Homework assignment II

Course: “Performance of Networked Systems”, Spring 2018

Lecturer: Prof.dr. Rob van der Mei

I. Dimensioning of cellular networks with video-conferencing services

A mobile operator of a cellular GSM network wants to determine how many base stations are needed to satisfy its customers’ Quality of Service (QoS) demands. To this end, the operator wants to determine the maximum size of a cell for which the call-blocking probability is still below some given threshold. Voice telephone calls are generated with rate 10 calls per minute *per square kilometer* (i.e., km^2), and the call duration has an exponential distribution with mean 1.5 minutes. Assume that each voice call requires a single channel to the nearest base station, and that each cell can support only 6 channels in parallel.

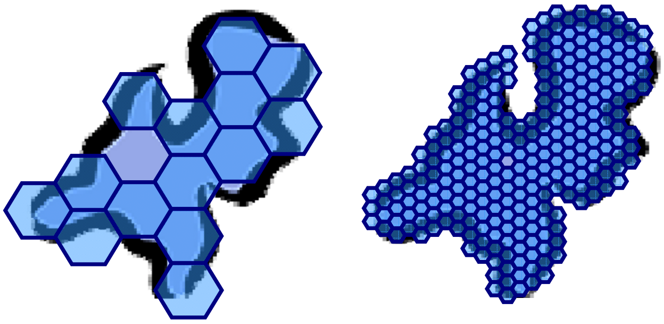


Figure 1. Illustration of cellular network dimensioning problem.

To make a proper decision on the number of base stations to be placed to offer good quality to its customers, the operator wants to understand the impact of the cell size (in km^2) on the call-blocking probability.

1. Formulate a simple model description for the problem. Be precise, motivate your assumptions and clearly define any notation!

2. Give a formula for the call blocking probability in terms of the model parameters,

3. Write your own ‘Erlang-B calculator” and calculate the blocking probability for cell sizes 0.5 km^2, 1 km^2 and 2 km^2.

4. The call blocking probability is known to be *insensitive* with respect to the distribution of the call duration. What *exactly* does that mean?

5. Is the call blocking probability also insensitive with respect to the inter-arrival time distribution of the calls? If so, why, if not so, give a counter-example.

Now suppose the service provider wants to offer a new *additional* service to its customers: *video conferencing*. Each conference call requires 3 parallel channels for each connection. Video conferencing calls arrive according to a Poisson process with rate 0.25 calls *per hour per km*^{2}, and the conference call duration is exponentially distributed with mean 15 minutes. Recall that each cell has 6 channels. Call attempts are blocked when there are not enough lines available. Assume throughout that the cell size is 1 km^2.

6. Formulate the evolution of the system as a two-dimensional continuous-time Markov chain, describing the numbers of calls at both call types (i.e., voice calls and video conferencing calls, respectively). Specify the state space and the transition rates and make picture of the Markov chain, as was done during the course (similar to page 19 of the background reading material).

7. Formulate and solve the balance equations for the Markov chain and calculate the equilibrium state probability for each state.

8. Use the product-form theorem discussed during the lecture of March 5 to given an explicit formula for the equilibrium state probabilities of the Markov chain.

9. Calculate the equilibrium state probabilities for the model (simply by filling out the proper parameter value in the formula obtained in question 6).

10. Use this result to calculate the blocking probabilities for both call classes.

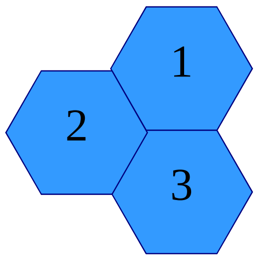
11. Formulate the Kaufman-Roberts recursion for the model.

12. Use the Kaufman-Roberts recursion to calculate the blocking probabilities for the voice calls and the conference calls (the answers should be the same as the ones obtained via the state probabilities in question 9).

13. What will happen to the results in questions 9 and 11 when the call durations of the two call types (voice and video conferencing) are constant instead of exponentially distributed?

II. Three-cell analysis

Consider a cellular voice network with three neighboring cells, as depicted below. The mean number of call attempts per minute for cells 1, 2 and 3 are 5, 10 and 15, respectively. We assume that the mean call duration is 2 minutes. There are a total of 32 channels, and there is a fixed number of channels per cell. However, to avoid interference neighboring cells cannot use the same channel.



14. Determine the distribution of the 32 channels over the three cells that minimizes the call blocking probability of an *arbitrary* call (regardless of the cell in which takes place. To this end, extend your Erlang-B calculator to the three-cell case, and sure it calculates the call blocking probabilities per cell and the overall call blocking probability.

15. Suppose we want to have an overall call blocking probability less than 5%. Do we need any additional channels? If so, how many additional channels are needed, and what would the optimal allocation of these channels then be?

HINT: In addressing all these questions, be clear and motivate the steps you are taking; do not only give plain answer, show ‘signs of thinking’.

The deadline is Monday, March 19, 2018. If you have questions, mail Rob at mei@cwi.nl.

Remarks:

1. An important aspect of the exercises is that you provide clear argumentations for your findings. Please make sure you formulate your answers carefully!
2. Good luck, and most importantly, have fun!