COURSE WORK

1. $E=?, V=?$

a) Bernoulli distribution

$$X \sim \begin{pmatrix} 0 & 1 \\ 1-p & p \end{pmatrix} \Rightarrow X^2 \sim \begin{pmatrix} 0^2 & 1^2 \\ 1-p & p \end{pmatrix}$$

$$E(X) = \sum_{i \in I} x_i \cdot P(X = x_i) = 0(1-p) + 1 \cdot p = p$$

$$V(X) = E(X^2) - (E(X))^2 = p - p^2$$

$$E(X^2) = \sum_{i \in I} x_i \cdot P(X^2 = x_i) = p$$

b) Binomial distribution

$$P(X=k) = C_m^k \, p^k (1-p)^{m-k}$$

$$E(X) = \sum_{k=1}^{m} k P(X=k) = \sum_{k=1}^{m} k \, C_m^k \, p^k (1-p)^{m-k}$$

$$= \sum_{k=1}^{m} \cancel{k} \frac{m!}{k\cancel{!}(m-k)!} p^k (1-p)^{m-k}$$

$$= m \sum_{k=1}^{m} \frac{(m-1)!}{(k-1)!(m-k)!} \cdot p^k (1-p)^{m-k}$$

$$= mp \underbrace{\sum_{k=1}^{m} C_{m-1}^{k-1} \cdot p^{k-1} (1-p)^{(m-1)-(k-1)}}_{1} = mp$$

$$E(X^2) = \sum_{k=1}^{m} k^2 C_m^k \, p^k (1-p)^{m-k}$$

$$= \sum_{k=1}^{m} [k + (k-1)k] \, C_m^k \, p^k (1-p)^{m-k}$$

$$= \sum_{k=1}^{m} k C_m^k \, p^k (1-p)^{m-k} + \sum_{k=1}^{m} \cancel{(k-1)k} \cdot \frac{m!}{\cancel{k!}(m-k)!} \cdot p^k (1-p)^{m-k}$$

$$= mp + \cancel{\sum_{k=1}^{m}} m(m-1) \cdot p^2 \underbrace{\sum_{k=2}^{m} C_{m-2}^{k-2} \, p^{k-2} (1-p)^{(m-2)-(k-2)}}_{1}$$

$$= mp + m^2p^2 - mp^2$$

$$\Rightarrow V(X) = mp + \cancel{m^2p^2} - mp^2 - \cancel{m^2p^2} = mp - mp^2 = mp(1-p)$$

c) Geometric distribution

$$E(x) = \sum_{k=1}^{\infty} k\, p(1-p)^k$$

$$= (1-p) \sum_{k=1}^{\infty} k(1-p)^{k-1} p$$

$$= (1-p) \sum_{k=0}^{\infty} (k+1)(1-p)^{k} p$$

$$= \underbrace{(1-p) \sum_{k=0}^{\infty} k(1-p)^{k} p}_{E(x)} + (1-p) \sum_{k=0}^{\infty} (1-p)^{k} p$$

$$= (1-p)E(x) + (1-p)p \sum_{k=0}^{\infty} (1-p)^{k}$$

$$= (1-p)E(x) + (1-p)\cdot \cancel{p} \cdot \frac{1}{\cancel{1-(1-p)}}$$

$$= (1-p)E(X) + (1-p)$$

$$\Rightarrow E(x) = (1-p)E(x) + 1 - p \Rightarrow E(x)(\cancel{1}-\cancel{1}+p) = 1-p \Rightarrow E(x) = \frac{1-p}{p}$$

$$E(x^2) = \sum_{k=0}^{\infty} k^2 p(1-p)^k$$

$$= \sum_{k=0}^{\infty} [k-(1-k)k]\, p(1-p)^k = \sum_{k=0}^{\infty} k p(1-p)^k - \sum_{k=0}^{\infty} (1-k)k\, p(1-p)^k$$

