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THE FLOW OF HUMAN CROWDS

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■ **Abstract** The modern study of a crowd as a flowing continuum is a recent development. Distinct from a classical fluid because of the property that a crowd has the capacity to think, interesting new physical ideas are involved in its study. An appealing property of a crowd in motion is that the nonlinear, time-dependent, simultaneous equations representing a crowd are conformably mappable. This property makes many interesting applications analytically tractable. In this review examples are given in which the theory has been used to provide possible assistance in the annual Muslim Hajj, to understand the Battle of Agincourt, and, surprisingly, to locate barriers that actually increase the flow of pedestrians above that when there are no barriers present. Modern developments may help prevent some of the approximately two thousand deaths that annually occur in accidents owing to crowding.The field of crowd motion, that is, the field of "thinking fluids," is an intriguing area of research with great promise.

1. INTRODUCTION

A large crowd moves with many of the attributes of a fluid. However, unlike a classical fluid, a crowd has the ability to think. There is a long-standing myth that all crowds are irrational and erratic, and by implication their behavior is unpredictable. However, unorchestrated crowds are rational. The idea that they are irrational is long standing and obtained academic standing from early sociological writings by aristocrats during the French Revolution of the 1790s. The present sociological view of crowds is that unorchestrated crowds are rational and can therefore be expected to abide by scientific rules of behavior (see McPhail 1991). Many sociological issues arise in studying the behavior of crowds, but the present study is concerned only with factors affecting their motion.

The study of crowd motion by discrete simulation of individual pedestrians is well established. Some fascinating work on this topic, with an emphasis on self-organizing crowds, is reported by Helbing (1997). Discrete simulation forms a very useful numerical tool for practical applications, but as a research tool, it suffers from a lack of analytical tractability that makes deriving general results difficult. Furthermore, such an approach does not see the flow as a whole (see Gaskell &

Benewick 1987). These difficulties are often avoided by using a continuum formulation. The modern study of the motion of crowds as a continuum, with equations specifically derived for the purpose, originated in the 1990s. No longer are the Navier-Stokes equations applied in their entirety as if ordained to be appropriate to all moving continua. New equations, using the concepts of fluid mechanics, have been derived in consultation with behavioral scientists. As presented here, it is remarkable that these nonlinear partial differential equations that govern the flow of a crowd of people are conformably mappable even in unsteady situations. This mapability makes the study of the dynamics of crowd motion highly amenable to analytic solution.

The present manuscript elucidates the governing equations (see Section 2). The manuscript also outlines some of the diversity of situations in which the study of the flow of pedestrians has been used (see Smith & Dickie 1993). The following are applications considered in Section 3:

(*a*) As representative of the many applications in engineering, an outline of a serious attempt to improve the flow of pilgrims over the Jamarat Bridge near Mecca during the annual Muslim Hajj is provided. This bridge was the site of several large, fatal disasters during the 1990s and has come to symbolize, apart from its religious significance, the huge problems involved in handling annually more than two million pilgrims to the Hajj.

(*b*) As representative of some of the novel phenomena that occur in the theory of crowd motion, an outline of the consequences of the theory relating to barrier location is given. Placing a barrier in a flow of pedestrians can decrease the travel time of all pedestrians involved. This startling result is known as Braess' paradox. The paradox may be inverted, as an application of the theory of crowd motion, to determine how barriers may be positioned to decrease the travel time of all pedestrians in a system.

A scan of the news services shows that each year approximately two thousand deaths occur as a direct result of crowding or crowd motion. Approximately half of these deaths occur at extremely high densities as a result of asphyxiation, and approximately half occur at lower densities as a result of percussion on fallen pedestrians. At very high crowd densities, for which asphyxiation may or may not occur, the equations of motion for a crowd resemble the two-dimensional Navier-Stokes equations but with a Rayleigh-like friction rather than the more usual Newtonian friction. In this case the advective accelerations are generally negligible. The consequences of such equations are studied in Section 4. To illustrate this theory, an outline of its novel application to the study of medieval battles is provided; that application of the theory to the Battle of Agincourt (1415) leads to a description of the battle that is consistent with contemporary chronicles but at variance with modern popular accounts.

