MATH-241 Calculus 1 | Created by P.-O. Parisé
Homework 05 | Fall 2021

Assigned date: 11/08/2021 9am
Due date: 11/15/2021 5pm

Last name: Solution.
First name:
Section:

Question:	1	2	3	4	5	6	Total
Points:	20	20	20	10	20	10	100
Score:							

Instructions: You must answer all the questions below and upload your solutions (in a PDF format) to Gradescope (go to `www.gradescope.com` with the Entry code GEK6Y4). Be sure that after you scan your copy, it is clear and readable. You must name your file like this: `LASTNAME_FIRSTNAME.pdf`. A homework may not be corrected if it's not readable and if it's not given the good name. No other type of files will be accepted (no PNG, no JPG, only PDF) and no late homework will be accepted.

Make sure to show all your work!

Good luck!

Question 1 **(20 points)**

Express the limits as a definite integral on the given interval. The points x_i are the right endpoint of your subintervals.

(a) (10 points) $\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=1}^{n} \frac{\sin x_i}{1+x_i}$ and $[0, \pi]$ $(\Delta x = \pi/n)$.

(b) (10 points) $\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} x_i\sqrt{1+x_i^3}$ and $[2, 5]$ $(\Delta x = 3/n)$.

(a) Here $f(x) = \dfrac{\sin x}{1+x}$. So, we get

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sin x_i}{1+x_i} \frac{\pi}{n} = \int_0^{\pi} \frac{\sin x}{1+x}\, dx$$

(b) Here, $f(x) = x\sqrt{1+x^3}$. So, we get

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i\sqrt{1+x_i^3} \cdot \frac{3}{n} = \int_2^5 x\sqrt{1+x^3}\, dx.$$

Question 2 **(20 points)**

Find the most general antiderivative of the following functions.

(a) (5 points) $f(x) = 4x + 7$

(b) (5 points) $f(x) = 7x^{2/5} + 8x^{-4/5}$

(c) (5 points) $f(u) = 3\cos u - 4\sin u$

(d) (5 points) $f(t) = 1 - \sec t \tan t$.

(a) $F(x) = 2x^2 + 7x + C$.

(b) $F(x) = \dfrac{7x^{7/5}}{7/5} + \dfrac{8x^{1/5}}{1/5} + C$

$= 5x^{7/5} + 40x^{1/5} + C$.

(c) $F(u) = 3\sin u + 4\cos u + C$.

(d) $F(t) = t - \sec t + C$.

QUESTION 3 **(20 points)**

Evaluate the following definite integrals.

(a) (10 points) $\int_{\pi/4}^{\pi/3} \csc^2\theta \, d\theta$.

(b) (10 points) $\int_1^2 \frac{v^5+3v^6}{v^4} \, dv$.

(a) An Antiderivative of $\csc^2\theta$ is

$$F(\theta) = \cot\theta.$$

So, by FTC, we have

$$\int_{\pi/4}^{\pi/3} \csc^2\theta \, d\theta = F(\pi/3) - F(\pi/4)$$

$$= \cot(\pi/3) - \cot(\pi/4)$$

$$\boxed{= \frac{1}{\sqrt{3}} - 1.}$$

(b) We see that $\frac{v^5+3v^6}{v^4} = v + 3v^2$.

Then, an antiderivative is

$$F(v) = \frac{v^2}{2} + v^3.$$

So,

$$\int_1^2 \frac{v^5+3v^6}{v^4} \, dv = \int_1^2 v + 3v^2 \, dv = F(2) - F(1).$$

So, $\int_1^2 \frac{v^5+3v^6}{v^4} \, dv = 2 + 8 - \frac{1}{2} - 1 = \boxed{8.5}$

QUESTION 4 **(10 points)**

Let $F(x) = \int_1^x f(t)\,dt$, where f is the function whose graph is shown below. Where is F concave downward?

