

(F3) Kap 6. Potenser (och logaritmer) s.154-162

Potens: består av en bas a och en exponent n : a^n

↑ exponent
↑
bas

$$a^n = \underbrace{a \cdot a \cdot a \dots a}_{n \text{ st a}} \quad a \in \mathbb{R}, n \in \mathbb{N}$$

Ex) $2^3 = 2 \cdot 2 \cdot 2 = 8$

$$\pi^4 = \pi \cdot \pi \cdot \pi \cdot \pi$$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

VIKTIG SKILLNAD

Ex) $(-2)^2 = (-2) \cdot (-2) = 2 \cdot 2 = 4$ basen är (-2)

$$-2^2 = -\underbrace{2 \cdot 2}_4 = -4 \quad \text{basen är } 2$$

$(-a)^n$ n =jämmtal, ger positivt svar
 n =udda tal, ger negativt svar

Ex) $(-2)^3 = (-2)(-2)(-2) = -2^3 = -8$ udda exponent

$$(-2)^4 = (-2)(-2)(-2)(-2) = 2^4 = 16 \quad \text{jämn exponent}$$

Ex) $(-27)^{18} = 27^{18}$ jämn exponent

$$(-27)^{19} = -27^{19} \quad \text{udda exponent}$$

Potenslagarna

$$a^x \cdot a^y = a^{x+y} \quad a, b > 0, x \text{ och } y \in \mathbb{R}$$

$$\frac{1}{a^x} = a^{-x} \quad \left(\frac{1}{a^{-x}} = a^x \right)$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1 \quad a \in \mathbb{R}, a \neq 0 \quad 0^0 \text{ ej def.}$$

$$(ab)^x = a^x \cdot b^x$$

$$\left(\frac{a}{b} \right)^x = \frac{a^x}{b^x}$$

$$\left(\frac{a}{b} \right)^{-n} = \left(\frac{b}{a} \right)^n$$

$$\left(\frac{a}{b} \right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a} \right)^n$$

$$\left(\frac{1}{a} \right)^n = \frac{1}{a^n}$$

$$\left(\frac{1}{a} \right)^n = \frac{1^n}{a^n} = \frac{1}{a^n}$$

Exempel

$$3^5 \cdot 3^4 = 3^{5+4} = 3^9$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}, \quad \frac{1}{2^{-1}} = 2^1 = 2$$

$$\frac{2^{50}}{2^{48}} = 2^{50-48} = 2^2 = 4$$

$$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$$

$$4^0 = 1, \quad (-4)^0 = 1, \quad -\underbrace{4^0}_1 = -1$$

$$2^2 \cdot 6^2 = (2 \cdot 6)^2 = 12^2 = 144$$

$$(5x)^3 = 5^3 \cdot x^3 = 125x^3$$

$$\frac{10^5}{5^5} = \left(\frac{10}{5} \right)^5 = 2^5 = 32$$

$$\left(\frac{2}{3} \right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

$$\left(\frac{2}{3} \right)^{-2} = \left(\frac{3}{2} \right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

$$\left(\frac{1}{2} \right)^8 = \frac{1}{2^8} = \frac{1}{256}$$

$$\frac{1}{1024} = \frac{1}{2^{10}} = \left(\frac{1}{2} \right)^{10}$$

$$\sqrt{a} = a^{\frac{1}{2}} \quad a > 0$$

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad n > 0, a > 0 \text{ om } n \text{ är jämn}$$

$$\sqrt{256} = \sqrt{2^8} = (2^8)^{\frac{1}{2}} = 2^{8 \cdot \frac{1}{2}} = 2^4 = 16$$

$$\sqrt[5]{32} = \sqrt[5]{2^5} = (2^5)^{\frac{1}{5}} = 2^{5 \cdot \frac{1}{5}} = 2^1 = 2$$

$$10^{-1} = \frac{1}{10} = 0,1$$

\uparrow \uparrow \uparrow
 potens tal bråk tal decimaltal

Grundpotensform

$$\text{Grundpotensform: } a \cdot 10^n \quad 1 \leq a < 10 \text{ och } n \in \mathbb{Z}$$