At very low densities the continuum hypothesis, as used throughout this study, becomes questionable. At these low densities it is constructive to consider discrete pedestrians.

This manuscript does not explore the wealth of discrete pedestrian modeling that is documented in the literature for all densities. However, Section 5 does provide an extremely brief introduction to this topic at very low densities, a situation where pedestrians may seek not to avoid each other but to move over well-worn paths established by earlier pedestrians; thus they deviate from a worn path only when it goes in the wrong direction. The continuum theory presented here might in the future be interpreted probabilistically, in which case it will likely meld with discrete models over the full range of pedestrian densities. A short summary of the present review and an indication of some likely extensions to the topic are presented in Section 6.

2. CONTINUUM BEHAVIOR

A crowd of pedestrians can generally be treated as a continuum, provided the characteristic distance scale between pedestrians is much less than the characteristic distance scale of the region in which the pedestrians move. In this section, and in Sections 3 and 4, we consider situations in which the crowd can be modeled simply as such. The density of the crowd is generally measured by the expected number of pedestrians found per unit area. In dense crowds pedestrians pack themselves in a body-centered square pattern aligned with the direction of motion and, except in the case of forming lines, may be regarded as locally homogeneous, as illustrated, for example, by Smith (1993). Furthermore, a crowd may freely orientate in any direction.

A group of pedestrians is often composed of many pedestrian types with different walking characteristics and various objectives. If we consider a single type of pedestrian who is walking up a slope, this pedestrian may have a different characteristic stride from those walking down the same slope. Here we neglect that the direction of motion often affects the walking characteristics of pedestrians. To derive the equations that govern the flow of a single type of pedestrian walking on isotropic topography, it is useful to combine the unsteady continuity equation with formulae resulting from the following three hypotheses governing the motion.

Hypothesis 1. The speed at which pedestrians walk is determined solely by the density of surrounding pedestrians, the behavioral characteristics of the pedestrians, and the ground on which they walk.

Hypothesis 2. Pedestrians have a common sense of the task (called potential) that they face to reach their common destination, such that any two individuals at different locations having the same potential would see no advantage to exchanging places.

Hypothesis 3. Pedestrians seek to minimize their (accurately) estimated travel time but temper this behavior to avoid extreme densities. This tempering is assumed to be separable, such that pedestrians minimize the product of their travel time as a function of density.

The evidence is highly supportive of these hypotheses. Hypothesis 1 is a restatement of the proposition by Greenshields (1934) for vehicular traffic as used by Lighthill & Whitham (1955) in their classic study of kinematic waves in the flow of vehicles on a highway (see also Whitham 1974). Over the years, numerous studies have confirmed the relevance of Hypothesis 1 to the flow of pedestrians (see, for example, Fruin 1971, Pushkarev & Zupan 1975, Daly et al. 1991). Virkler & Elayadath (1994) provided a very useful summary of many forms of the speed function. However, this hypothesis is not appropriate in very high-density flows where contact forces between pedestrians are significant.

The value of Hypothesis 2 is dependent on how well the pedestrians are visually informed. This hypothesis does not hold for vehicular traffic where a visual assessment of the situation is generally impossible. However, it does appear to hold if pedestrians can visually assess the situation. If there is a variation in the height of pedestrians in the crowd, short pedestrians with a blocked view must obtain information about the desired direction of motion from observing the behavior of the taller members of the crowd. This hypothesis has become commonly accepted in the field as a working hypothesis even though it is not appropriate in all situations.

Hypothesis 3 is dependent on the reason for the motivation of pedestrians. It is only appropriate to goal-directed pedestrians (see McFarland 1989) who have a clearly identified geographical target as their destination. Such pedestrians walk toward a well-defined objective. Furthermore, this hypothesis requires that reaching the objective and avoiding extremely high densities be mathematically separable. There is no observational support for this clause: It is assumed for mathematical convenience. However, the nonmathematical consequences of this assumption are generally thought to be slight.

