Polonium problem

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Theoretical and numerical aspects of nuclear physics
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Theoretical aspects

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Theoretical aspects

Polonium, element number 84 on the periodic chart, was discovered by Marie Curie in 1898. She obtained Polonium from pitchblend, a material that contains Uranium, after noticing that unrefined pitchblend was more radioactive than the Uranium that it was separated from.

Polonium, symbol Po, has atomic number 84 and atomic weight 209. Metallic Polonium is a silvery metal which is soluble in dilute acids. It has a low melting temperature (254°C), but is fairly volatile at temperatures as low as 55°C. Over 25 isotopes of it are known, none of which are stable. Polonium-210 is the predominant naturally occurring isotope and most widely used.

What's Po-210 used for?

Po-210 can be used to eliminate static charges in machinery where it can be caused by processes such as paper rolling, manufacturing sheet plastics and spinning synthetic fibers. It's also used in brushes that remove accumulated dust from photographic films and lenses.

Po-210 can be mixed with Beryllium to be used as a neutron source. Po-210 emits an alpha particle as it decays. Beryllium absorbs that alpha and emits a neutron. This combination has been used as a neutron source for nuclear weapons.

It has also been used as a lightweight heat source to power thermo-electric cells.

Where does Po-210 come from?

Polonium is a decay product of radium in the Uranium decay chain. It can be made by bombarding Bismuth-209 with neutrons in a nuclear reactor. This forms Bismuth-210 which has a half-life of 5 days. Thus, Bismuth-210 decays to Po-210 through decay β.
For further properties go to the url: https://doh.wa.gov/sites/default/files/legacy/Documents/Pubs//320-091_po210_fs.pdf.

Bateman's equations

Bateman's equations

Bateman's equations

Bateman's equations is a system of coupled differential equations that describes a decay process. Harry Bateman was the inventor and the first man who solved this system of ode (ordinary differential equations).

The easiest system we can build is:

$$dN_1 = -\lambda_1 N_1 dt$$

$$dN_2 = \lambda_1 N_1 dt - \lambda_2 N_2 dt$$

where the number of parent nuclei decreases with time according to the first eq, while the number of daughter nuclei grows as result of decay of parents and decreases as result of its own decay, as the second eq. shows.

If we assume there are several succeeding generations of radioactive nuclei, we can easily generalize the last equation (daughter radioactive nuclei) in this way

$$dN_i = \lambda_{i-1} dN_{i-1} dt - \lambda_i N_i dt$$

. The general solution is given by the Bateman equations, in which the activity of the nth number of the chain is given in terms of the decay constants of all preceding members:

$$A_n = N_0 \sum_{i=1}^{n} c_i e^{\lambda_i t}$$

where

$$c_m = \frac{\prod_{i=1}^{n} \lambda_i}{\prod_{i=1}^{n'} (\lambda_i - \lambda m)}$$

These formulas are taken from Introductory Nuclear Physics by Kenneth Krane: https://faculty.kfupm.edu.sa/PHYS/aanaqvi/Introductory-Nuclear-Physics-new-Krane.pdf

For the process we deal with $^{209}Bi \rightarrow {}^{210}Bi \rightarrow {}^{210}Po$ Bateman's equations are:

$$\frac{dn_{Bi209}}{dt} = -d_{Bi209}n_{Bi209}$$

$$\frac{dn_{Bi210}}{dt} = d_{Bi209}n_{Bi209} - d_{Bi210}n_{Bi210}$$
$$\frac{dn_{Po210}}{dt} = d_{Bi210}n_{Bi210} - d_{Po210}n_{Po210}$$

Hereby represents $n_{nuclide}(t)$ the nuclide concentration, $d_{nuclide}$ the decay constant of the nuclide and $\frac{dn_{nuclide}}{dt}$ the change in nuclide concentration over time.