Ex) Skriv 2300 och 0,00023 på grundpotensform

$$2300 = \underline{2,3 \cdot 10^3} \quad \text{decimalkommat flyttas 3 steg åt väster}$$

$$0,00023 = \underline{2,3 \cdot 10^{-4}} \quad \text{decimalkommat flyttas 4 steg åt höger}$$

6008d) Beräkna $\frac{2 \cdot 10^4}{8 \cdot 10^{-5}}$ och svara i grundpotensform

$$\frac{2 \cdot 10^4}{8 \cdot 10^{-5}} = \frac{2}{8} \cdot 10^{4-(-5)} = \frac{1}{4} \cdot 10^9 = 0,25 \cdot 10^9 = 2,5 \cdot 10^{-1} \cdot 10^9 = 2,5 \cdot 10^{-1+9} = \underline{2,5 \cdot 10^8}$$

Ex) $f(x) = x^2$ bestäm $f(-2)$

$$f(-2) = (-2)^2 = 4$$

$$f(-2) \neq -2^2 \neq 4$$

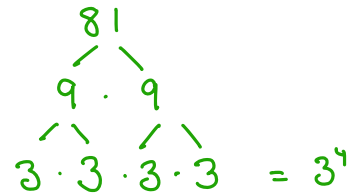
$$f(x) = -x^2 \quad f(2) = -2^2 = -4$$

Ex) Beräkna $(-2)^{-4}$ och -2^{-4}

$$\left. \begin{aligned} (-2)^{-4} &= \frac{1}{(-2)^4} = \frac{1}{16} \\ -2^{-4} &= -\frac{1}{2^4} = -\frac{1}{16} \end{aligned} \right\} \text{viktig skillnad}$$

Ex) Skriv $\frac{1}{81}$ i potensform med basen 3 a^n

$$\frac{1}{81} = \frac{1}{3^4} = \underline{3^{-4}}$$



Ex) Beräkna $(0,2)^4$, svara på bråkform

$$(0,2)^4 = \left(\frac{2}{10}\right)^4 = \left(\frac{1}{5}\right)^4 = \frac{1}{5^4} = \underline{\frac{1}{625}}$$

Ex) Beräkna $\left(\frac{2}{7}\right)^{-2}$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^2 = \frac{7^2}{2^2} = \underline{\frac{49}{4}}$$

Ex) Beräkna $\frac{2^{-2}}{2^2}$ och ge svaret i både bråkform och potensform

Bråkform: $\frac{2^{-2}}{2^2} = \frac{1}{2^2 \cdot 2^2} = \frac{1}{4 \cdot 4} = \underline{\frac{1}{16}}$

Potensform: $\frac{2^{-2}}{2^2} = 2^{-2} \cdot 2^{-2} = 2^{-2-2} = \underline{2^{-4}}$

Ex) Beräkna $(2^3)^2$ och 2^{3^2}

$$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = \underline{64}$$

$$2^{3^2} = 2^9 = \underline{512}$$

$$(2^3)^2 \neq 2^{3^2}$$

Ex) Beräkna $(2 \cdot 10^{-4})^3$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$(2 \cdot 10^{-4})^3 = 2^3 \cdot (10^{-4})^3 = 8 \cdot 10^{-4 \cdot 3} = \underline{8 \cdot 10^{-12}}$$

6011d) Skriv $\frac{(2^{n+2})^3}{8^{n+4}}$ på enklaste form

$$\frac{(2^{n+2})^3}{8^{n+4}} = \frac{(2^{n+2})^3}{(2^3)^{n+4}} = \frac{2^{3(n+2)}}{2^{3(n+4)}} = \frac{2^{3n+6}}{2^{3n+12}} = 2^{3n+6-(3n+12)} = 2^{3n+6-3n-12} = \underline{2^{-6}}$$

$$\text{alt. } 2^{-6} = \frac{1}{2^6} = \underline{\frac{1}{64}}$$

[Paus]

