
Econometrics for Causal Inference

URP Part 2: Difference-in-Difference

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- Machine Learning and Econometrics -

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Last Time...

- ▶ Econometrics and Causal Inference
- ▶ What is Causal Inference
- ▶ Counterfactual
- ▶ Difference-in-Difference

Card and Kruger (1994)

- ▶ Treatment: minimum wage increase in October 1996
- ▶ Treatment Group: PA
- ▶ Control Group: NJ

- ▶ $(E[Y|s = PA, t = 1] - E[Y|s = NJ, t = 1])$
 $-(E[Y|s = PA, t = 0] - E[Y|s = NJ, t = 0])$
 $= (23.33 - 20.44) - (21.17 - 21.03) = 2.76$

- ▶ $Y = \alpha PA + \beta T + \tau PA \times T + \epsilon$
 $\hat{\tau} = 2.76$

Card and Kruger (1994)

- ▶ Conditions for proper DiD
 1. Well-defined control group (good counterfactual)
 2. Treatment effect must occur after treatment

- ▶ If both state had different trends before treatment...

→ research design in this paper failed.

No Parallel Trend

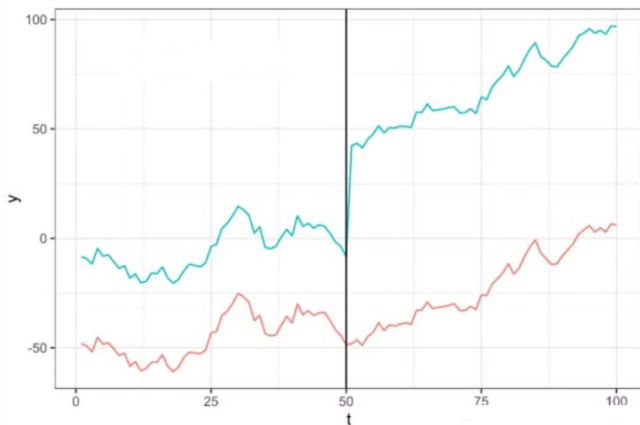


Figure: Difference-in-Difference

No Parallel Trend

- ▶ i : treatment group / j : control group
- ▶ $t = 1$: after treatment / $t = 0$: before treatment

$$\begin{aligned}\tau^{\hat{DD}} &= (Y_{i,t=1}(1) - Y_{j,t=1}(0)) - (Y_{i,t=0}(0) - Y_{j,t=0}(0)) \\ &= (Y_{i,t=1}(1) - Y_{i,t=0}(0)) - Y_{j,t=1}(0) - Y_{j,t=0}(0) \\ &\quad + (Y_{i,t=1}(0) - Y_{i,t=0}(0)) \\ &= (Y_{i,t=1}(1) - Y_{i,t=0}(0)) + [(Y_{i,t=1}(0) - Y_{i,t=0}(0)) - (Y_{j,t=1}(0) - Y_{j,t=0}(0))] \\ &\approx E[Y_{i,t=1}(1) - Y_{i,t=1}(0)] + E[Y_{i,t=1}(0) - Y_{i,t=0}(0)] \\ &\quad - E[Y_{j,t=1}(0) - Y_{j,t=0}(0)] \\ &= \tau + \text{CounterfactualTrend} - \text{UntreatedTrend}\end{aligned}$$

- ▶ Parallel Trend: Counterfactual Trend = Untreated Trend

No Parallel Trend

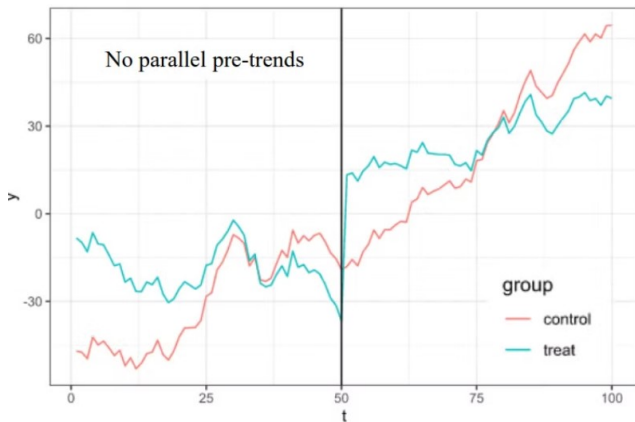


Figure: Violate Parallel Trend Assumption

Anticipation Effect

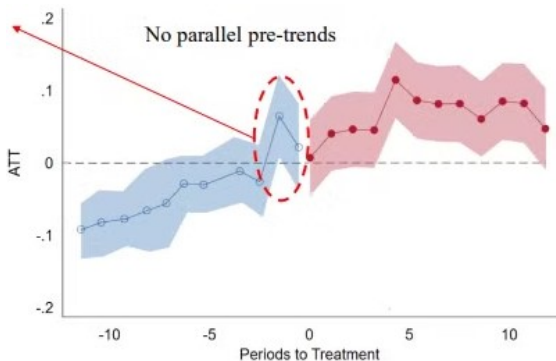


Figure: Anticipation Effect

Adding Covariates

- ▶ Control for important variables
- ▶ Build Ceteris Paribus as close as possible

