SAMPLE MIDTERM FALL 2022

1. (15 points) Let $f(x) = x^3 - x$.

(a) List the intervals where the graph of $f(x)$ is increasing and decreasing.

(b) Find the local maximum and minimum values of $f(x)$.

(c) List the intervals where the graph is concave up and concave down.

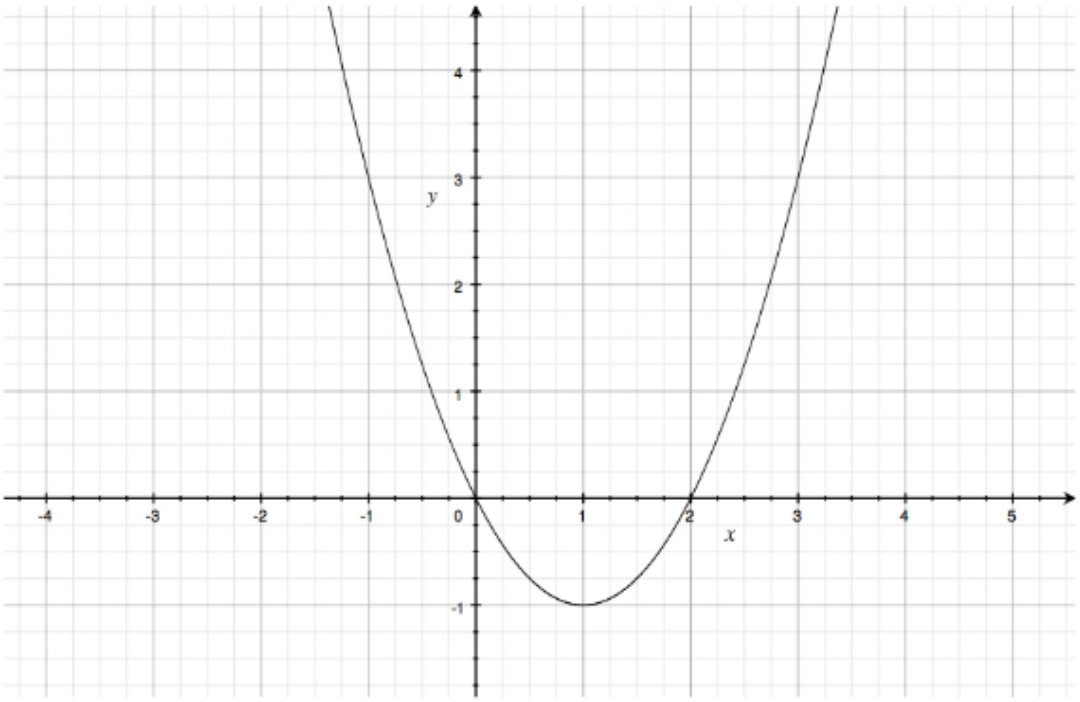
(d) Sketch the graph of the function.

2. (10 points) Emma wants to enclose a rectangular field with total area $200m^2$. Along one side of the field, she will use a pre-existing straight wall, but on the other three sides, she needs to buy fence.

   If it costs \$2 for each meter of fence, what is the least amount she can spend to enclose her field? (Simplify your answer.)

3. (7 points) A right triangle is changing shape. If the base is 3 meters and expanding at 0.2 meters per minute, and the height is 4 meters and shrinking at 0.1 meters per minute, at what rate is the length of the hypotenuse changing?