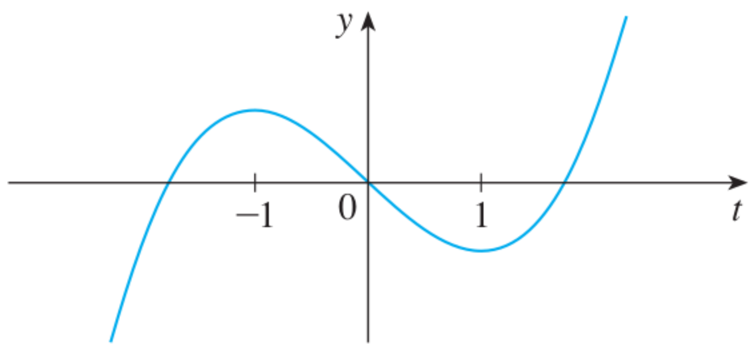

Figure 1: Graph of the function $f(x)$

By the fundamental Theorem of calculus, we know that

$$F'(x) = f(t).$$

So,

- $(-\infty,-1)$, & $(1,\infty)$ the slopes to the graph of f' are positive $\to f'' > 0$
- $(-1,1)$, the slopes to the graph of f' are negative. $\to f'' < 0$.

So, f is concave upward on $(-\infty,-1)$ & $(1,\infty)$.

So, f is concave downward on $(-1,1)$.

Question 5 (20 points)

Compute the following indefinite integrals.

(a) (5 points) $\int \sqrt{t}(t^2 + 3t + 2)\, dt$.

(b) (5 points) $\int u^2 + 1 + \frac{1}{u^2}\, du$.

(c) (5 points) $\int \frac{1 - \sin^3 t}{\sin^2 t}\, dt$.

(d) (5 points) $\int \sec t(\sec t + \tan t)\, dt$.

(a) $\int \sqrt{t}\,(t^2+3t+2)dt = \int t^{1/2}\cdot t^2 + 3t^{1/2}t + 2t^{1/2}\,dt$

$= \int t^{5/2} + 3t^{3/2} + 2t^{1/2}\,dt$

$= \frac{2t^{7/2}}{7} + \frac{6}{5}t^{5/2} + \frac{4}{3}t^{3/2} + C$

(b) $\int u^2+1+u^{-2}\,du = \frac{u^3}{3} + u - u^{-1} + C$

(c) $\int \frac{1-\sin^3 t}{\sin^2 t}\,dt = \int \frac{1}{\sin^2 t} - \sin t\,dt$

$= \text{cotan}(t) + \cos t + C.$

(d) $\int \sec t(\sec t + \tan t)\,dt = \int \sec^2 t + \sec t \tan t\,dt$

$= \tan t + \sec t + C.$

Question 6 (10 points)

Evaluate the following definite integrals.

(a) (5 points) $\int_0^4 x\sqrt{x^2+16}\,dx$.

(b) (5 points) $\int_{1/2}^1 \frac{\cos(\pi x^{-2})}{x^3}\,dx$.

(a) Put $u = x^2+16 \rightarrow du = 2x\,dx \rightarrow x\,dx = \frac{du}{2}$.

So

$$\int_0^4 x\sqrt{x^2+16}\,dx = \int_0^4 \sqrt{x^2+16}\;\underbrace{x\,dx}_{\downarrow}$$

$$= \int_{16}^{32} \sqrt{u}\;\frac{du}{2}$$

$$= \frac{2u^{3/2}}{3}\Bigg|_{16}^{32} = \frac{2(32)^{3/2}}{3} - \frac{2\cdot 4^3}{3}$$

$$\Rightarrow \int_0^4 x\sqrt{x^2+16}\,dx \;\boxed{\cong 78.0129.}$$

(b) Put $u = \frac{1}{x^2} \rightarrow du = \frac{-2}{x^3}dx \rightarrow -\frac{du}{2} = \frac{dx}{x^3}$.

So,

$$\int_{1/2}^1 \frac{\cos(\pi x^{-2})}{x^3}\,dx = \int_4^1 \cos(\pi u)\;-\frac{du}{2}$$

$$= \frac{1}{2}\int_1^4 \cos(\pi u)\,du$$

$$= \frac{1}{2}\;\frac{\sin(\pi u)}{\pi}\Bigg|_1^4 \;\boxed{= 0.}$$