CSCB63 Summer 2018 – Assignment 2
Due: Monday June 25 6PM

1. (a) [5 marks] Prove that in a tree (undirected) with at least 2 vertices, there exists some vertex of degree 1.

   Suggestion: This is short if you do some counting and use the right theorem.

   (b) [5 marks] Prove that a non-empty tree is a chain if every vertex's degree is at most 2. More precisely:

   For all $n \geq 1$, for all $G = (V, E)$ such that it is a tree, $|V| = n$, and every vertex has degree at most 2: V can be ordered into a simple path, call it $\langle v_1, \ldots, v_n \rangle$, such that $E = \{\{v_i, v_{i+1}\} \mid i \in \{1, \ldots, n-1\}\}$.

   Suggestion: Use induction, and use part (a) to help.

   Suggestion: The induction step does **not** look like this: "IH: True for some graph of $n-1$ vertices. WTP: Still true if I add a vertex and such-and-such edges."

2. [10 marks] Sam thought up this algorithm to compute the edges of a minimum spanning tree. Prove that it is correct, or give a counterexample (and prove that it is a counterexample).

   $E :=$ list of edges ordered by increasing weight
   $V :=$ set of all vertices
   for each edge $\{u, v\}$ in E:
   if $u \in V$ or $v \in V$:
   $V := V - \{u, v\}$
   output $\{u, v\}$

3. [12 marks] Chromorpher is a monster that travels on a game board—a directed graph; its name is because it changes colour every time it moves one step. It can be in one of 4 colours, represented by the numbers 0, 1, 2, and 3. When Chromorpher moves one step, its colour increases by 1, modulo 4 (i.e., 3 "increases" to 0). One step means moving through one edge that points from the current location to a next location.

   Chromorpher wants to start at a given location s in colour 0, and move until it reaches a given location t and become colour 0 again. Can it be done? And what is the minimum number of steps needed? (It is OK to revisit any location in intermediate steps, even s and/or t.) Implement an algorithm to solve this.

   Locations on the game board are represented by non-negative integers below 2^{18}. The game board is represented by an array of adjacency lists.

   Suggestion: Yes you will want to perform a graph search; no the graph will not be the game board verbatim.

   4 marks for code that looks like a general algorithm; 8 marks for passing test cases, provided the code looks like a general algorithm. Testing will be done with python3 on a BV lab computer, with 2 seconds per test case—make sure your code works and is fast enough there.

   The test driver uses Python's standard unittest library. Example usage:

```
python3 testcmp.py
python3 testcmp.py TestCMP.test_0
```

End of questions.