The above hypotheses lead to the basic governing equations for the flow of a single pedestrian type. These equations are

$$-\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho g(\rho)\, f^2(\rho)\,\frac{\partial \varphi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho g(\rho)\, f^2(\rho)\,\frac{\partial \varphi}{\partial y}\right) = 0$$

and

$$g(\rho)\, f(\rho) = \frac{1}{\sqrt{\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2}},$$

where φ is the remaining travel time, which is a measure of the remaining task (called the potential), ρ is the density of the crowd, $f(\rho)$ is the speed of pedestrians as a function of density, $g(\rho)$ is a factor related to the discomfort of the crowd at a given density, and (x, y, t) denotes the horizontal space and time coordinates. The discomfort factor is equal to the distance that a pedestrian would be prepared to walk uninhibited in order to avoid walking a greater unit distance in the crowd. With these governing equations, it is necessary to specify the forms of the speed and discomfort functions, $f(\rho)$ and $g(\rho)$, respectively. A full derivation of these equations may be found in Hughes (2002a).

Appropriate choice of the speed and discomfort functions makes the above equations of motion conformably mappable in space, despite their time dependence and nonlinearity (see, for example, Hughes 2002a). This remarkable result simplifies the solution of the equations governing the flow of crowds. (However, the density must be rescaled according to the Jacobian of the conformal map, and in the case of the high densities, the equations are only conformably mappable when the dependent density is rewritten as the density of holes, by analogy with the flow of holes in semiconductor in solid state physics.) As noted in the above discussion of Hypothesis 1, there have been many proposals for the speed function, but there have been few for the discomfort function. Most studies have not separated the role of these two functions even though observations sometimes strongly suggest a sense of panic within crowds, as noted by Hankin & Wright (1958). The use of both functions of the form proposed for conformal mapability greatly simplifies the analysis and may increase applicability.

Using numerical simulation, Blue & Adler (2001) have studied the motion of crowds, with possible cross-flow, with an emphasis on the detailed behavior of discrete pedestrians in a long passage. Although their geometry is limited, the detail that they have exposed in this and earlier articles is very valuable. If the long passage can be conformably mapped into some geometry to be studied, then the detailed behavior found in the channel might also be expected to map in the same way. There is need to explore the mapping of the detailed structure.

The above formulation leads to two flow regimes, represented by subcritical flow and supercritical flow, as expected by analogy with traffic flow on a highway (see Lighthill & Whitham 1955). Thus there is clearly a strong parallel with open channel flow, as described in most engineering hydraulics texts (see, for example, Nalluri & Featherstone 2001). Unfortunately, the terminology of a fast, low-density flow being supercritical and a slow, high-density flow being subcritical is not shared between transport engineers and fluid dynamists, resulting in some confusion.

It may seem from the above equations that there are strong similarities and few differences between the flow of crowds and the flow of a liquid governed by the frictionless, shallow water equations. However, it is remarkable that, whereas the shallow water equations are only conformably mappable under restricted steady-state conditions, the above equations are amenable to conformal mapping even in the unsteady case.

Unparalleled in a classical fluid, it is possible to have a crowd of various types of pedestrians walking toward different objectives or with various speed relationships, $f(\rho)$. The hypotheses given earlier generalize readily to such flows. Hypothesis 1 is the only one with a surprising generalization. It is important to note that, when multiple types of pedestrians are present, the speed of each type of pedestrian is given by its speed-density relationship; the appropriate density is thus the total density (see, for example, Toshiyuki 1993). This behavior appears surprising and was for many years neglected in the Western literature because, at the time, it seemed inexplicable. Such behavior occurs because pedestrians walk through each other in bands, as illustrated in Figure 1. Each band consists only of