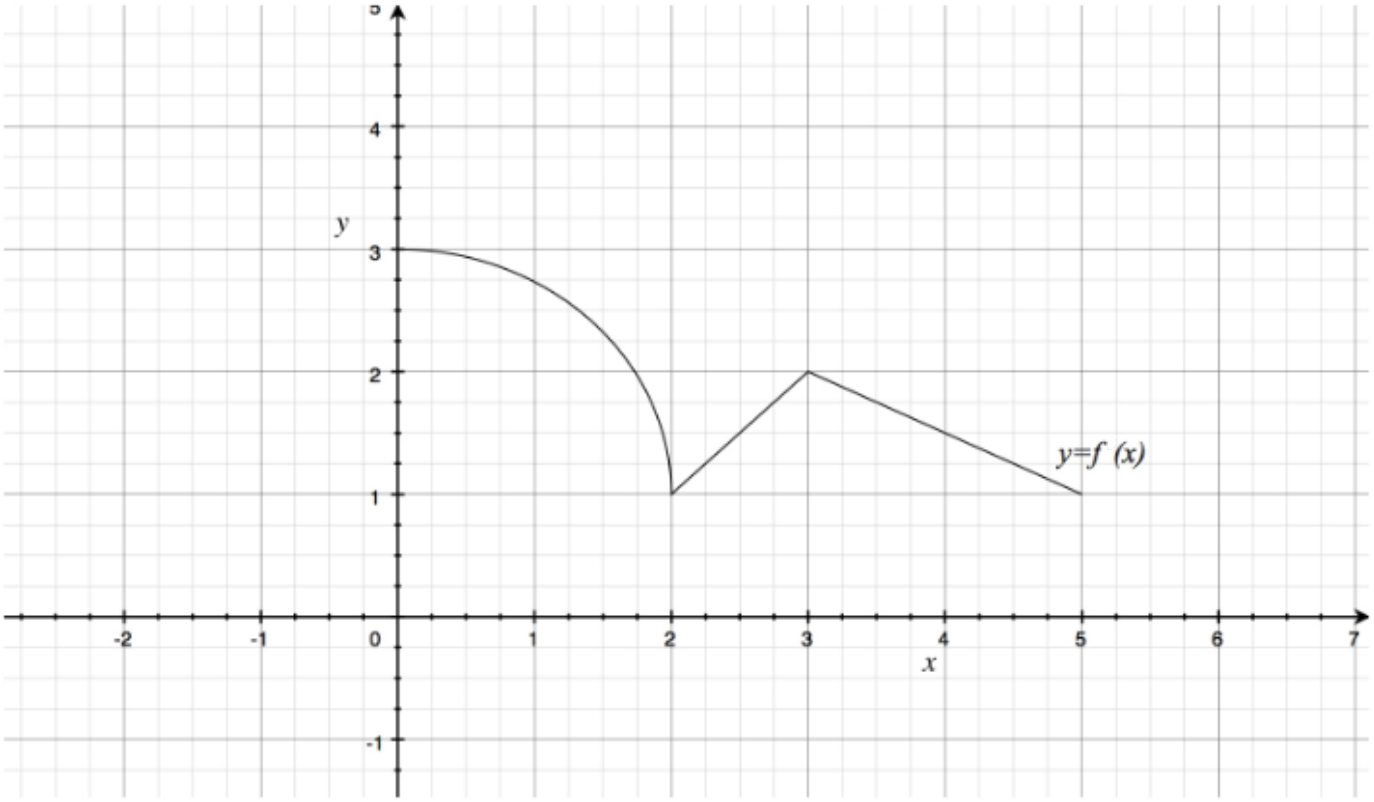
4. Consider the function $f(x) = x^2 - 2x$ as pictured below.



(a) (6 points) Compute a Riemann sum for this function that approximates the integral $\int_1^3 f(x)dx$. Use four equal-width intervals for your Riemann sum, and use the right endpoint of each interval to determine the height of the corresponding rectangle. You do not have to simplify your answer.

(b) (2 points) Sketch the rectangles that correspond to part (a) on the graph above.

5. A function f of a single variable x is defined on the interval $[0, 5]$
The following picture shows the graph of $f(x)$.



In the picture, the portion of the graph on the interval $(0, 2)$ identifies with a quarter-circle of radius 2 and center $(0, 1)$; the portions of the graph on the intervals $[2, 3]$ and $[3, 5]$ are line segments.

(a) (2 points) What is the value of $\int_0^0 f(x)\,dx$?

(b) (4 points) What is the value of $\int_0^3 f(x)\,dx$?

(c) (4 points) What is the value of $\int_3^5 f(x)\,dx$?

6. (6 points) Use linear approximation to estimate the number $(.95)^{10}$.

7. Compute the following limits.

(a) $\lim_{x\to\infty} \dfrac{4-7x^2}{(x+5)^2}$.

(b) $\lim_{x\to\infty} \dfrac{7-\sqrt{x}}{7+\sqrt{x}}$.

8. Answer the following.

(a) Given that $\frac{1}{2} \leq \frac{x}{x+1} \leq \frac{2}{3}$ for any x such that $1 \leq x \leq 2$, give an estimate of the following integral:

$$\int_1^2 \frac{x}{x+1}\, dx.$$

(b) Find the value of the following integral by intepreting it geometrically:

$$\int_0^1 \sqrt{1-x^2}\, dx.$$