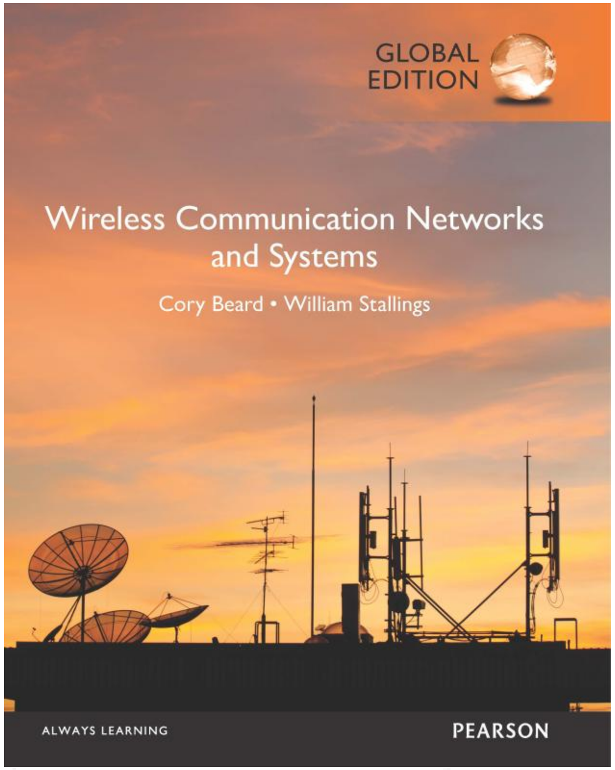
MOBILE SYSTEM-HT25
LECTURE 7: TRANSMISSION FUNDAMENTALS (CTFT, DTFT, DFT/FFT)

Azra Abtahi

Email: azra.abtahi-fahliani@mau.se

Faculty of Technology and Society Department of Computer Science and Media Technology Malmö University

Most slides are primarily adapted from Beard & Stallings (2016), Wireless Communication Networks and Systems (Chapter 2)



Wireless Communication Networks and Systems

1st edition, Global edition

Cory Beard, William Stallings

OUTLINE

- Review on Frequency-Domain Concepts

- Transforms:
  - CTFT: Continuous Time ↔ Continuous Frequency

$$s(t) \xleftrightarrow{\text{CTFT}} S(f)$$

  - DTFT: Discrete Time ↔ Continuous Frequency

$$\mathrm{s}[n] = s(nT_s) \xleftrightarrow{\text{DTFT}} S\left(e^{j2\pi f}\right)$$

  - DFT/FFT: Discrete Time ↔ Discrete Frequency

$$s[n] \xleftrightarrow{\text{DFT}} S[k]$$

Next Lecture

FREQUENCY-DOMAIN CONCEPTS

- Spectrum –

Concept	What It Describes	Typical Notation	Meaning
Signal Spectrum	What frequencies are present in the signal	$\boxed{S(f)=F\{s(t)\}}$	The representation of a signal in the frequency domain, showing how its power or amplitude is distributed across different frequencies.
Channel Spectrum	How the channel affects each frequency	$\boxed{H(f)=F\{h(t)\}}$	The **frequency response** — describes gain and phase shift vs. frequency

Impulse Response

CONTINUOUS TIME FOURIER TRANSFORM (CTFT):

Continuous Time $\leftrightarrow$ Continuous Frequency

$$s(t) \xleftrightarrow{\text{CTFT}} S(f)$$

$$S(f) = \int_{-\infty}^{\infty} s\,(t)\, e^{-j2\pi f t}\, dt$$

$$s(t) = \int_{-\infty}^{\infty} S\,(f)\, e^{j2\pi f t}\, df$$

CTFT

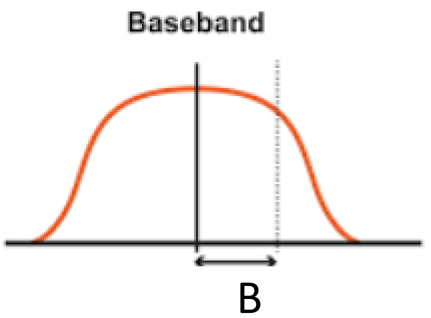
Property	Mathematical Form	Meaning / Use
Linearity	$a_1 s_1(t) + a_2 s_2(t) \leftrightarrow a_1 S_1(f) + a_2 S_2(f)$	Superposition of signals
Time Shift	$s(t - t_0) \leftrightarrow e^{-j2\pi f t_0} S(f)$	Delay introduces a phase shift
Frequency Shift	$e^{j2\pi f_0 t} s(t) \leftrightarrow S(f - f_0)$	Modulation or carrier frequency translation
Convolution	$s_1(t) * s_2(t) \leftrightarrow S_1(f) S_2(f)$	Filtering – channel response in frequency domain
Multiplication	$s_1(t) s_2(t) \leftrightarrow S_1(f) * S_2(f)$	Amplitude modulation / windowing effect

