University of Hawai'i

MATH-241 Calculus I
Midterm 03

Created by Pierre-O. Parisé
Fall 2021, 12/01/2021

Last name: ____________________
First name: ____________________

Question:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

Instructions: Make sure to write your complete name on your copy. You must answer all the questions below and write your answers directly on the questionnaire. At the end of the 50 minutes, hand out your copy.

No devices such as a smart phone, cell phone, laptop, or tablet can be used during the exam. You are not allowed to use the lecture notes, the textbook, or any other notes.

You must show ALL your work to have full credit. An answer without justification worth no point.

Good luck!

Pierre-Olivier Parisé

Your Signature: ____________________

Question 1 **(10 pts)**

(a) (5 points) Compute the sum $\sum_{i=3}^{7}(i-2)$.

Solution: We replace i by 3, 4, 5, 6, and 7 respectively:

$$\sum_{i=3}^{7}(i-2) = 1+2+3+4+5 = 15.$$

(b) (5 points) Express the limit as a definite integral:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\sqrt{\frac{2i}{n}}.$$

Solution: We see that $\Delta x = (b-a)/n = 2/n$ and $a + i\delta x = 2i/n$ and so $a = 0$, $b = 2$, and $f(x) = \sqrt{x}$. We then obtain

$$\int_0^2 \sqrt{x}\,dx.$$

Question 2 **(10 pts)**

Evaluate the following definite integrals.

(a) (5 points) $\int_0^1 (u+2)(u-3)\,du$.

Solution: We have $(u+2)(u-3) = u^2 - u - 6$. So

$$\int_0^1 u^2 - u - 6\,du = \left(\frac{u^3}{3} - \frac{u^2}{2} - 6u\right)\Bigg|_0^1 = -\frac{37}{6}.$$

(b) (5 points) $\int_1^2 x(x^2-1)^3\,dx$.

Solution: Let $u = x^2 - 1$ then $du = 2xdx$ and so $xdx = du/2$. Then we get

$$\int_1^2 x(x^2-1)^3\,dx = \int_0^3 u^3 \frac{du}{2} = \frac{1}{2}\left(\frac{u^4}{4}\right)\Bigg|_0^3 = \frac{81}{8}.$$

QUESTION 3 **(10 pts)**

The acceleration of a mo'o is $a(t) = 3t^2 - 2t$ on the period $0 \leq t \leq 4$. Answer the following questions using the integral.

(a) (5 points) Find the velocity $v(t)$ of the mo'o if $v(0) = 0$.

Solution: From the properties of the integral, we have

$$\int 3t^2 - 2t\,dt = t^3 - t^2 + C.$$

Since $v(0) = 0$, we get $C = 0$. So $v(t) = t^3 - t^2$.

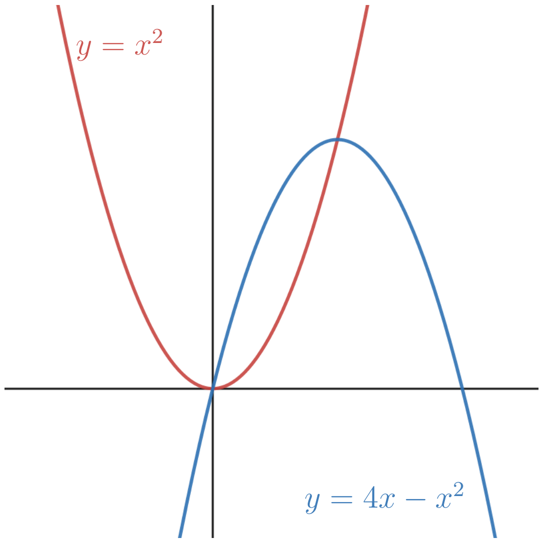
(b) (5 points) Find the displacement of the mo'o during this period of time (from 0 to 4 seconds).

Solution: The displacement is given by

$$\int_0^4 v(t) = \frac{t^4}{4} - \frac{t^3}{3}\bigg|_0^4 = 4^3 - \frac{4^3}{3} = \frac{3 \cdot 64 - 64}{3} = \frac{128}{3}.$$

Question 4 (10 pts)

Find, using the integral, the area of the region enclosed by the curves $y = x^2$ and $y = 4x - x^2$.

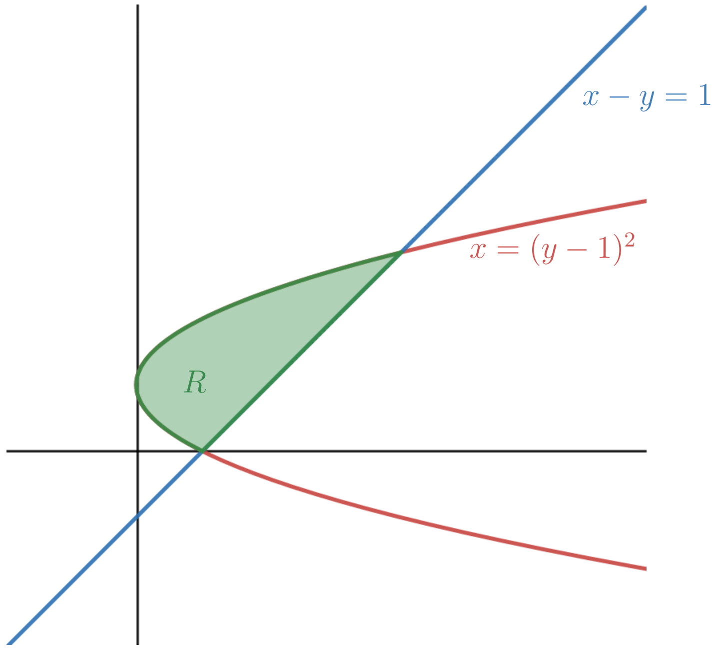


Solution: We have $x^2 = 4x - x^2$ if $x^2 = 2x$ and so if $x = 0$ or $x = 2$. The curve $y = 4x - x^2$ lies above the curve $y = x^2$ when x is between 0 and 2. So the area enclosed by the two curves is

$$\int_0^2 4x - x^2 - x^2\, dx = \left(2x^2 - \frac{2}{3}x^3\right)\Big|_0^2 = 8 - \frac{16}{3} = \frac{8}{3}.$$

Question 5 (10 pts)

Let R be the region enclosed by the curves $x = (y-1)^2$ and $x - y = 1$. The region R is illustrated below in green.



Find the volume of the solid obtained by rotating the region R about the line $x = -1$. You choose one of the method seen in class: the method with disks/washers or the method with cylindrical shells.

Solution: First we see that the intersects between the curves are $y = 0$ and $y = 3$ because we have $x = (y-1)^2$ and $x = y+1$, so $(y-1)^2 = y+1$. Solving this equation for y gives

$$y^2 - 2y + 1 = y + 1 \iff y^2 - 3y = 0 \iff y = 0 \text{ or } y = 3.$$

Let's denote by $x_1 = (y-1)^2$ and $x_2 = (y+1)$.

We will use the washer method. The inner radius is

$$r_{in} = 1 + x_1 = 1 + (y-1)^2.$$

The outer radius is

$$r_{out} = 1 + x_2 = 1 + y + 1 = y + 2.$$

So the area function $A(y)$ is

$$A(y) = \pi r_{out}^2 - \pi r_{in}^2 = \pi(y+2)^2 - \pi(1 + (y-1)^2)^2.$$

So the volume of the solid is

$$V = \int_0^3 A(y)\, dy = 117\pi/5.$$