## FROM SYSTEMS to FUNCTIONS

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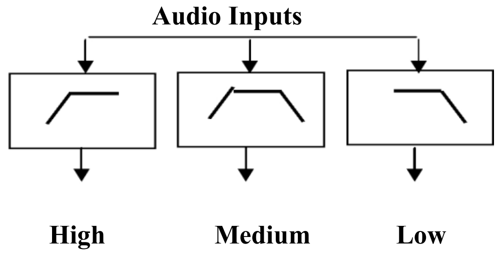
**Project: DIGITAL FILTERS SYNTHESIS**

## ***First session : Programming under Matlab***

***Student are taught and trained to implement a set of filters for the treatment of audio signals. More specifically it allows the extraction/separation of the low, medium and high frequencies through the three different types of filters.***




***This first session is dedicated to get familiar with a software tool (Matlab) and the programming/implementing three basic filters using and obtaining their step response.***

***The step response of a filter is the response of a filter to a input signal e(t) which is a step signal. A step signal is a signal equals to 1 for t ≥0 and 0 for t < 0. This response gives information on the speed of a filter (k) , through its time constant ω0=1/k and cut-off frequency.***
***In each program, it is necessary to program the line (or lines) of the filters to test at specific frequencies.***
***No report is required to hand in for this session but it is recommended that students compile all their data and results by frequently saving them in order to reproduce them at the end of the project in a detailed report.***

## I. MODELING and MATLAB COMPUTATIONS

**THEORY**

***Programming the derivative***

First, let's estimate a derivative numerically using a variation rate:

$$s(t) = k.\frac{d\ e(t)}{dt} = k.\lim_{\tau \to 0} \frac{e(t) - e(t-\tau)}{\tau}$$

The smallest time interval we can estimate using a computer is τ which is the time between two signal samples ( sampling rate). Time is now discrete, and the estimate of the derivative will be based on τ=1 sample and s(t) will be calculated as follow :

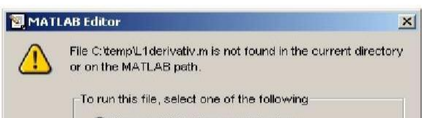
$$s(t) = \frac{k.[e(t) - e(t-1)]}{1} = k.[e(t) - e(t-1)] \quad \text{with} \quad t: \text{ integer}$$

**MATLAB**

Q1. Programming the calculation of a derivative (see fig.2)

Launch Matlab and open a new folder following these steps:

File/New/M-file to set up the function L1derivativ in the editor mode (or download it via Campus)
Save and execute command: Debug/Run (F5) If needed set-up the following option with



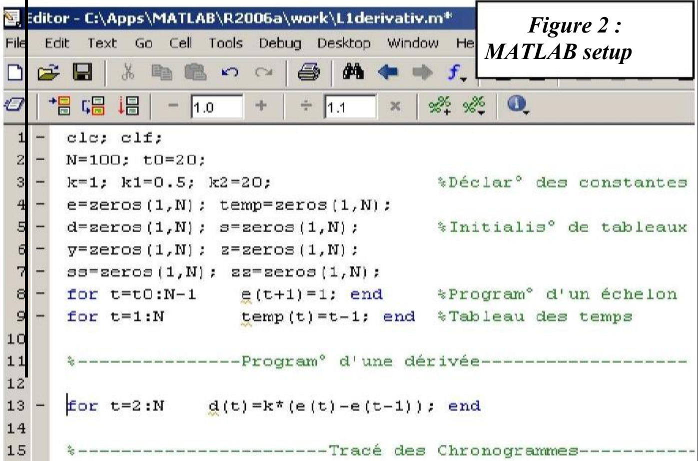




***Figure 2 : MATLAB setup***

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Ecole d'ingénieurs du numérique

- Record the results graphically and plot Input/output of the differentiator.
- Program the input signal e(t): `e = [zeros(1,t0) ones(1,N-t0)];` compare the result with line 8.

