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$$|x_0| = \frac{|z_0| \cdot a^2}{c^2} \Rightarrow x_0 = \pm \frac{z_0 a^2}{c^2}$$

$$|y_0| = \frac{|z_0| b^2}{c^2} \Rightarrow y_0 = \pm \frac{z_0 b^2}{c^2}$$

$$\frac{z_0^2 a^{\cancel{4}}}{c^4 \cancel{a^2}} + \frac{z_0^2 b^{\cancel{4}}}{c^4 \cancel{b^2}} + \frac{z_0^2}{c^2} = 1$$

$$\frac{z_0^2}{c^2}\left(\frac{a^2}{c^2} + \frac{b^2}{c^2} + 1\right) = 1 \ \ldots \ z_0 = \pm \frac{c^2}{\sqrt{a^2+b^2+c^2}}$$

...

$$P = \frac{1}{\sqrt{a^2+b^2+c^2}}\left(\pm a^2, \pm b^2, \pm c^2\right)$$

Seminar 14

1) $S: \begin{cases} x = 4\cos(s)\cos t \\ y = 4\sin s \cos t \\ z = 2\sin t \end{cases}$, $s \in [0, 2\pi)$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right)$

a) give an eq for S

b) find the params of $P(3, \sqrt{3}, 1)$

c) find a param of the tangent plane T_pS using P.

d) find an eq of T_pS

a) $\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} = 1$

b) $\begin{cases} 4\cos s \cos t = 3 \\ 2\cos s \cos t = \sqrt{3} \\ 2\sin t = 1 \Rightarrow \sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6} \Rightarrow \cos t = \frac{\sqrt{3}}{2} \end{cases}$

$\Rightarrow 2\sqrt{3}\cos s = 3 \Rightarrow s = \frac{\pi}{6}$

c) $\sigma(s, t)$

$$\frac{\partial}{\partial s}\sigma(s,t) = (-4\sin s \cos t, \ 4\cos s \cos t, 0)$$

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$\frac{\partial}{\partial t}\sigma(s,t)=(-4\cos s\sin t,\ -4\sin s\sin t,\ 2\cos t)$

$\frac{\partial}{\partial s}\sigma\left(\frac{\pi}{6},\frac{\pi}{6}\right)=\left(-\cancel{2}\frac{\sqrt{3}}{\cancel{2}},3,0\right)$

$\frac{\partial}{\partial t}\sigma\left(\frac{\pi}{6},\frac{\pi}{6}\right)=(-\sqrt{3},-1,\sqrt{3})$

$$T_pS:\begin{cases}x=3+(-\sqrt{3})u+(-\sqrt{3})v\\ y=\sqrt{3}+3u+(-1)v\\ z=1+0u+\sqrt{3}v\end{cases}$$

d) $\begin{vmatrix}x-3 & y-\sqrt{3} & z-1\\ -\sqrt{3} & 3 & 1\\ -\sqrt{3} & -1 & \sqrt{3}\end{vmatrix}=0\ \ldots$

$\cancel{\sqrt{3}x-3\sqrt{3}+\sqrt{3}z}$

14. $H_{2,3,1}:\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{1}=1$

a) Find the tg plane of $H_{2,3,1}$ in the point $M(2,3,1)$.

b) Show that T_MH intersects H in 2 ≠ lines

17. a) Determine the intersection of the surface $P_{2,\frac{1}{2}}: x^2-4y^2=4z$ with the line $l:\begin{bmatrix}2\\0\\3\end{bmatrix}+\left\langle\begin{bmatrix}2\\1\\-2\end{bmatrix}\right\rangle$

b) Write the equations of tangent planes in the intersection points

a) $l:\begin{cases}x=2+2t\\ y=t\\ z=3-2t\end{cases}$

$\Rightarrow (2+2t)^2-4t^2=4(3-2t)$

$4+8t+\cancel{4t^2}-\cancel{4t^2}=12-8t$

$16t=8\Rightarrow t=\frac{1}{2}\Rightarrow\begin{cases}x=3\\ y=\frac{1}{2}\\ z=2\end{cases}$

b) $\frac{3x}{1}-4\cdot\frac{1}{2}y-\cancel{4/2}z-\bar{z}=0\Rightarrow 3x-2y-z=2$

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$|x_0| = \frac{|z_0| \cdot a^2}{c^2} \Rightarrow x_0 = \pm \frac{z_0 a^2}{c^2}$

$|y_0| = \frac{|z_0| b^2}{c^2} \Rightarrow y_0 = \pm \frac{z_0 b^2}{c^2}$

$\frac{z_0^2 a^4}{c^4 \cancel{a^2}} + \frac{z_0^2 b^4}{c^4 \cancel{b^2}} + \frac{z_0^2}{c^2} = 1$

$\frac{z_0^2}{c^2}\left(\frac{a^2}{c^2} + \frac{b^2}{c^2} + 1\right) = 1 \ldots z_0 = \pm \frac{c^2}{\sqrt{a^2+b^2+c^2}}$

...

