L1 - year 2019-2020

From Systems to Functions
Lab session 2 : Digital Filtering

Make sure to have finished project session 1.

The topic of interest is to study audio signals digitized at a sampling frequency f_S=44,1 kHz on a 1s time sequence. (Standard frequency in audio following the Shannon criterium)
To validate the digital observations as it relates to the different types of filtering, students will be able to evaluate and appreciate the analysis (for various types of filters) using function soundsc(x, f_E) which enables the listening (optional) the sound produced by the values stored in an array (table) using samples spaced by 1/ f_S second.
Students may use headphones NB : audible frequencies are between 20Hz and 20KHz.
Student will establish the relationship between the impulse response (time domain) and the frequency response of the filters studied in lab session 1. Data and results should be carefully compiled to be used in the next lab session and included in the final report.

These results are essentially illustrated by the timing diagrams and the spectra of the inputs and outputs to the filters studied. The spectrum of a periodic signal x(t) can be evaluated under MATLAB using a sub-program of the form:

```
Fx=fft(x);   Sx=sqrt(power(real(Fx),2)+  power(imag(fx),2));
                              Or
          Fx=fft(x);   Sx=sqrt(Fx.*conj(Fx));
```

where fft(x) designate the fast Fourier transform of x, power(.,2), squaring, real(.) et imag(.) real and imaginary part of a complex number and conj(.), the complex conjugate.

NB: a) ***No report is required but students must save, compile all data and results to include in details in the final project report.***
b) in the frequency domain an impulse gives a signal which contains all frequencies of the same amplitude.
c) White noise is a signal that contains all frequencies with random amplitudes

I. SIGNAL OBSERVATION AND SPECTRAL ANALYSIS

A few computation techniques under MATLAB

Q0. Pick two vectors x and y in the command window of MATLAB. Example : x= [1 2 3] and y=[4 5 6].
- Compute one at a time the products x.*y, x*y′ et x*y in the command window.
- Which operation correspond to a scalar product? to a term by term product? to nothing?
- In the command window, record all the elements of vector z constructed as : z=x+y*i.
- Repeat for vector conj(z). Comments.
- How, under MATLAB, can we calculate the magnitude of all vector elements of a complex vector like z?

Sampling and time windows of signals

Q1*. Assuming that the sampling frequency of the signals is 44,1 kHz
- How many samples N are necessary to represent a signal in a 1s time window?
- Waht should the maximum frequency f_{MAX} of the signals we can analyze (process) without losses ? (Shannon-Nyquist theorem/criterium). Is the sampling without losses for the sound signal ? Explain ?

Applications

Q2. Complete the program **qEcoute.m** (in the dashed fields) to generate a sinusoidal signal at a 50Hz real frequency (lines 3,11).
- Justify the normalization of the frequencies (line 4). (Sampling).
- Plot the timing diagram (lines 24) and its spectrum (line 27).
- Repeat the above steps for frequencies up to 50kHz and for each frequency record (using the cursor) the low frequencies f_l (low) and f_h (high) which appear on the signal spectrum. Fill out the table below :

v1

Validation1

f(Hz)	20		200		1000		10000		15000		16000		17000		18000		20000		40000		50000	
f_l(Hz)																						
F_h(Hz)																						
Audible	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N

- If necessary, adjust the scale of your coordinates to optimize the plots.
- What do f_l et f_h correspond to? Compare these frequencies to f and to $|f_l-f|$ or to $|n.f_l-f|$ with n= integer).
- What is the real frequency heard at 40kHz and at 50kHz (ultrasounds !!!) ?

Q3. Program a white noise.

- Carefully observe its spectrum and its audible perception.
- In a similar way record the spectrum of an impulse. What do you notice?

Programme qEcoute.m: Observation des signaux

```
clc; clf;
N=.....;            %- Nombre d'échantillons=?
frequence=50;       %- Fréquence réelle en Hertz
f=frequence/...;    %- Fréquence normalisée=?

%---- L'échelle des temps et des fréquences----------
temp=0:(N-1); freq=0:(N-1);

%---------------------- Programmation de l'entrée ---
% 1°) ENTREE SINUSOIDALE
      e=sin(2*pi*f*temp);
% 2°) ENTREE BRUIT BLANC
      %e=(rand(1,N)-0.5)*2;
% 3°) ENTREE IMPULSIONNELLE
      %e=zeros(1,N); t0=3; e(t0)=N;
% 4°) ENTREE INDICIELLE
      %t0=3; e=[zeros(1,t0) ones(1,N-t0)];

Fe=fft(e); Se=sqrt(Fe.*conj(Fe))/N; %Calcul du Spectre--
soundsc(e,N);                       %Ecoute du signal --
%pause;
figure(1); %------------Tracé de Chronogrammes---------
temp=temp/N;
subplot(4,1,1); plot(temp,e,'r'); title('CHRONOGRAMME');
xlabel('Temps: : sec'); ylabel ('Amplitide'); grid;
%----------------------Tracé de Spectres--------------
subplot(4,1,2); semilogx(freq,Se,'r'); title('SPECTRE');
xlabel('Fréquence: : Hz'); ylabel('Se'); grid;
%xlim([20 20000]);
```

