# Föreläsning 1

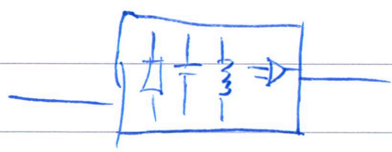
Förberedelser: Ett installerat EQ-program +
Lämplig låt

---

→ Signalbehandling: Att påverka eller förändra frekvens och amplitud innehållet i en signal.

→ Spela låt: Ändra på EQ
Högpass / Lågpassfilter.

→ Komponenter:



Passiva:
- Resistorer
- Kapasitanser
- Induktanser

Aktiva:
- Förstärkare

→ Tidskontinuerligt ⟷ Tidsdiskret.
- IRL

- I datorn
- Tyndpunkt i kursen

Förändra

In → A/D → Lagra → [ ] → Lagra → D/A → Ut

→ Ex. Signalbehandling

· Ljud

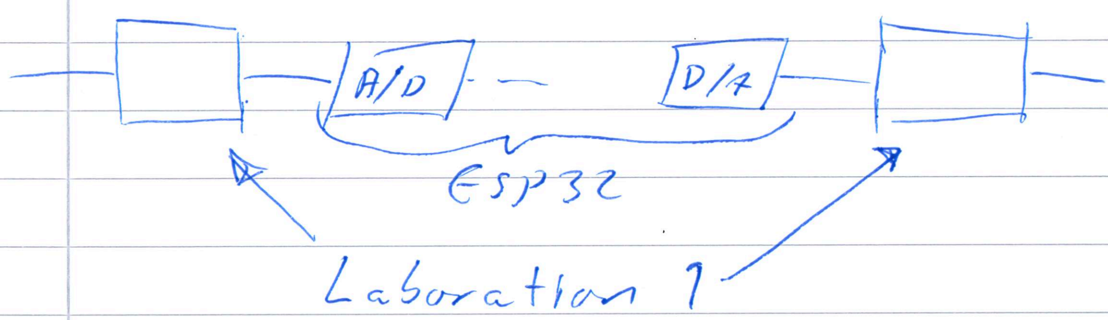
· Sensorer: Steg – När har man fått ett steg

Puls – ~~När~~ hur gör man när pulsen ändras.

→ Kursinfo

Boken. ny upplaga på G.

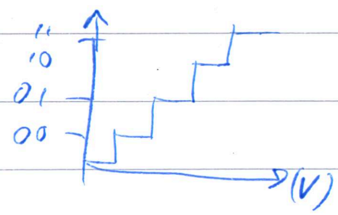
Laborationer




Mycket räkna, mycke komplexa tal

**j istället för i**

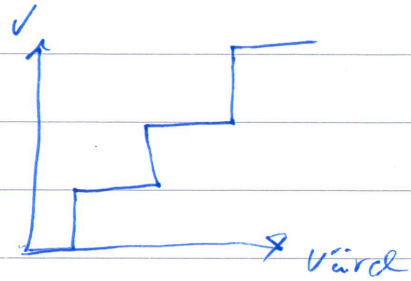
A/D-omvandling




ESP- 12 bitar

0–3,3V (<u>0,2–3,1V</u>)

D/A-omvandling:




ESP: 8 biter

0–3,3V (0,–{2,8–3,0})

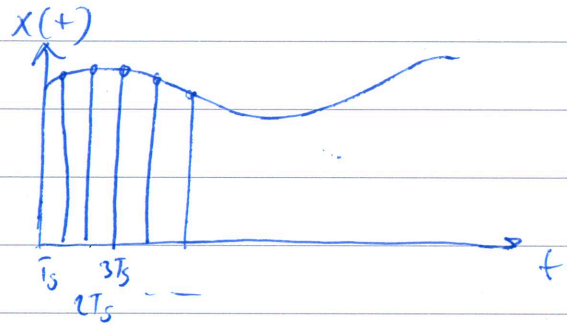
→ Sampling:

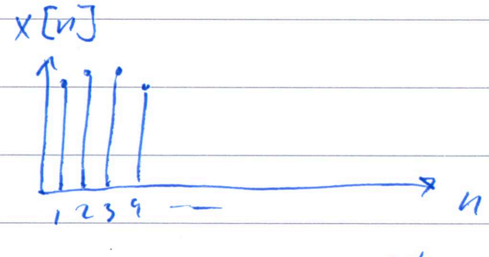
Rita "fin" positiv signal

$T_s$ = Samplingstid

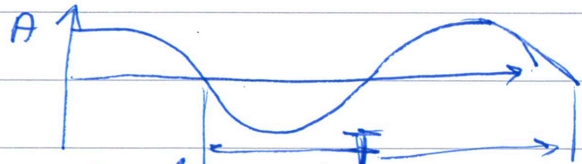
$\frac{1}{T_s} = f_s$ = Samplingsfrekvens.

$x[n] = x(n \cdot T_s)$




→ Exempel: $x(t) = A\cos(\underbrace{2\pi f}_{\omega} t + \phi) = A\cos(\omega t + \phi)$




$x[n] = x(nT_s) = A\cos(2\pi f \cdot n T_s + \phi) =$

$f_s = \frac{1}{T_s}$

$= A\cos\left(2\pi \frac{f}{f_s} n + \phi\right) = A\cos(\hat{\omega} n + \phi)$

$\hat{\omega}$

## Analogt / Tidskontinuerligt system

$x(t) \longrightarrow \boxed{\quad} \longrightarrow y(t)$

- Enkel filtrering → billigt
- Blir snabbt komplext → dyrt
- Stora feltoleranser

## Digitalt / Tidsdiskretsystem

$x[n] \longrightarrow \boxed{\quad} \longrightarrow y[n]$

~~Enkla~~ Kräver mycket hårdvara → dyrt
mjukvara

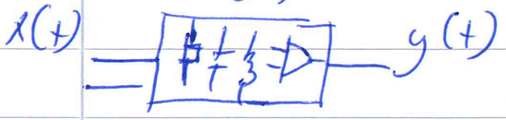
Ökad komplexitet → billigt

Lätt att ändra

Exakta värden.

# Föreläsnig 2

Analogt / Tidskont.

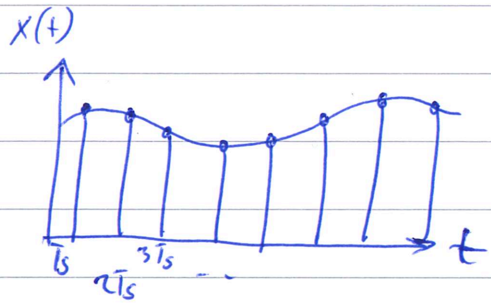



Digitalt / Tidsdiskret

$x(t)$ — [A/D] — $x[n]$ — [Kod] — $y[n]$ — [D/A] — $y(t)$

[ Se föregående föreläsnings / sida ]

Sampling:

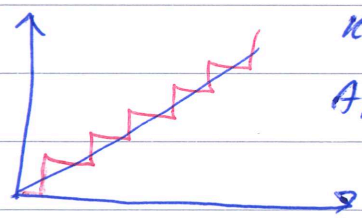



$T_s$ - samplingstid

$\frac{1}{T_s} = f_s$ = samplingsfrekvens

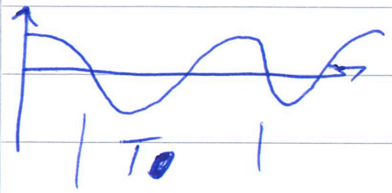
$x[n] = x(n \cdot T_s)$

Borde vara "$\approx$" då kvantiseringsfel A/D-omvandling



~~Exempel:~~

Exempel: $\quad w = 2\pi f \quad\quad f = \frac{1}{T}$




$$x(t) = A \cdot \cos(\cancel{w}\, 2\pi f \cdot t + \phi)$$
$$x[n] = A \cdot \cos(2\pi f \cdot n \cdot T_s + \phi)$$
$$x[n] = A \cdot \cos\left(2\pi \frac{f}{f_s} \cdot n + \phi\right) \quad f_s = \frac{1}{T_s}$$
$$\hat{w} = 2\pi \frac{f}{f_s}$$

(1) $f = 1\,kHz \quad \phi = 0 \quad f_s = 8\,kHz \quad A = 0$

$$x[n] = 1 \cdot \cos\left(2\pi \cdot 1 \cdot \frac{1}{8} \cdot n + 0\right)$$
$$x[n] = 1 \cos\left(2\pi \frac{1}{8} \cdot n\right) = \cos\left(\frac{\pi}{4} \cdot n\right)$$

$x[0] = \cos(0) = 1$

$x[1] = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x[2] = \cos\left(\frac{\pi}{2}\right) = 0$

$x[3] = \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$x[4] = \cos(\pi) = -1$

$x[5] = \cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$x[6] = \cos\left(\frac{3\pi}{2}\right) = 0$

$x[7] = \cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$x[8] = \cos(2\pi) = 0$

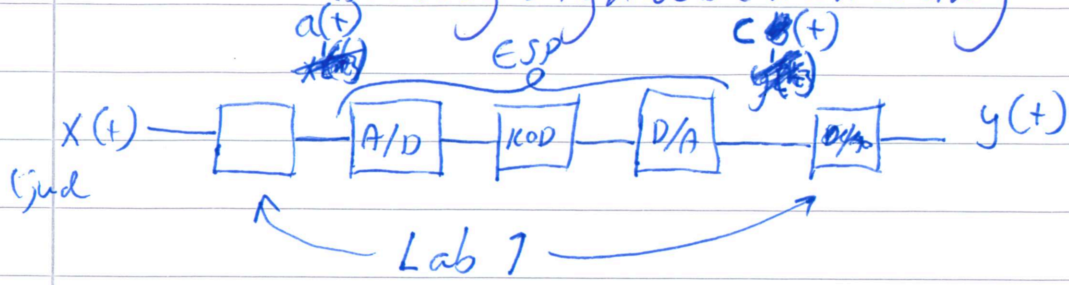
(2) $f = 5\,kHz \quad f_s = 40\,kHz \quad A = 1 \quad \phi = 0$

$$x[n] = 1 \cdot \cos\left(2\pi \frac{5}{40} n\right) = \cos\left(\frac{\pi}{4} \cdot n\right)$$

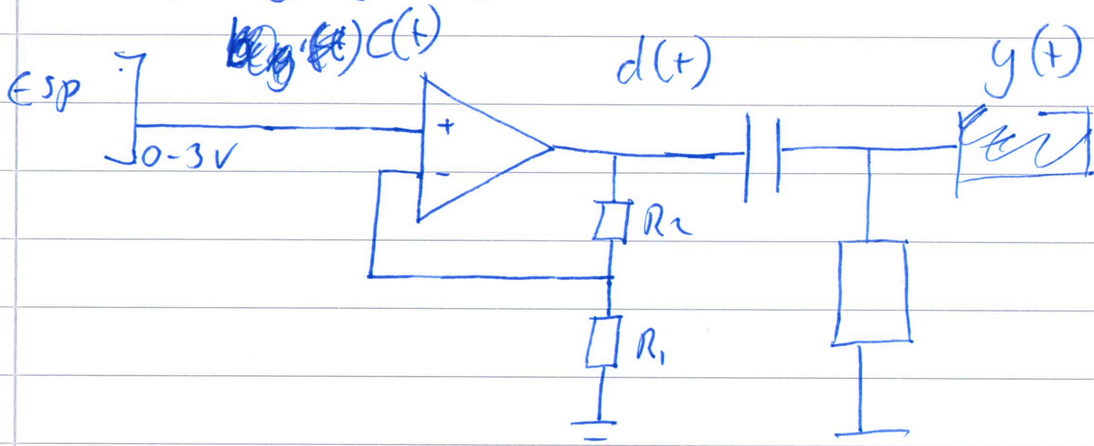
(1) $\frac{f}{f_s} = \frac{1}{8} \Rightarrow \hat{w} = \frac{\pi}{4}$

(2) $\frac{f}{f_s} = \frac{5}{40} = \frac{1}{8} \Rightarrow \hat{w} = \frac{\pi}{4}$

# Analog signalbehandling



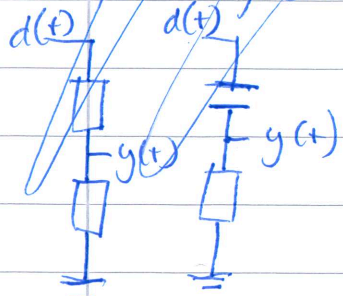

## ~~Utsteg~~ Utgångssteg



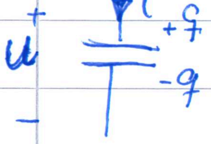

Repetition:
- resistorer i serie och parallellt
- Spänningsdelning
- Superposition
- OP-förstärkare

$d(t) = c(t)\frac{R_1+R_2}{R_1}$ $y(t) = ?$ Kondensator

~~exempel~~ Kondensator stoppar likström!



