MATH 131: Numerical Methods for Scientists and Engineers

Homework 4 Solutions

Porfirio Montoya

1.) The complete table is the following:

i	1	2	3	4	5	6	7
x_i	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
y_i	0	$1/2$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/2$	0
$y_i^{'}$	1	$\sqrt{3}/2$	$1/2$	0	$-1/2$	$-\sqrt{3}/2$	-1
$y_i^{'\text{FWD}}$	0.954930	0.699057	0.255873	-0.255873	-0.699057	-0.954930	$\times$
$y_i^{'\text{CTR}}$	$\times$	0.8269934	0.477465	0.000000	-0.477465	-0.826993	$\times$
$y_i^{'\text{BWD}}$	$\times$	0.954930	0.699057	0.255873	-0.255873	-0.699057	-0.954930

For additional reference, I have included the true value of the derivative of $y'(x) = \cos x$.

I observe that the values in the second order function ($y_i^{'\text{CTR}}$) have less accuracy than the first order ones ($y_i^{'\text{FWD}}$ & $y_i^{'\text{BWD}}$) by a noticeable amount. If you take a look at the approximations of 1/2 and 0, you can see that there is a 0.25 or near 0.2 absolute error from the true value. The absolute error for the first order approximations becomes nearly 0.1 when the approximating $\sqrt{3}/2$, but that is as low as it gets for this data.

2.) In the error plots you can see that larger iterations of these methods leads to lower error. Lower error also decreases as the order increases. It is more noticeable when moving from Figure 1 to Figure 2, but the difference also exists at about a margin of 10^{-4} from Figure 2 to Figure 3. There are odd drops of error that go down to nearly zero in the graph as well. These drops are consistent between Figures 2 and 3, but they are not in Figure 1. When it comes to Figure 1, the drops occur at approximately 1.17, 4.32, and 7.45. As for the others, the sudden drops are at their lowest at approximately 2.55, 5.69, and 8.82. It's important to note that these dips do not ever reach zero, but the error decreases significantly at these locations.