Problem 2

Don't get confused, π is a constant (it does not depend on x). So π^2 is a constant. Then $f'(x) = 0$.

Problem 4

Using the fact that the derivative of a sum is the sum of the derivatives and the power rule, we have

$$\begin{aligned} g'(x) = \frac{dg}{dx} &= \frac{d}{dx}\left(\frac{7}{4}x^2\right) - 3\frac{d}{dx}(x) + \frac{d}{dx}(12) \\ &= (7/4)2x - 3(1) + 0 \\ &= (7/2)x - 3. \end{aligned}$$

Therefore, $g'(x) = (7/2)x - 3$.

Problem 18

Using the quotient rule, we obtain

$$y' = \frac{(\sqrt{x} + x)'x^2 - (\sqrt{x} + x)(x^2)'}{x^4}.$$

We have, from the power rule,

$$(\sqrt{x} + x)' = 1/2\sqrt{x} + 1 \quad \text{and} \quad (x^2)' = 2x$$

and so replacing that in y', we obtain

$$y' = \frac{(1/2\sqrt{x} + 1)x^2 - (\sqrt{x} + x)2x}{x^4} = \frac{x^{3/2}/2 + x^2 - 2x^{3/2} - 2x^2}{x^4} = \frac{-3x^{3/2}/2 - x^2}{x^4}.$$

Finally, we get $y' = -3x^{-5/2}/2 - x^{-2}$.

There is another approach. By letting $x \neq 0$, we can rewrite the expression as

$$y = x^{-3/2} + x^{-1}$$

and by the sum and quotient rules, we obtain

$$y' = -3x^{-5/2}/2 - x^{-2}.$$

Problem 26

Using the product rule, we have

$$B'(u) = (2u^2 - 4u - 2)\frac{d}{du}(u^3 + 1) + (u^3 + 1)\frac{d}{du}(2u^2 - 4u - 2).$$

Then, using the sum rule and the power rule for derivatives, we get

$$\frac{d}{du}(u^3 + 1) = 3u^2$$

and

$$\frac{d}{du}(2u^2 - 4u - 2) = 4u - 4.$$

Plugging in back in $B'(u)$, we get

$$\begin{aligned} B'(u) = (2u^2 - 4u - 2)3u^2 + (u^3 + 1)(4u - 4) &= 6u^4 - 12u^3 - 6u^2 + 4u^4 - 4u^3 + 4u - 4 \\ &= 10u^4 - 16u^3 - 6u^2 + 4u - 4. \end{aligned}$$

Therefore, $B'(u) = 10u^4 - 16u^3 - 6u^2 + 4u - 4$.

Problem 30

Using the quotient rule, we get

$$\begin{aligned} h'(t) &= \frac{(6t-1)\frac{d}{dt}(6t+1) - (6t+1)\frac{d}{dt}(6t-1)}{(6t-1)^2} \\ &= \frac{(6t-1)6 - (6t+1)6}{(6t-1)^2} \\ &= -\frac{36}{(6t-1)^2}. \end{aligned}$$

Problem 54

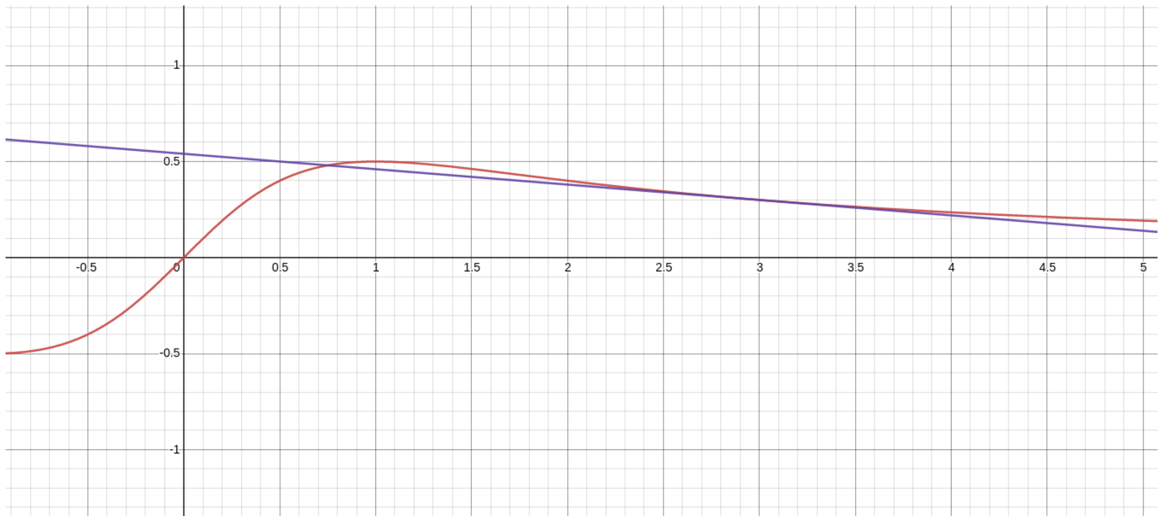
(a) We first find the derivative. We get

$$y' = \frac{1 + x^2 - 2x^2}{1 + x^2} = \frac{1 - x^2}{(1 + x^2)^2}.$$

The equation of the tangent line is given by the equation $y - 0.3 = y'(3)(x - 3)$. So, pugging in the numbers, with $y' = -8/10 = -4/5$, we obtain

$$y - 0.3 = (-4/5)(x - 3) \iff y = -0.8x + 2.4.$$

(b) Using Desmos, we get the following picture.



Problem 58

The equation of the tangent line at $(4, 0.4)$ is

$$y - 0.4 = f'(0.4)(x - 4).$$

We have $f(x) = \sqrt{x}/(x+1)$. Using the quotient rule, we get

$$f'(x) = \frac{\frac{x+1}{2\sqrt{x}} - \sqrt{x}}{(x+1)^2} = \frac{1-x}{2\sqrt{x}(x+1)^2}.$$

Therefore, we have $f'(4) = -0.03$. Plugging this into the equation of the tangent line and after simplifying, we get

$$y = 0.52 - 0.03x.$$

Problem 66

Using the power rule for derivatives, we see that

$$S'(A) = (0.882)(0.842)A^{-0.158} = (0.742644)A^{-0.158}.$$

Using the formula for $S'(A)$, we find that

$$S'(100) = (0.882)(0.842)(100)^{-0.158} \approx 0.35874 \text{ trees/m}^2$$

where m^2 means square meters.

Problem 72

Since the function $f(x) = x$ is not zero at $x = 2$, we can use the quotient rule. We obtain

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right) = \frac{h'(x)x - h(x)}{x^2}$$

and then, at $x = 2$, we get

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} = \frac{h'(2) \times 2 - h(2)}{4} = \frac{(-3) \times 2 - 4}{4} = -5/2.$$