MATH-331 Intro. to Real Analysis | Created by Pierre-O. Parisé
Midterm 01 | Fall 2021, 10/01/2021

Last name: ______________________________

First name: ______________________________

Question:	1	2	3	4	Total
Points:	10	10	10	10	40
Score:					

Instructions: Make sure to write your complete name on your copy. You must answer all the questions below and write your answers directly on the questionnaire. At the end of the 50 minutes, hand out your copy.

No devises such as a smart phone, cell phone, laptop, or tablet can be used during the exam. You are not allowed to use the lecture notes and the textbook also.

Make sure to show all your work. State clearly any theorem or definition you are using in your proofs or your calculations.

Good luck!

Pierre-Olivier Parisé

Question 1 **(10 pts)**

Let $S \subseteq \mathbb{R}$ be a subset of real numbers bounded from below.

(a) (5 points) Prove that S has an infimum.

(b) (5 points) Let $x := \inf S$. Prove that for each $\varepsilon > 0$, there exists an element $s \in S$ such that $x \leq s < x + \varepsilon$.

Question 2 **(10 pts)**

Show that if a sequence $(a_n)_{n=0}^{\infty}$ is a Cauchy sequence, then $(a_n^2)_{n=1}^{\infty}$ is a Cauchy sequence.

Question 3 **(10 pts)**

Let $(a_n)_{n=1}^{\infty}$ be a sequence of non-negative real numbers ($a_n \geq 0$, for any $n \geq 1$). Define the sequence $b_n := a_1 + a_2 + \cdots + a_n$. Show that if $(b_n)_{n=1}^{\infty}$ is bounded from above, then the sequence (b_n) converges.

Question 4 **(10 pts)**

Let $(a_n)_{n=1}^{\infty}$ be the sequence of non-negative real numbers defined recursively by

$$a_n = \sqrt{a_{n-1}} + \sqrt{a_{n-2}} \quad (n \geq 3)$$

Assume that the sequence (a_n) converges to a limit A, that is $a_n \to A$ for some $A \in \mathbb{R}$. Find the possible values of A.