Ex) $f(x) = x^2$ Bestäm $f(a^4)$

$$f(x) = x^2 \quad f(a^4) = (a^4)^2 = a^{4 \cdot 2} = a^8$$

$$f(a^4) \neq a^{4^2} \neq a^8$$

6013d) Förenkla $\left(\frac{a^2}{27}\right)^{\frac{1}{3}} \cdot \left(\frac{64}{a}\right)^{\frac{2}{3}}$

$$\left(\frac{a^2}{27}\right)^{\frac{1}{3}} \cdot \left(\frac{64}{a}\right)^{\frac{2}{3}} = \left(\frac{a^2}{3^3}\right)^{\frac{1}{3}} \cdot \left(\frac{4^3}{a}\right)^{\frac{2}{3}} = \frac{(a^2)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}} \cdot \frac{(4^3)^{\frac{2}{3}}}{a^{\frac{2}{3}}} = \frac{a^{\frac{2}{3}}}{3^{3 \cdot \frac{1}{3}}} \cdot \frac{4^{3 \cdot \frac{2}{3}}}{a^{\frac{2}{3}}} = \frac{a^{\frac{2}{3}} \cdot 4^2}{3 \cdot a^{\frac{2}{3}}} = \underline{\frac{16}{3}}$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad n > 0, a > 0 \text{ om } n \text{ är jämn}$$

Ex) Beräkna a) $\sqrt[3]{8}$ b) $\sqrt[4]{625}$ c) $\sqrt[3]{-8}$ d) $\sqrt[4]{-16}$

$$a) \sqrt[3]{8} = 8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3 \cdot \frac{1}{3}} = 2^1 = \underline{2}$$

$$b) \sqrt[4]{625} = 625^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{4 \cdot \frac{1}{4}} = 5^1 = 5$$

$$c) \sqrt[3]{-8} = (-8)^{\frac{1}{3}} = (-2^3)^{\frac{1}{3}} = (-2)^{3 \cdot \frac{1}{3}} = (-2)^1 = \underline{-2} \quad (-2)^3 = -8$$

udda n

$$d) \sqrt[4]{-16} \quad \text{saknar lös}$$

jämnt n

Tänk på att skriva tydligt! Det är stor skillnad på:

$$\sqrt[4]{16} \quad \text{och} \quad 4\sqrt{16}$$

$$\sqrt[4]{16} = (2^4)^{\frac{1}{4}} = 2$$

$$4\sqrt{16} = 4 \cdot \sqrt{16} = 4 \cdot 4 = 16$$

2.23d) Analys Förenkla $\frac{\sqrt[4]{a^3 \cdot \sqrt{a}}}{\sqrt[8]{1/a}}$

$$3 + \frac{1}{2} = \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\frac{\sqrt[4]{a^3 \cdot \sqrt{a}}}{\sqrt[8]{1/a}} = \frac{(a^3 \cdot a^{\frac{1}{2}})^{\frac{1}{4}}}{(\frac{1}{a})^{\frac{1}{8}}} = \frac{(a^{3+\frac{1}{2}})^{\frac{1}{4}}}{(a^{-1})^{\frac{1}{8}}} = \frac{(a^{\frac{7}{2}})^{\frac{1}{4}}}{a^{-\frac{1}{8}}} = \frac{a^{\frac{7}{2} \cdot \frac{1}{4}}}{a^{-\frac{1}{8}}} = \frac{a^{\frac{7}{8}}}{a^{-\frac{1}{8}}} = a^{\frac{7}{8} - (-\frac{1}{8})} = a^{\frac{7}{8} + \frac{1}{8}} = a^{\frac{8}{8}} = a^1 = \underline{a}$$

- Börja "innifrån"
- små steg

Tips:

Beräkna och svara som tal.

$$8^3 = (2^3)^3 = 2^{3 \cdot 3} = 2^9 = \underline{512}$$

$$\frac{8^2}{4^6} = \frac{(2^3)^2}{(2^2)^6} = \frac{2^{3 \cdot 2}}{2^{2 \cdot 6}} = \frac{2^6}{2^{12}} = \frac{1}{2^{12-6}} = \frac{1}{2^6} = \underline{\frac{1}{64}}$$

