

$$|\lambda_1| = \frac{|z_0 \cdot \alpha^2|}{c^2} \Rightarrow z_0 = \pm \frac{z_0 \alpha^2}{c^2}$$

$$|\lambda_2| = \frac{|z_0 \beta^2|}{c^2} \Rightarrow z_0 = \pm \frac{z_0 \beta^2}{c^2}$$

$$\frac{z_0^2 \alpha^4}{c^4 \alpha^2} + \frac{z_0^2 \beta^4}{c^4 \beta^2} + \frac{z_0^2}{c^2} = 1$$

$$\frac{z_0^2}{c^2} \left( \frac{\alpha^2}{c^2} + \frac{\beta^2}{c^2} + 1 \right) = 1 \quad \dots \quad z_0 = \pm \frac{c^2}{\sqrt{2\beta^2 + c^2}}$$

...

$$P = \frac{1}{\sqrt{\alpha^2 + \beta^2 + c^2}} (\pm \alpha^2, \pm \beta^2, \pm c^2)$$

## Sommerar 14

II S:  $\begin{cases} x = 4 \cos(\alpha) \cos t \\ y = 4 \sin \alpha \cos t \\ z = 2 \sin t \end{cases}, \quad \alpha \in [0, 2\pi), \quad t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

- give an eq for S
- find the points of  $P(3, \sqrt{3}, 1)$
- find a point of the tangent plane  $T_P S$  using P1
- find an eq of  $T_P S$

a)  $\frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} = 1$

b)  $\begin{cases} 4 \cos \alpha \cos t = 3 \\ 4 \sin \alpha \cos t = \sqrt{3} \end{cases}$

$2 \sin t = 1 \Rightarrow \sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6} \Rightarrow \cos t = \frac{\sqrt{3}}{2}$

$\Rightarrow 4\sqrt{3} \cos \alpha = 3 \Rightarrow \alpha = \frac{\pi}{6}$

c)  $\tilde{v}(\alpha, t)$

$\frac{\partial}{\partial \alpha} \tilde{v}(\alpha, t) = (4 \sin \alpha \cos t, 4 \cos \alpha \cos t, 0)$

$$\frac{\partial}{\partial t} \sigma(s, t) = (-s \cos s \sin t, -s \sin s \sin t, s \cos t)$$

$$\frac{\partial}{\partial s} \sigma\left(\frac{\pi}{6}, \frac{\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, 3, 0\right)$$

$$\frac{\partial}{\partial t} \sigma\left(\frac{\pi}{6}, \frac{\pi}{6}\right) = (-\sqrt{3}, -1, \sqrt{3})$$

$$\text{Tp S: } \begin{cases} x = 3 + 1.5t \\ y = \sqrt{3} + 3t \\ z = 1 + 0.5t + \sqrt{3}t \end{cases}$$

d)  $\begin{vmatrix} x-3 & y\sqrt{3} & z-1 \\ -\sqrt{3} & 3 & 1 \\ \sqrt{3} & -1 & \sqrt{3} \end{vmatrix} = 0 \quad \dots$

~~$\rightarrow 3\sqrt{3} + \sqrt{3}z$~~

14.  $\mathcal{H}_{2,3,1}: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{1} = 1$

- a) Find the tangent plane of  $\mathcal{H}_{2,3,1}$  in the point  $M(2,3,1)$   
 b) Show that  $T_M \mathcal{H}$  intersects  $\mathcal{H}$  in 2 lines