$$= \frac{1-p}{p} + \sum_{k=0}^{\infty} k(k-1)p(1-p)^{k-1}$$

$$= \frac{1-p}{p} + p(1-p)^2 \sum_{k=0}^{\infty} k(k-1)(1-p)^{k-2}$$

$$f(x) = \sum_{k=0}^{\infty} (1-p)^k = \frac{1}{1-(1-p)} = \frac{1}{p}$$

$$f'(x) = \sum_{k=1}^{\infty} k(1-p)^{k-1} = -\frac{1}{p^2}$$

$$f''(x) = \sum_{k=2}^{\infty} k(k-1)(1-p)^{k-2} = \frac{2}{p^3}$$

$$E(X^2) = \frac{1-p}{p} + \cancel{p}(1-p)^2 \cdot \frac{2}{p^{\cancel{3}}} = \frac{p-p^2+2p^2-4p+2}{p^2} = \frac{+p^2-3p+2}{p^2}$$

$$\Rightarrow V(x) = \frac{+p^2-3p+2}{p^2} - \frac{1-2p+p^2}{p^2} = \cancel{\frac{-2p^2+4p-3}{p^2}} = \frac{1-p}{p^2}$$

d) Poisson distribution

$$E(x) = \sum_{K=1}^{\infty} K \cdot \frac{\lambda^K}{K!} e^{-\lambda} = \sum_{K=1}^{\infty} \cancel{K} \cdot \frac{\lambda^K}{(K-1)!\cancel{K}} \cdot e^{-\lambda}$$

$$= \lambda \underbrace{\sum_{K=1}^{\infty} \frac{\lambda^{K-1}}{(K-1)!} e^{-\lambda}}_{1} = \lambda$$

$$E(x^2) = \sum_{K=1}^{\infty} [K-(1-K)K] \frac{\lambda^K}{K!} e^{-\lambda}$$

$$= \lambda - \sum_{K=1}^{\infty} (1-\cancel{K})\cancel{K} \cdot \frac{\lambda^K}{K!} \cdot e^{-\lambda}$$

$$= \lambda + \lambda^2 \underbrace{\sum_{K=2}^{\infty} \frac{\lambda^{K-2}}{(K-2)!} e^{-\lambda}}_{1} = \lambda + \lambda^2$$

$$\Rightarrow V(x) = \cancel{\lambda} + \lambda^2 - \cancel{\lambda} = \lambda$$

2)

a) Uniform distribution

$$E(x) = \int_a^b x \cdot \frac{1}{b-a} dx = \frac{1}{b-a} \cdot \frac{x^2}{2}\Big|_a^b = \frac{1}{b-a}\left(\frac{b^2}{2} - \frac{a^2}{2}\right) = \frac{1}{\cancel{b-a}} \cdot \frac{(\cancel{b-a})(b+a)}{2}$$

$$= \frac{b+a}{2}$$

$$E(x^2) = \int_a^b x^2 \frac{1}{b-a} dx = \frac{1}{b-a} \cdot \frac{x^3}{3}\Big|_a^b = \frac{1}{b-a}\left(\frac{b^3}{3} - \frac{a^3}{3}\right) = \frac{1}{\cancel{b-a}} \frac{(\cancel{b-a})(b^2+ab+a^2)}{3}$$

$$V(x) = \frac{^{4)}b^2+ab+a^2}{3} - \frac{^{3)}b^2+2ab+a^2}{4} = \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$$

b) Exponential distribution

$$E(x) = \frac{1}{\lambda}\int_0^{\infty} x \lambda e^{-\lambda x} \lambda dx$$

$t = \lambda x \,|\, ()'$
$dt = \lambda dx$
$t(0) = 0$
$t(\infty) = \infty$

$$\Rightarrow E(x) = \frac{1}{\lambda}\int_0^{\infty} te^{-t} dt = \frac{1}{\lambda}\int_0^{\infty} t(-e^{-t})' dt = \frac{1}{\lambda}\left(-te^{-t}\Big|_0^{\infty} + \int_0^{\infty} e^{-t} dt\right)$$

$$= \frac{1}{\lambda}\left(-te^{-t}\Big|_0^{\infty} - \frac{e^{-t}}{\underbrace{\ln e}_{1}}\Big|_0^{\infty}\right) = -\frac{1}{\lambda}\left(\lim_{t\to\infty}\frac{t}{e^t} - 0 + \lim_{t\to\infty}\frac{1}{e^t} - 1\right)$$