Hur fungerar en kondensator?




$$i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} = \frac{d}{dt} C \cdot u = C \frac{d}{dt} u$$

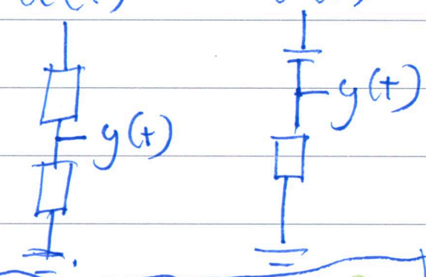
$\boxed{q = C \cdot u}$

$\Downarrow$

Differential ekvation
Jippi!

$$d(t) = \frac{R_1 + R_2}{R_1} \cdot c(t)$$

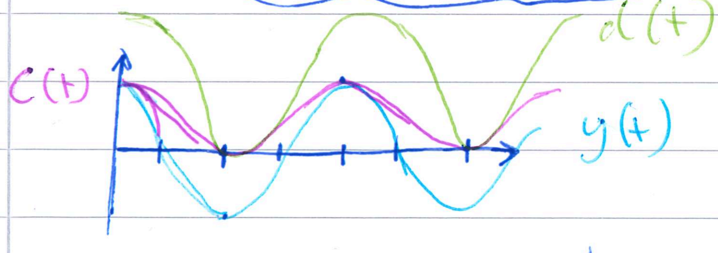
$y(t)$




kondensator stoppar likström

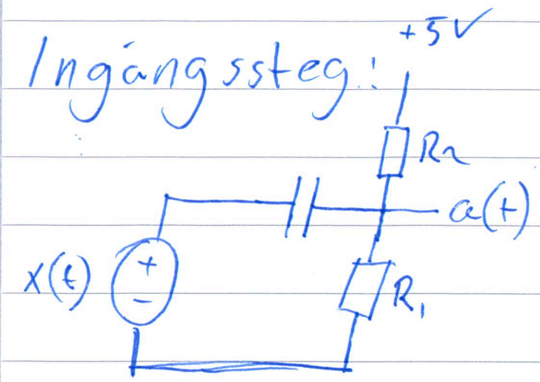
$\Downarrow$

$y(t) = d(t) - \text{likström}$



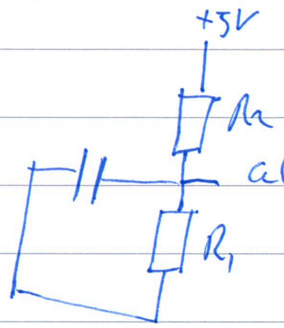

$c(t) = 1{,}5 + 1{,}5 \cos(2\pi f \cdot t)$

Ingångssteg:



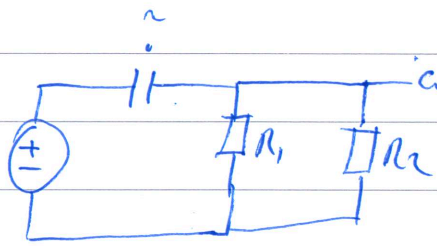

Superposition:

fall I



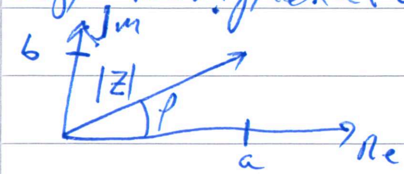

$$a(t) = 5\frac{R_1}{R_1+R_2}$$

fall II




$$a(t) = \frac{R_1 // R_2}{? + R_1//R_2}$$

Rep Komplexatal




$$z = a + jb \qquad |z| = \sqrt{a^2+b^2}$$

$$a = |z|\cos(\varphi) \qquad b = |z|\sin(\varphi)$$

$$z = |z|\cos(\varphi) + j\,|z|\sin(\varphi) =$$

$$= |z|\left(\cos(\varphi) + j\sin(\varphi)\right) =$$

[ Eulers formel $e^{j\varphi} = \cos(\varphi) + j\sin(\varphi)$ ]

$$= |z|\,e^{j\varphi}$$

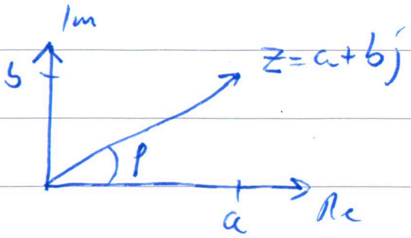
Undersök:

$$e^{j0} \qquad e^{j\frac{\pi}{2}} \qquad e^{j\pi} \qquad e^{j\frac{3\pi}{2}}$$

$$\tan(\varphi) = \frac{b}{a} \Rightarrow \operatorname{atan}\left(\frac{b}{a}\right) = \varphi \quad a \geq 0$$

# Föreläsning 3

Rep:




$\tan(\varphi)=\frac{b}{a}$

$\varphi=\operatorname{atan}\left(\frac{b}{a}\right)$ $a \geq 0$

$a=|z|\cos(\varphi)$

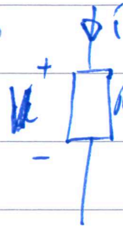
$b=|z|\sin(\varphi)$

$|z|=\sqrt{a^2+b^2}$

$z=|z|\cos(\varphi)+j|z|\sin(\varphi)=$

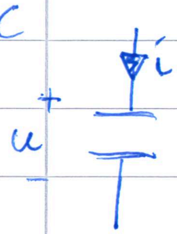
$=|z|(\cos(\varphi)+j\sin(\varphi))=|z|e^{j\varphi}$

R



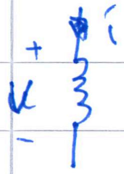

$u=R\cdot i$

C




$i=C\frac{d}{dt}u(t)$

L



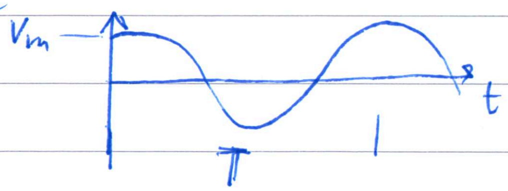

$u=L\frac{d}{d(t)}i(t)$

Impedans

$\frac{d}{dt} \Rightarrow$ differential ekvation [Ej kul]

m: magnitude

Sinusformad signal




$f=\frac{1}{T}$

$v=V_m\cos(2\pi f\cdot t)=V_m\cos(\omega t)$

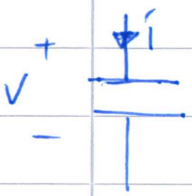
[$\omega=2\pi f$ - vinkelfrekvens]

forts

Inför komplexspänning:

komplextal (ej konjugat)

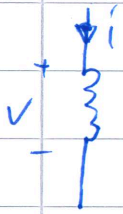
$$\bar{v} = V_m \cos(\omega t) + j V_m \sin(\omega t) = V_m e^{j\omega t}$$



inre derivata

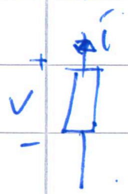
$$\bar{i} = C \frac{d}{d(t)} \bar{v}(t) = C \cdot V_m \, j\omega \cdot e^{j\omega t} = j\omega C \cdot V_m \cdot e^{j\omega t} =$$

$$= j\omega C \cdot \bar{v}(t) \Rightarrow \bar{v}(t) = \frac{1}{j\omega C} \cdot \bar{i} \qquad \boxed{Z_C = \frac{1}{j\omega C}}$$



$$\bar{i} = I_m e^{j\omega t}$$

$$\bar{v} = L \frac{d}{dt} \bar{i} = j\omega L \, I_m e^{j\omega t} = j\omega L \, \bar{i} \qquad \boxed{Z_L = j\omega L}$$



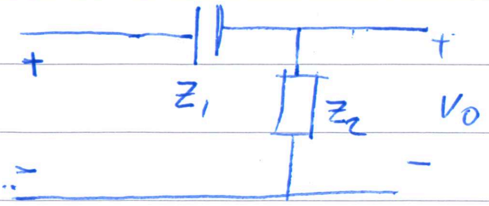
$$\bar{v} = R \cdot \bar{i} \qquad \boxed{Z_R = R}$$

Impedans

enhet: $\Omega$

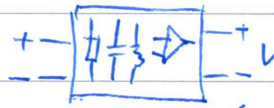
# Första delen i vårt ingångssteg:

$V_s$




$$V_s = V_{ms}\cos(\omega t)$$

$$\bar{V}_s = V_{ms}\cos(\omega t) + j V_{ms}\sin(\omega t) =$$
$$= V_{ms}\, e^{j\omega t}$$




$$\bar{V}_0 = \frac{Z_2}{Z_1+Z_2}\cdot \bar{V}_s = \overbrace{\boxed{\frac{R}{\frac{1}{j\omega C}+R}}}^{H(j\omega)}\bar{V}_s =$$

$$V_0 = H(j\omega)\, V_s$$

$H(j\omega)$ – frekvensfunktion

$$= \frac{\frac{1}{R}\, R}{\frac{1}{R}\left(\frac{1}{j\omega C}+R\right)}\bar{V}_s = \frac{1}{\frac{1}{j\omega CR}+1}\bar{V}_s = \frac{1}{1-j\frac{1}{\omega RC}}\cdot \bar{V}_s =$$

$$= \frac{1}{1+j\left(-\frac{1}{\omega RC}\right)}\bar{V}_s = \frac{1}{\underbrace{\sqrt{1^2+\left(\frac{1}{\omega RC}\right)^2}}_{|Z|}\cdot e^{j\,\mathrm{atan}\left(\frac{-\frac{1}{\omega RC}}{1}\right)}}\bar{V}_s =$$

$$\varphi = \mathrm{atan}\left(\frac{b}{a}\right)$$

$$\boxed{\frac{1}{e^x} = e^{-x}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}\, e^{-j\,\mathrm{atan}\left(\frac{-1}{\omega RC}\right)}\cdot \overbrace{V_{ms}\, e^{j\omega t}}^{\bar{V}_s} =$$

$$e^x\cdot e^y = e^{(x+y)}$$

$$-\mathrm{atan}(-x) = \mathrm{atan}(x)$$

$$= \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}\, V_{ms}\, e^{j\left(\omega t + \mathrm{atan}\left(\frac{1}{\omega RC}\right)\right)} =$$