Programme Q5harmonic.m: Réponse à un sinus

```
clc; clf;
N=.....;
frequence=...;                   %Fréquence réelle en Hertz
frcoupure=...;                   %Fréq. de coupure des I°ordre en Her
frcentral=...; deltafreq=...;    %Fréq. centrale & LB du passe-bande

%------ Normalisations et calcul des constantes de temps ------
f=frequence/..; fc=frcoupure/..; f0=frcentral/..; Df=deltafreq/..;
k =1/(2*pi*fc); k2=1/(2*pi*Df); k1=Df/(2*pi*f0*f0); %-----Ctes de t

temp=0:(N-1); freq=0:(N-1); %-- L'échelle des temps et des fréquenc

%---------------------- Initialisation des tableaux ---------------
s=zeros(1,N); z=zeros(1,N); ss=zeros(1,N); zz=zeros(1,N); y=zeros(:
%-----------------------------------------------------------------
e=sin(2*pi*f*temp);                   %-- Exemple d'entrée sinus
soundsc(e,N); pause;
Fe=fft(e); Se=sqrt(Fe.*conj(Fe))/N;   %-- Calcul du spectre de l'e

%-------------------Programmation des filtres --------------------
for t=2:N
    s(t)=...; z(t)=...;               % --- filtres du 1er ordre
end;

for t=3:N
    ss(t)=...; zz(t)=...; y(t)= ...;  % --- filtres du 2er ordre
end;

figure(1);  %----------Tracé des Chronogrammes-------------------
subplot(4,1,1); plot(temp,e,'r'); grid; ylabel('Entrée'); title('CH
```

II. FILTER SYNTHESIS

Cut-off frequencies, Bandwidth and center frequency.

Q4. In the previous session, the relationship between the frequency responses and the time constants k, k_1 et k_2 for each type of filter had been established:

$$k = \frac{1}{\omega_c} \qquad k1 = \frac{1}{q.\omega_0} \qquad k2 = \frac{q}{\omega_0} \qquad q = \frac{\omega_0}{\Delta\omega}$$

where fc (ω_c) is the cut-off frequency of the first order low-pass of high-pass filters, f_0 (ω_0) the central frequency of the bandpass filter, q its quality factor and Δf its bandwidth .

- Express the different time constants as a function of the frequencies.   NB: $\Delta\omega = 2.\pi.\Delta f$

Specifications

Q5. Complete the programming of the 5 filters by filling out the qHarmonic.m. program. Take:

- 1 kHz for the first order low pass and high pass filters cut-off frequency
- 2 kHz as the central frequency and a 20Hz bandwidth for the bandpass filter.
- The second order lowpass and high pass filters will be implemented using first order filters in series The output s(t) must be expressed as a function of the inputs e(t), e(t-1) … and its previous values s(t), s(t-1), s(t-2) … and eliminating the intermediate output.
- What is the advantage of such programming of the outputs?

II. APPLICATION TO AUDIO-FREQUENCIES SIGNALS

Q6. Use the composite signal

$e(t)=\sin(2\pi f_1 t) + \sin(2\pi f_2 t)$ with f_1=50Hz et f_2=10kHz

as the inputs to the lowpass and high pass filters

- Explain the shape and the spectrum of the output signals of each filters.
- V Listen to the output signals and note the audible perception of the second-order low-pass and high pass filters and the discrimination of the low and high pitch components of the input signal. Validation2

Q7. Replace the input with a uniform white noise signal: `e=(rand(1,N)-0.5)*2;` et record the spectrum of the filter output signals for all the second order filters. (White noise includes all frequencies).
What do you observe ?

III. IMPULSE RESPONSE

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Q8. Generate an impulse signal using a command of the form: `e=zeros(1,N); e(10)=N` as inputs to the filters

- Comment on the timing diagrams. Measure the frequency of the pseudo oscillations at the output of the bandpass filter…
- Integrate the command in the next window to observe the spectra on a log scale.
- Record the spectra of the inputs and outputs signals for all filters.

```
%-------------- Tracé des Spectres---------------------
figure(2);
subplot(4,1,1); semilogx(freq,Se,'r'); grid;
ylabel('Entrée'); title('SPECTRES');
xlim([20 10000]);

subplot(4,1,2);
semilogx(freq,20*log10(Ss),'b',freq,20*log10(Sss),'c');
grid; ylabel ('Passe-bas I et II'); xlim([20 10000]);

subplot(4,1,3);
semilogx(freq,20*log10(Sz),'b',freq,20*log10(Szz),'c');
grid; ylabel ('Passe-bas I et II'); xlim([20 10000]);

subplot(4,1,4); semilogx(freq,20*log10(Sy),'g'); grid;
ylabel ('Passe-bande'); xlabel('Fréq.(Hz)');
xlim([20 10000]);
```

· Compare your observations to the Bode plots for each of the filters measuring the cutoff frequencies , the center frequency and bandwidth as well as the slope of the asymptote in low frequencies (LF) and in high frequencies (HF). **Validation3**