$$= \frac{1}{\lambda}$$

$$E(x^2) = \int_0^{\infty} x^2 \lambda e^{-\lambda x} dx$$

$$= \frac{1}{\lambda}\int_0^{\infty} (x\lambda)^2 \cdot e^{-\lambda x} dx = \frac{1}{\lambda^2}\int_0^{\infty} t^2 \cdot e^{-t} dt = \frac{1}{\lambda^2}\int_0^{\infty} t^2(-e^{-t})' dt$$

$$= \frac{1}{\lambda^2}\left(-t^2 e^{-t}\Big|_0^{\infty} + 2\underbrace{\int_0^{\infty} t e^{-t} dt}_{1} \right) = \frac{1}{\lambda^2}\left(-\lim_{t\to\infty} \frac{t^2}{e^t} + 0 + 2 \right) = \frac{2}{\lambda^2}$$

$$V(X) = \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

c) Gamma distribution

$$E(X) = \int_0^{\infty} x \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \cdot x^{\alpha-1} e^{-\lambda x} dx$$

$$= \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} x^{\alpha} e^{-\lambda x} dx$$

$$= \frac{1}{\Gamma(\alpha)} \int_0^{\infty} (\lambda x)^{\alpha} e^{-\lambda x} dx$$

$\lambda x = t \,|\, ()'$
$\lambda dx = dt$
$t(0) = 0$
$t(\infty) = \infty$

$$\Rightarrow \frac{1}{\lambda \Gamma(\alpha)} \underbrace{\int_0^{\infty} t^{\alpha} e^{-t} dt}_{\Gamma(\alpha+1)} = \frac{\Gamma(\alpha+1)}{\lambda \Gamma(\alpha)} = \frac{\alpha \cancel{\Gamma(\alpha)}}{\lambda \cancel{\Gamma(\alpha)}} = \frac{\alpha}{\lambda}$$

$$E(X^2) = \int_0^{\infty} x^2 \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \cdot x^{\alpha-1} e^{-\lambda x} dx$$

$$= \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} x^{\alpha+1} e^{-\lambda x} dx$$

$$= \frac{1}{\lambda \Gamma(\alpha)} \int_0^{\infty} (x\lambda)^{\alpha+1} \cdot e^{-\lambda x} dx$$

$$= \frac{1}{\lambda^2 \Gamma(\alpha)} \underbrace{\int_0^{\infty} t^{\alpha+1} e^{-t} dt}_{\Gamma(\alpha+2)} = \frac{(\alpha+1)\Gamma(\alpha+1)}{\lambda^2 \Gamma(\alpha)} = \frac{(\alpha+1)\alpha \cancel{\Gamma(\alpha)}}{\lambda^2 \cancel{\Gamma(\alpha)}} = \frac{(\alpha+1)\alpha}{\lambda^2}$$

$$V(X) = \frac{\alpha(\alpha+1)}{\lambda^2} - \frac{\alpha^2}{\lambda^2} = \frac{\alpha}{\lambda^2}$$

d) Normal distribution

$$E(X) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$t = \frac{x-\mu}{\sigma} \,|\, ()'$
$dt = \frac{dx}{\sigma}$
$t(0) = 0$
$t(\infty) = \infty$

$$\Rightarrow E(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma t + \mu) e^{-\frac{t^2}{2}} dt$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t e^{-\frac{t^2}{2}} dt + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt$$

$$= -\frac{\sigma}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\Big|_{-\infty}^{\infty} + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt$$

$$= -\frac{\sigma}{\sqrt{2\pi}} \left(\underbrace{\lim_{t\to\infty} \frac{1}{e^{\frac{t^2}{2}}}}_{0} - \underbrace{\lim_{t\to-\infty} \frac{1}{e^{\frac{t^2}{2}}}}_{0} \right) + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt$$

$$J = \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}}\,dt = 2\int_{0}^{\infty} e^{-\frac{t^2}{2}}\,dt$$