$P = \frac{1}{\sqrt{a^2+b^2+c^2}}\left(\pm a^2, \pm b^2, \pm c^2\right)$

Seminar 14

1) $S: \begin{cases} x = 4\cos(s)\cos t \\ y = 4\sin s \cos t \\ z = 2\sin t \end{cases}$, $s \in [0, 2\pi)$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right)$

a) give an eq for S

b) find the params of $P(3, \sqrt{3}, 1)$

c) find a param of the tangent plane T_pS using $P\omega$

d) find an eq of T_pS

a) $\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} = 1$

b) $\begin{cases} 4\cos s\cos t = 3 \\ 2\cos s\cos t = \sqrt{3} \\ 2\sin t = 1 \Rightarrow \sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6} \Rightarrow \cos t = \frac{\sqrt{3}}{2} \end{cases}$

$\Rightarrow 2\sqrt{3}\cos s = 3 \Rightarrow s = \frac{\pi}{6}$

c) $\bar{r}(s,t)$

$\frac{\partial}{\partial s}\bar{r}(s,t) = (-4\sin s\cos t, 4\cos s\cos t, 0)$

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$$\frac{\partial}{\partial t}\sigma(s,t)=(-4\cos s\sin t,\ -4\sin s\sin t,\ 2\cos t)$$

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$$T_pS:\begin{cases}x=3+(-\sqrt{3})u+(-\sqrt{3})v\\ y=\sqrt{3}+3u+(-1)v\\ z=1+0u+\sqrt{3}v\end{cases}$$

d) $$\begin{vmatrix}x-3 & y-\sqrt{3} & z-1\\ -\sqrt{3} & 3 & 1\\ -\sqrt{3} & -1 & \sqrt{3}\end{vmatrix}=0\ \dots$$

~~$\sqrt{3}x-3\sqrt{3}+\sqrt{3}z$~~

14. $\mathcal{H}_{2,3,1}: \frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{1}=1$

a) Fing the tg plane of $\mathcal{H}_{2,3,1}$ in the point $M(2,3,1)$.

b) Show that $T_M\mathcal{H}$ intersects $\mathcal{H}$ in 2 $\nparallel$ lines

17. a) Determine the intersection of the surface $P_{2,\frac{1}{2}}: x^2-4y^2=4z$ with the line $\ell: \begin{bmatrix}2\\0\\3\end{bmatrix}+\left\langle\begin{bmatrix}2\\1\\-2\end{bmatrix}\right\rangle$

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a) $$\ell:\begin{cases}x=2+2t\\ y=t\\ z=3-2t\end{cases}$$

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$$16t=8\Rightarrow t=\frac{1}{2}\Rightarrow\begin{cases}x=3\\ y=\frac{1}{2}\\ z=2\end{cases}$$

b) $\frac{3x}{1}-4\cdot\frac{1}{2}y=\cancel{4\cdot}2z-\frac{z}{1}=0 \Rightarrow 3x-2y-z=2$

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$$\frac{x^2}{2} - 2y^2 - 2z = 0$$

$\Rightarrow a = \sqrt{2},\ b = \frac{1}{\sqrt{2}}$

$$\frac{x_0 x}{a} - \frac{y_0 y}{b} - z_0 - z = 0$$

$$\frac{3x}{\sqrt{2}} - \frac{\sqrt{2} y}{2} - 2 - z = 0$$

6.2. $\mathcal{C} : x^2 + y^2 = R^2 \ | : R^2$

$$\underbrace{\frac{x^2}{R^2} + \frac{y^2}{R^2}}_{f(x,y)} = 1$$

$(x_0, y_0) \in \mathcal{C}$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{2}{R^2} x \\ \frac{2}{R^2} y \end{bmatrix} = \frac{2}{R^2} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\nabla f \big|_{(x_0, y_0)} = \frac{2}{R^2} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$\vec{v_0} = \frac{2}{R^2} \begin{bmatrix} y_0 \\ -x_0 \end{bmatrix} \Rightarrow$ this is the tangent vector

8.24.

a) $S : 2xz + 2yz + 2xy = 1$

b) $S : 3x^2 + 3y^2 + xz = 1$

c) $S : 2xy + 2yz + y + z = 1$

Which is a hyperboloid

a) $G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\det (G - I_3 \lambda) = 0$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0 \iff \lambda^3 + 1 + 1 + 3\lambda = 0$$

$$\lambda^3 + 3\lambda + 2 = 0$$

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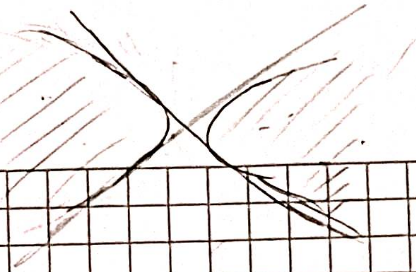

$$\Rightarrow \begin{array}{c|cccc} & \lambda^3 & \lambda^2 & \lambda^1 & \lambda^0 \\ & 1 & 0 & 3 & 2 \\ \hline & & & & \end{array}$$

$\Rightarrow \cancel{x_{1,2}}\ \lambda_{1,2} = -1,\ \lambda_3 = 1 \Rightarrow \mathcal{H}$

b) $S:\ 3x^2 + 3y^2 + xz = 1$

$3\left(x + \frac{1}{6}z\right)^2 + 3y^2 - \frac{1}{12}z^2 = 1$

not $x' = x + \frac{1}{6}z$

$3x'^2 + 3y^2 - \frac{1}{12}z^2 = 1 \Rightarrow \mathcal{H}$

6.19 $\frac{x^2}{4} - \frac{y^2}{9} = 1$

$P(x_P, y_P)$ s.t. P does not belong to any tangent of $\mathcal{H}$

$\cancel{x=}\ y = \frac{b}{a}x$

$y > \frac{b}{a}x$

$y < -\frac{b}{a}y$