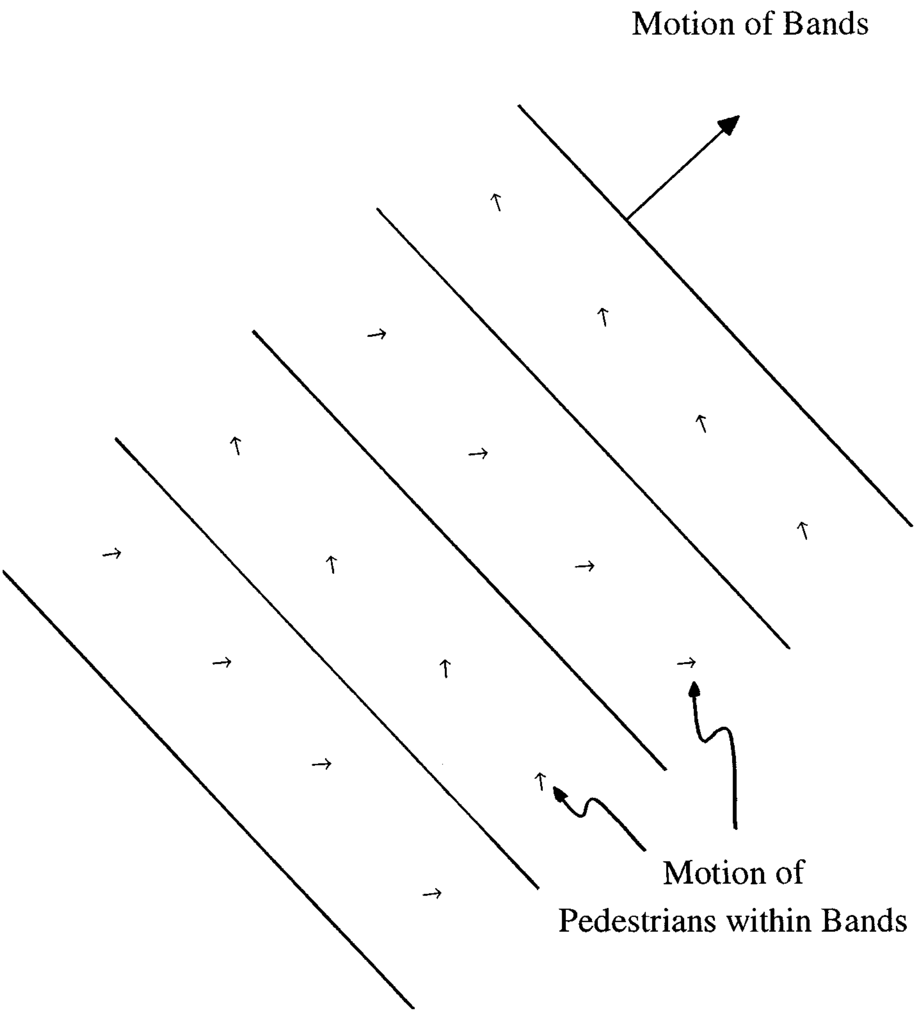

Figure 1 Diagrammatic representation of the banding that may occur in a crowd when it is composed of two different types of pedestrians, both with the same walking habits and flow rate but walking in directions at right angles to each other. Each band moves at an angle to both flows, such that everyone remains within a band of their own type of pedestrian.

pedestrians of a particular type. The density of pedestrians in each band is equal to the density of the entire flow as if there were no bands (see Ando et al. 1988). The resulting equations continue to be conformably mappable with two additional equations for each type of pedestrian in the crowd (see Hughes 2002a).

It is interesting to note that in a crowd consisting of multiple types of pedestrians at the same location there is either a subcritical or a supercritical flow for all types.

It is not possible to have a subcritical flow for some types of pedestrians and a supercritical flow for other types at the same location. Thus a control section for one type of pedestrian is a control section for all types (see Hughes 2002a).

Numerous extensions to the above are possible. An interesting extension is that of allowing pedestrians to tire as they walk (see Rose & Gamble 1994). If the speed of pedestrians falls with this tiredness, the density of pedestrians increases as they move along a passage. Thus a supercritical flow may become critical with distance. Dunn (2001) has considered the density of pedestrians in an initially empty passage subjected subsequently to a constant flow of entering pedestrians. For small times the well-known fan structure for the characteristics of the solution develops as described by Whitham (1974). At these small times a few pedestrians are located at a substantial distance down the passage, with density increasing with the distance behind them. However, as these leading pedestrians tire, their density starts to increase and critical conditions are encountered. A kinematic wave then moves back up the passage toward the entry to the passage where it limits the number of pedestrians entering.

