

Potens: består av en bas a och en exponent n :  $a^n$

↑  
bas

$$a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n \text{ sta}} \quad a \in \mathbb{R}, n \in \mathbb{N}$$

Ex)  $2^3 = 2 \cdot 2 \cdot 2 = 8$        $\pi^4 = \pi \cdot \pi \cdot \pi \cdot \pi$        $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$

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### VIKTIG SKILLNAD

Ex)  $(-2)^2 = (-2) \cdot (-2) = 2 \cdot 2 = 4$  basen är  $(-2)$

$-2^2 = -\underbrace{2 \cdot 2}_4 = -4$  basen är  $2$

$(-a)^n$     n=jämnt tal, ger positivt svar  
n=udda tal, ger negativt svar

Ex)  $(-2)^3 = (-2)(-2)(-2) = -2^3 = -8$  udda exponent

$(-2)^4 = (-2)(-2)(-2)(-2) = 2^4 = 16$  jämn exponent

Ex)  $(-27)^{18} = 27^{18}$  jämn exponent

$(-27)^{19} = -27^{19}$  udda exponent

# Potenslagarna

$$a^x \cdot a^y = a^{x+y} \quad a, b > 0, x \text{ och } y \in \mathbb{R}$$

$$\frac{1}{a^x} = a^{-x} \quad \left( \frac{1}{a^{-x}} = a^x \right)$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$a^0 = 1 \quad a \in \mathbb{R}, a \neq 0 \quad 0^\circ \text{ ej def.}$$

$$(ab)^x = a^x \cdot b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$$

$$\left(\frac{1}{a}\right)^n = \frac{1}{a^n}$$

$$\left(\frac{1}{a}\right)^n = \frac{1^n}{a^n} = \frac{1}{a^n}$$

## Exempel

$$3^5 \cdot 3^4 = 3^{5+4} = 3^9$$

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}, \quad \frac{1}{2^{-1}} = 2^1 = 2$$

$$\frac{2^{50}}{2^{48}} = 2^{50-48} = 2^2 = 4$$

$$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$$

$$4^0 = 1, \quad (-4)^0 = 1, \quad -\underbrace{4^0}_{1} = -1$$

$$2^2 \cdot 6^2 = (2 \cdot 6)^2 = 12^2 = 144$$

$$(5x)^3 = 5^3 \cdot x^3 = 125x^3$$

$$\frac{10^5}{5^5} = \left(\frac{10}{5}\right)^5 = 2^5 = 32$$

$$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

$$\left(\frac{1}{2}\right)^8 = \frac{1}{2^8} = \frac{1}{256}$$

$$\frac{1}{1024} = \frac{1}{2^{10}} = \left(\frac{1}{2}\right)^{10}$$

$$\sqrt{a} = a^{\frac{1}{2}} \quad a > 0$$

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad n > 0, \quad a > 0 \quad \text{om } n \text{ är jämn}$$

$$\sqrt{256} = \sqrt{2^8} = (2^8)^{\frac{1}{2}} = 2^{\frac{8}{2}} = 2^4 = 16$$

$$\sqrt[5]{32} = \sqrt[5]{2^5} = (2^5)^{\frac{1}{5}} = 2^{\frac{5}{5}} = 2^1 = 2$$

$$10^{-1} = \frac{1}{10} = 0,1$$

↑  
Potensstal      ↑  
bråktal      ↑ decimaltal

## Grundpotensform

$$\text{Grundpotensform: } a \cdot 10^n \quad 1 \leq a < 10 \text{ och } n \in \mathbb{Z}$$

Ex) Skriv 2300 och 0,00023 på grundpotensform

$$2300 = 2,3 \cdot 10^3 \quad \text{decimalkommat flyttas 3 steg åt väster}$$

$$0,00023 = 2,3 \cdot 10^{-4} \quad \text{decimalkommat flyttas 4 steg åt höger}$$

