

# Chapter 4

## Integrals

4.4 Indefinite Integrals and the Net Change Theorem

# Indefinite Integral.

Previously on Calc I:

Fondamental Theorem  
of Calculus

$$\int_a^b f(x) dx = F(b) - F(a).$$

We introduce a notation for the antiderivatives:

$$\int f(x) dx = F(x) \quad \text{means} \quad F'(x) = f(x)$$

**Example.**

a)  $\int x^2 dx =$

b)  $\int \cos x dx =$

c)  $\int \sec^2 x dx =$

## Table of Indefinite integrals

$$\int cf(x) dx = c \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

Remark: We adopt the convention that the general indefinite integral is valid on a given interval. Thus we write

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

with the understanding that it is valid on the interval  $(0, \infty)$  or on the interval  $(-\infty, 0)$ .

**EXAMPLE 1** Find the general indefinite integral

$$\int (10x^4 - 2 \sec^2 x) dx$$

---

**EXAMPLE 2** Evaluate  $\int \frac{\cos \theta}{\sin^2 \theta} d\theta$ .

**EXAMPLE 4** Find  $\int_0^{12} (x - 12 \sin x) \, dx$ .

---

**EXAMPLE 5** Evaluate  $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \, dt$ .

**Net Change Theorem** The integral of a rate of change is the net change:

$$\int_a^b F'(x) dx = F(b) - F(a)$$

a) Displacement:

b) Total distance traveled:

c) Acceleration:

---

**EXAMPLE 6** A particle moves along a line so that its velocity at time  $t$  is  $v(t) = t^2 - t - 6$  (measured in meters per second).

- (a) Find the displacement of the particle during the time period  $1 \leq t \leq 4$ .  
(b) Find the distance traveled during this time period.