$$(S_1 * S_2)(t) = \int_{-\infty}^{\infty} S_1(\tau)\, S_2(t - \tau)\, d\tau$$

DISCUSSION TIME I

If the spectrum of $x(t)$ is $X(f)$,what is the spectrum of the modulated signal $x(t)\cos(2\pi f_0 t)$? Sketch it for the following example.

Hint: $\cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$



Property	Mathematical Form	Meaning / Use
Linearity	$a_1 s_1(t) + a_2 s_2(t) \leftrightarrow a_1 S_1(f) + a_2 S_2(f)$	Superposition of signals
Time Shift	$s(t - t_0) \leftrightarrow e^{-j2\pi f t_0} S(f)$	Delay introduces a phase shift
Frequency Shift	$e^{j2\pi f_0 t} s(t) \leftrightarrow S(f - f_0)$	Modulation or carrier frequency translation
Convolution	$s_1(t) * s_2(t) \leftrightarrow S_1(f) S_2(f)$	Filtering – channel response in frequency domain
Multiplication	$s_1(t) s_2(t) \leftrightarrow S_1(f) * S_2(f)$	Amplitude modulation / windowing effect

CLASS DISCUSSION I

$$y(t) = x(t)\cos(2\pi f_0 t) \;\; and \;\; \cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

$$y(t) = x(t)\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} = \frac{1}{2}\, x(t) e^{j2\pi f_0 t} + \frac{1}{2} x(t) e^{-j2\pi f_0 t}$$

Property	Mathematical Form	Meaning / Use
Linearity	$a_1 s_1(t) + a_2 s_2(t) \leftrightarrow a_1 S_1(f) + a_2 S_2(f)$	Superposition of signals
Time Shift	$s(t - t_0) \leftrightarrow e^{-j2\pi f t_0} S(f)$	Delay introduces a phase shift
Frequency Shift	$e^{j2\pi f_0 t} s(t) \leftrightarrow S(f - f_0)$	Modulation or carrier frequency translation
Convolution	$s_1(t) * s_2(t) \leftrightarrow S_1(f) S_2(f)$	Filtering – channel response in frequency domain
Multiplication	$s_1(t) s_2(t) \leftrightarrow S_1(f) * S_2(f)$	Amplitude modulation / windowing effect

CLASS DISCUSSION I

$$y(t) = x(t)\cos(2\pi f_0 t) \;\; and \;\; \cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

$$y(t) = x(t)\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} = \frac{1}{2}\, x(t)e^{j2\pi f_0 t} + \frac{1}{2} x(t)e^{-j2\pi f_0 t}$$

$$Y(f) = \frac{1}{2} CTFT\{\, x(t)e^{j2\pi f_0 t}\} + \frac{1}{2} CTFT\{\, x(t)e^{-j2\pi f_0 t}\}$$

Property	Mathematical Form	Meaning / Use
Linearity	$a_1 s_1(t) + a_2 s_2(t) \leftrightarrow a_1 S_1(f) + a_2 S_2(f)$	Superposition of signals
Time Shift	$s(t - t_0) \leftrightarrow e^{-j2\pi f t_0} S(f)$	Delay introduces a phase shift
Frequency Shift	$e^{j2\pi f_0 t} s(t) \leftrightarrow S(f - f_0)$	Modulation or carrier frequency translation
Convolution	$s_1(t) * s_2(t) \leftrightarrow S_1(f) S_2(f)$	Filtering – channel response in frequency domain
Multiplication	$s_1(t) s_2(t) \leftrightarrow S_1(f) * S_2(f)$	Amplitude modulation / windowing effect

CLASS DISCUSSION I

$$\mathrm{y}(t) = x(t)\cos(2\pi f_0 t) \;\; and \;\; \cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

