Model 1

11.04.2024

Part 1

~~$K = (O, i, j) \to$ right oriented~~

~~$K' = (O', i', j')$~~

~~$[O']_K$~~ ~~P1. $\vec{CA}(-2,-2)$~~

~~$\vec{BA}(-4,-2)$~~

~~$\langle \vec{CA}; \vec{BA} \rangle = +8+4 = 12 \neq 0 \Rightarrow \vec{CA} \not\perp \vec{BA}$~~

P1. $\vec{CA}(2,2)$

$\vec{BA}(4;2)$

$\langle \vec{CA}; \vec{BA} \rangle = 8+4 = 12 \neq 0 \Rightarrow \vec{CA} \not\perp \vec{BA}$

P2. $c \in l$

$\sphericalangle(AB, l) = 45^\circ$

take $v(v_x, v_y)$ d.v. of l

$$l: \begin{cases} x = x_c + t \cdot v_x \\ y = y_c + t \cdot v_y \end{cases}$$

$\vec{AB}(-4;-2)$

$$\cos \sphericalangle(\vec{AB}, v) = \frac{\sqrt{2}}{2} = \frac{\langle \vec{AB}, v \rangle}{|\vec{AB}| \cdot |v|} = \frac{-4v_x - 2v_y}{2\sqrt{5} \cdot \sqrt{v_x^2 + v_y^2}}$$

~~we can take~~

$$2\sqrt{10} \cdot \sqrt{v_x^2 + v_y^2} = -8v_x - 4v_y$$

$$\sqrt{5} \cdot \sqrt{v_x^2 + v_y^2} = -2\sqrt{2} \cdot v_x - \sqrt{2} \cdot v_y$$

$$5v_x^2 + 5v_y^2 = 8v_x^2 + 8v_x v_y + 2v_y^2$$

$$3v_x^2 + 8v_x v_y - 3v_y^2 = 0$$

we can take v_x as $1 \Rightarrow$ $3 + 8v_y - 3v_y^2 = 0$

$3v_y^2 - 8v_y - 3 = 0$

$\Delta = 100$

$v_y = \frac{8 \pm 10}{6} \in \{-\frac{1}{3} ; 3\}$

we can take v_y as 3

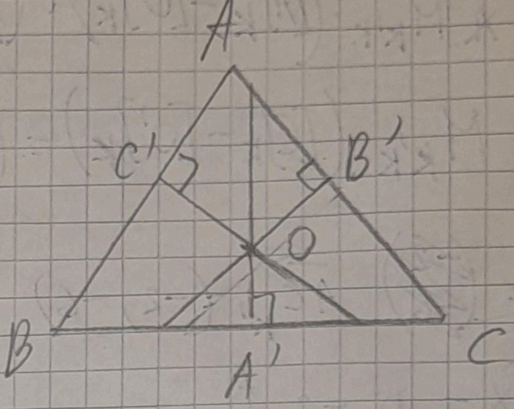
$l: \begin{cases} x = -1 + t \\ y = 1 + 3t \end{cases}$

$\frac{x+1}{1} = \frac{y-1}{3}$

$3x + 3 = y - 1$

$l: 3x - y + 4 = 0$

P_3: B' middle of AC
C' middle of AB
A' middle of BC



$\vec{AC}(-2; -2)$

$\vec{AB}(-4; -2)$

~~B' E~~ $B'(0, 2)$
$C'(-1, 2)$
$A'(-2, 1)$

~~$\vec{B'O} \perp \vec{AC} \Rightarrow$~~ $J(\vec{AC}) = -\wedge_B \vec{AC} = -\begin{vmatrix} i & j \\ -2 & -2 \end{vmatrix} = 2i - 2j$

$B'O: \begin{cases} x = 0 + 2t \\ y = 2 - 2t \end{cases}$

$J(\vec{AB}) = -\wedge_B \vec{AB} = -\begin{vmatrix} i & j \\ -4 & -2 \end{vmatrix} = 2i - 4j$