17. a) Determine the intersection of the surface

$$P_{2, \frac{1}{2}}: x^2 - 4y^2 = 4z \text{ with the line } l: \begin{Bmatrix} 2 \\ 0 \\ 3 \end{Bmatrix} + t \begin{Bmatrix} 1 \\ -1 \\ -2 \end{Bmatrix}$$

b) Write the equations of tangent planes in the intersection points

$$a) l: \begin{cases} x = 2 + t \\ y = -t \\ z = 3 - 2t \end{cases}$$

$$\Rightarrow (2+t)^2 - 4(-t)^2 = 4(3-2t)$$

$$4 + 8t + 4t^2 - 4t^2 = 12 - 8t$$

$$16t = 8 \Rightarrow t = \frac{1}{2} \Rightarrow \begin{cases} x = 3 \\ y = -\frac{1}{2} \\ z = 2 \end{cases}$$

$$b) \frac{3x}{4} - 4 \cdot \frac{1}{2}y = 4/2z - 8 = 0 \Rightarrow 3x - 2y - 2z = 0$$

$$|x_0| = \frac{|z_0| \cdot a^2}{c^2} \Rightarrow x_0 = \pm \frac{z_0 \cdot a^2}{c^2}$$

$$|f_0| = \frac{|z_0| \theta^2}{c^2} \Rightarrow f_0 = \pm \frac{z_0 \theta^2}{c^2}$$

$$\frac{Z_0^2 \alpha^4}{C^4 \alpha^2} + \frac{Z_0^2 \beta^2}{A^4 \alpha^2} + \frac{Z_0^2}{\alpha^2} = 1$$

$$\frac{z_0^2}{c^2} \left( \frac{a^2}{c^2} + \frac{b^2}{c^2} + 1 \right) = 1 \quad \therefore z_0 = \pm \sqrt{\frac{c^2}{a^2 + b^2 + c^2}}$$

$$P = \frac{1}{\sqrt{a^2 + b^2 + c^2}} \left( \pm a^2, \pm b^2, \pm c^2 \right)$$

Seminar 14

$$\text{II} \quad \text{Si} \begin{cases} x = 4 \cos(s) \cos t \\ y = 4 \sin(s) \cos t \\ z = 2 \sin t \end{cases}, \quad s \in [c, 2\pi], \quad t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

- c) give an ex for S
  - b) find the param. of  $P(3, \sqrt{3}, 1)$
  - c) find a form of the tangent plane  $T_P S$  using  $P$
  - d) find an ex of  $T_P S$

$$a) \frac{x^2}{16} + \frac{y^2}{16} + \frac{z^2}{4} = 1$$

$$b) \begin{cases} 4 \cos \Delta \cos t = 3 \\ 2 \cos \Delta \cos t = \sqrt{3} \end{cases}$$

$$\left\{ \begin{array}{l} 2 \sin t = 1 \Rightarrow \sin t = \frac{1}{2} \Rightarrow t = \frac{\pi}{6} \Rightarrow \cos t = \frac{\sqrt{3}}{2} \end{array} \right.$$

$$\Rightarrow 2\sqrt{3} \cos A = 2 \Rightarrow A = \frac{\pi}{6}$$

$$o) \quad \Gamma(s, t)$$

$$\frac{\partial}{\partial \rho} \tilde{v}(r, \omega) = (\sin \text{east}, \cos \text{east}, 0)$$

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$$\frac{\partial}{\partial t} \vec{v}(s, t) = (-s \cos \alpha \sin \beta, -s \sin \alpha \sin \beta, s \cos \beta)$$

$$\frac{\partial}{\partial s} \vec{v}\left(\frac{\pi}{6}, \frac{\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, 3, 0\right)$$

$$\frac{\partial}{\partial t} \vec{v}\left(\frac{\pi}{6}, \frac{\pi}{6}\right) = (-\sqrt{3}, -1, \sqrt{3})$$

$$\text{Tp S: } \begin{cases} x = 3 + \sqrt{3}s + (-1)t \\ y = \sqrt{3} + 3s + (-1)t \\ z = 1 + 0s + \sqrt{3}t \end{cases}$$

$$\text{d)} \quad \begin{vmatrix} x-3 & y-\sqrt{3} & z-1 \\ -\sqrt{3} & 3 & 1 \\ -\sqrt{3} & -1 & \sqrt{3} \end{vmatrix} = 0$$