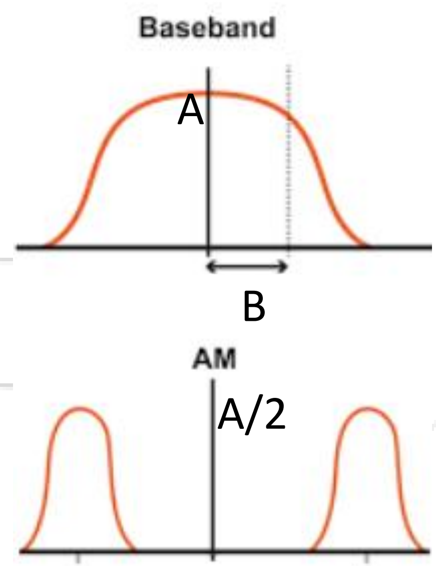
$$\mathrm{y}(t) = x(t)\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} = \frac{1}{2}\,x(t)e^{j2\pi f_0 t} + \frac{1}{2}x(t)e^{-j2\pi f_0 t}$$

$$Y(f) = \frac{1}{2}CTFT\{\,x(t)e^{j2\pi f_0 t}\} + \frac{1}{2}CTFT\{\,x(t)e^{-j2\pi f_0 t}\}$$

$$=\frac{1}{2}X(f + f_0) + \frac{1}{2}X(f - f_0)$$

$X(f) \rightarrow$

$Y(f) \rightarrow$



$-f_0$ f_0

CTFT

1. **Zero everywhere except at $t = 0$**

$$\delta(t) = 0 \quad \text{for } t \neq 0$$

2. **Infinite at $t = 0$**

At the exact instant $t = 0$, the impulse is considered **infinitely large**, but in such a way that the *area* under the spike equals 1.

3. **Unit area**

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Time-Domain Signal $s(t)$	Frequency-Domain Spectrum $S(f)$	Notes / Relevance
$\delta(t)$	1	Impulse → flat spectrum
1	$\delta(f)$	Constant signal → DC
$e^{j2\pi f_0 t}$	$\delta(f - f_0)$	Complex exponential ↔ single tone
$\cos(2\pi f_0 t)$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$	Real sinusoid ↔ symmetric lines
$\mathrm{rect}(t/T)$	$T\,\mathrm{sinc}(fT)$	Time-limited pulse ↔ frequency spread (Nyquist pulse, OFDM subcarrier)
$\mathrm{sinc}(t/T)$	$T\,\mathrm{rect}(fT)$	Band-limited signal ↔ infinite time support
$s(t)\cos(2\pi f_c t)$	$\frac{1}{2}[S(f - f_c) + S(f + f_c)]$	Modulation → frequency shift

Time localization ↔ frequency spreading
Frequency localization ↔ time spreading.
So, we have bandwidth vs. symbol duration trade-off

DISCRETE-TIME FOURIER TRANSFORM (DTFT)

Discrete Time ↔ Continuous Frequency

$$s[n] = s(nT_s) \leftrightarrow S\left(e^{j\omega}\right), \omega = 2\pi f$$

$$S\left(e^{j2\pi f}\right)$$

$$S\left(e^{j2\pi f}\right) = \sum_{n=-\infty}^{\infty} s\,[n]\, e^{-j2\pi f n}$$

$$\mathrm{s}[n] = \int_{-0.5}^{0.5} S\left(e^{j2\pi f}\right) e^{j2\pi f n}\, df$$

DTFT is periodic with the period of 1

DTFT

Property	Mathematical Form (using f)	Meaning / Use
Linearity	$a_1 x_1[n] + a_2 x_2[n] \;\leftrightarrow\; a_1 X_1(e^{j2\pi f}) + a_2 X_2(e^{j2\pi f})$	Superposition of signals
Time Shift	$x[n - n_0] \;\leftrightarrow\; e^{-j2\pi f n_0} X(e^{j2\pi f})$	Delay introduces phase shift
Frequency Shift	$e^{j2\pi f_0 n} x[n] \;\leftrightarrow\; X(e^{j2\pi (f - f_0)})$	Modulation / carrier shift
Convolution	$x_1[n] * x_2[n] \;\leftrightarrow\; X_1(e^{j2\pi f})\, X_2(e^{j2\pi f})$	Filtering in frequency domain
Multiplication	$x_1[n] x_2[n] \;\leftrightarrow\; (X_1 * X_2)(f)$ where convolution is over periodic f	Windowing / amplitude modulation
Periodicity	$X(e^{j2\pi (f+1)}) = X(e^{j2\pi f})$	DTFT repeats every 1 in normalized frequency

