MATH 131: Numerical Methods for Scientists and Engineers

Homework 5 Solutions

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1.) a.) $E_{L,R} \leq \max_{[a,b]} |f'(x)| \frac{(b-a)^2}{2n} \leq 10^{-4}$

$\equiv \max_{[1,2]} |\ln x + 1| \frac{(2-1)^2}{2n} \leq 10^{-4}$

$\equiv \max_{[1,2]} |\ln 2 + 1| \frac{1}{2n} \leq 10^{-4}$

$\equiv (1.693...) \frac{1}{2n} \leq 10^{-4}$

$\equiv n \geq 8465.736...$

$\equiv n \geq 8466$

At least 8466 iterations.

b.) $E_M \leq \max_{[a,b]} |f''(x)| \frac{(b-a)^3}{24n} \leq 10^{-4}$

$\equiv \max_{[1,2]} |\frac{1}{x}| \frac{(2-1)^3}{24n} \leq 10^{-4}$

$\equiv (1) \frac{1}{24n} \leq 10^{-4}$

$\equiv n \geq 416.66...$

$\equiv n \geq 417$

At least 417 iterations.

c.) $E_T \leq \max_{[a,b]} |f''(x)| \frac{(b-a)^3}{12n} \leq 10^{-4}$

$\equiv \max_{[1,2]} |\frac{1}{x}| \frac{(2-1)^3}{12n} \leq 10^{-4}$

$\equiv (1) \frac{1}{12n^2} \leq 10^{-4}$

$\equiv \frac{1}{n^2} \leq \frac{12}{10^4}$

$\equiv n^2 \geq \frac{10^4}{12}$

$\equiv n^2 \geq \sqrt{\frac{10^4}{12}}$

$\equiv n \geq 28.8675...$

$\equiv n \geq 29$

At least 29 iterations.

d.) $E_S \leq \max_{[a,b]} |f^4(x)| \frac{(b-a)^5}{180n^4} \leq 10^{-4}$

$\equiv \max_{[1,2]} |\frac{2}{x^3}| \frac{(2-1)^5}{180n^4} \leq 10^{-4}$

$\equiv (2) \frac{1}{180n^4} \leq 10^{-4}$

$\equiv \frac{1}{n^4} \leq \frac{90}{10^4}$

$\equiv n^4 \geq \frac{10^4}{90}$

$\equiv n \geq 3.24668...$

$\equiv n \geq 4$

At least 4 iterations.

2.) a.) Each subsequent method seems to have a steeper slope than the next.

b.) This is hard to determine for Simpson's rule, but it looks like the other slopes are accurate for their order of convergence. The slopes for right Riemann and middle Riemann are different in that they are 'swapped'.