$\vec{C'O}: \begin{cases} x = -1 + 2t \\ y = 2 - 4t \end{cases}$

$B'O: \frac{x}{2} = \frac{2-y}{2} \Rightarrow x = 2 - y \Rightarrow x + y - 2 = 0$

$C'O: \frac{x+1}{2} = \frac{2-y}{4} \Rightarrow 2x + 2 = 2 - y \Rightarrow 2x + y = 0$

$\vec{C'O} \cap \vec{B'O} : \begin{cases} x+y-2=0 \\ 2x+y=0 \end{cases}$

$x+2=0$

$\left.\begin{matrix} x_0=-2 \\ y_0=4 \end{matrix}\right\} \Rightarrow O(-2,4)$

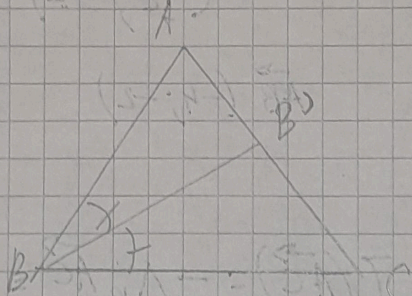
P4. $M_{K'K} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$

$[A]_{K'} = \cancel{A}\, M_{K'K} \cdot \left([A]_K - [O']_K\right) = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

$[C]_{K'} = M_{K'K} \cdot \left([C]_K - [O']_K\right) = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$\cancel{[AC]_{K'} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}}$ $\vec{AC}_{K'} = (-2,-6)$

$\vec{AC} : \begin{cases} x = 0-2t \\ y = 5-6t \end{cases}$



Part 2

P5.

$\vec{AB}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\vec{BC}\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{AC}\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$\cos\measuredangle(BB', AB) = \cos\measuredangle(BB', BC) = \frac{\langle \vec{AB}, \vec{BB'} \rangle}{|AB| \cdot |BB'|} = \frac{\langle \vec{BC}, \vec{BB'} \rangle}{|BC| \cdot |BB'|} \Rightarrow$

$\Rightarrow \frac{(x_{B'}-1)\cdot 0 + (y_{B'}-1)\cdot 0 + 1\cdot(z_{B'}+1)}{1} = \frac{(-1)\cdot(x_{B'}-1) + 1\cdot(y_{B'}-1) + 1\cdot(z_{B'}+1)}{\sqrt{3}}$

$z_{B'} = \frac{-x_{B'} + y_{B'} + z_{B'}}{\sqrt{3}}$

$x_{B'} - y_{B'} - (1+\sqrt{3}) \cdot z_{B'} = 0$

$AC: \begin{cases} x = 0 - t \\ y = 0 + t \\ z = 1 + 2t \end{cases}$

$AC: \quad -x = y = \dfrac{z-1}{2}$

$-x_{B'} = y_{B'} = \dfrac{z_{B'} - 1}{2} \Rightarrow z_{B'} = 1 - 2x_{B'}$

$x_{B'} + x_{B'} - (1+\sqrt{3}) \cdot (1 - 2x_{B'}) = 0$

$\cancel{x_{B'}(2+2+2\sqrt{3}) = 1+\sqrt{3} \Rightarrow x_{B'} = \dfrac{1+\sqrt{3}}{4+2\sqrt{3}}}$

$x_{B'}(2+2-2\sqrt{3}) = 1-\sqrt{3} \Rightarrow x_{B'} = \dfrac{1-\sqrt{3}}{4-2\sqrt{3}}$

$y_{B'} = \dfrac{\sqrt{3}-1}{4-2\sqrt{3}}$

$z_{B'} = 1 - \dfrac{1-\sqrt{3}}{2-\sqrt{3}} = \dfrac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

$\vec{BB'}\left(\dfrac{\sqrt{3}-3}{4-2\sqrt{3}}, \dfrac{3\sqrt{3}-5}{4-2\sqrt{3}}, \dfrac{2}{4-2\sqrt{3}}\right)$