IMPULSE RESPONSE

- $h(t)$: describes how the channel reacts to an impulse (**Impulse Response**)

$$s(t)=\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t)$$

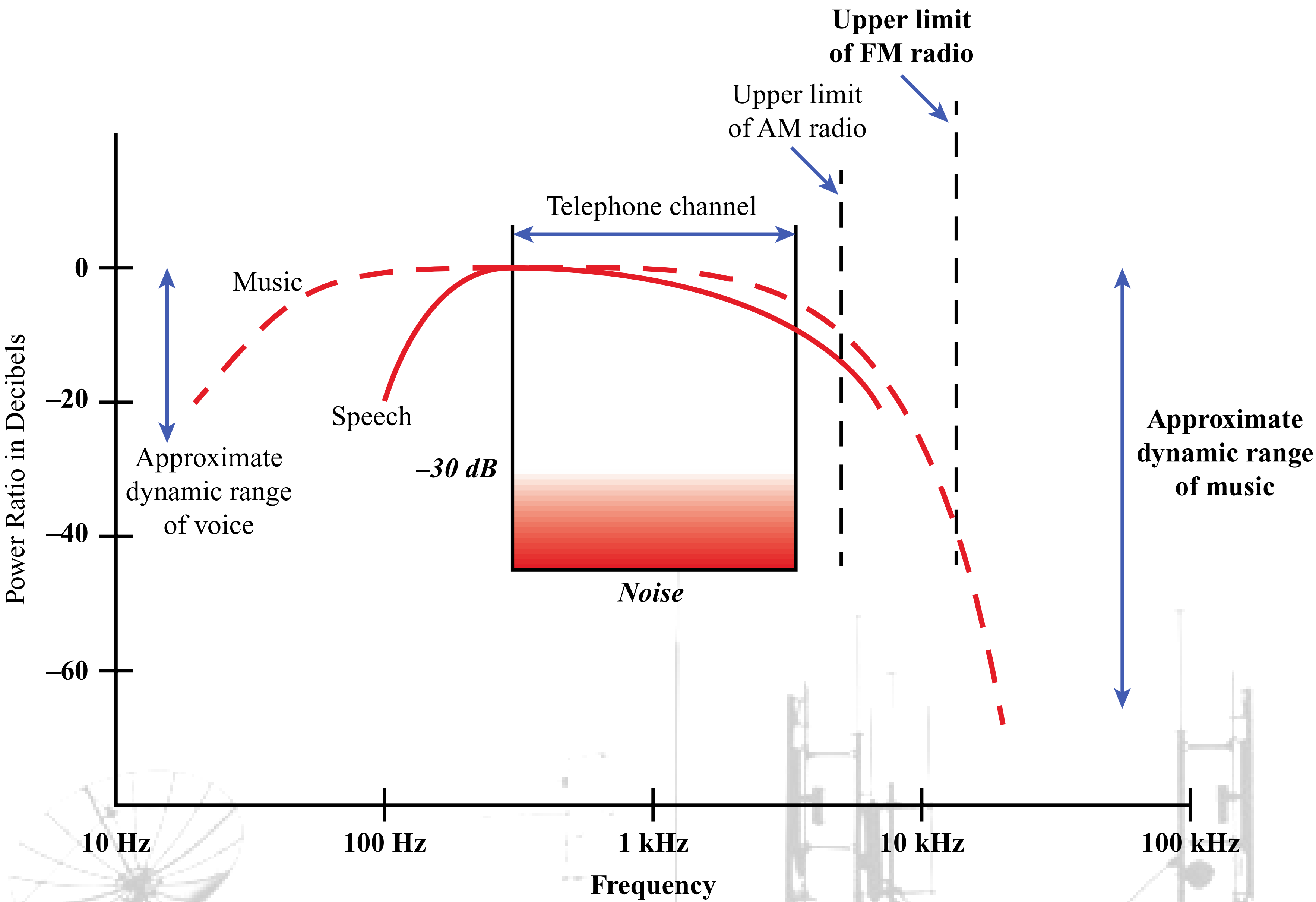
$$y(t) = s(t) * h(t)$$

FREQUENCY RESPONSE

- $H(f)$=CTFT$\{h(t)\}$:**Frequency response** shows how the channel modifies each frequency

