

Probability Theory - Coursework

Spring 2024

Problem 1. Compute the expectation and the variance for a random variable X with the following discrete distributions, using, respectively, the probability mass functions provided in the lecture notes:

- a. Bernoulli distribution
- b. Binomial distribution
- c. Geometric distribution
- d. Poisson distribution

Problem 2. Compute the expectation and the variance for a random variable X with the following continuous distributions, using, respectively, the density functions provided in the lecture notes:

- a. Uniform distribution
- b. Exponential distribution
- c. Gamma distribution
- d. Normal distribution

Problem 3. A computer program undergoes compilation in 5 sequential stages (one after the other), each stage operating independently from the other 4. The time taken for each stage follows an Exponential distribution with mean of 7 minutes.

- a. Determine the expected total compilation time and its variance.
- b. Calculate the probability that the entire program is compiled in under 20 minutes.

Problem 4. A computer program consists of two modules. The first module has a 30% chance of containing an error. The second module is independent from the first one and its probability to contain an error is 50%. An error in the first module alone leads to a program crash with a 55% probability. For the second module alone, this probability is 90%. In the scenario where errors exist in both modules, the program crashes with a probability of 95%. Given that the program has crashed, what is the probability that errors exist in both modules?