A note of caution needs to be sounded in regard to all of the above modeling because it is based on the assumption that pedestrians in the crowd can accelerate to their desired speed instantaneously. There are two cases where such acceleration is not possible. The first case is when the crowd is blocked and a very high density forms (see Section 4). The second case is when the distance scale involved in the flow is so small that insufficient time elapses between directional changes for a pedestrian in the crowd to adjust to a new direction before another change is imminent. There are two reasons for a finite adjustment time: the time needed for mental adjustment to unexpected changes in geometry (as studied by Sidaway et al. 1996 for vehicular traffic) and the physical requirements on both nonslip contact with the ground and the strength of the pedestrian to handle the acceleration in an adjustment. This second case, where an instantaneous adjustment is not possible, requires a model that allows for the time involved in acceleration. Such a model is the excellent, discrete one presented by Hoogendoorn & Bovy (2000). Provided the distance scale of the problem is greater than a few meters, there is no need to include this acceleration, and Hypotheses 1, 2, and 3 remain valid.

3. APPLICATIONS

To illustrate the above theory, two examples chosen from the author's own work are considered. The first example is that of the flow over the Jamarat Bridge near Mecca on the last day of the annual Muslim Hajj (see Stewart 1980). This bridge has been the site of numerous major accidents, most notably the 1990 Hajj when 1426 pilgrims lost their lives in a single accident. While crossing this bridge, between sunrise and sunset on a single day, more than two million pilgrims must stone each of three pillars distributed along the bridge. In doing so pilgrims are reenacting the stoning of the devil by Mohammed, who in turn was following in Abraham's footsteps. The pillars are protected by barriers that are presently arranged in a circle around each individual pillar. Within the Muslim community there has been discussion of the optimum shape necessary to avoid disasters such as the 1990 incident.

The study of the flow of pilgrims is largely a study of which boundary conditions are appropriate at each of the barriers around the pillars. The equations described in the previous section must be solved with two pedestrian types, one type corresponding to pedestrians approaching a barrier around a pillar and the other type corresponding to pedestrians leaving the same barrier and moving toward the next pillar. If approaching pilgrims see the stoning of the pillar at hand as their objective, the barrier around that pillar is an equipotential, that is, the potential is constant on the barrier. By contrast, the pilgrims leaving the barrier have the barrier of the next pillar as an equipotential. The result is that pilgrims accumulate at the presenting section of the barrier. This pattern is very similar to the behavior observed at the Jamarat Bridge.

If, however, pilgrims were to take an overview of their current task and aim not at stoning the pillar at hand in the minimum time but at stoning all three pillars in minimum time, the normal derivative of the potential rather than the potential itself would have to be specified at the pillar for approaching pilgrims. The resulting flow and density of pilgrims is very different from that described above. Pilgrims concentrate near the sides of the barrier, throwing their stones while en route to the next pillar. This more-efficient behavior is not observed at the Jamarat Bridge. Pilgrims clearly take a local view, raising an important question as to what pedestrians in general see as the task at hand. The way the task is perceived may strongly influence the behavior of pedestrians or, in this case, pilgrims. There is obvious room for significant research on human behavior in this area.

As noted above, the flow of pilgrims is not optimum; there is considerable scope for improving the flow of pilgrims by changing the shape of the barrier around each of the pillars. If pilgrims can be induced into thinking globally rather than locally about the stoning of the pillars, they may be persuaded to stone the pillars as they are walking. Thus rather than stand at least half a dozen deep at each barrier, pilgrims would walk past the barrier without stopping. However, the region immediately in front and behind the barrier would be void of pilgrims as there is no advantage to lengthening their path when there is a maximum speed at which they can walk no matter how low the density. Because the nonlinear equations of motion are conformably mappable as noted above, free streamline theory from fluid mechanics (see, for example, Batchelor 1967) may be used to calculate the form of the flow. Both upstream and downstream separation is predicted. Hence pilgrims may be prevented from taking a local view of the stoning by placing a barrier in the region upstream of each pillar that is never occupied by those pilgrims who take a global view of the task. The interested reader is referred to a paper by Hughes (2002a) for a complete description.