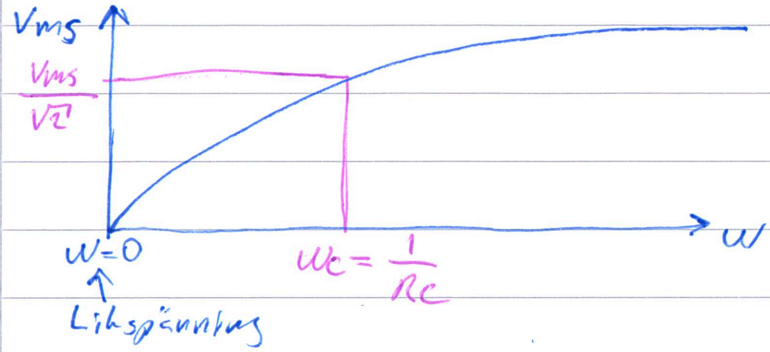
$$= \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}\, V_{ms}\left(\cos\left(\omega t+\mathrm{atan}\left(\frac{1}{\omega RC}\right)\right) + j\sin\left(\omega t+\mathrm{atan}\left(\frac{1}{\omega RC}\right)\right)\right)$$

$$V_0 = \mathrm{Re}(\bar{V}_0) = \overbrace{\frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}\cdot V_{ms}}^{V_{m0}}\cdot \cos\left(\omega t + \overbrace{\mathrm{atan}\left(\frac{1}{\omega RC}\right)}^{\text{fasförskjutning}}\right)$$

$$V_{mo} = \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}} \cdot V_{ms}$$

$f \to 0 \Leftrightarrow \omega \to 0 \Rightarrow$ stor nämnare $\Rightarrow V_{mo} \to 0$

$f \to \infty \Leftrightarrow \omega \to \infty$ ~~liten~~ $\Leftrightarrow \frac{1}{\omega} \to 0 \Rightarrow V_{mo} \to V_{ms}$



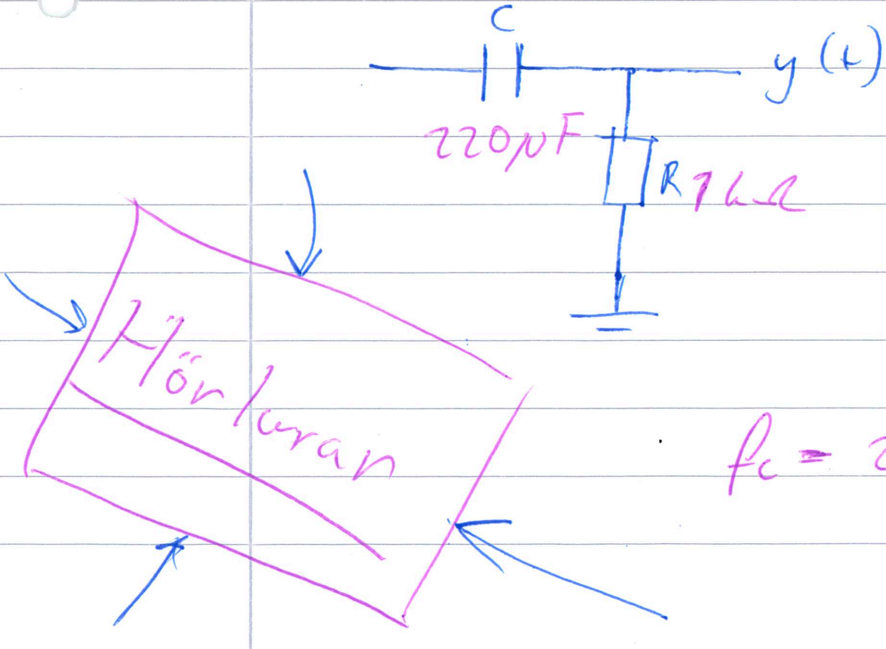

$$V_{mo} = \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}} V_{ms}$$

om $\omega = \frac{1}{RC} \Rightarrow \frac{1}{\omega RC} = 1$

$$V_{mo} = \frac{1}{\sqrt{1+1}} V_{ms} = \frac{1}{\sqrt{2}} V_{ms}$$

$\omega = \frac{1}{RC} \leftarrow$ cut off freq.
Brytfrekvens

# Utgångssteget




$$\omega_c = \frac{1}{RC}$$

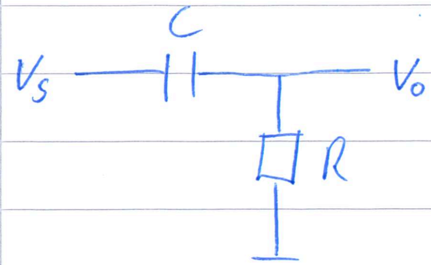
$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC}$$

$f_c = 20$ Hz
gränsen för hörbart.

# Föreläsning 4

Rep




$V_s = V_{ms} \cdot \cos(\omega t)$

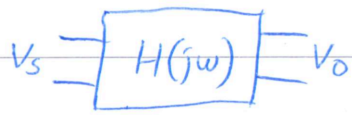
$$\bar{V}_0 = \underbrace{\boxed{\frac{R}{R+\frac{1}{j\omega C}}}}_{H(j\omega)} \bar{V}_s$$

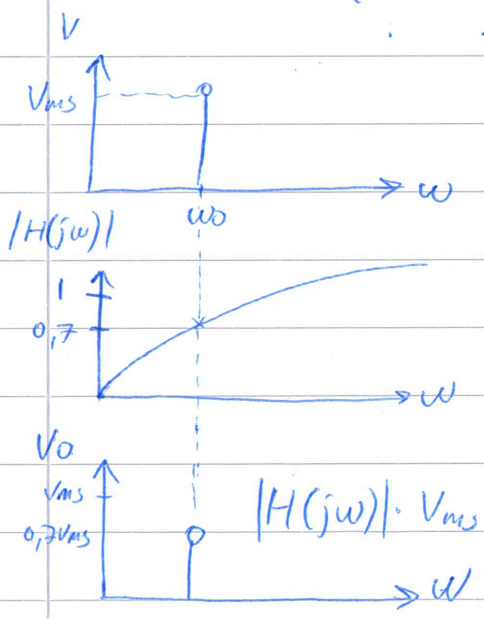
$$V_0 = \underbrace{\boxed{\frac{1}{\sqrt{1+\left(\frac{1}{\cancel{j}\omega R C}\right)^2}}}}_{|H(j\omega)|} \cdot V_{ms} \cos\Big(\omega t + \underbrace{\operatorname{atan}\left(\frac{1}{\cancel{j}\omega R C}\right)}_{\arg(H(j\omega))}\Big)$$

$$V_0 = \underbrace{|H(j\omega)| \cdot V_{ms}}_{= V_{m0}} \cdot \cos(\omega t + \arg(H(j\omega)))$$

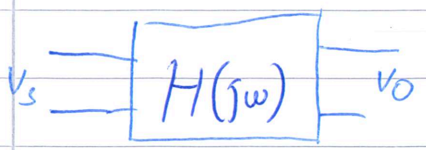
Spektrum

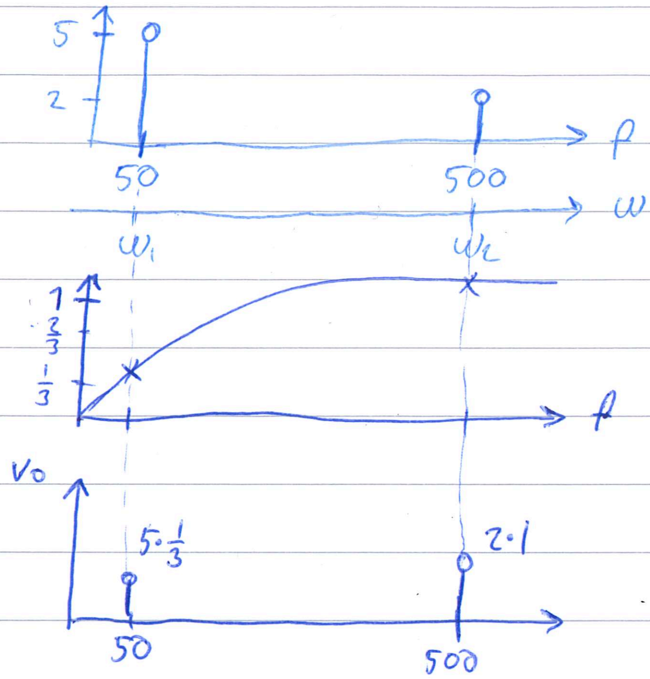
$V_s = V_{ms} \cdot \cos(\omega_0 t)$



Exempel: $V_s = 5\cos(\underbrace{2\pi\cdot \overset{f_1}{50}\cdot t}_{\omega_1}) + 2\cos(\underbrace{2\pi\cdot \overset{f_2}{500}\cdot t}_{\omega_2})$



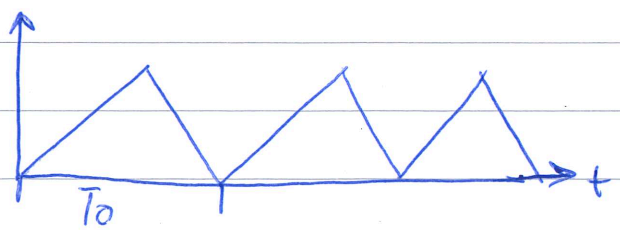

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{1}{j\omega C}\right)^2}}$$

$$V_{m0} = |H(j\omega)|\cdot V_{ms}$$

Grafiskmultiplikation
[Amplitud men ej fas]

# Fourierserier:

Alla periodiska och fysikaliska signaler kan skrivas som samman av sinusformade signaler.



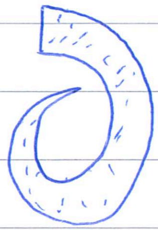

$T_0$ - period tid
$f_0 = \frac{1}{T_0}$ - grundfrekvens

$$V_s = V_{DC} + \sum_{k=1}^{\infty} V_{m_k}\cos(2\pi\cdot k f_0\cdot t + \phi_k)$$

$\omega_0 = 2\pi f_0$

Varje k ger en "delton" som är en multipel av grundfrekvensen $f_0, 2f_0, 3f_0 \dots$

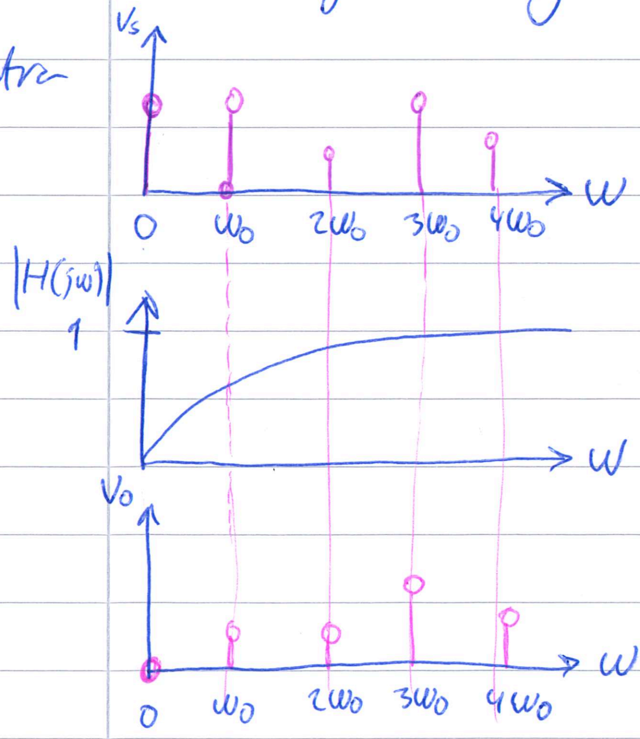
- Slå på ett glas, jämför olika instrument med samma grundton låter olika pga olika övertoner. "frekvenser"
- Visa Triangel & Fyrkantsvåg.