DiD Event Study

- ▶ Check parallel trend
- ▶ Analyze dynamic effect

Adding Covariates

$$Y_{it} = \alpha + \tau \textit{Treat}_i \times \textit{Post}_t + \beta \textit{Treat}_i + \delta \textit{Post}_t + \gamma X_{it} + \epsilon_{it}$$

$$E[Y_1(0) - Y_0(0)|\textit{Post} = 1] \neq E[Y_1(0) - Y_0(0)|\textit{Post} = 0]$$

► But it can be....

$$E[Y_1(0) - Y_0(0)|\textit{Post} = 1, X] = E[Y_1(0) - Y_0(0)|\textit{Post} = 0, X]$$

DiD Event Study

$$Y_{it} = \alpha + \sum_{\tau=0}^m \delta_{-\tau} D_{i,t-\tau} + \sum_{\tau=1}^q \delta_{+\tau} D_{s,t+\tau} + X'_{it}\beta + \epsilon_{it}$$

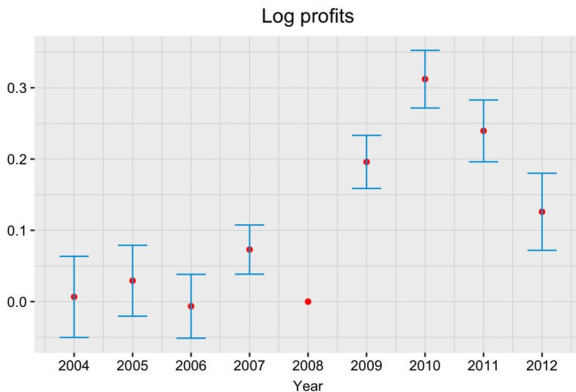


FIGURE 4

Event-study coefficients $\{\beta_s\}$ for log profits, estimated using the event-study specification in (19).

Why DiD Event Study

- ▶ Check parallel trend (Rambachan, 2023)
- ▶ Dynamic treatment effect

The Effect of Value-added Taxes

- ▶ Treatment: 2009 VAT Reform
 - ▶ Before: a standard rate of 19.6% applied to sit-down meals and a reduced rate of 5.5% for take-away meals
 - ▶ After: decrease in the price of meals consumed in sit-down restaurants and equalization between the VAT rate between sit-down meals and take-away meals
- ▶ Treatment group: sit-down restaurants

$$\log Y_{idt} = \eta \cdot 1\{i \in T\} \times \text{After} + \lambda_t + \omega_i + \epsilon_{idt}$$

- ▶ Dynamic Effects

$$\log Y_{idt} = \sum_{\nu=-k}^q \eta_{\nu} \cdot 1\{i \in T\} \times 1\{t = \nu\} + \lambda_t + \omega_i + \epsilon_{idt}$$

Dynamic Effects

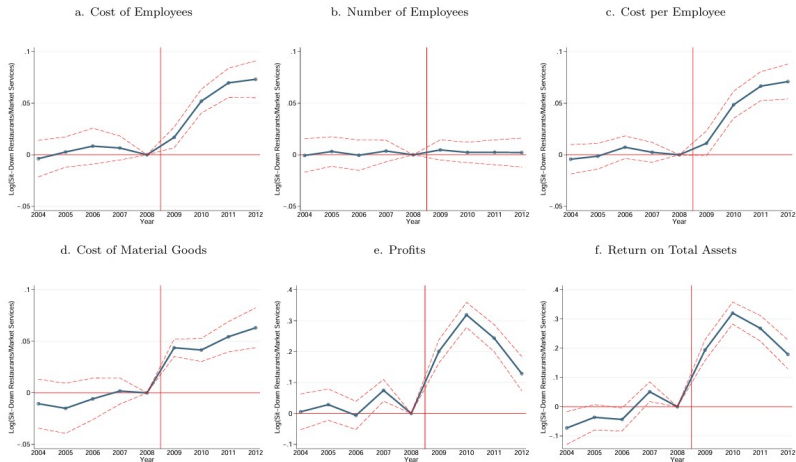
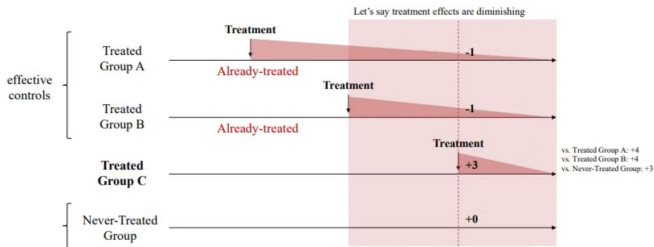


Figure 2. : Dynamic Effects of the VAT Cut: Sit-Down Restaurants vs. Market Services

Staggered DiD



Continuous Treatment Effect

- ▶ Binary treatment: treatment or not
- ▶ Continuous treatment: treatment level
 - ▶ Example: receive regular income from government in proportion to income level.

- ▶ The impact of regulations on firm size on firm growth
- ▶ Treatment: regulations on firm size
- ▶ Treatment Effect: firm growth (employment, revenue)
- ▶ Legal amendment in 2015
 - ▶ The criteria for Small-and-Mid-Sized firm have changed
 - ▶ "treatment group" is made

Billboards really work?

- ▶ Data: experiment result for billboard (hypothetical)
- ▶ Treatment on July
- ▶ Work
 - ▶ Make regression model using variables in data
 - ▶ Conduct DiD analysis
 - ▶ Check parallel trend
 - ▶ Report and plot the result
 - ▶ 1-2 pages report and code