MTH 220-01 **Quiz 4** **Due: 10/6/2009, 5:30pm** **Name:** Key

A. I did not receive help from other people except perhaps Dr. Han when I did this quiz. **Signature:** ____________________.

B. Show your work. All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. [4 pts] Do # 12 on Page 71.

Consider $\left[\begin{array}{ccc|c} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -6 & 8 & 0 \\ 0 & -5 & h+16 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (echelon form.)

From this echelon form, we see that the system:

$$x_1 \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 7 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 8 \\ h \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

always has nontrivial solutions for all h values.

Thus, the three vectors are linearly dependent for all real h.

2. [3 pts] Do # 16, # 18, # 22(c) on Page 71.

#16: $\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$ are linearly dependent since $\begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix} = 1.5 \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$.

#18. They're linearly dependent since they are 4 vectors in R^2 and $4 > 2$.

#22(c). True. Reason: Because $\vec{z}$ is in span$\{\vec{x}, \vec{y}\}$, there exist scalars α, β such that $\vec{z} = \alpha\vec{x} + \beta\vec{y}$. Thus, $\alpha\vec{x} + \beta\vec{y} - \vec{z} = \vec{0}$.

This implies that $\vec{x}, \vec{y}, \vec{z}$ are linearly dependent (with $c_1 = \alpha$, $c_2 = \beta$, $c_3 = -1 \neq 0$.)

MTH 220-01 Quiz 3 Due: 9/29/2009, 5:30pm Name: Key

A. I did not receive help from other people except perhaps Dr. Han when I did this quiz. **Signature:** ____________________.

B. Show your work. All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. [6 pts] Let

$$A = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 2 \\ 2 & 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -2 \\ 4 & -1 & -6 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}.$$

Compute: (a) $3\mathbf{u} - 4\mathbf{v}$ (b) $A\mathbf{u} - 3B\mathbf{v}$ (c) $B(2\mathbf{v} - \mathbf{u})$

(a) $3\vec{u} - 4\vec{v} = 3\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} - 4\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ -3 \end{bmatrix} - \begin{bmatrix} -8 \\ 0 \\ 4 \end{bmatrix} = \boxed{\begin{bmatrix} 17 \\ 0 \\ -7 \end{bmatrix}}$

(b) $A\vec{u} - 3B\vec{v} = \begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 2 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -2 \\ 4 & -1 & -6 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} -2*3 + 0*0 + (-1)(-1) \\ 1*3 + (-1)*0 + 2*(-1) \\ 2*3 + 1*0 + (-3)*(-1) \end{bmatrix} - 3\begin{bmatrix} 1*(-2) + 2*0 + 3*1 \\ 1*(-2) + 0*0 + (-2)*1 \\ 4*(-2) + (-1)*0 + (-6)*1 \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 1 \\ 9 \end{bmatrix} - 3\begin{bmatrix} 1 \\ -4 \\ -14 \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 9 \end{bmatrix} - \begin{bmatrix} 3 \\ -12 \\ -42 \end{bmatrix} = \boxed{\begin{bmatrix} -8 \\ 13 \\ 51 \end{bmatrix}}$

(c) $B(2\vec{v} - \vec{u}) = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -2 \\ 4 & -1 & -6 \end{bmatrix}\left(2\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & -2 \\ 4 & -1 & -6 \end{bmatrix}\begin{bmatrix} -7 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1*(-7) + 2*0 + 3*3 \\ 1*(-7) + 0*0 + (-2)*3 \\ 4*(-7) + (-1)*0 + (-6)*3 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 2 \\ -13 \\ -46 \end{bmatrix}}$

2. [3 pts] Do # 26 on Page 48 (in Section 1.4).

Since $3\vec{u} - 5\vec{v} - \vec{w} = \vec{0}$,

$3\vec{u} - 5\vec{v} = \vec{w}$. i.e. $3\begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix} - 5\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$.

This implies that $\boxed{x_1 = 3 \text{ and } x_2 = -5}$ satisfy

$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$.

MTH 220-01 **Quiz 7** **Due: 11/3/2009 at 5:30pm** **Name:** Key

P1

A. I did not receive help from other people except perhaps Dr. Han when I did this quiz. **Signature:** ____________________.

B. Show your work. All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. [3 pts] Do # 6 on page 180.

2. [3 pts] Do # 10 on Page 181.

3. [2 pts] Do # 16 on page 181.

4. [2 pts] Do # 24 on Page 182.

#1. We need to find scalars c_1 & c_2 such that $c_1\begin{bmatrix}-3\\1\\-4\end{bmatrix}+c_2\begin{bmatrix}7\\5\\-6\end{bmatrix}=\begin{bmatrix}11\\0\\7\end{bmatrix}$.