~~$\sqrt{3}x - 3\sqrt{3} + \sqrt{3}z$~~

$$14. \quad \text{Fl}_{2,3,1}: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{1} = 1$$

a) Find the tangent plane of  $\text{Fl}_{2,3,1}$  in the point  $M(2,3,1)$

b) Show that  $T_M \text{Fl}$  intersects  $\text{Fl}$  in 2 lines

15. a) Determine the intersection of the surface

$$\text{P}_{2, \frac{1}{2}}: x^2 - 4y^2 = 4z \text{ with the line } l: \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

b) Write the equations of tangent planes in the intersection points

$$\text{a) } l: \begin{cases} x = 2 + 2t \\ y = -t \\ z = 3 - 2t \end{cases}$$

$$\Rightarrow (2+2t)^2 - 4(-t)^2 = 4(3-2t)$$

$$4 + 8t + 4t^2 - 4t^2 = 12 - 8t$$

$$16t = 8 \Rightarrow t = \frac{1}{2} \Rightarrow \begin{cases} x = 3 \\ y = -\frac{1}{2} \\ z = 2 \end{cases}$$

$$\text{b) } \frac{3x}{1} - 4 \cdot \frac{1}{2}y = 4(3z - 2) = 0 \Rightarrow 3x - 2y - 8z = 0$$

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$$\frac{x^2}{3} - \frac{2y^2}{2} - 2z = 0$$

$$\Rightarrow a = \sqrt{2}, b = \frac{1}{\sqrt{2}}$$

$$\frac{x_0 x}{a} - \frac{y_0 y}{b} - z_0 - z = 0$$

$$\frac{sx}{\sqrt{2}} - \frac{\sqrt{2}y}{2} - 2 - z = 0$$

6.2.  $\mathcal{C}: x^2 + y^2 - R^2 / R^2$

$$\frac{x^2}{R^2} + \frac{y^2}{R^2} = 1$$

$$\underbrace{f(x, y)}$$

$$(x_0, y_0) \in \mathcal{C}$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{2}{R^2} x \\ \frac{2}{R^2} y \end{bmatrix} = \frac{2}{R^2} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\nabla f|_{(x_0, y_0)} = \frac{2}{R^2} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\vec{n}_0 = \frac{2}{R^2} \begin{bmatrix} y_0 \\ -x_0 \end{bmatrix} \Rightarrow \text{this is the tangent vector}$$

8.24.

a)  $S: 2x^2 + 2y^2 + 2xz = 1$

b)  $S: 3x^2 + 3y^2 - xz = 1$

c)  $S: 2xy + 2y^2 + y + z = 1$

Which is a hyperboloid

or  $G = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

$$\det(G - \lambda I_3) = 0$$

$$\begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = 0 \Leftrightarrow \begin{aligned} x^3 + 1 - 1 + 3x &= 0 \\ 2x^3 + 3x - 2 &= 0 \end{aligned}$$

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$$\Rightarrow \begin{vmatrix} 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & 3 & 2 \end{vmatrix}$$

$$\Rightarrow -x_1/x_2 \geq_{1,2} -1, x_3 = 1 \Rightarrow \mathcal{H}$$

b) S:  $3x^2 + 3y^2 + xz = 1$

$$3\left(x + \frac{1}{6}z\right)^2 + 3y^2 - \frac{1}{12}z^2 = 1$$

$$\text{met } x' = x + \frac{1}{6}z$$

$$3x'^2 + 3y^2 - \frac{1}{12}z^2 = 1 \rightarrow \mathcal{H}$$

6.19.  $\frac{x^2}{4} - \frac{y^2}{9} = 1$

$P(x_p, y_p)$  s.t.  $P$  does not belong to any tangent of  $\mathcal{H}$

~~$x \neq$~~   $y = \frac{b}{a}x$

$y > \frac{b}{a}x$

$y < -\frac{b}{a}x$