$BB': \begin{cases} x = 1 + \dfrac{\sqrt{3}-3}{4-2\sqrt{3}} \cdot t \\ y = 1 + \dfrac{3\sqrt{3}-5}{4-2\sqrt{3}} \cdot t \\ z = 0 + \dfrac{2}{4-2\sqrt{3}} \cdot t \end{cases}$

$BB': \dfrac{x-1}{\frac{\sqrt{3}-3}{4-2\sqrt{3}}} = \dfrac{y-1}{\frac{3\sqrt{3}-5}{4-2\sqrt{3}}} = \dfrac{z}{\frac{2}{4-2\sqrt{3}}} \Rightarrow \dfrac{x-1}{\sqrt{3}-3} = \dfrac{y-1}{3\sqrt{3}-5} = \dfrac{z}{2}$

P6. - n-am facut

P7. $(\vec{AB} \times \vec{AC}) \times k =$

$(\vec{AB} \times \vec{AC}) \times k + (\vec{AC} \times k) \times \vec{AB} + (k \times \vec{AB}) \times \vec{AC} = 0 \Rightarrow$

$\Rightarrow (\vec{AB} \times \vec{AC}) \times k + (\vec{AC} \times k) \times \vec{AB} = -(k \times \vec{AB}) \times \vec{AC} =$

$= -(\langle k, \vec{AC} \rangle \cdot \vec{AB} - \langle \vec{AB}, \vec{AC} \rangle \cdot k) =$

$= -(2 \cdot \vec{AB} - 2 \cdot k) = -(0,0,0) = (0,0,0)$

P8. $\vec{BA}\ k = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$\vec{AC}\ \ \vec{CD} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

$M_{K',K} = \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & -2 \\ -1 & 2 & -1 \end{pmatrix}$

$\det(M_{K',K}) = (-1) \cdot \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = (-1) \cdot (3-1) = -2 \Rightarrow$

$\Rightarrow K'$ left oriented

P9. $[C]_{K'} = M_{K',K} \cdot ([C]_K - [B]_K) = \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & -2 \\ -1 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} =$

$= (0\ \ -1\ \ 2)$

~~$\vec{ACD}$: $x = 0$~~

$\vec{AC}_{K'}(0,1,0)$

$\vec{CD}_{K'}(0,0,1)$

$\vec{ACD} : \begin{cases} x = 0 + 0 \cdot t + 0 \cdot s \\ y = -1 + 1 \cdot t + 0 \cdot s \\ z = 2 + 0 \cdot t + 1 \cdot s \end{cases}$

16. BCD: $\begin{vmatrix} x-1 & y & z \\ -1 & 1 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 0$ $\qquad \vec{BC}(-1,1,1)$

$\cancel{(-1)}$ $\cancel{x/y}$ $\begin{vmatrix} x-1 & z \\ -1 & 1 \end{vmatrix} = 0$

BCD: $x + z - 1 = 0$

$n(1,0,1)$ normal of BCD

$d(A, BCD) = \frac{|1-1-1|}{\sqrt{1+1}} = \frac{+1}{\sqrt{2}}$

$AA': \begin{cases} x = 1+t \\ y = 1 \\ z = \cancel{1} -1+t \end{cases}$

$\cancel{d(A, A} \; |\vec{AA'}| = \frac{2}{\sqrt{2}} = \sqrt{2}$

$d(A', BCD) = \frac{1}{\sqrt{2}}$

$\frac{|1 \cdot x_{A'} + 1 \cdot z_{A'} \cancel{+} -1|}{\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow |x_{A'} + z_{A'} - 1| = 1$

$A' \neq \cancel{A} \, A \Rightarrow x_{A'} + z_{A'} - 1 = 1 \Rightarrow x_{A'} = 2 - z_{A'}$

$AA': \frac{x-1}{1} = \frac{y}{1} = \frac{z+1}{1}$

$x_{A'} - 1 = z_{A'} + 1 = y_{A'}$

$1 - z_{A'} = z_{A'} + 1 \Rightarrow z_{A'} = 0 \Rightarrow y_{A'} = 1$

$x_{A'} = 2$

$A'(2,1,0)$