$$y(t) = s(t) * h(t) \Leftrightarrow Y(f) = S(f)\,H(f)$$

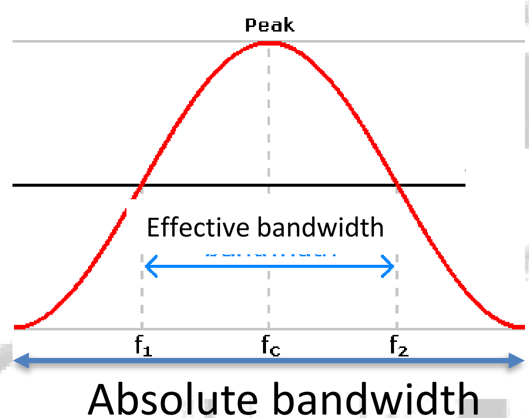
Convolution in time = multiplication in frequency



2.6 ACOUSTIC SPECTRUM OF SPEECH AND MUSIC

FREQUENCY-DOMAIN CONCEPTS

- Spectrum
- Absolute bandwidth - width of the spectrum of a signal
- Effective bandwidth (or just bandwidth) - narrow band of frequencies that most of the signal's energy is contained in



EFFECTIVE BANDWIDTH

If $S(f)$ is the Fourier transform of $s(t)$ (the signal's spectrum) the **power spectral density (PSD)** is:

$$|S(f)|^2$$

The **total signal power** is the area under the PSD curve:

$$P = \int_{-\infty}^{\infty} |S(f)|^2 \; df$$

$$\boxed{\int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df}$$

The **effective bandwidth** B_{eff} is defined as the bandwidth that contains the same total power as the actual signal spectrum, if the spectrum were rectangular and centered at the frequency where $|S(f)|^2$ is maximum:

$$B_{eff} = \frac{1}{|S(f_0)|^2} \int_{-\infty}^{\infty} |S(f)|^2 \; df$$

GROUP DISCUSSION II

A mobile base station transmits a **baseband signal** $s(t)$ that is **band-limited to 200 kHz**.

The wireless channel has an **impulse response** $h(t)$ that causes **multipath fading**, and its **frequency response** $H(f)$ looks as shown below:

- For frequencies below **150 kHz**, the magnitude of $H(f) \approx 1$ (flat response).
- Between **150–250 kHz**, $H(f)$decreases linearly to 0.
- Above **250 kHz**, $H(f) = 0$.

The received signal is $y(t) = s(t) * h(t)$,and $Y(f) = S(f)H(f)$.

- How does the **frequency-domain multiplication** $Y(f) = S(f)H(f)$ affect the received signal?
- What is the spectrum of the received signal if the transmitted signal is $\cos(20\pi t)$. Hint: $\boxed{\cos(2\pi f_0 t) \quad \leftrightarrow \quad \frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]}$
- What happens to components of $S(f)$that fall in the region where $H(f) = 0$?
- If the **effective bandwidth** of $S(f)$is 250 kHz, Do you think the received signal's effective bandwidth will be the same, wider, or narrower than 250 kHz? Why?
- What observable effects might this have on the received waveform in **time domain**? (e.g., distortion, delay spread, symbol interference)

CLASS DISCUSSION

In the **frequency domain**, multiplication means that each frequency component of the transmitted signal $S(f)$is **scaled** and **phase-shifted** by the channel's response $H(f)$.

- For frequencies below **150 kHz**, where $H(f)$ $1 \approx$ $\rightarrow$ frequencies pass **undistorted**.
- Between **150–250 kHz**, where $H(f) \rightarrow 1 >$ signal components are **attenuated**.
- Above **250 kHz**, where $H(f) \rightarrow 0 =$ those frequencies are **completely removed**.