örat - Hörselsnäcka
spektrumanalysator  resonans på olika ställen.

$$V_s = V_{DC} + \sum_{k=1}^{\infty} V_{mk} \cdot \cos\left(2\pi \cdot kf_0 + \phi_k\right) \qquad \omega_0 = 2\pi f_0$$

Varje ~~ke~~ k ger en "delton" som är en mutipel av grundtonen. $f_0, 2f_0, 3f_0 \dots$

spektra

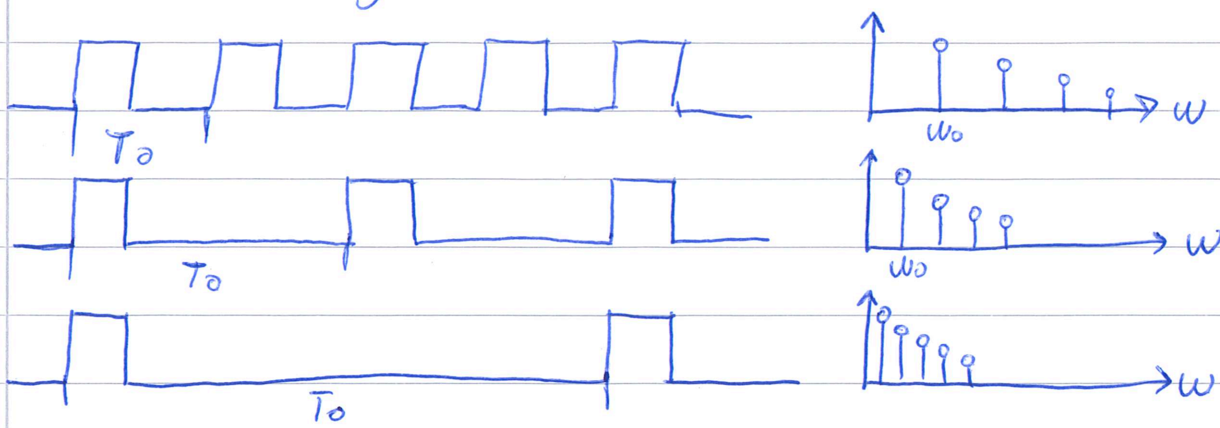



Koppla till Labb
"stopp av låga frekvenser"
dämpning

Oktav - vad gäller.

Periodisk signaler Spektrum

$\omega_0 = 2\pi f_0 = 2\pi \frac{1}{T_0}$




Längre period → ~~högre~~ lägre grundfrekvens → mer packat spektrum

Om man har jätte stort $T_0$, frekvenserna kommer jätte tätt.

Icke periodiska signaler kan beskrivas som en summa av sinusformade signaler där <u>alla</u> frekvenser är med.

# Dubbelsidigt spektrum

$$v = V_m \cdot \cos(\omega_0 t) = \frac{V_m}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) =$$

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$e^{-j\varphi} = \cos(-\varphi) + j\sin(-\varphi) = \cos(\varphi) - j\sin(\varphi)$$

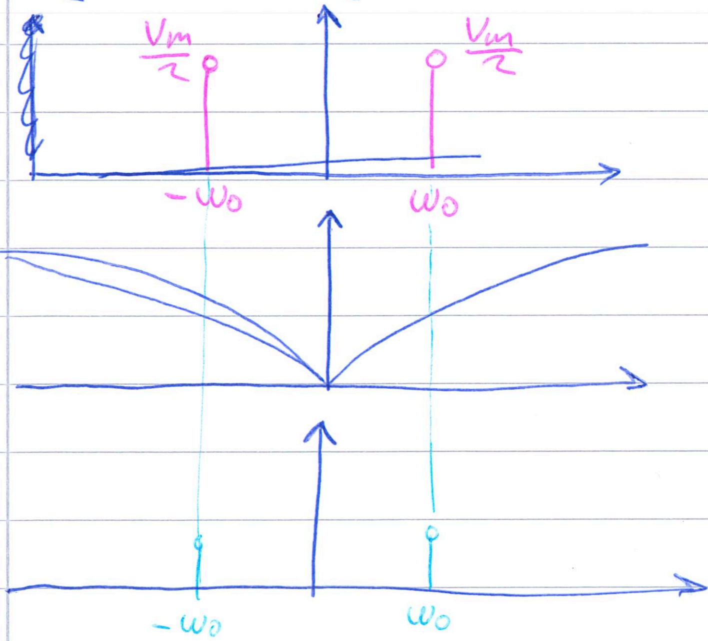
cos jämn funk $\cos(x) = \cos(-x)$

sin udda funk $\sin(-x) = -\sin(x)$

$$e^{j\varphi} + e^{-j\varphi} = \cos(\varphi) + j\sin(\varphi) + \cos(\varphi) - j\sin(\varphi) = 2\cos(\varphi) + 0$$

$$\cos\varphi = \frac{1}{2}\left(e^{j\varphi} + e^{-j\varphi}\right)$$

$$v = \frac{V_m}{2}e^{j\omega_0 t} + \frac{V_m}{2}e^{j(-\omega_0)t}$$




$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{1}{j\omega C}\right)^2}}$$

# Föreläsning 5

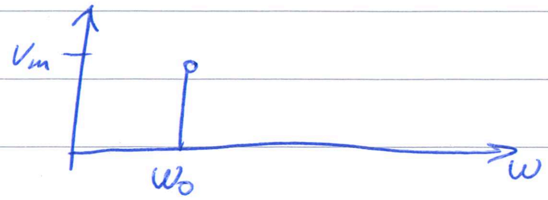
Rep: Impedans

$Z_R = R$

$Z_C = \frac{1}{j\omega C}$

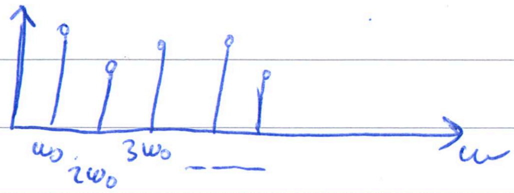
$Z_L = j\omega L$

Sinus



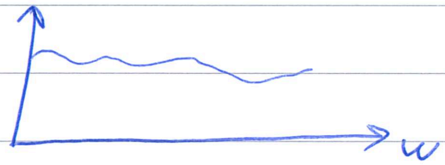

$V_m \cos(j\omega_0 t)$

Periodisk




Begränsat antal

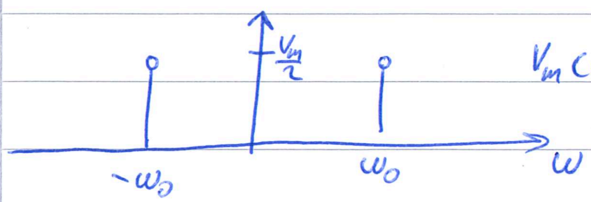
Icke Periodisk




Kontinuerligt spektra

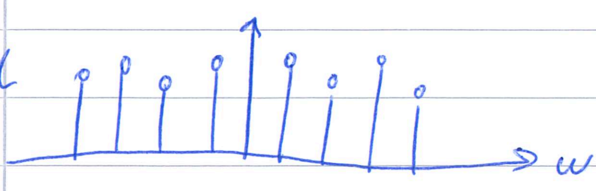
Dubbelsidigt spektra

sinus




$V_m \cos(j\omega_0 t) = \frac{V_m}{2} e^{j\omega_0 t} + \frac{V_m}{2} e^{-j\omega_0 t}$

Periodisk

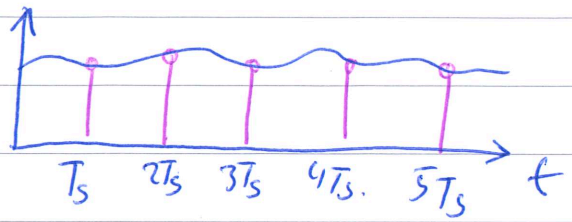



Icke-periodisk

# Sampling

$$x(t) \longrightarrow \boxed{A/D} \longrightarrow x[n] \qquad x[n] = x(nT_s)$$

$$f_s = \frac{1}{T_s}$$



ex. $x(t) = A\cos(\omega_0 t)$

$$x[n] = A\cos(\omega_0 n \cdot T_s) =$$
$$= A\cos\left(2\pi \frac{f}{f_s} \cdot n\right) = x[n]$$

Visa Bild 2

$A = 1$

$$f_0 = \frac{1}{0{,}5} = 2$$

$$f_s = \frac{1}{0{,}1} = 10$$

Påstående 1: $y(t) = A\cos(2\pi(f_0 + l\cdot f_s)\cdot t)$ ger $y[n] = x[n]$

Bevis $y[n] = y(nT_s) = A\cos(2\pi(f_0 + l\cdot f_s)\cdot n\cdot T_s) =$

$$= A\cos\left(2\pi(f_0 + l\cdot f_s)\cdot \frac{n}{f_s}\right) =$$

$$= A\cos\left(2\pi\left(\frac{f_0 + l\cdot f_s}{f_s}\right)\cdot n\right) =$$

$$= A\cos\left(2\pi\left(\frac{f_0}{f_s} + l\right)\cdot n\right) =$$

heltal

$$= A\cos\left(2\pi \frac{f_0}{f_s}\cdot n + 2\pi \cdot l \cdot n\right) =$$

$$y[n] = A\cos\left(2\pi \frac{f_0}{f_s}\cdot n\right) = x[n] \quad \text{då}$$

$$\cos(x) = \cos(x + 2\pi \cdot n)$$

Påstående 2:

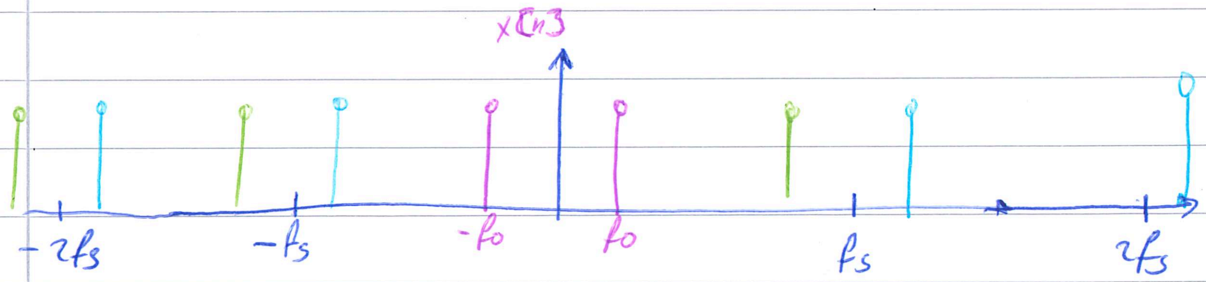
$$z(t) = A\cos\left(2\pi\left(l\cdot f_s - f_0\right)\cdot t\right) \text{ ger } z[n] = x[n]$$

$$z[n] = z\left(\frac{n}{f_s}\right) = A\cos\left(2\pi\left(l\cdot f_s - f_0\right)\frac{n}{f_s}\right) =$$

$$= A\cos\left(-2\pi \frac{f_0}{f_s} n + 2\pi \cdot l \cdot n \cdot \frac{f_s}{f_s}\right) =$$

$$= A\cos\left(-2\pi \frac{f_0}{f_s}\cdot n\right) = \qquad \cos(x) = \cos(-x)$$

$$z[n] = A\cos\left(2\pi \frac{f_0}{f_s}\cdot n\right) = x[n]$$




| | | | | | | |
|---|---|---|---|---|---|---|
| ① | $-2f_s+f_0$ | $-f_s+f_0$ | | $f_0$ | $f_s+f_0$ | $2f_s+f_0$ |
| ② | $-2f_s-f_0$ | $-f_s-f_0$ | $-f_0$ | | $f_s-f_0$ | $2f_s-f_0$ |