Matrix exponential method

Matrix exponential method

Bateman's equations

The system of Bateman's equations can be expressed in matrix form as follows:

$$\frac{d}{dt}\begin{bmatrix} n_{Bi209} \\ n_{Bi210} \\ n_{Po210} \end{bmatrix} = \begin{bmatrix} -d_{Bi209} & 0 & 0 \\ d_{Bi209} & -d_{Bi210} & 0 \\ 0 & d_{Bi210} & -d_{Po210} \end{bmatrix} \begin{bmatrix} n_{Bi209} \\ n_{Bi210} \\ n_{Po210} \end{bmatrix}$$

Here the decay constants are shown:

$d_{Bi209} = 1.83163 * 10^-12 s^-1$
$d_{Bi210} = 1.60035 * 10^-6 s^-1$
$d_{Po210} = 5.79764 * 10^-8 s^-1$

The theoretical notion about matrix exponential method is briefly discussed in this chapter.
The basic ingredient is a square matrix A whose elements can be real or complex eventually. Since the matrix is square, the operation of raising to a power is defined and we're able to calculate

$$A^0 = I, A^1 = A, A^2 = A * A, ...$$

.
We form the infinite matrix power series and we express it by an exponential: $e^{tA} = \sum_{k=0}^{\infty} \frac{t^k}{k!} A^k$.
If we consider a system like $X'(t) = AX(t)$ the general solution is represented in terms of the matrix exponential as $X(t) = e^{tA}C$ where C is an arbitrary n-dim column vector.
For the Cauchy problem, the components of C are expressed in terms of the initial conditions. In this case, the solution of the system can be written as $X(t) = e^{tA}X_0$ where $X_0 = X(t = t_0)$.

Matrix exponentiation can be successfully used for solving systems of differential equations. Consider a system of linear homogeneous equations, which in matrix form can be written by using a square matrix containing the constant coefficients. If that matrix is diagonalizable then its exponential can be obtained by exponentiating each entry on the main diagonal. In formulas we have

$$A = UDU^{-1}$$

and D is diagonal, then

$$e^A = Ue^DU^{-1}.$$

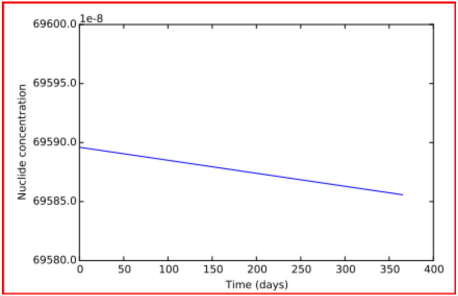
Anyway there are several other methods to compute matrix exponential such as Jordan decomposition, Laurent-Mc Laurin series...but everyone uses Padè approximation algorithm to represent the exponential of a matrix, but it's not the topic of this paper, so you're suggested to go on that by your own. It's recommended to look at the case where the matrix is not diagonalizable, where Vandermonde matrix is suited for the case.

Conclusions

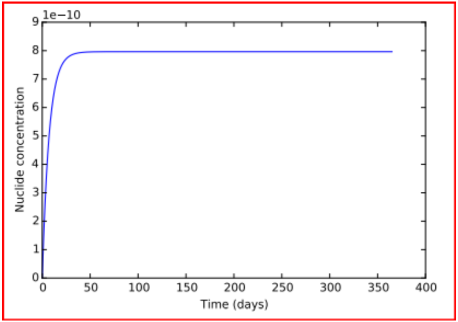
Conclusions

Results

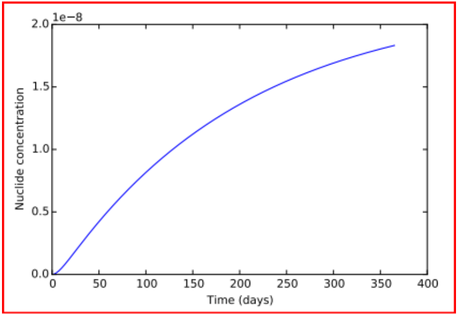
Here the results taken from
file:///C:/Users/checc/OneDrive/Desktop/Masterproef_Maren_Vranckx.pdf
(pages 4-6)



Decaying of Bi209



Decaying of Bi210



Decaying of Po210