WHAT IS THE SPECTRUM OF THE RECEIVED SIGNAL IF THE TRANSMITTED SIGNAL IS $\cos(20\pi t)$.

$$s(t) = \cos(20\pi t) = \cos(2\pi \cdot 10\, t)$$

the frequency is

$$f_0 = 10 \text{ Hz}.$$

Therefore its frequency-domain representation is

$$S(f) = \tfrac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right] = \tfrac{1}{2}\left[\delta(f - 10) + \delta(f + 10)\right].$$

The channel $H(f)$ is:

- ≈ 1 for $|f| < 150 \text{ kHz}$
- Decreases linearly to 0 between 150–250 kHz
- 0 above 250 kHz

Since the signal's frequency components are at ±10 Hz, which is far below 150 kHz,

$$H(\pm 10 \text{ Hz}) \approx 1.$$

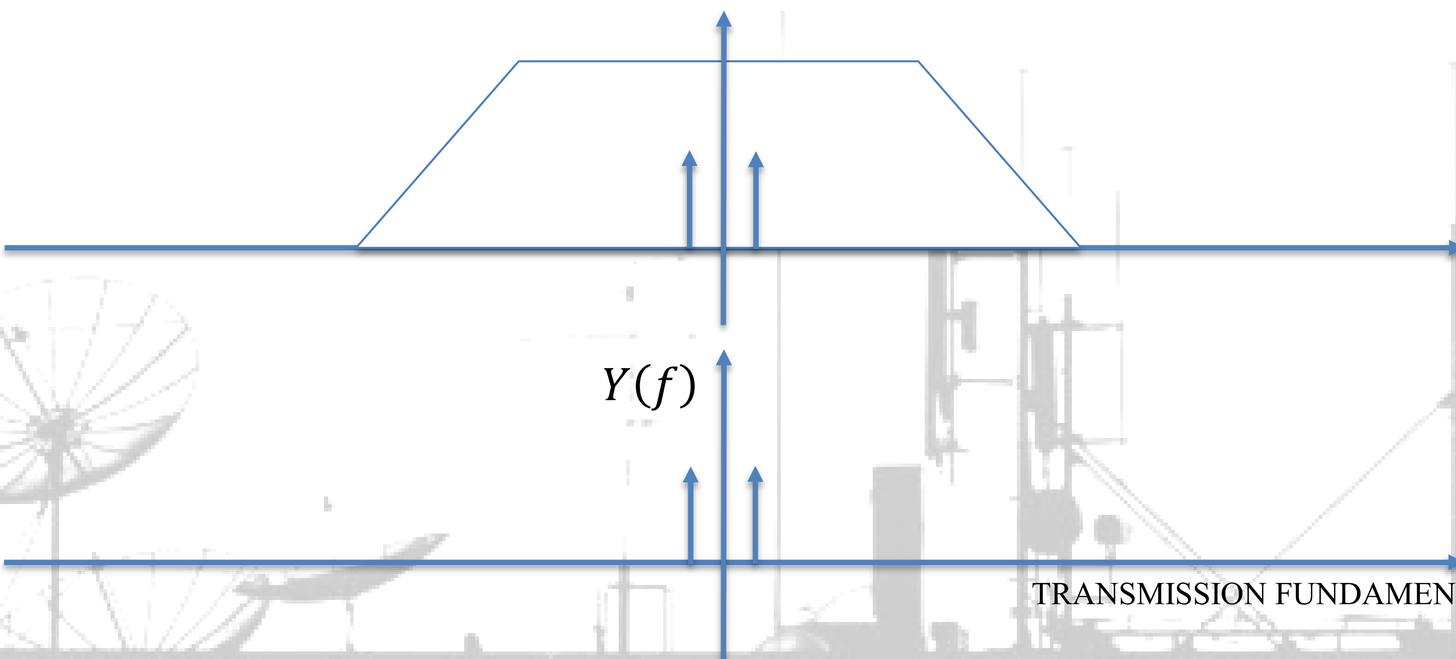
WHAT IS THE SPECTRUM OF THE RECEIVED SIGNAL IF THE TRANSMITTED SIGNAL IS $\cos(20\pi t)$.

Spectrum of the received signal

$$Y(f) = S(f)H(f) = \tfrac{1}{2}\left[H(f_0)\delta(f - f_0) + H(-f_0)\delta(f + f_0)\right].$$

Because $H(\pm 10 \text{ Hz}) \approx 1$, the received spectrum is:

$$\boxed{Y(f) \approx \tfrac{1}{2}\left[\delta(f - 10) + \delta(f + 10)\right]}$$



CLASS DISCUSSION

$$Y(f) = S(f)\,H(f)$$

- **Effective bandwidth:** Narrower than 250 kHz because the channel is flat only up to 150 kHz and fades to zero by 250 kHz.
- **Time-domain effects:** Smoother waveform, delay spread , and inter-symbol interference (ISI) as the channel is frequency selective.

QUESTIONS?