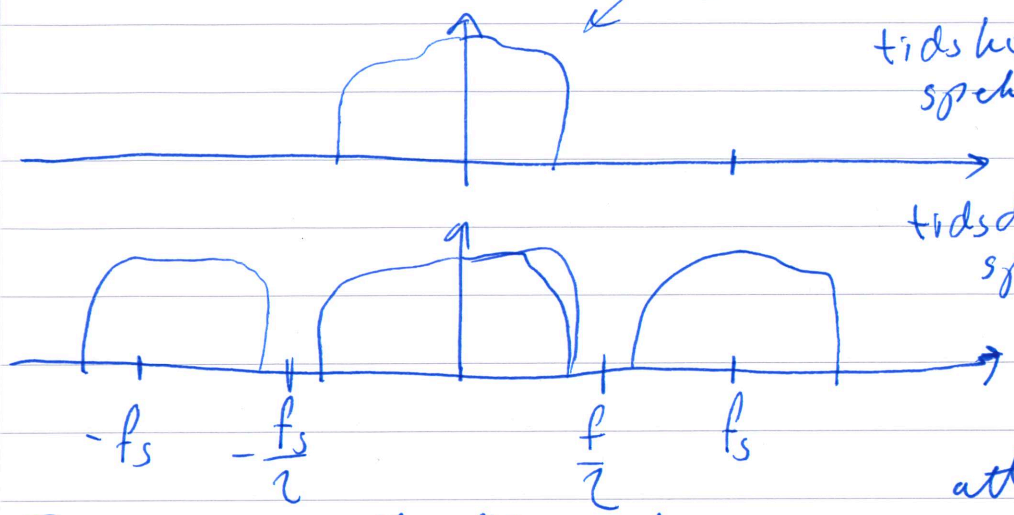
$f_0 = 2$    ① $f = f_s + f_0 = 2 + 10 = 12$

$f_s = 10$    2 $f = f_s - f_0 \; 10 - 2 = 8$

# Aliasing / vikning

signal ickeperiodisk

tidskontinuerligt spektra

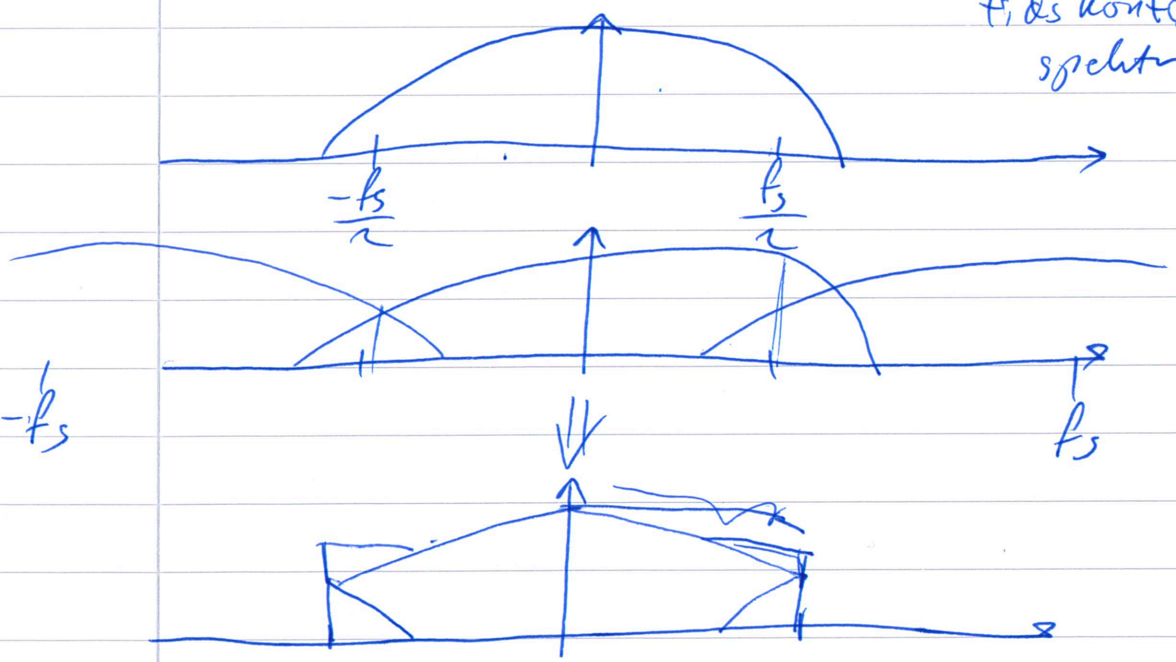



tidsdiskret spektra

Bör gå att återskapa

Bör gå att återskapa

tidskontinuerligt spektr



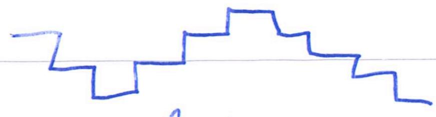

Vikningseffekt gör att man ej kan återskapa signal.

Fix: Lågpassfilter, antivikningsfilter

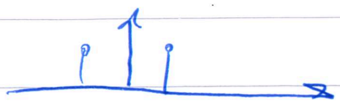
# Samplingsteoremet

om $x(t)$ <u>inte</u> innehåller frekvenser över $\frac{f_s}{2}$ kan signalen rekonstrueras exakt ur $x[n]$. Men ~~då~~ för detta krävs att alla värden för $x[n]$ är kända. [se boken]. Dvs fungerar ej i realtid då framtida värden ej är kända.

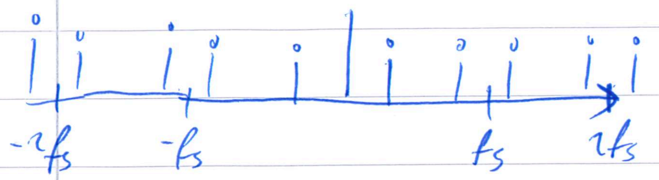
# D/A-omvandlaren



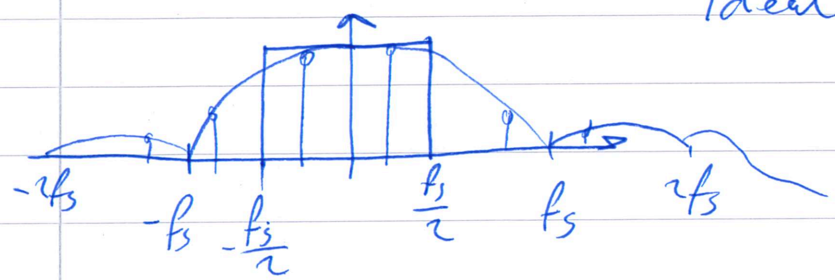
Många frekvenser.



sinusformad insignal

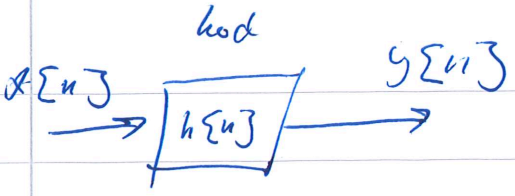



Ideal rekonstruktion




$$S(f) = \left| \frac{\sin(\pi \frac{f}{f_s})}{\pi \frac{f}{f_s}} \right|$$

Större förhållande mellan $f_s$ och $f$ ger bättre upplösning ⇒ bättre rekonstruktion.




LTI - system:

L - Linjärt, proportionellt mellan $x[n]$ och $y[n]$

TI - Tidsinvariant, fungerar på samma sätt oavsett när. Systemet förändras inte.

signal, 500 Hz, störning, 5000 Hz

Ex $$x(t) = \underbrace{\sin(2\pi\cdot\overbrace{500}^{500\text{ Hz}}\cdot t)}_{\text{signal}} + \underbrace{0{,}25\cdot\sin(2\pi\cdot\overbrace{5000}^{5000\text{ Hz}}\cdot t)}_{\text{störning}} =$$

$$f_s = 40000 \text{ Hz}$$

$$x[n] = x(n\cdot T_s) = x\left(n\cdot\frac{1}{f_s}\right) =$$

$$= \sin\left(2\pi\cdot 500\cdot n\cdot\frac{1}{f_s}\right) + 0{,}25\sin\left(2\pi\cdot 5000\cdot n\cdot\frac{1}{f_s}\right) =$$

$$= \sin\left(2\pi\cdot\frac{500}{40000}\cdot n\right) + 0{,}25\sin\left(2\pi\cdot\frac{5000}{40000}\cdot n\right) =$$

$$= \sin\left(2\pi\cdot\frac{1}{80}\cdot n\right) + 0{,}25\sin\left(2\pi\frac{1}{8}\cdot n\right)$$

i Matlab:

```
n = 0:1:80000;
fs = 40000;
x1 = sin(2·pi·500·n/fs);    stem(n,x1) sound(x1,fs)
x2 = sin(2·pi·5000·n/fs);   stem(n,x2) sound(x2,fs)
x = x1 + x2
stem(n,x)
sound(x,fs)
```

# Glidande medelvärde / Moving Average

$$y[n] = \frac{1}{M+1}\left(x[n] + x[n-1] + x[n-2] + \dots + x[n-M]\right)$$

exempel! $M=3 \Rightarrow y[n] = \frac{1}{3+1}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

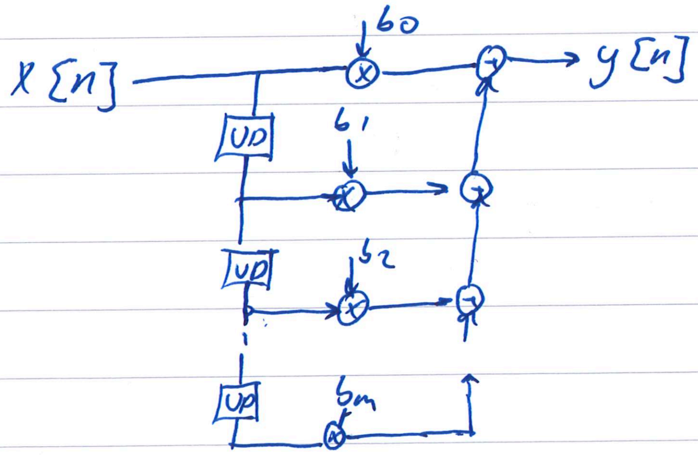
## Matlab exempel : ma.m.