$$J^2 = J\cdot J = 2\int_{0}^{\infty} e^{-\frac{t^2}{2}}\,dt \int_{0}^{\infty} e^{-\frac{u^2}{2}}\,du$$

$$= 4\int_{0}^{\infty}\int_{0}^{\infty} e^{-\frac{(t^2+u^2)}{2}}\,du\,dt$$

$u = vt \;|\,(\,)'$

$du = t\,dv$

$\Rightarrow v = \frac{u}{t}$

$v(0)=0$

$v(\infty)=\infty$

$$\Rightarrow J^2 = 4\int_{0}^{\infty}\int_{v=0}^{v=\infty} e^{-\frac{(t^2+v^2t^2)}{2}}\cdot t\,dv\,dt$$

$$= 4\int_{0}^{\infty}\underbrace{\int_{0}^{\infty} t\,e^{-\frac{t^2(1+v^2)}{2}}\,dt}\,dv$$

$w = -t^2(1+v^2)\;|\,(\,)'$

$dw = -2t(1+v^2)\,dt$

$$\Rightarrow \frac{1}{-2(1+v^2)}\underbrace{\int_{0}^{-\infty} e^{\frac{w}{2}}\,dw}_{2e^{\frac{w}{2}}\big|_{0}^{-\infty} = -2} = \frac{-\cancel{2}}{-\cancel{2}(1+v^2)} = \frac{1}{(1+v^2)}$$

$$= 4\int_{0}^{\infty}\frac{1}{\cancel{2}(1+v^2)}\,dv$$

$$= 4\int_{0}^{\infty}\frac{1}{1+v^2}\,dv = 4\,\mathrm{arctg}\,v\Big|_{0}^{\infty} = \cancel{2}\,\frac{\pi}{\cancel{2}} - 0 = 2\pi$$

$\Rightarrow J = \sqrt{2\pi}$

$\Rightarrow E(X) = \frac{\mu}{\sqrt{2\pi}}\cdot\sqrt{2\pi} = \mu$

$$E(X^2) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} x^2 e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

$t = \frac{x-\mu}{\sigma}\;|\,(\,)'$

$dt = \frac{1}{\sigma}\,dx$

$$\Rightarrow E(X^2) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(\sigma t+\mu)^2 e^{-\frac{t^2}{2}}\,dt$$

$$= \frac{1}{\sqrt{2\pi}}\left(\sigma^2\int_{-\infty}^{\infty} t^2 e^{-\frac{t^2}{2}}\,dt + 2\sigma\mu\underbrace{\int_{-\infty}^{\infty} t\,e^{-\frac{t^2}{2}}\,dt}_{0} + \mu^2\underbrace{\int_{-\infty}^{\infty} e^{-\frac{t^2}{2}}\,dt}_{\sqrt{2\pi}}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(+\sigma^2\underbrace{\int_{-\infty}^{\infty} t^2 e^{-\frac{t^2}{2}}} + \mu^2\sqrt{2\pi}\right)$$

$$J = \int_{-\infty}^{\infty} t^2 e^{-\frac{t^2}{2}}\,dt = 2\int_{0}^{\infty} t^2 e^{-\frac{t^2}{2}}\,dt = \int_{0}^{\infty} 2u\cdot e^{-u}\cdot\frac{1}{\sqrt{2u}}\,du = \sqrt{2}\int_{0}^{\infty} u^{\frac{1}{2}} e^{-u}\,du$$

$u = \frac{t^2}{2}$

$t = \sqrt{2u}$

$du = t\,dt = \sqrt{2u}\,dt$

$dt = \frac{1}{\sqrt{2u}}\,du$

$$\cancel{\Rightarrow \int_{-\infty}^{\infty} u\,e^{-\frac{u}{2}}\cdot\frac{1}{2\sqrt{u}}\,du = \frac{1}{2}\int_{0}^{\infty} u^{\frac{1}{2}} e^{-\frac{u}{2}}\,du = \frac{1}{2\sqrt{2}}\int_{-\infty}(2v)^{\frac{1}{2}}\cdot e^{-v}\cdot 2\,dv}$$

