Problem 1: True or False [20 points, 4 points per question]
For each of the **claims** below, circle True or False. **NO justification is needed.**

1. **Claim:** Every tree is a bipartite graph.

   True / False

2. **Claim:** There is an algorithm to compute the edit distance between two strings of length n in $O(n \log n)$ time.

   True / False

3. **Claim:** For a weighted undirected graph G, there can be different minimum spanning trees.

   True / False

4. **Claim:** Edit Distance $\leq_P$ Independent Set

   True / False

5. **Claim:** If a NP-complete problem can be solved in linear time, then all NP-complete problems can be solved in linear time.

   True / False

Problem 2: Short Answers [18 points]

1. [8 points, 2 points per question]

   The following are a few of the design strategies we followed in class to solve several problems.

   a. Dynamic programming.

   b. Greedy strategy.

   c. Divide-and-conquer.

   For each of the following problems, mention which of the above design strategies was used (in class).

   i. Knapsack.

   ii. Counting Inversions.

   iii. Single source shortest path for undirected graphs with positive edge weights.

   iv. Negative weight shortest path with no negative cycle.

2. [5 points] Draw the dynamic programming table of the following instance of the knapsack problem: There are 5 items with weight 1, 2, 6, 7, 8 and value 1, 3, 11, 18, 20 respectively and the size of your knapsack is 11.

3. [5 points] Let A and B be two computational problems. Suppose $A \leq_P B$. If A cannot be solved in polynomial time, then what can you infer about problem B?

Problem 3: Dynamic programming [20 points]
Given an integer value n and a set of integers $S = \{s_1, s_2, .., s_m\}$, if we want to make change for n cents, and we have infinite supply of each of S valued coins, what is the **smallest number** of coins to make n cents?
For example, if $n = 34$ and $S = \{1, 5, 10, 25\}$, it requires at least 6 coins to make 34 ($34 = 25 + 5 + 1 + 1 + 1 + 1$).

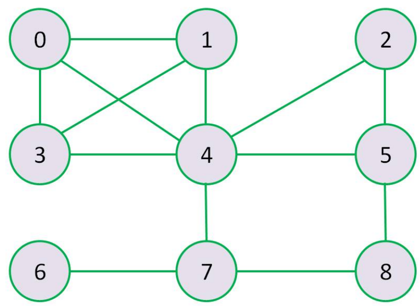
1. [2 points] Assume $S = \{1, 10, 21, 33, 70, 100\}$ and $n = 142$. How many coins are used in the optimal solution? Give an optimal solution (a set of coins with total value 142 such that the value of each coin equals to some element of S).

2. [15 points] Design an algorithm that finds the smallest number of coins required for given S and n. You can assume S contains 1 so that the solution always exists. No justification is needed.

3. [3 points] What is the running time of your algorithm (as a function of n and $|S|$)? No justification is needed.

Problem 4: NP-Complete [22 points]
COMPLETE-SUBGRAPH problem is defined as follows: Given a graph $G = (V, E)$ and an integer k, output yes if and only if there is a subset of vertices $S \subseteq V$ such that $|S| = k$, and every pair of vertices in S are adjacent (there is an edge between any pair of vertices).

For example, for the following graph and $k = 4$, the answer is yes, because $S = \{0, 1, 3, 4\}$ satisfies the requirement. But if $k \leq 5$, the answer is no.

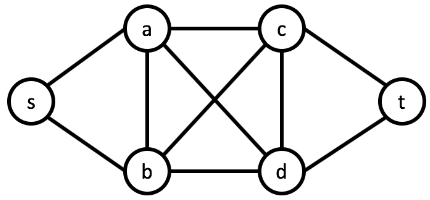


1. [6 points] Show that COMPLETE-SUBGRAPH problem is in NP.
2. [16 points] Show that COMPLETE-SUBGRAPH problem is NP-Complete.

   (Hint 1: **INDEPENDENT-SET** problem is a NP-Complete problem.)

   (Hint 2: You can also use other NP-Complete problems to prove NP-Complete of COMPLETE-SUBGRAPH.)

Problem 5: Dynamic Programming [20 points]
Let $G = (V, E)$ be a connected, unweighted and undirected graph. Given two vertices s and t in G, in general there may be multiple shortest paths connecting s and t. For example, the following graph have four distinct shortest paths connecting s and t: $s - a - c - t$, $s - a - d - t$, $s - b - c - t$, $s - b - d - t$.



1. [16 points] Give an algorithm that computes the **number** of distinct shortest paths connecting s and t. **Do not enumerate, just count them.**

2. [4 points] What is the running time of your algorithm?