This second example is chosen for its surprising nature. However, its application is limited. Braess' paradox is a classic in the study of vehicular traffic. This paradox describes the counter-intuitive result that addition of a road link to a road network may cause all vehicles on the network to take longer to traverse the network. To understand the basis of the paradox, consider a simple road network consisting

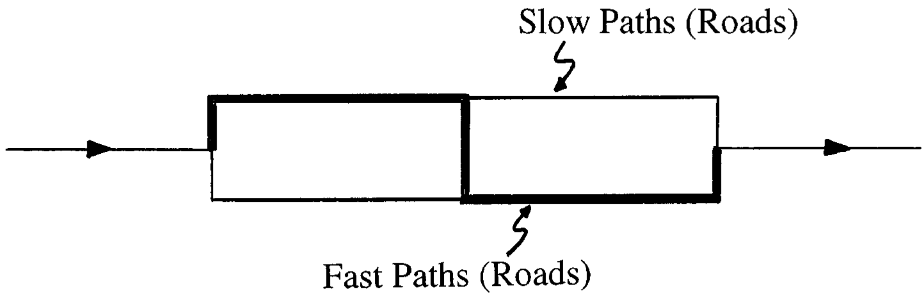

Figure 2 Representation of the network considered. The network consists of a fast path followed by a slow path or visa versa, with very slow terrain off the paths. The construction of a joining path may disadvantage all pedestrians. These paths may be broadened to fill the region but with regional variations in speed to achieve similar behavior.

of two roads of equal length from a shared start to a shared finish (illustrated in Figure 2). One of the roads starts with a fast section and ends with a slow section, whereas the other road starts with a slow section and ends with a fast section. In both cases the fast and slow sections are half the length of the road. By symmetry the traffic, if well informed, divides equally between the two roads, introducing the same amount of traffic congestion into each road. However, if a short, fast road link from one of these roads to the other is added halfway along their lengths, all of the traffic along the fast links may be diverted. The increased traffic on the fast links causes the traffic to be slower on these links than before the additional link was added; as a result the whole network is slower to pass through than before the additional short, fast link was added. No equivalent paradox occurs in a pipe network carrying a fluid because the path usually taken by a particle of fluid is such as to avoid a discontinuity in pressure at locations where pipes join rather than, as here, a discontinuity in travel time. Steinberg & Zangwill (1983) claim that the occurrence of Braess' paradox in road networks is common. The reader is referred to their paper, that by Pas & Principio (1997), and the references cited therein for a fuller appreciation of the paradox.

If the paradox can occur in road networks, it may possibly do so in a less-constrained environment involving pedestrians. Inverting the paradox, we would like to know where a barrier might be placed to decrease the travel time of pedestrians. Using variational calculus and the equations of motion given in Section 2, it is possible to show that if the density at any point in the domain decreases when the flow through the domain increases, a barrier should be placed at that point. As variational calculus is only concerned with small (infinitesimal) variations, it is necessary to grow barriers iteratively. Nevertheless, the procedure works well when combined with direct simulation (see Hughes 2002b). It needs to be emphasized that in practical situations there is possibly little value in adding barriers both because many flows of pedestrians vary substantially with time and because of legal consequences.

4. HIGH-DENSITY BEHAVIOR

The behavior of crowds at high densities is even less well explored than that of crowds at more-common densities. At high pedestrian densities there has been some success in matching observed behavior with that expected from the Navier-Stokes equations (see, for example, Bradley 1993). However, four significant objections to this approach exist. First, such an approach does not exhibit a natural transition to the more readily verifiable lower-density theory presented in Section 2. Such a transition is considered imperative in this newly established field. Second, the assumption that a crowd obeys Newton's second law of motion without regard to the horizontal shear stress exerted on pedestrians by the pavement is not sensible. Third, previous researchers have assumed that any dampening of motion may be represented by a viscous analogue. However, because of the high densities involved in such studies, the viscous terms in the equations of motion have been minor in comparison with pressure forces, and so studies have not tested this representation. Fourth is a minor objection regarding inconsistent scaling. Studies using the Navier-Stokes equations have retained all terms, including the advective acceleration terms, despite their unimportance in all crowds, irrespective of the crowd density, except at very small distance scales as studied by Hoogendoorn & Bovy (2000).