6008d) Beräkna  $\frac{2 \cdot 10^4}{8 \cdot 10^{-5}}$  och svara i grundpotensform

$$\frac{2 \cdot 10^4}{8 \cdot 10^{-5}} = \frac{2}{8} \cdot 10^{4-(-5)} = \frac{1}{4} \cdot 10^9 = 0,25 \cdot 10^9 \left( = 2,5 \cdot 10^{-1} \cdot 10^9 = 2,5 \cdot 10^{1+9} \right) = 2,5 \cdot 10^8$$

Ex)  $f(x) = x^2$  bestäm  $f(-2)$

$$f(-2) = (-2)^2 = 4$$

$$f(-2) \neq -2^2 \neq 4$$

$$f(x) = -x^2 \quad f(2) = -2^2 = -4$$

Ex) Beräkna  $(-2)^{-4}$  och  $-2^{-4}$

$$\left. \begin{array}{l} (-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16} \\ -2^{-4} = -\frac{1}{2^4} = -\frac{1}{16} \end{array} \right\} \text{Viktig skillnad}$$

Ex) Skriv  $\frac{1}{81}$  i potensform med basen 3

$$\frac{1}{81} = \frac{1}{3^4} = \underline{3^{-4}}$$

$a^n$

$$\begin{array}{c} 81 \\ \diagdown \quad \diagup \\ 9 \cdot 9 \\ \diagdown \quad \diagup \\ 3 \cdot 3 \cdot 3 \cdot 3 \end{array} = 3^4$$

Ex) Beräkna  $(0,2)^4$ , svara på bråkform

$$(0,2)^4 = \left(\frac{2}{10}\right)^4 = \left(\frac{1}{5}\right)^4 = \frac{1}{5^4} = \underline{\frac{1}{625}}$$

Ex) Beräkna  $\left(\frac{2}{7}\right)^{-2}$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{2}\right)^2 = \frac{7^2}{2^2} = \underline{\frac{49}{4}}$$

Ex) Beräkna  $\frac{2^{-2}}{2^2}$  och ge svaret i både bråkform och potensform

Bråkform:  $\frac{2^{-2}}{2^2} = \frac{1}{2^2 \cdot 2^2} = \frac{1}{4 \cdot 4} = \underline{\frac{1}{16}}$

Potensform:  $\frac{2^{-2}}{2^2} = 2^{-2} \cdot 2^{-2} = \underline{2^{-2-2}} = \underline{2^{-4}}$

Ex) Beräkna  $(2^3)^2$  och  $2^{3^2}$

$$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = \underline{64}$$

$$2^{3^2} = 2^9 = \underline{512}$$

$$(2^3)^2 \neq 2^{3^2}$$

Ex) Beräkna  $(2 \cdot 10^{-4})^3$

$$(a \cdot b)^n = a^n \cdot b^n$$

$$(2 \cdot 10^{-4})^3 = 2^3 \cdot (10^{-4})^3 = 8 \cdot 10^{-4 \cdot 3} = \underline{8 \cdot 10^{-12}}$$

6011d) Skriv  $\frac{(2^{n+2})^3}{8^{n+4}}$  på enklaste form

$$\frac{(2^{n+2})^3}{8^{n+4}} = \frac{(2^{n+2})^3}{(2^3)^{n+4}} = \frac{2^{3(n+2)}}{2^{3(n+4)}} = \frac{2^{3n+6}}{2^{3n+12}} = 2^{3n+6-(3n+12)} = 2^{3n+6-3n-12} = \underline{2^{-6}}$$

$$\text{alt. } 2^{-6} = \frac{1}{2^6} = \underline{\frac{1}{64}}$$

[Paus]

Ex)  $f(x) = x^2$  Bestäm  $f(a^4)$

$$f(x) = x^2 \quad f(a^4) = (a^4)^2 = a^{4 \cdot 2} = a^8 \quad f(a^4) \neq a^{4^2} \neq a^8$$

