6.1220 LECTURE 24

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Contents

1. Research

1.1. Markov Chains: Extended

Problem 1.1

Given a complicated and high-dimensional probability distribution π efficiently sample from π.

We can perhaps use Markov Chains. Design a walk matrix P such that the distribution $\pi_t = \pi_0 P^t$ of X_t converges to π as $t \to \infty$. We need to ensure that P has π as its stationary distribution, can be simulated efficiently, and be both strongly connected and aperiodic for this idea to work.
A general question is how can we bound the rate of convergence? We want to bound it by $n \log n$ to ensure feasibility.

Definition 1.2: Hardcore Gas Model

$G = (V, E), \lambda \geq 0, \pi(I) \propto \lambda^{|I|}$ for independent sets $I \subseteq V$.

Theorem 1.3

$\forall \delta \in \mathbb{N}, \exists \lambda_c(\delta)$ such that for the task of sampling from the hardcore model given any graph with max-degree δ, the following hold:

- If $\lambda < \lambda_c(\delta)$, then the Gibbs sampler succeeds in $O(n \log n)$ time.
- If $\lambda > \lambda_c(\delta)$ then sampling is NP hard.

Conjecture 1.4

If $q \geq \delta + 1 **$ where δ is the max degree, then the Metropolis-Hastings for q coloring succeeds in $O(n \log n)$ time.

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1.2. Multi-Agent Learning

Recently, machine learning has been able to understanding information of different modalities and do things with them. There are correspondingly many multi-agent applications, split roughly into four categories:

- Multi-player Game Playing: Poker, Starcraft
- Multi-robot interactions: Autonomous drivinng, Economic policy design
- Generative Adversarial Networks (GANs): synthetic data generation
- Adversarial Training: robustifying models against adversarial attacks

So why do we train modes using multi-agent learning?

(1) Strategic Behavior does not emerge from standard training
(2) Naively trained models can be manipulated
(3) AI Interaction may lead to unpredictable/undesirable consequence
(4) The optimization workhorse of deep learning struggles in multi-agent setting (gradient descent)
(5) Game Theory breaks