1 PLDA within class and across class analysis

Assume that the within class covariance matrix is I, and the across class covariance matrix is Ψ .

For a test utterance u^{te}, the probability of this utterance belonging to a specific speaker can be described as Eq. 1, where $u^{tr}_{1...n}$ is n training utterance of this speaker. It's distributed with some mean and some covariance. For simplicity, this mean is denoted with m, see Eq. 4.

Let n equals to 0, the probability of test utterance with no class assumption is as Eq. 2.

Then the objective likelihood ratio of an utterance belonging to a specific speaker denotes as Eq. 3.

$$P(u^{te}|u^{tr}_{1...n}) = N(u^{te}|\tfrac{n\Psi}{n\Psi+I}\bar{u}^{tr}, I + \tfrac{\Psi}{n\Psi+I}). \tag{1}$$

$$P(u^{te}) = N(u^{te}|0, I + \Psi). \tag{2}$$

$$N(u^{te}|\tfrac{n\Psi}{n\Psi+I}\bar{u}^{tr}, I + \tfrac{\Psi}{n\Psi+I}) \quad / \quad N(u^{te}|0, I + \Psi). \tag{3}$$

$$m = (n\Psi)/(n\Psi + I)\bar{u}^{tr}. \tag{4}$$

The objective log likelihood ratio taking reference of Eq. 3 is expanded as Eq. 5

$$\begin{aligned} -0.5[(u^{te} - m)(I + \Psi/(n\Psi + I))^{-1}(u^{te} - m) + logdet(I + \Psi/(n\Psi + I))] + \\ 0.5[u^{te}(I + \Psi)u^{te} + logdet(I + \Psi)]. \end{aligned} \tag{5}$$

2 Reference

This analysis comes from Kaldi toolkit, which is taking reference of "Probabilistic Linear Discriminant Analysis" by Sergey Ioffe, ECCV 2006. They are the pioneers.