6013d) Förenkla  $\left(\frac{a^2}{27}\right)^{\frac{1}{3}} \cdot \left(\frac{64}{a}\right)^{\frac{2}{3}}$

$$\left(\frac{a^2}{27}\right)^{\frac{1}{3}} \cdot \left(\frac{64}{a}\right)^{\frac{2}{3}} = \left(\frac{a^2}{3^3}\right)^{\frac{1}{3}} \cdot \left(\frac{4^3}{a}\right)^{\frac{2}{3}} = \frac{(a^2)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}} \cdot \frac{(4^3)^{\frac{2}{3}}}{a^{\frac{2}{3}}} = \frac{a^{2 \cdot \frac{1}{3}}}{3^{3 \cdot \frac{1}{3}}} \cdot \frac{4^{3 \cdot \frac{2}{3}}}{a^{\frac{2}{3}}} = \frac{\cancel{a^{\frac{2}{3}}} \cdot \cancel{4^2}}{3 \cdot \cancel{a^{\frac{2}{3}}}} = \underline{\frac{16}{3}}$$

$$\sqrt{a} = a^{\frac{1}{2}}$$

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad n > 0, a > 0 \text{ om } n \text{ är jämn}$$

Ex) Beräkna a)  $\sqrt[3]{8}$  b)  $\sqrt[4]{625}$  c)  $\sqrt[3]{-8}$  d)  $\sqrt[4]{-16}$

a)  $\sqrt[3]{8} = 8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{\frac{3 \cdot \frac{1}{3}}{3}} = 2^1 = \underline{2}$

b)  $\sqrt[4]{625} = 625^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\frac{4 \cdot \frac{1}{4}}{4}} = 5^1 = \underline{5}$

c)  $\sqrt[3]{-8} = (-8)^{\frac{1}{3}} = (-2^3)^{\frac{1}{3}} = (-2)^{\frac{3 \cdot \frac{1}{3}}{3}} = (-2)^1 = \underline{-2}$   $(-2)^3 = -8$   
udda n

d)  $\sqrt[4]{-16}$  saknar lsg  
jämnt n

Tänk på att skriva tydligt! Det är stor skillnad på:

$$\sqrt[4]{16} \text{ och } 4\sqrt{16}$$

$$\sqrt[4]{16} = (2^4)^{\frac{1}{4}} = 2 \quad 4\sqrt{16} = 4 \cdot \sqrt{16} = 4 \cdot 4 = 16$$

2.23d) Analys Förenkla  $\frac{\sqrt[4]{a^3 \cdot \sqrt{a}}}{\sqrt[8]{1/a}}$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$\frac{\sqrt[4]{a^3 \cdot \sqrt{a}}}{\sqrt[8]{1/a}} = \frac{\left(a^3 \cdot a^{\frac{1}{2}}\right)^{\frac{1}{4}}}{\left(\frac{1}{a}\right)^{\frac{1}{8}}} = \frac{\left(a^{3+\frac{1}{2}}\right)^{\frac{1}{4}}}{\left(a^{-1}\right)^{\frac{1}{8}}} = \frac{\left(a^{\frac{7}{2}}\right)^{\frac{1}{4}}}{a^{-\frac{1}{8}}} = \frac{a^{\frac{7}{2} \cdot \frac{1}{4}}}{a^{-\frac{1}{8}}} = \frac{a^{\frac{7}{8}}}{a^{-\frac{1}{8}}} = a^{\frac{7}{8} - (-\frac{1}{8})} = a^{\frac{7}{8} + \frac{1}{8}} = a^{\frac{8}{8}} = a^1 = \underline{a}$$

- Börja "innifrån"
- Små steg

Tips:

$$8^3 = (2^3)^3 = 2^{3 \cdot 3} = 2^9 = \underline{512}$$

Beräkna och svara som tal.

$$\frac{8^2}{4^6} = \frac{(2^3)^2}{(2^2)^6} = \frac{2^{3 \cdot 2}}{2^{2 \cdot 6}} = \frac{2^6}{2^{12}} = \frac{1}{2^{12-6}} = \frac{1}{2^6} = \underline{\frac{1}{64}}$$