Augmented matrix: $\left[\begin{array}{cc|c}-3&7&11\\1&5&0\\-4&-6&7\end{array}\right]\sim\left[\begin{array}{cc|c}1&5&0\\-3&7&11\\-4&-6&7\end{array}\right]\sim\left[\begin{array}{cc|c}1&5&0\\0&22&11\\0&14&7\end{array}\right]$

$\sim\left[\begin{array}{cc|c}1&5&0\\0&22&11\\0&0&0\end{array}\right]\sim\left[\begin{array}{cc|c}1&5&0\\0&1&1/2\\0&0&0\end{array}\right]\sim\left[\begin{array}{cc|c}1&0&-5/2\\0&1&1/2\\0&0&0\end{array}\right]$

Solution is $c_1=-5/2$, $c_2=1/2$. Thus, $\boxed{[\vec{x}]_{\mathcal{B}}=\begin{bmatrix}-5/2\\1/2\end{bmatrix}}$

#2 (1°) From the echelon form, a basis for Col A is

$$\boxed{\left\{\begin{bmatrix}1\\1\\-2\\4\end{bmatrix},\begin{bmatrix}-2\\1\\0\\1\end{bmatrix},\begin{bmatrix}9\\6\\-6\\9\end{bmatrix}\right\}}$$

(2°)

$[A\,|\,\vec{0}\,]\sim\left[\begin{array}{ccccc|c}1&-2&9&5&4&0\\0&1&-3&0&-7&0\\0&0&0&1&-2&0\\0&0&0&0&0&0\end{array}\right]\sim\left[\begin{array}{ccccc|c}1&-2&9&0&14&0\\0&1&-3&0&-7&0\\0&0&0&1&-2&0\\0&0&0&0&0&0\end{array}\right]$

$\sim\left[\begin{array}{ccccc|c}1&0&3&0&0&0\\0&1&-3&0&-7&0\\0&0&0&1&-2&0\\0&0&0&0&0&0\end{array}\right]$ (This is reduced echelon form)

MTH 220-01 **Quiz 6** **Due: 10/27/2009 at 5:30pm** **Name:** Key P1

A. I did not receive help from other people except perhaps Dr. Han when I did this quiz. **Signature:** ____________________.

B. Show your work. All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. [3 pts] Is it possible that a 7×7 matrix to be invertible when its columns do not span R^7? Why or why not?

No.

By IMT (a, h), a 7×7 matrix is invertible if and only if its columns span R^7.

2. [3 pts] Assume that A and B are both $n \times n$ matrices. If AB is invertible, show that B is invertible.

Since the matrix AB is invertible, by IMT (j), there exists matrix P such that $P(AB) = I$ i.e. $(PA)B = I$.

Denote $Q = PA$. Then $QB = I$. Using IMT (j) again, B is invertible

4. [9 pts] Let

$$A = \begin{bmatrix} 1 & -1 & 5 & 0 \\ 2 & 0 & 7 & 1 \\ -3 & -5 & -2 & 2 \end{bmatrix}, \qquad \vec{u} = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}.$$

(a). Find a basis for the column space of A: Col A.
(b). Find a basis for the null space of A: Nul A.
(c). Is the vector $\vec{u}$ in Nul A? Justify your answer.

(a) $A = \begin{bmatrix} 1 & -1 & 5 & 0 \\ 2 & 0 & 7 & 1 \\ -3 & -5 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 & 0 \\ 0 & 2 & -3 & 1 \\ 0 & -8 & 13 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 & 0 \\ 0 & 2 & -3 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$

From the echelon form, columns 1, 2, and 3 are pivot. Thus, a basis for Col A is $\boxed{\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ -2 \end{bmatrix} \right\}}$

(b) $[A \mid \vec{0}] \sim \left[\begin{array}{cccc|c} 1 & -1 & 5 & 0 & 0 \\ 0 & 2 & -3 & 1 & 0 \\ 0 & 0 & 1 & 6 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & -30 & 0 \\ 0 & 2 & 0 & 19 & 0 \\ 0 & 0 & 1 & 6 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & -1 & 0 & -30 & 0 \\ 0 & 1 & 0 & 19/2 & 0 \\ 0 & 0 & 1 & 6 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{41}{2} & 0 \\ 0 & 1 & 0 & \frac{19}{2} & 0 \\ 0 & 0 & 1 & 6 & 0 \end{array}\right]$. Thus, general solution to $A\vec{x} = \vec{0}$ is

$$\begin{cases} x_1 = -\frac{41}{2} x_4 \\ x_2 = -\frac{19}{2} x_4 \\ x_3 = -6x_4 \\ x_4 \text{ is free} \end{cases}$$

i.e. $\vec{x} = x_4 \begin{bmatrix} -41/2 \\ -19/2 \\ -6 \\ 1 \end{bmatrix}$. So a basis for Nul A is $\boxed{\left\{ \begin{bmatrix} -41/2 \\ -19/2 \\ -6 \\ 1 \end{bmatrix} \right\}}$

(c) No. Every vector in Nul A must have 4 entries. However, $\vec{u}$ has only 3 entries. Thus, $\vec{u}$ can not be in Nul A.

MTH 220-01 **Quiz 5** **Due: 10/20/2009 at 5:30pm** **Name:** Key

A. I did not receive help from other people except perhaps Dr. Han when I did this quiz. **Signature:** ________________.