$$\cancel{= \frac{\sqrt{2}}{\sqrt{2}}\underbrace{\int_{-\infty}(2v)^{\frac{3}{2}-1} e^{-\frac{2v}{2}}\,du} = \Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\int_{0}^{\infty} x^{-\frac{1}{2}} e^{-x}\,dx}$$

$$\cancel{\int_0^\infty \frac{e^{-x}}{\sqrt{x}}\,dx}\ \int_0^\infty \sqrt{x}\,e^{-x}\,dx = \underbrace{-\sqrt{x}\,e^{-x}\Big|_0^\infty}_{0} + \underbrace{\frac{1}{2}\int_0^\infty \frac{1}{\sqrt{x}}\,e^{-x}\,dx} = \frac{1}{2}\sqrt{\pi} \Rightarrow J = 2\cdot\sqrt{2}\cdot\frac{1}{2}\sqrt{\pi} = \sqrt{2\pi}$$

$$\left.\begin{aligned} &\sqrt{x} = t \\ &x = t^2 \,|\, (\,)' \\ &dx = 2t\,dt \\ &t(0) = 0 \\ &t(\infty) = \infty \end{aligned}\right| \Rightarrow \int_0^\infty \frac{e^{-t^2}}{\cancel{t}}\cdot 2\cancel{t} = 2\int_0^\infty e^{-t^2}\,dt = \cancel{\int_{-\infty}^{\infty} e^{-t^2}\,dt} = \sqrt{\pi}$$

$$\Rightarrow E(X^2) = \frac{1}{\sqrt{2\pi}}\left(\sigma^2\cdot\frac{\sqrt{2\pi}}{1} + \mu^2\sqrt{2\pi}\right) = \frac{\sigma^2}{1} + \mu^2$$

$$V(X) = \sigma^2 + \cancel{\mu^2} - \cancel{\mu^2} = \sigma^2$$

3) $T \sim Exp(\lambda) \Leftrightarrow f_T(x) = \begin{cases} 0, & t \le 0 \\ \lambda e^{-t\lambda}, & t > 0 \end{cases}$

a) $E(S_i) = 7$

$$E(T) = \sum_{k=1}^{5} E(S_k) = 7+7+7+7+7 = 35$$

$$E(S_i) = \frac{1}{\lambda} \Rightarrow \frac{1}{\lambda} = 7 \Rightarrow \lambda = \frac{1}{7} \Rightarrow V(S_i) = \frac{1}{\lambda^2} = 49$$

$$V(T) = \sum_{k=1}^{5} V(S_k) = 49\cdot 5 = 245$$

b) $P(T < 20) = \frac{1}{\sqrt{2\pi}}\int_0^{20} e^{-\frac{t^2}{2}}\,dt = \frac{1}{\sqrt{2\pi}}\int_0^{20} e^{-\left(\frac{t}{\sqrt{2}}\right)^2}\,dt$

$$\left.\begin{aligned} &u = \frac{t}{\sqrt{2}} \Rightarrow \sqrt{2}u = t \,|\, (\,)' \\ &\sqrt{2}\,du = dt \\ &u(0) = 0 \\ &u(20) = \frac{20}{\sqrt{2}} = 10\sqrt{2} \end{aligned}\right| \Rightarrow \frac{1}{\sqrt{2\pi}}\int_0^{10\sqrt{2}} e^{-u^2}\cdot\sqrt{2}\,du = \frac{1}{\sqrt{\pi}}\int_0^{10\sqrt{2}} e^{-u^2}\,du$$

$$\cancel{= \frac{1}{2}\int_0^{10\sqrt{2}} \frac{2e^{-u^2}}{\sqrt{\pi}}\,du = \frac{1}{2}\Gamma(u)\Big|_0^{10\sqrt{2}}}$$