To overcome these difficulties, pedestrians try to achieve the speed $f(\rho)$ by applying a shear force to the ground underneath. Therefore the equations of motion of a crowd are constructed by use of a force balance on an element of the crowd with the shear force of the underlying ground on the flow parameterized in terms of the difference in speed between the desired and the achieved speed of pedestrians. The third and fourth objections, noted above, are eliminated by neglecting the small terms involved, in the absence of information about them. This representation appears to work well in explaining observations. A general description of the formulation may be found in an appendix to Hughes (2002a).

To illustrate the effectiveness of this representation, it is informative to consider the intriguing application of the use of the formulation in exploring medieval battle records. The battle of Agincourt (1415) (described in some detail by Burne 1999) is considered here. Accounts vary, but during the main stage of the battle men-at-arms from both the English and French camps faced each other in conflict. (Neither archery nor mounted knights were crucially involved in this phase of the battle.) The English, weakened by dysentery, met a confident, vastly superior French force along a linear battlefront. The result, according to modern accounts, was a "wall of bodies" composed mainly of slain French. Modeling the encounter using the equations described above yielded an unstable battlefront, with the French force more strongly motivated than the English force. French forces in the model pushed back the English line in a spatially varying manner along the battlefront. This suggests that the French, surrounded by the English in various places along the battlefront, might have contributed not to a "wall of bodies" but to mounds of bodies. Clearly, either the mathematical model or modern accounts of the battle are wrong. The issue is further confused by the old German, and hence Saxon, word “wal,” meaning “those who are left dead on the field” [as discussed in the English translation of the Nobel Prize–winning 1962 book by Canetti (2000)]. A search of the literature, mainly chronicles in French or Latin, shows that there was no confusion in the language used (see, for instance, one of the English translations of the *Chronique de Enguerrand de Monstrelet*). Three “mounds of bodies” were left after the battle rather than a “wall of bodies” as appears in modern literature (see Clements & Hughes 2002). Modern military historians, presumably copying each other’s interpretations, have been incorrectly describing the battle.

The above example is regarded as encouraging support for the present mathematical representation of very dense crowds. This example, like most others, is quasi-steady. However, the time-dependent waves described by Bradley (1993) are also described by the above formulation, provided that the timescale associated with accelerations in the wave is short enough so that inertial forces dominate the shear force on the underlying pavement.

5. LOW-DENSITY BEHAVIOR

Little work has been done on the use of continuum modeling at low densities presumably because of complications associated with statistically interpreting the results. However, some very interesting work has been done with discrete modeling at low densities. The numerical model by Hoogendoorn & Bovy (2000) (noted above) is well suited to studying such crowds. Earlier models of pedestrian behavior were based on an analogy with the Boltzmann’s formulation for gases, superimposed on a mean speed (see, for example, Henderson 1974). However, because Boltzmann-type models do not represent the large-scale flow well, such models have faded from use in favor of models such as that by Hoogendoorn & Bovy (2000).

To illustrate the interesting behavior possible at low densities, we consider a study by Helbing et al. (1997). This example concerns the paths taken by pedestrians crossing the central green on the university campus at Stuttgart-Vaihingen, Germany. At very low densities pedestrians may show preference toward paths that are worn and well established while crossing the grassed area. Helbing et al. (1997) constructed a simple model for simulating the paths of individual pedestrians. Each pedestrian was influenced by their desired destination and the attraction of walking on an already established path. In the early stages of the simulation pedestrians walked directly toward their desired destination. As the simulation progressed the pedestrians on a less-traveled path that was similar to a more-traveled path for part of the journey deviated to the more-traveled path. Furthermore, frequently used paths joined to create yet more frequently used paths. Thus the pattern of paths evolved with time (as illustrated in Figure 3).