Mer generellt

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_M x[n-M]$$

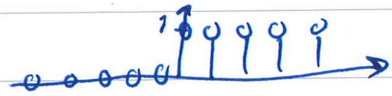
kallas FIR-filter.




exempel

$x[n] = 0 \quad n < 0$

$x[n] = 1 \quad n \geq 0$




$x[-1] = 0$

$x[0] = 1$

$x[1] = 1$

$\vdots$

$b_0 = b_1 = b_2 = \frac{1}{3}$

$y[n]$

$y[0] = \frac{1}{3}x[0] + \frac{1}{3}x[-1] + x[-2] =$
$\frac{1}{3} + 0 + 0 = \frac{1}{3}$

$y[1] = \frac{1}{3}x[1] + \frac{1}{3}x[0] + \frac{1}{3}x[-1] = \frac{1}{3} + \frac{1}{3} + 0 = \frac{2}{3}$

$y[2] = \frac{1}{3}x[2] + \frac{1}{3}x[1] + \frac{1}{3}x[2] = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$\vdots$

```
n = 0:1:80000;
fs = 40000;
x1 = sin(2·pi·500·n/fs);
x2 = sin(2·pi·5000·n/fs);
x = x1 + x2;
stem(n,x)
hold on
y = ma(10,x);        medelvärde
                     med 10
stem(n,y,'r');
y1 = ma1(x);
stem(n,y1,'g')
plot(n,x,'k')
```

# Föreläsning 7: Designa filter

Rep:

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_M x[n-M] =$$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

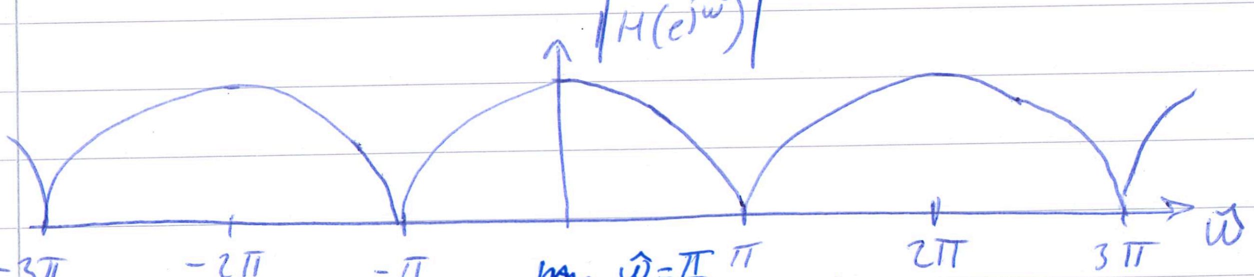
ex) $y[n] = \frac{1}{2} x[n] + \frac{1}{2} x[n-1]$ ~~$x(t) = A\cos(2\pi f t)$~~

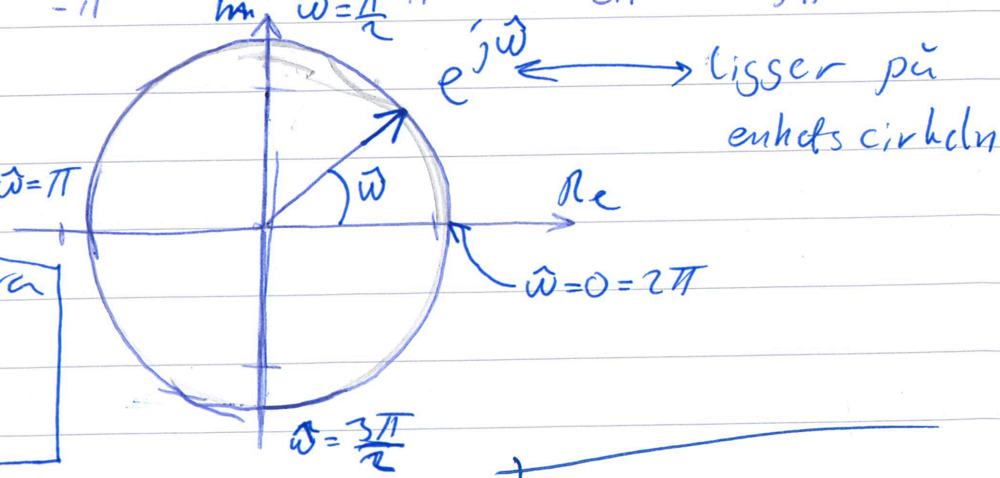
$$x(t) = A\cos(2\pi f t)$$

$$x[n] = A\cos\left(2\pi \frac{f}{f_s} \cdot n\right) = A\cos(\hat{\omega} n)$$

$$y[n] = \left|H(e^{j\hat{\omega}})\right| A\cos\left(\hat{\omega} n + \arg\left(H(e^{j\hat{\omega}})\right)\right)$$

där $H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k \left(e^{j\hat{\omega}}\right)^{-k}$




Designa filter genom att placera nollställen. Tittar vi på

Visa GCF
LP
enhetscirkel

Utvidga def. området till hela komplexa talplanet

$z = r e^{j\omega} \qquad 0 \leq r \leq \infty$

$H(z)$ - överföringsfunktion

$y[n] = \frac{1}{2} x[n] + \frac{1}{2} x[n-1]$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k \left(e^{j\hat{\omega}}\right)^{-k} = \frac{1}{2} e^{j\hat{\omega}\cdot 0} + \frac{1}{2} e^{-j\hat{\omega}}$$

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{1}{2} z^{0} + \frac{1}{2} z^{-1} =$$

$$= \frac{1}{2} z^{-1} \left(z^{1} + 1\right) = \frac{1}{2} \frac{z^{1}+1}{z^{1}}$$

$$|H(z)| = \frac{1}{2} \frac{|z+1|}{|z|}$$

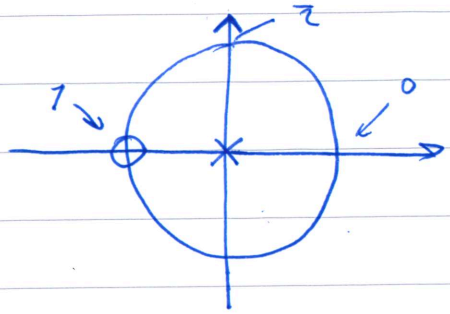
$z = -1 \Rightarrow$ nollställe (täljaren)

$z = 0 \Rightarrow$ pol (nämnare)

$\frac{1}{|z|} \to \infty$ när $z \to 0$

$\frac{|z+1|}{|z|} \to 1$ när $z \to \infty$

$H(z) \to \frac{1}{2}$ när $z \to \infty$




Visa GCF
LP
med filter.

Allmänt FIR-filter

$$y[n] = H(e^{j\omega})\, x[n] = H(z)\, x[n]$$

$$y[n] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

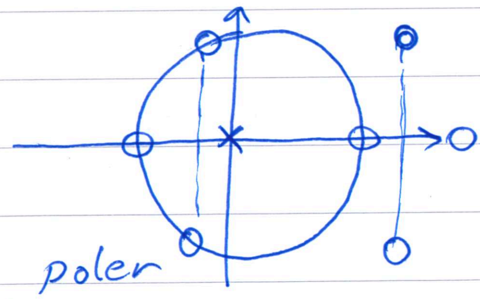
$$H(z) = b_0 z^{-0} + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M} =$$

$$= b_0 z^{-M} \underbrace{\left( z^M + \frac{b_1}{b_0} z^{M-1} + \frac{b_2}{b_0} z^{M-2} + \dots + \frac{b_M}{b_0} z^{\overset{M-M}{0}} \right)}_{\text{Polynom med M-st nollställen}} =$$

$$= \frac{b_0}{z^M} (z - z_1)(z - z_2)(z - z_3) \dots (z - z_M)$$

$$|H(z)| = \frac{b_0}{|z^M|} |z - z_1||z - z_2||z - z_3| \dots |z - z_M|$$

Alla poler i origo



Lika många poler som nollställen

Alla komplexa nollställe kommer i komplexkonjugerade par. Om inte så blir $b_0, b_1, b_2 \dots$ inte reela.

Exempel

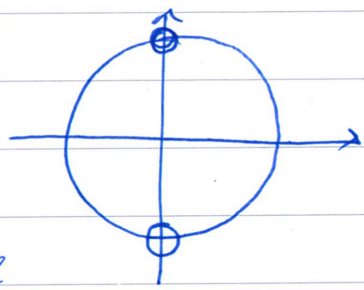
Vi vill konstruera ett filter som spärrar en viss frekvens ("Bandspärrfilter).

$f = 2{,}5$ kHz att spärra
$f_s = 10$ kHz

$$\hat{\omega} = 2\pi \frac{f}{f_s} = 2\pi \frac{2{,}5}{10} = \frac{\pi}{2}$$

· placera ett nollställe

$$z = e^{j\frac{\pi}{2}} = j$$

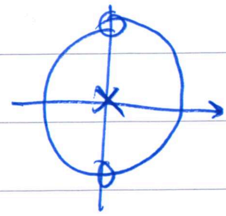


· Detta ger ett nollställe till

$$z = e^{j\frac{3\pi}{2}} = -j$$

$$H(z) = (z-j)(z-(-j))$$

· Två nollställe ger två poler



$$H(z) = \frac{1}{z^2}(z-j)(z-(-j)) =$$

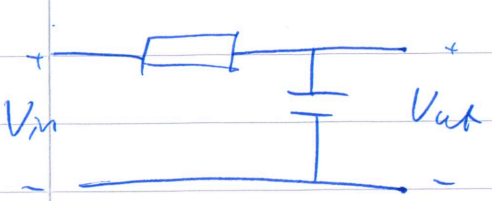
$$= \frac{z^2 - jz + jz + 1}{z^2} = \frac{z^2+1}{z^2} = \underset{b_0}{1} \cdot z + \underset{b_1}{0} \cdot z^{-1} + \underset{b_2}{1} \cdot z^{-2}$$

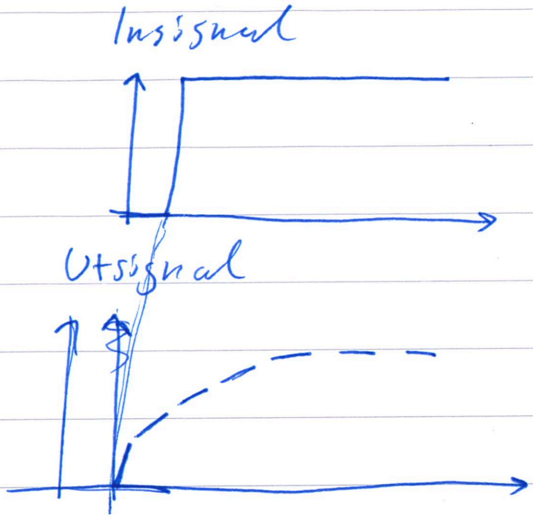
$$y[n] = x[n] + (0 \cdot x[n-1] +)\, x[n-2]$$

Normering $\frac{1}{k}(b_0 + b_1 + b_2 + \dots b_m) = 1$

$$y[n] = \frac{1}{2}(x[n] + x[n+2]) = \frac{1}{2}x[n] + \frac{1}{2}x[n-2]$$

# Föreläsning 8




Utsignalen beror på nuvarande värde + insignalen

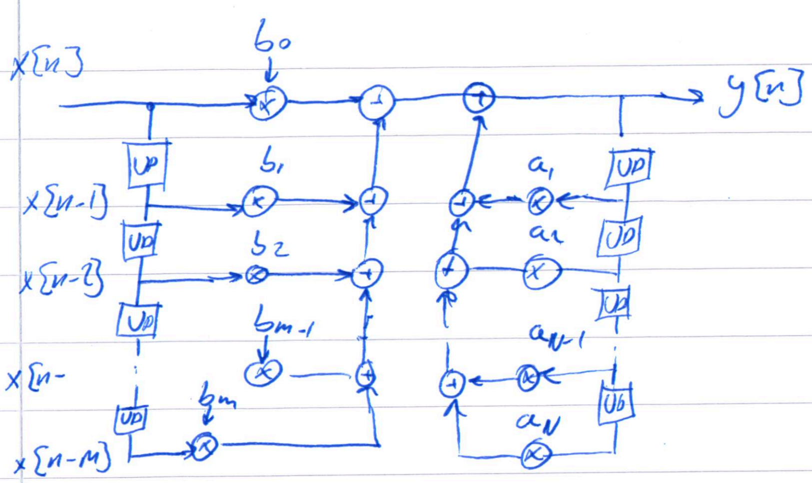
$$y[n] = b_0 x[n] + a_1 y[n-1]$$

ex. ~~g.~~ $y[n] = 0,1\, x[n] + 0,9\, y[n-1]$

Matlab: Lecture 08, script
10-poligt FIR-filter mha y=ma(10,x)
1poligt IIR-filter mha y=ma1(x)

IIR-filter (generell)

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_M x[n-M]$$
$$+ a_1 y[n-1] + a_2 y[n-2] + \dots + a_N x[n-N]$$




## FIR-filter

$$H(e^{j\omega}) = \sum_{k=0}^{M} b_k (e^{j\omega})^{-k}$$ frekvensfunktion

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$ {överföringsfunktion / Systemfunktion