$$\left.\begin{aligned} &u^2 = v \,|\, (\,)' \ \cancel{\Rightarrow \sqrt{v} = u} \\ &2u\,du = dv \Rightarrow du = \frac{1}{2\sqrt{v}}\,dv \\ &v(0) = 0 \\ &v(10\sqrt{2}) = 200 \end{aligned}\right| \Rightarrow \frac{1}{\sqrt{\pi}}\int_0^{200} e^{-v}\cdot\frac{1}{2\sqrt{v}}\,dv = \frac{1}{2\sqrt{\pi}}\int_0^{200} v^{-\frac{1}{2}}e^{-v}\,dv$$

$$\left.\begin{aligned} &\mu = 35 \\ &\sigma = \sqrt{245} = 15{,}65 \end{aligned}\right| \Rightarrow \cancel{z = \frac{20-\mu}{\sigma} = \frac{20-35}{15{,}65} = -0{,}95}$$

$$\cancel{P(T \le 20) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{t^2}{2}}\,dt}$$

$$P(0 < S_1 + \dots + S_5 < 20) = P\left(\frac{0 - 5\mu}{\sigma\cdot\sqrt{5}} < \frac{S_1 + \dots + S_5 - 5\cdot 35}{15{,}65\cdot\sqrt{5}} < \frac{20 - 5\cdot 35}{15{,}65\cdot\sqrt{5}}\right) = F_{N(0,1)}(4{,}4) - F_{(0,1)}(-5{,}01)$$

4) e_{m_1} = error in the first module

e_{m_2} = error in the second module

$P(e_{m_1}) = 30\%$

$P(e_{m_2}) = 50\%$

$P(C|e_{m_1}) = 55\%$

$P(C|e_{m_2}) = 90\%$

$P(C|(e_{m_1}\cap e_{m_2})) = 95\%$

$P((e_{m_1}\cap e_{m_2})|C) = ?$

$$P((e_{m_1}\cap e_{m_2})|C) = \frac{\overbrace{P(C|(e_{m_1}\cap e_{m_2}))}^{\frac{95}{100}}\cdot \overbrace{P(e_{m_1}\cap e_{m_2})}^{P(e_{m_1})\cdot P(e_{m_2})}}{P(C)}$$

$$P(C|e_{m_1}) = \frac{P(C\cap e_{m_1})}{P(e_{m_1})} \Rightarrow P(C\cap e_{m_1}) = \frac{55}{100}\cdot\frac{30}{100}$$

$$P(C|e_{m_2}) = \frac{P(C\cap e_{m_2})}{P(e_{m_2})} \Rightarrow P(C\cap e_{m_2}) = \frac{90}{100}\cdot\frac{50}{100}$$

$$P(C\cap(e_{m_1}\cap e_{m_2})) = P(C|(e_{m_1}\cap e_{m_2}))\cdot P(e_{m_1}\cap e_{m_2}) = \frac{95}{100}\cdot\frac{30}{100}\cdot\frac{50}{100}$$

$$\Rightarrow \cancel{P(C\cap(e_{m_1}))}\; P(C) = \frac{55}{100}\cdot\frac{30}{100}+\frac{90}{100}\cdot\frac{50}{100}$$

$$\Rightarrow P((e_{m_1}\cap e_{m_2})|C) = \frac{\frac{95}{100}\cdot\frac{30}{100}\cdot\frac{50}{100}}{\frac{55}{100}\cdot\frac{30}{100}+\frac{90}{100}\cdot\frac{50}{100}} = 0,23$$

3.

b) $\int_0^{20} g(x)\,dx$

$\cancel{g(x) = \frac{\lambda^{5}}{\Gamma(\alpha)}x}$

$$g(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1}e^{-\lambda x} = \frac{\left(\frac{1}{7}\right)^5}{\Gamma(5)} x^4 e^{-\frac{\cancel{5}x}{7}}$$

$$\Rightarrow \frac{1}{7^5\Gamma(5)}\int_0^{20} x^4 e^{-\frac{\cancel{5}x}{7}}dx = \cancel{\frac{1}{7^5\Gamma(5)}\int_0^{20} x^4\left(\frac{e^{-5x}}{-5}\right)'dx}$$

$$\cancel{= \frac{1}{7^5\Gamma(5)}\left(x^4\frac{e^{-5x}}{-5}\Big|_0^{20} + \frac{4}{5}\int_0^{20}x^3e^{-5x}dx\right)}$$

$$= \frac{1}{7^{\cancel{5}}\Gamma(5)}\int_0^{20}\left(\frac{x}{7}\right)^4 e^{-\frac{x}{7}}dx$$