B. Show your work. All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. [3 pts] Use the method introduced in Section 2.2 to find the inverse matrix of

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$$

Since $\det A = (1)(2) - (3)(-1) = 5 \neq 0$, A^{-1} exists.

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} 2 & -(-1) \\ -3 & 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -\frac{3}{5} & \frac{1}{5} \end{bmatrix}}$$

2. [5 pts] Use the algorithm introduced in Section 2.2 to find the inverse matrix of

$$B = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & 0 \end{bmatrix}$$

$$[B \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & 2 & 2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 4 & 10 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5/2 & -\frac{1}{2} & \frac{1}{4} \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -3/2 & 1/2 & -1/4 \\ 0 & 1 & 0 & -3/2 & 1/2 & 1/4 \\ 0 & 0 & 1 & \frac{5}{2} & -\frac{1}{2} & \frac{1}{4} \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -9/2 & 3/2 & 1/4 \\ 0 & 1 & 0 & -3/2 & 1/2 & 1/4 \\ 0 & 0 & 1 & 5/2 & -1/2 & 1/4 \end{array}\right]$$

Thus, B^{-1} exists and $B^{-1} = \begin{bmatrix} -9/2 & 3/2 & 1/4 \\ -3/2 & 1/2 & 1/4 \\ 5/2 & -1/2 & 1/4 \end{bmatrix}$

3. [3 pts] Find matrix C if $C^{-1} = \begin{bmatrix} 3 & -9 \\ 2 & 6 \end{bmatrix}$.

$$C = (C^{-1})^{-1} = \frac{1}{(3)(6)-2(-9)} \begin{bmatrix} 6 & -(-9) \\ -2 & 3 \end{bmatrix} = \frac{1}{36} \begin{bmatrix} 6 & 9 \\ -2 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 1/6 & 1/4 \\ -1/18 & 1/12 \end{bmatrix}}$$

4. [3 pts] Suppose that $AD = BD$, where A and B are $m \times n$ matrices and D is invertible. Show that $A = B$.

$AD = BD$

Multiply both sides by D^{-1} from the right:

$ADD^{-1} = BDD^{-1}$

i.e $AI_n = BI_n$

So $A = B$.

5. [6 pts] Consider the matrices:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}, \quad C = \begin{bmatrix} 8 \\ 10 \\ -2 \end{bmatrix}, \quad D = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}.$$

Evaluate (a) BA (b) $C^T D$ (c) CD^T

(a) $$BA = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8\cdot 1 + 3(-3) + 1\cdot 2 & 8\cdot 0 + 3\cdot 1 + 1(-3) & 8\cdot(-2) + 3\cdot 4 + 1\cdot 4 \\ 10\cdot 1 + 4(-3) + 1\cdot 2 & 10\cdot 0 + 4\cdot 1 + 1(-3) & 10(-2) + 4\cdot 4 + 1\cdot 4 \\ \frac{7}{2}\cdot 1 + \frac{3}{2}(-3) + \frac{1}{2}\cdot 2 & \frac{7}{2}\cdot 0 + \frac{3}{2}\cdot 1 + \frac{1}{2}(-3) & \frac{7}{2}(-2) + \frac{3}{2}(4) + \frac{1}{2}\cdot 4 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

(b) $$C^T D = \begin{bmatrix} 8 & 10 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 8\cdot 3 + 10(-1) + (-2)\cdot 4 \end{bmatrix} = \begin{bmatrix} 6 \end{bmatrix} = \boxed{6}$$

(c) $$CD^T = \begin{bmatrix} 8 \\ 10 \\ -2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 4 \end{bmatrix} =_2 \begin{bmatrix} 8(3) & 8(-1) & 8(4) \\ 10(3) & 10(-1) & 10(4) \\ (-2)(3) & (-2)(-1) & (-2)(4) \end{bmatrix} = \boxed{\begin{bmatrix} 24 & -8 & 32 \\ 30 & -10 & 40 \\ -6 & 2 & -8 \end{bmatrix}}$$

MTH 220-01 Quiz 8 Due: 11/10/2009 at 5:30pm Name: Key

A. I did not receive help from other people except perhaps Dr. Han when I did this quiz. **Signature:** ________________________.

B. Show your work. All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. [9 pts] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \\ 1 & 3 & 6 & 6 \\ 1 & 3 & 6 & 9 \end{bmatrix}.$$

(a). Find the determinant of A: det A (show the steps).
(b). Find the determinant of A^4: det A^4.

(a)
$$\det A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 \\ 1 & 3 & 6 & 6 \\ 1 & 3 & 6 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 2 & 5 & 5 \\ 0 & 2 & 5 & 8 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 3 & 6 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 3 \end{vmatrix}$$

$$= 1 \cdot 2 \cdot 3 \cdot 3 = \boxed{18}$$

(b) Using the property $\det A^k = (\det A)^k$, we have

$$\det A^4 = (\det A)^4 = \boxed{18^4} \text{ or } \boxed{104976}$$