## IIR-filter

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{l=1}^{N} a_l z^{-l}}$$

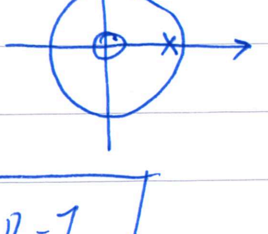
Man kan härleda detta men det är krångligt. Så vi tar den färdiga...

ex. fortsättningen: $y[n] = \underset{b_0}{0{,}1}\, x[n] + \underset{a_1}{0{,}9}\, y[n-1]$

$$H(z) = \frac{0{,}1 \cdot z^0}{1 - 0{,}9 z^{-1}} = \frac{0{,}1}{z^{-1}(z - 0{,}9)} = \frac{z \cdot 0{,}1}{z - 0{,}9}$$

nollställe: $z = 0$

polställe: $z = 0{,}9$



$\boxed{\text{GCF: fil IIR-1}}$

$$\cancel{H(z) = \frac{\sum_{k=0}^{M} b_k x[n-k]}{1 - \sum_{l=0}^{N} a_l y[n-l]} = \frac{b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]}{1 - a_1 y[n-1] + a_2 y[n-2] + \dots + a_N y[n-N]} =}$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{l=0}^{N} a_l z^{-l}} = \frac{b_0 z^0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 - a_1 z^{-1} - a_2 z^{-2} + \dots + a_N z^{-N}} =$$

$$= \frac{b_0 z^{-M} \left(z^{M} + \frac{b_1}{b_0} z^{M-1} + \frac{b_2}{b_0} z^{M-2} + \dots + \frac{b_M}{b_0} z^0\right)}{z^{-N} \left(z^N - a_1 z^{N-1} - a_2 z^{N-2} - \dots - a_N z^0\right)} =$$

$$= b_0 \, z^{N-M} \frac{(z-z_1)(z-z_2)\dots(z-z_M)}{(z-p_1)(z-p_2)\dots(z-p_N)}$$

M st nollställen

N st poler

$z_{n_1}$

$z_{p_1}$

om $N > M$ nollställen i origo

om $M > N$ poler i origo

Både poler och nollställen utanför den reella axlen måste vara komplex-konjugerade.

IIR-filter kan

# Föreläsning 9

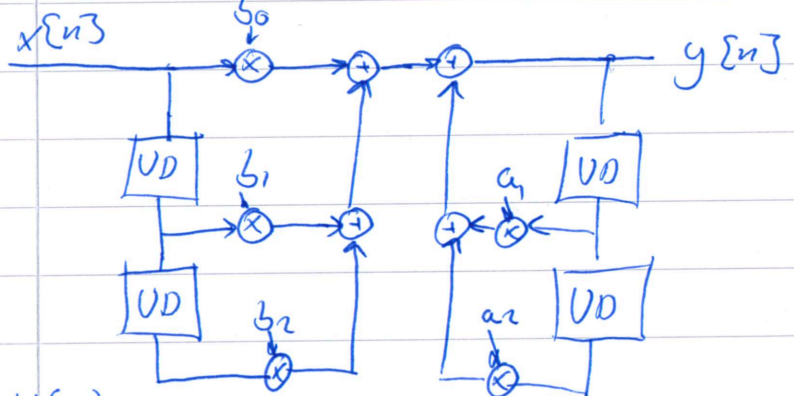
Rep: $\bar{x}[n] \rightarrow \boxed{H_1(e^{j\bar{\omega}})} \xrightarrow{\bar{\alpha}[n]} \boxed{H_2(e^{j\bar{\omega}})} \rightarrow \bar{y}[n]$

$$\bar{y}[n] = H_1(e^{j\bar{\omega}}) \cdot H_2(e^{j\bar{\omega}})\, \bar{x}[n] =$$

$$= H_2(e^{j\bar{\omega}}) \cdot H_1(e^{j\bar{\omega}})\, \bar{x}[n] \Rightarrow$$

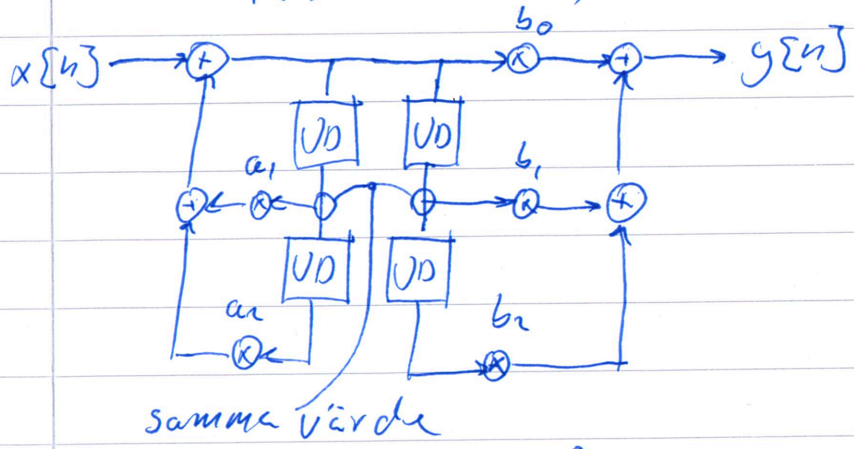
$$\Rightarrow H(z) = H_1(z)\, H_2(z) = H_2(z)\, H_1(z)$$

Systemet måste vara stabilt!

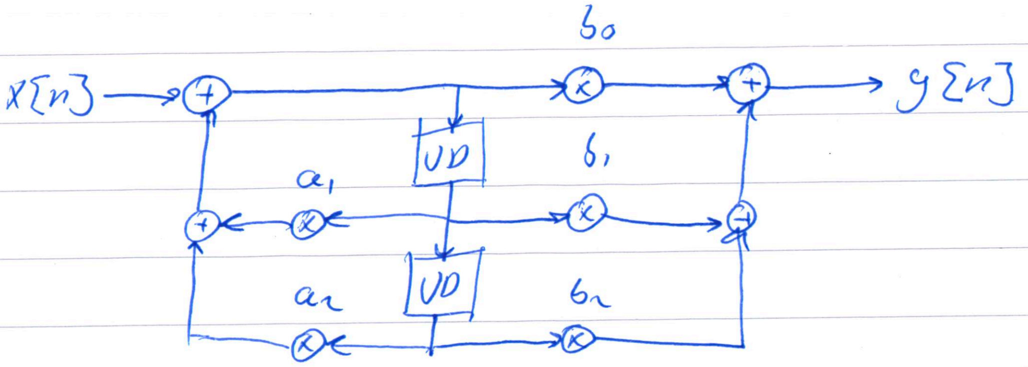



Direktform I

$H(z)$ $H_1(z)$ $H_2(z)$




$H(z)$ $H_2(z)$ $H_1(z)$



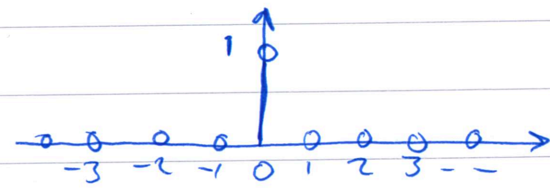

## Direktform II

Slå på glas ⟶ Impuls

Svårt att beskriva i det analoga, i frekvensplanet är det lättare.

Enhets impuls (Kroneckers deltafunktion)

$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$



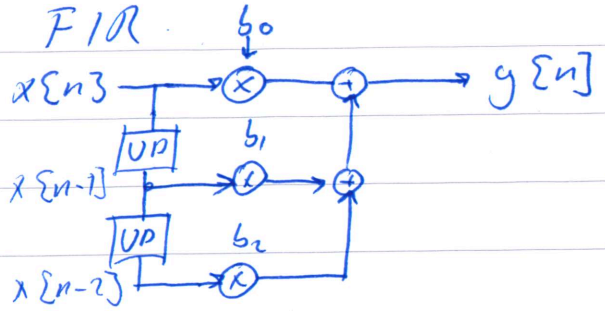

$x[n]=\delta[n] \longrightarrow$ LTI $\longrightarrow y[n]=h[n] \leftarrow$ impulssvar

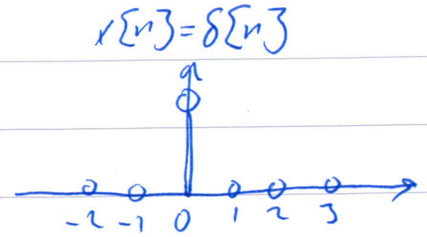
L - Linjär

T - Tids

I - Invariant

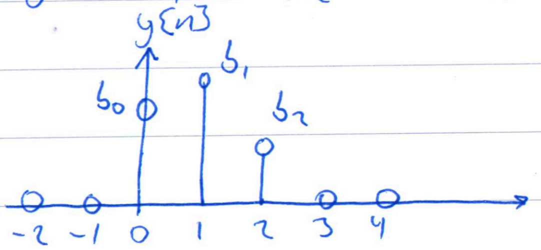
ex) FIR



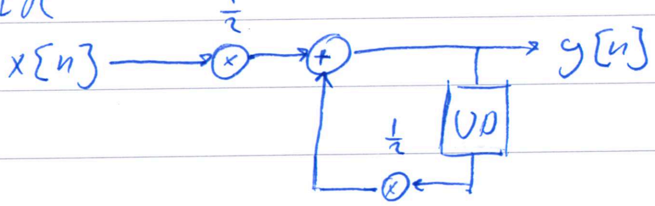

$$y\{n\} = b_0\, x[n] + b_1\, x\{n-1\} + b_2\, x\{n-2\}$$

$$y\{0\} = b_0 \cdot 1 + b_1 \cdot 0 + b_2 \cdot 0 = b_0$$
$$y\{1\} = b_0 \cdot 0 + b_1 \cdot 1 + b_2 \cdot 0 = b_1$$
$$y\{2\} = b_0 \cdot 0 + b_1 \cdot 0 + b_2 \cdot 1 = b_2$$

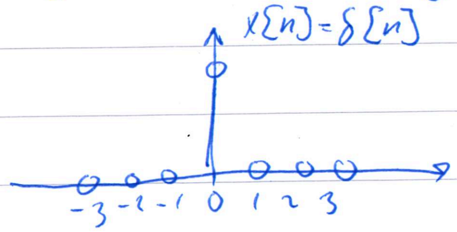



F - Finit
I - Impulse
R - Response

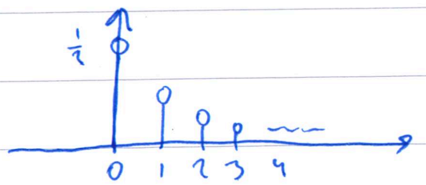
ex IIR




$$y\{n\} = \frac{1}{2}\, x\{n\} + \frac{1}{2}\, y\{n-1\}$$



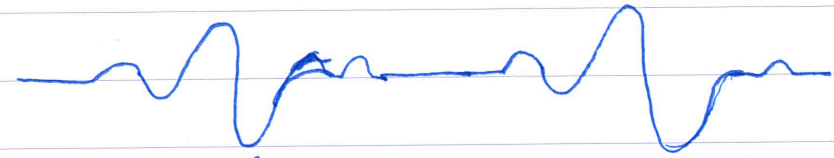

$$y\{0\} = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2}$$
$$y\{1\} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$
$$y\{2\} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$
$$\vdots$$




I - Infinite
I - Impulse
R - Response

# Linjärfas

ex. EKG - små spänningar



Kurvansform är vihtig vid analys.