*The response of a linear system to a step function is called the step response of the system.*

Q2. Eliminate the last semi-colon of line 2 and observe the obtained result (Command Window). **Comment?**

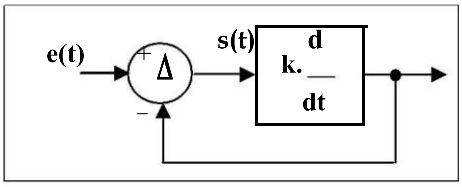
## II. FILTER PROGRAMMING

**LOW_PASS FILTER**

Q3. Show that the output of a first order lowpass filter (next figure) can be programmed using the recurring equation:

$s(t) = \alpha.e(t) + \beta.s(t - 1)$

*Identify $\alpha$ et $\beta$ as a function of the gain k of the differentiator.*




Q4. Program this low pass filter and record its step-response for several values of k (at least 3)
0.1, 0.01, 0.001

- Plot the tangent at t=0 for 3 values of k (Insert/Arrow) and measure its slope.
- What is the relationship between k and the cut-off frequency of the filter? What role does k play on the filter step response?

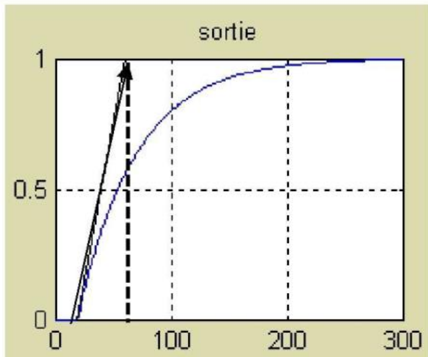



*Figure 3:*
*Step response of a first order low pass filter*

**HIGH-PASS FILTER**

Q5. Where does the output which corresponds to a high pass filter lie (See Lab2 et Lab3)

- Program this filter and obtain its step response as in Q4. **Validation1**

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**SECOND ORDER**

**Q6*. Based on the theoretical analysis presented in the introduction, show how the second derivative of a signal can be numerical calculated (using a computer)**

**Give its expression ? Theoretically only without experiment.**

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**Q7. Implement a $2^{nd}$ order band pass filter by putting two first order filters in series**

**How can the output of this filter can be mathematically modeled as a function of its input**

**Program the filter and compare its step response for k=10 to that of a first order filter plotting the timing diagrams on the same set of coordinates.**

**Repeat the same steps for a high pass filter. Conclusion?** **Validation2**

**BAND PASS FILTER**

**Q8. Remember (see previous labs) that $k_1$ et $k_2$ define the quality factor q and the intrinsic (natural) frequency $\omega_0$ of the band pass filter by the following equations :**

$$k_1 = \frac{1}{q.\omega_0} \qquad k_2 = \frac{q}{\omega_0}$$

**Program a bandpass filter with avec $k_1$=0,1 et $k_2$=20 and obtain q and $\omega_0$ . Use figure 6.**
**Record its step response and measure the pseudo period of the filtered output signal oscillations.**
**How does q affect the step response of the filter.** **Validation3**

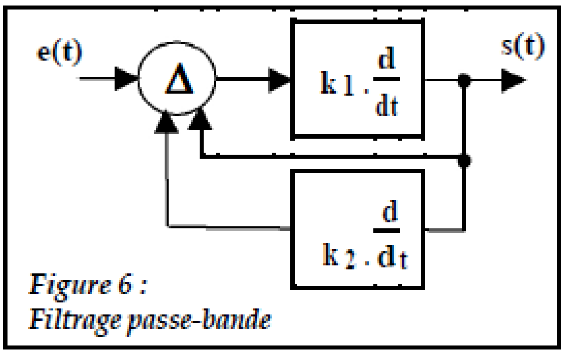


*Figure 6 :*
*Filtrage passe-bande*

**Compile all your data and results very carefully to include them in your final project report.**

**NB : Matlab does not allow for negative arguments**