In this case the density of pedestrians is extremely low. It is clear that at low densities on grassed surfaces, as here, pedestrians are attracted toward regions of

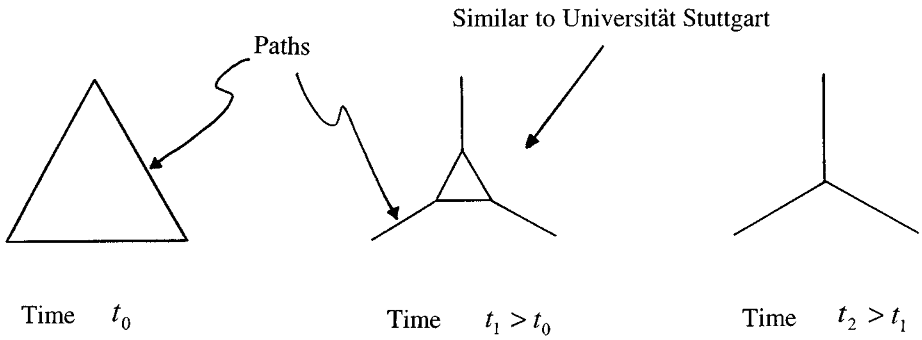

Figure 3 Redrawn paths of those calculated by Helbing et al. (1997), illustrating the possible evolution of the paths with time depending on the flow between the different origins and destinations.

high pedestrian density. Such behavior is consistent with the discomfort function, $g(\rho)$, increasing sharply at very low densities with pedestrians preferring paths that contain a higher pedestrian traffic. To the author's knowledge, no continuum representation has been used to study such low-density flows. If such a study were undertaken, it would be essential to interpret the density of the crowd as a statistical quantity.

6. CONCLUSIONS

In recent years there has been substantial progress in the theory of the flow of human crowds. Earlier modeling was based on assumed paths for the pedestrians with, in some cases, a prescription for modification to these assumed pedestrian paths for congestion. Because this research field is in its infancy, there is much room for innovative research. Modern theory calculates the particle paths. The theory is based on continuum modeling and its relationship with discrete modeling. There is a strong similarity to classical fluid dynamics, but the present flows "think," which gives them some intriguing properties. Present continuum modeling of this "thinking fluid" is based on well-defined hypotheses, which are supported by observations but which may be refined in the years to come as our knowledge of the behavioral sciences expands.

Much needs to be learned about the behavioral science of crowds, and as our knowledge of them develops many modifications will no doubt need to be accommodated in the theory. The presently understood, basic equations governing continuum modeling of crowds are coupled, nonlinear, partial differential equations. The remarkable thing about these equations is that they are conformably mappable and as such may be easily solved in simple geometries. Many problems require modification of these basic equations for specific features of the crowd's behavior. Such features may be easily included in the equations of motion, but depending on the modification, they may destroy the invariance of the equations under conformal mapping.

Numerical simulation of crowds, both as a continuum and as discrete pedestrians, is extremely useful in situations with complicated geometry and when complicated behavioral effects are included. At present, numerical simulation is the only realistic method of solution at very low crowd density. At such densities the continuum hypothesis is not valid. However, there is room for reinterpreting continuum models in a probabilistic framework such that the density calculated is the expected value of the density. Such an extension has not been pursued to date but holds great promise.

The important study of the motion of human crowds is in its infancy; as such there are many unstudied, important problems at all densities. As noted above, there is need to extend in a probabilistic manner the continuum theory of pedestrian flows to low-density flows. Such an extension would provide an important interface between discrete and continuous models of human flows. At higher densities there is a great need to understand the transition from behavior at the macroscopic level of the continuum to the microscopic level of the individual. The nature of this transition is important in studying the "turbulent" nature of the flow behind objects. At very high densities there is a need to determine what macroscopic conditions are likely to be dangerous to pedestrians that move as microscopic identities. A dearth of information on which to base such a study is clearly present. However, data on the nature of accidents in the form of descriptions of locations where bodies are found, as, for example, in coroner's reports and police reports, exist. The problem becomes an inverse one of determining the conditions of the accident from the results they caused. There are important phenomena, at all densities, in need of study.

Understanding the interface between civil engineering and behavioral science is crucial. The above field of work spans only one of many such topics relevant to this interface, but it is a field of work that may be described aptly as fluid mechanics with an "unclassical fluid," a "thinking fluid" that we are only just beginning to understand.

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