Då man studerar kurvan så får den inte förändras. T.ex för att filtrera bort 50 Hz

Vid EKG sitter man elektroder på benen då kroppen fungerar som en antenn.

periodisk signal: $f_0$ - grundfrekvensen

$$x(t) = \sum_{k=1}^{\infty} A_k \cos(2\pi f_0 \cdot k + \phi_k)$$

Deltoner

Deltonerna får inte fasföskjutas!
Vi behöver göra ett filter där alla deltoner fördröjs lika lång tid, detta gör att kurvformen inte att förändras.

$$x[n] \longrightarrow \boxed{H(e^{j\hat{\omega}})} \longrightarrow y[n]$$

$$x[n] = A\cos(\hat{\omega} \cdot n)$$

$$y[n] = |H(e^{j\hat{\omega}})|\, A \cdot \cos(\hat{\omega} \cdot n + \arg(H e^{j\hat{\omega}}))$$

[Om vi gör $\arg(H(e^{j\hat{\omega}}))$ konstant, räcker det?
Nej, då det är olika mycket av en period beroende på frekvensen.]

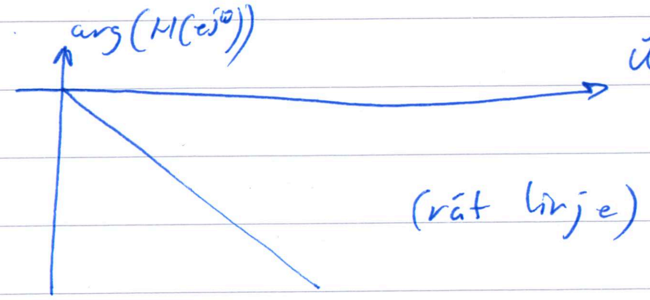
$$\cancel{|H| A \cos\left(\hat{\omega}\left(n + \frac{\arg(H(e^{j\hat{\omega}}))}{\hat{\omega}}\right)\right)}$$

$$\cos(\hat{\omega} \cdot n + \arg(H(e^{j\hat{\omega}}))) = \cos\Big(\hat{\omega}\big(\underset{\text{tid}}{n} + \underbrace{\frac{\arg(H(e^{j\hat{\omega}}))}{\hat{\omega}}}_{\text{tidsförskjutning}}\big)\Big)$$

Man vill att alla frekvenser ska ha samma tidsfördröjning/förskjutning.
Vilket ger:

$$\frac{\arg(H(e^{j\hat{\omega}}))}{\hat{\omega}} = k \text{ (är konstant)}$$

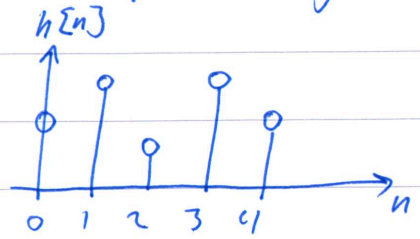
$$\arg(H(e^{j\hat{\omega}})) = k \cdot \hat{\omega}$$




negativt värde svarar mot tidsfördröjning

Vi har gått från att ha samma förskjutning i ~~tid~~ grader till att ha samma förskjutning i tid

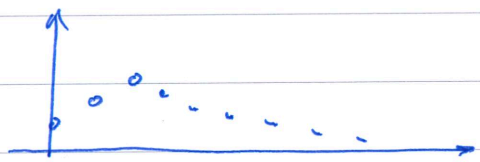
Man kan visa [se boken] att om $h[n]$, impulssvaret, är symetriskt kring något n-värde fås linjär fas.




FIR

symetri kring $n=2$

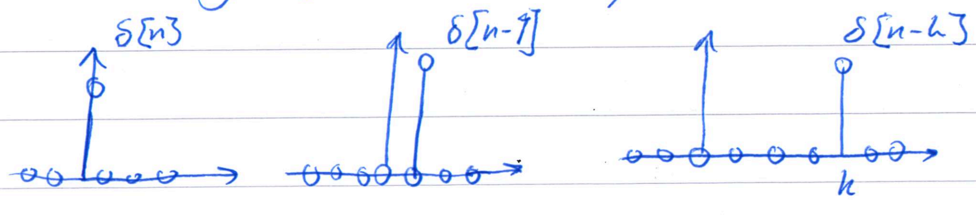
IIR - går inte att göra symetriskt.
Men det går nästan

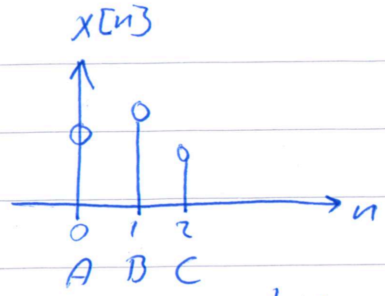


Alla design metoder för FIR-filter ger symetriskt $h[n]$ och där med linjärsvar.

---

Faltning (convolution)






Delar upp x[n] i tre delar A, B och C

Insignal — Bidrag till utsignal

(A) $\underbrace{x[0] = x[0]\cdot\delta[n]}^{\text{Insignal}} \Rightarrow x[0]\cdot h[n]$

(B) $x[1] = x[1]\cdot\delta[n-1] \Rightarrow x[1]\cdot h[n-1]$

(C) $x[2] = x[2]\cdot\delta[n-2] \Rightarrow x[2]\cdot h[n-2]$

$$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

$$y[n] = x[0]\cdot h[n] + x[1]\cdot h[n-1] + x[2]\cdot h[n-2] =$$

$$= \sum_{l=0}^{2} x[l]\cdot h[n-l]$$

$l \to \infty \Rightarrow k \to -\infty$
$l \to -\infty \Rightarrow k \to \infty$

Allmänna fallet.

$$y[n] = \sum_{l=-\infty}^{\infty} x[l]\cdot h[n-l] = \begin{bmatrix} \text{subs} \\ n-l=k \\ l=n-k \end{bmatrix} =$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = h * x$$

"h faltat med x"

I realtidssystem känner vi nuvarande och tidigare värden $x[n], x[n-1], x[n-2]$ ...

Vi känner inte till "framtiden" dvs. systemet är kausalt.

$$y[n] = \sum_{k=0}^{\infty} h[k]\, x[n-k]$$

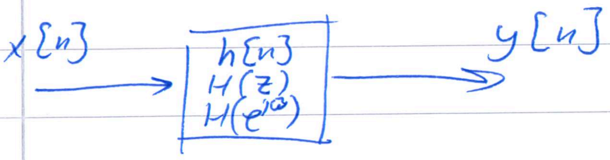
# Föreläsning 10

## Kausala system

Orsak → Verkan    realtidssystem

$$y[n] = \sum_{k=0}^{\infty} b_k \, x[n-k]$$

Titta på sambandet




Sinusform

$$x[n] = A\cos(\hat{\omega} n)$$

$$\bar{x}[n] = A\left(\cos(\hat{\omega} n) + j\sin(\hat{\omega} n)\right) = A e^{j\hat{\omega} n}$$

$$\bar{y}[n] = H(e^{j\hat{\omega}}) \cdot \bar{x}[n]$$

Generella fallet

$$\bar{y}[n] = \sum_{k=-\infty}^{\infty} h[k]\,\bar{x}[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, A \cdot e^{j\hat{\omega}(n-k)} =$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\hat{\omega} k} \cdot \underbrace{A e^{j\hat{\omega} n}}_{\text{inget } k} = \underbrace{A \cdot e^{j\hat{\omega} n}}_{x[n]} \underbrace{\sum_{k=-\infty}^{\infty} h[k]\, e^{-j\hat{\omega} k}}_{H(e^{j\hat{\omega}})}$$

$$\Rightarrow H(e^{j\hat{\omega}}) = \sum_{k=-\infty}^{\infty} h[k] (e^{j\hat{\omega}})^{-k}$$

enhetscirkeln

Utvidgar till hela komplexa talplanet.

$$\Rightarrow H(z) = \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}$$

Def: Z-transform (är ett hjälpmedel)

$$Z\{x[n]\} = X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

[jämför med $H(z)$]

dvs. $H(z)$ är $Z\{h[n]\}$

Man kan visa (se boken s. 213) att

$$y = h * x \iff Y(z) = H(z) \cdot X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)}$$

Finns färdiga tabeller!

Egenskap

$$x[n] = X(z)$$
$$x[n-1] = X(z) z^{-1}$$
$$x[n-2] = X(z) z^{-2}$$
$$x[n-k] = X(z) z^{-k}$$

Exempel: FIR-filter

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$Z$

$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) =$$
$$= X(z)(b_0 + b_1 z^{-1} + b_2 z^{-2})$$

$$\frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} = H(z)$$

jämför med $H(z) = \sum_{k=-\infty}^{\infty} b_k z^{-k}$

IIR-filter

$$y[n] = 0{,}1\, x[n] + 0{,}9\, y[n-1]$$

$\mathcal{Z}$

$$Y(z) = 0{,}1\, X(z) + 0{,}9\, z^{-1} Y(z)$$

$$Y(z) - 0{,}9 z^{-1} Y(z) = 0{,}1\, X(z)$$

$$Y(z)\left(1 - 0{,}9 z^{-1}\right) = 0{,}9\, X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0{,}9}{1 - 0{,}9 z^{-1}}$$

---

$$Y(z) = H(z) \cdot X(z)$$

$$Y(e^{j\hat{\omega}}) = H(e^{j\hat{\omega}}) \cdot X(e^{j\hat{\omega}})$$

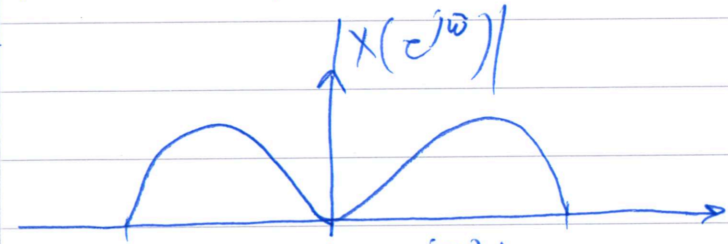
$$|Y(e^{j\hat{\omega}})| = |H(e^{j\hat{\omega}})|\, |X(e^{j\hat{\omega}})|$$

specialfall
$z = e^{j\hat{\omega}}$
vi är på enhetscirkeln

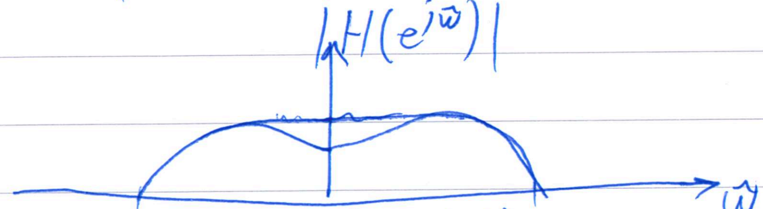
H och X är
frekvens representation
av $h[n]$ och $x[n]$

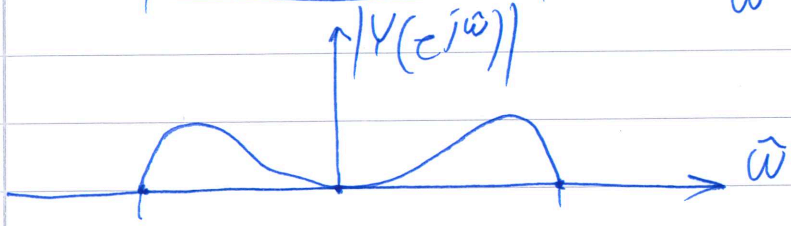
Visa

②




①



$$Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad (\text{låt } z = e^{j\hat{\omega}}$$

$$DTFT\{x[n]\} = X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{\infty} x[n] (e^{j\hat{\omega}})^{-n}$$

D- Diskret
T- Tid
F- Fourier
T- Transform

Datorberäkningar kräver begränsningar. Om man nöjer sig med ändliga värden dvs räknar på en del av signalen x[n] får man en DFT (Diskret Fourier Transform). En snabbare variant är FFT