$\frac{x}{7}=t \Rightarrow x=7t$

$dx = 7dt$

$t(0)=0$

$t(20)=\frac{20}{7}$

$$\Rightarrow \frac{1}{\cancel{7}\Gamma(5)}\int_0^{\frac{20}{7}} t^4e^{-t}\cancel{7}dt = \frac{1}{\Gamma(5)}\int_0^{\frac{20}{7}} t^4(-e^{-t})'dt$$

$$= \frac{1}{\Gamma(5)}\left(-t^4e^{-t}\Big|_0^{\frac{20}{7}} + 4\int_0^{\frac{20}{7}}t^3e^{-t}dt\right) = \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4e^{\frac{20}{7}}} + 4\int_0^{\frac{20}{7}}t^3(-e^{-t})'dt\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} + 4\left(-t^3 e^{-t}\Big|_0^{\frac{20}{7}} + 3\int_0^{\frac{20}{7}} t^2 e^{-t}\,dt\right)\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} + 4\left(-\frac{20^3}{7^3 e^{\frac{20}{7}}} + 3\int_0^{\frac{20}{7}} t^2 (-e^{-t})'\,dt\right)\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} - \frac{4\cdot 20^3}{7^3 e^{\frac{20}{7}}} + 12\left(-t^2 e^{-t}\Big|_0^{\frac{20}{7}} + 2\int_0^{\frac{20}{7}} t e^{-t}\,dt\right)\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} - \frac{4\cdot 20^3}{7^3 e^{\frac{20}{7}}} - \frac{12\cdot 20^2}{7^2 e^{\frac{20}{7}}} + 24\int_0^{\frac{20}{7}} t(-e^{-t})'\,dt\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} - \frac{4\cdot 20^3}{7^3 e^{\frac{20}{7}}} - \frac{12\cdot 20^2}{7^2 e^{\frac{20}{7}}} + 24\left(-te^{-t}\Big|_0^{\frac{20}{7}} + \int_0^{\frac{20}{7}} e^{-t}\,dt\right)\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} - \frac{4\cdot 20^3}{7^3 e^{\frac{20}{7}}} - \frac{12\cdot 20^2}{7^2 e^{\frac{20}{7}}} - \frac{24\cdot 20}{7\cdot e^{\frac{20}{7}}} + 24\left(-e^{-t}\right)\Big|_0^{\frac{20}{7}}\right)$$

$$= \frac{1}{\Gamma(5)}\left(-\frac{20^4}{7^4 e^{\frac{20}{7}}} - \frac{{}^{7/}4\cdot 20^3}{7^3 e^{\frac{20}{7}}} - \frac{{}^{7^2/}12\cdot 20^2}{7^2 e^{\frac{20}{7}}} - \frac{{}^{7^3/}24\cdot 20}{7 e^{\frac{20}{7}}} - \frac{{}^{7^4/}24}{e^{\frac{20}{7}}} + 24\right)$$

$$\cancel{= \frac{1}{120}\left(-\frac{160\,000 + 32000 + 4800 + 480 + 24}{e^{20}} + 24\right)}$$

$$\cancel{= \frac{1}{120}\left(-\frac{197\,304}{e^{20}} + 24\right) = \cancel{23{,}94}\ 0{,}19}$$

$$= \frac{1}{24}\left(-\frac{160\,000 + 224000 + 235\,200 + 164640 + 57624}{7^4\cdot e^{\frac{20}{7}}} + 24\right)$$

$$= \frac{1}{24}\left(-\frac{841\,464}{7^4\cdot e^{\frac{20}{7}}} + 24\right) = \frac{1}{24}\left(-\frac{350{,}46}{e^{\frac{20}{7}}} + 24\right) = \frac{1}{24}\left(-\frac{350{,}46}{17{,}41} + 24\right)$$

$$= 0{,}16$$