Chapter 2 Derivatives

2.3 Differentiation Formulas

$$f(x) = c - D \qquad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{c - c}{h} = 0$$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

Power Functions.

The Power Rule If n is a positive integer, then

$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

EXAMPLE 4 Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal. $y' = (x^4)' - 6(x^2)' + (4)'$ $= 4x^3 - 12x + 0$

Goal: Find while y' is zero. =
$$4x^3 - 12x + 0$$
 $y' = \lim_{h \to 0} \frac{(x+h)^4 - (e(x+h)^2 + 4 - x^4 + 6x^2 - 4)}{h}$
 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4 - (e(x+h)^2 + 6x^2 + 4 - 4)}{h}$
 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h} + \lim_{h \to 0} - (e(x+h)^2 - x^2)$
 $+ \lim_{h \to 0} \frac{4-4}{h}$
 $= \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h} - \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} + \lim_{h \to 0} \frac{4-4}{h}$

$$= \frac{d}{dx}(x^4) - 6 \frac{d}{dx}(x^2) + \frac{d}{dx}(4)$$

$$= 4x^3 - 6.2x + 0$$

$$\Rightarrow y' = 4x^3 - 12x$$

$$y' = 0 + 0 + 1x^3 - 17x = 0$$

$$(4x^2 - 12) = 0$$

$$4-D \left(4\chi^{2}-12\right) \chi = 0$$

$$4-D \left(2\chi^{2}-12\right) \chi = 0$$

$$\chi = 2\sqrt{3}$$

Multiplication by a constant.

The Constant Multiple Rule If c is a constant and f is a differentiable function,

$$\overbrace{\frac{d}{dx}}[cf(x)] = c \frac{d}{dx}f(x)$$

Sum.

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Difference.

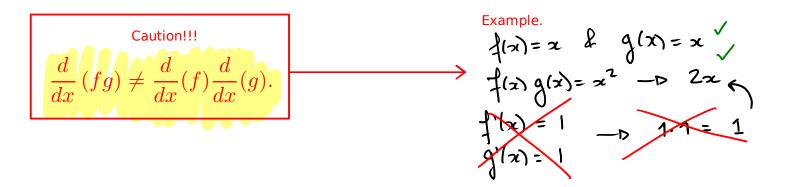
The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Product.

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$



Example. Find the derivative of the function $f(x) = (5x^2 - 2)(x^3 + 3x)$.

$$f'(x) = (5x^{2}-2) \frac{d}{dx} (x^{3}+3x) + (x^{3}+3x) \frac{d}{dx} (5x^{2}-2)$$

$$= (5x^{2}-2) (3x^{2}+3) + (x^{3}+3x) (10x-0)$$

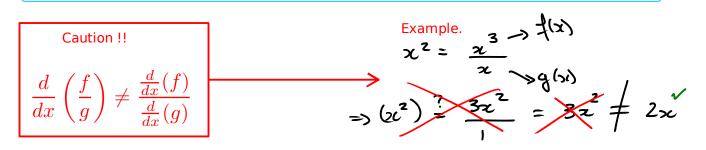
$$= (5x^{2}-2) (3x^{2}+3) + 10x (x^{3}+3x).$$

$$L_{b} = 15x^{4} + 39x^{2} - 6$$

Quotient.

The Quotient Rule If
$$f$$
 and g are differentiable, then

$$\frac{d}{dx} \left[\underbrace{\frac{f(x)}{g(x)}} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$



EXAMPLE 8 Let
$$y = \frac{x^2 + x - 2}{x^3 + 6}$$
. Compute the derivative.

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$$f(x) = x^2 + x - 2 \qquad \qquad f'(x) = 2x + 1$$

$$g(x) = x^3 + 6 \qquad \qquad g'(x) = 3x^2$$

$$y' = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2}$$

$$= \frac{(x^3 + b)(2x + 1) - (x^2 + x - 2)(3x^2)}{(x^3 + b)^2}$$

$$= \frac{2x^4 + x^3 + 12x + 6 - 3x^4 - 3x^3 + 6x^2}{(x^3 + 6)^2}$$

$$= \frac{-x^{4} - 2x^{3} + 6x^{2} + 17x + 6}{(x^{3} + 6)^{2}}$$

The Power Rule (General Version) If n is any real number, then $(n \neq 0)$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Case n = 0:

$$\frac{d}{dx}(x^{\circ}) = \frac{d}{dx}(1) = 0$$

Example. Find the derivative of the function
$$f(x) = x^{2/3}$$
.
$$f'(x) = \frac{2}{3}x^{-1} = \frac{2}{3}x^{-1/3}$$
$$= \frac{2}{3}x^{1/3}$$

$$\frac{2}{3} - 1 = \frac{2}{3} - \frac{3}{3}$$

$$= -\frac{1}{3}$$

$$g(x) = x^{-1/2}$$
 $-x$ $g'(x) = \left(-\frac{1}{2}\right) x^{\frac{-1}{2}-1} = -\frac{1}{2} x^{-\frac{3}{2}}$

EXAMPLE 13 At what points on the hyperbola xy = 12 is the tangent line parallel to the line 3x + y = 0?

Goal: Find x p.t.
$$f'(x)$$
 is equal to the slope of $3x+y=0$.

If
$$u_1$$
 $3x+y=0 y = -3x -1> slope = -3.$

Hu, $y = \frac{12}{x}$ $-> y' = -12x^{-1-1} = -12x^{-2}$
 $= 12x^{-1}$ $\Rightarrow y' = -\frac{12}{x^2}$

Now,
$$\frac{-12}{x^2} = y' = -3$$

$$= 3(x^{2})^{2} - 12 = -3 \Rightarrow 4 = x^{2} \Rightarrow x = \pm 2$$

Answer: the tangent is parallel to
$$3x+y=0$$
 at $(-z,-6)$ & $(2,6)$.

Summary of Differentiation Formulas.

Table of Differentiation Formulas

$$\frac{d}{dx}(c) = 0 \qquad \qquad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$(cf)' = cf' \qquad \qquad (f+g)' = f'+g' \qquad \qquad (f-g)' = f'-g'$$

$$(fg)' = fg' + gf' \qquad \qquad \left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$