Chapter 2

Derivatives

2.9 Linear Approximations and Differentials.

An observation:

A curve $y = f(x)$ lies very close to its tangent line near the point of tangency. Linearization

https://www.desmos.com/calculator/1sp51krlae

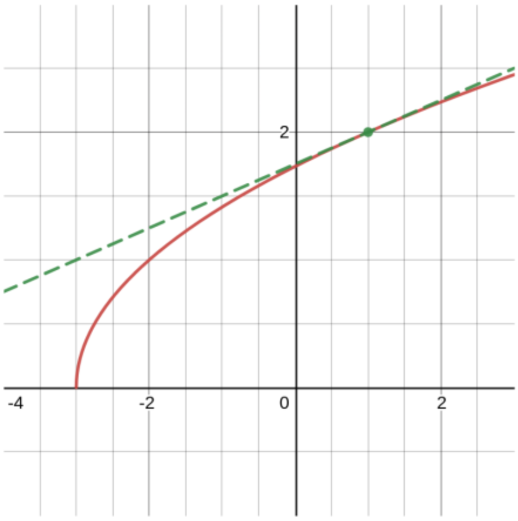


Figure: Linearization near the point of tangency

This suggests to approximate the values of f by the tangent line. This is a really useful procedure because $f(x)$ may be difficult to compute!

Approximation by the tangent line:

$$f(x) \approx f(a) + f'(a)(x - a)$$

So the linearization is

$$L(x) = f(a) + f'(a)(x - a)$$

EXAMPLE 1 Find the linearization of the function $f(x) = \sqrt{x + 3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$. Are these approximations overestimates or underestimates?

Differentials.

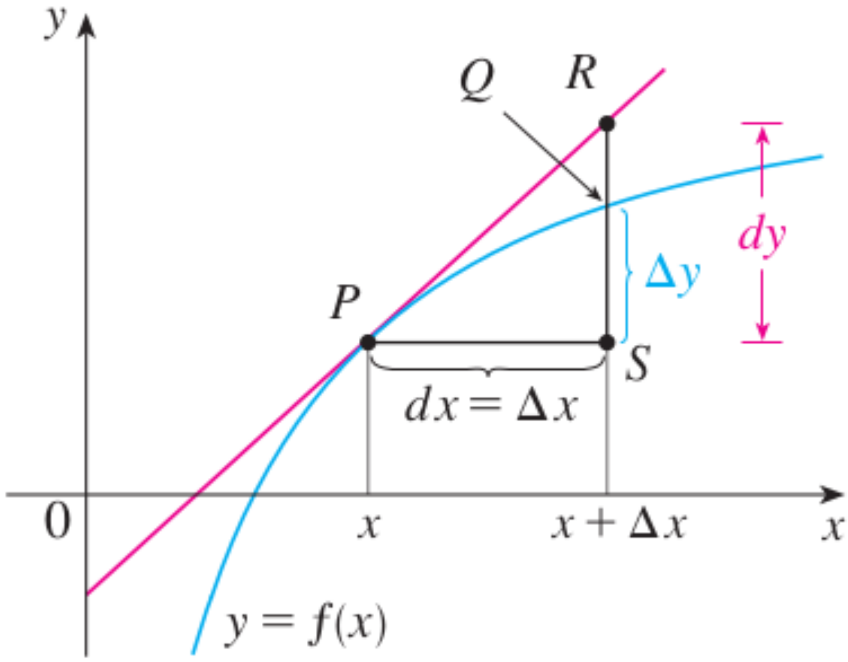
If $y = f(x)$, then

- dx is the differential of x. It's a little increment in the variable x.
- dy is the differential of y and dy is the approximate increment in the variable y given by

$$\boxed{dy = f'(x)dx.}$$

Remark:

Geometric interpretation.



EXAMPLE 3 Compare the values of Δy and dy if $y = f(x) = x^3 + x^2 - 2x + 1$ and x changes (a) from 2 to 2.05 and (b) from 2 to 2.01.

EXAMPLE 4 The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of at most 0.05 cm. What is the maximum error in using this value of the radius to compute the volume of the sphere?

Relative Error.

$$\frac{\Delta V}{V} \approx$$