = 0.$$

Problem 10

Notice that all of the functions are analytic on $\mathbb{C} \setminus \{2\}$, $\mathbb{C} \setminus \{-2\}$, $\mathbb{C} \setminus \{2i\}$, and $\mathbb{C} \setminus \{-2i\}$. The unit circle and its interior are contained in all of these regions. Hence, by Cauchy's Theorem, each integral should be zero.